Effects of Directed Learning Groups upon Students' Ability to Understand Conceptual Ideas

Karen Gabrielle Johnson
Benjamin Jason Galluzzo
Shippensburg University

Abstract

Mathematical modeling and directed learning groups were employed in a terminal mathematics course to encourage university students to conceptualize real-world mathematics problems. Multiple assessments were utilized to determine whether students' conceptual development is enhanced by participating in directed learning groups conducted in a learning center. Instituting directed learning groups early in a semester can have long-term effects on students' ability to apply concepts to future problems, feel comfortable participating in groups, increase their understanding of real-world applications, and maintain their confidence and self-efficacy in understanding mathematical concepts.

Common curricular goals for many disciplines in higher education include student development of critical thinking skills and application to real-world situations. Even though mathematics is often considered purely algorithmic because of the large amount of such content in college textbooks, developing conceptual understanding of foundational principles is crucial to deeper learning and application to real-world problems. Difficulties in grappling with conceptual understanding are not limited to developmental students; even good students who ask for help in understanding math problems seek the necessary, formulaic equations from instructors so they can simply insert the correct numbers without having to process the conceptual foundations underlying the algorithmic processes. By

Karen Gabrielle Johnson | Shippensburg University | kgjohnson@ship.edu

incorporating directed learning groups — small study groups that consist of three to five students led by a peer tutor — instructors can provide student support that can help students develop conceptual understanding and apply algorithmic applications to real-world problems.

Literature Review

Teaching mathematics in a manner that encourages conceptual development requires approaches that employ application processes. Mathematical modeling is a pedagogy whereby instructors present real-world problems as a model for situating the study of mathematical concepts. Houston and Lazenbatt (1996) describe models as a mathematical description of a simplification of a phenomenon. They explain that a mathematical model is the result of the process where mathematical entities presented with statements describe how a modeler moved from a phenomenon to an abstract representation.

In practice, students in higher education may struggle with such methods because they may not have practiced mathematical modeling in secondary schools. Making the transition from solving equations to determining which equations are needed in mathematical models can challenge students whose prior educational experiences may have been characterized by instruction followed by independent completion of calculation exercises. Such practices may encourage memorization of steps or skills that require procedural knowledge, problems that are solved through one-step operations. However, successful completion of mathematical modeling exercises requires conceptual knowledge, a process that requires students to make a connection between a described practical event or activity and mathematics in order to determine the appropriate mathematical operation for use. University students may need alternative teaching methods and supplemental supports for helping them develop the conceptual thinking necessary for mathematical modeling.

Biddlecomb (2005) suggests that learning centers should develop courses or workshops to help students prepare for math modeling courses where tutors and staff can help students build on their current mathematical knowledge and learn to apply their understanding to modeling courses. Such tutoring models can help students make sense of new forms of application because language is the primary means tutors employ for developing conceptual understanding. Vygotsky's (1978) theoretical underpinnings emphasize the importance of language to individual and social learning. Vygotsky explains that an essential feature of learning is using language to create a zone of proximal development where learning can occur when individuals interact with people in their environment and in cooperation with their peers. According to Vygotsky, once new concepts are internalized, independent achievement can take place. The value of courses or workshops exists in the facilitator's ability to use language interactions, which serve as scaffolds for students' development of conceptual knowledge, helping them develop associations and apply concepts to new experiences; nevertheless, helping students construct meaning should not be limited to courses or workshops.

Assisting students in mastering mathematics concepts can take place in a number of spaces, as long as a facilitator incorporates an effective process for learning. Valkenburg (2010) stresses the importance of communication as the primary means for learning. Valkenburg highlights the importance of communication because it is language that "allows humans to construct reality and to describe and define their experience" (p. 35). Valkenburg explains that language interactions allow learners to develop associations to improve their learning. Once a facilitator has identified the independent level of an individual, scaffolding, a technique to help students learn new concepts, can help students develop independence in applying new knowledge (Bruner, 1960). Valkenburg believes that tutors can serve as the means to help students learn by utilizing scaffolding to help students independently solve future problems.

For scaffolding to be successful and advance new learning, facilitators must intentionally connect new information to already-known information (Valkenburg & Dzuback, 2009). Valkenburg and Dzuback suggest that tutors work as translators by changing the language into one that students can understand, thereby intentionally creating contexts for formulating new ideas. Furthermore, tutors can help clarify content by presenting information in a different setting where students can freely ask questions (Laskey & Hetzel, 2011). Laskey and Hetzel suggest that students often feel more comfortable asking a tutor questions because the tutor has no power to influence their grades. The comfort in asking a tutor questions exists in individual sessions or in small group tutoring sessions where tutors direct learning experiences. In a small group model, group discussions provide an open environment for discussing work with peers (Solomon et al., 2010) where tutors can lead discussions and intentionally scaffold conceptual knowledge.

Aside from serving as facilitators in students' learning processes, tutors' work with students is not limited to students' increased understanding of concepts. Tutors can help students improve their self-efficacy, confidence, and the ability to do well in school, which can help students connect to university life (Tinto, 1999). Retention may be an additional benefit of tutoring, especially for at-risk students. A number of studies have found that at-risk students who regularly attend tutoring sessions can also experience higher grades and increased confidence, which can lead to achievement and retention (Dowling & Nolan, 2006; Hodges, 2001; Laskey & Hetzel, 2011; Rheinheimer et al., 2010).

Even though many students may experience higher grades and increased confidence, researchers find it challenging to find reliable methods for directly measuring the impact of math tutoring upon students' achievement. To discover how institutions of higher education measure the effectiveness of mathematical support services, Gillard, Robathan, and Wilson (2011) conducted an email survey of 21 higher education institutions. Their results revealed that formal measurement of math tutoring effectiveness is very difficult, and most institutions were focused on assessing students' perceptions of math support. In the collective records from the institutions, anecdotal evidence indicated a positive impact on students who utilized support, leading administrators to conclude that math support is a valuable resource for students' academic development. Even though anecdotal evidence can be useful and compelling, learning center directors need more rigorous forms of assessment and evaluation of math support services.

Learning center directors can benefit from understanding the impact of tutorials on students' development of conceptual knowledge because this form of understanding can improve students' critical thinking skills. Simply helping students gain proficiency with procedural knowledge does not require students to utilize higher-level reasoning skills for their computations. In contrast, guiding students to make gains in conceptual knowledge will challenge students to move beyond procedural steps, integrate higher-order reasoning skills, practice deep reflection on the underlying meaning of mathematical concepts, and apply mathematical operations to real-world problems. Discovering methods to measure gains in conceptual knowledge during tutorials could help learning center directors assess tutorials and provide more effective training for tutors.

Some researchers have utilized exam results to measure effectiveness of tutoring models. Bamforth et al. (2007) compared the passing rates of engineering students who used additional support to those who did not utilize support services. Their findings revealed that students who attended support sessions progressed to pass their mathematical modules while those who did not utilize the additional support failed the same mathematical modules. However, these results did not provide a clear explanation of whether gains in conceptual understanding contributed to the students' ability to pass the exams. One's ability to pass a math exam may be an indication of improvement in procedural knowledge, rather than gains in conceptual knowledge.

In addition to understanding whether individual tutorials contribute to development of conceptual knowledge, learning center directors and instructors could benefit from understanding whether small group tutoring contributes to the development of conceptual thinking. Group tutoring models can be more complex to evaluate because interactions between group members will be influenced by the composition of a group, which is crucial to a group's success. Houston and Lazenbatt's (1996) group tutoring model discovered that a majority of students surveyed reported a reluctance to form peer learning groups and did not find it a valuable experience. For groups to be beneficial, these students felt that groups should be selected by the instructor to reflect a mix of males and females and a variety of abilities, instead of allowing groups to self-select members on the basis of friendships. Despite students' reluctance to join a peer learning group, most agreed that they had developed better communication skills and appreciated the presence and advice of math tutors. Student reluctance to rate group tutoring as valuable while valuing math tutors' advice seems contradictory, revealing a need to investigate dynamics within tutorials and whether gains in conceptual knowledge were made.

Webb's (1991) research of interactions within study groups—small groups directed by peer to help students master academic material—provides understanding about the importance of verbal exchanges. In Webb's study, verbal interaction and achievement were positively correlated when students received content-related explanations and listened to others. Thus, the success for small group tutoring appears to be dependent on a leader's ability to initiate and maintain productive conversations. For successful implementation in a learning center, learning center directors must provide direct training on how to lead discussions in small groups so a trained peer tutor can lead productive verbal exchanges.

The directed learning group model appears to offer opportunities for students to engage in language actions designed to improve their conceptual knowledge of math. However, formal measurements of conceptual growth and controlled experimental models that help measure conceptual growth are difficult to construct. Furthermore, first-year students may not understand the value of group tutoring models, so learning more about the impact of study groups can help professors determine ways to incorporate study groups into their courses. Understanding students' perceptions of study groups and any short-term effects of directed learning groups can help learning centers and math instructors develop effective directed learning group strategies to enhance students' development of conceptual knowledge in math. This study seeks to determine whether students' conceptual development is enhanced by participating in directed learning groups conducted in a learning center.

Research Questions

1. Will students who participate in directed learning groups at the Learning Center score significantly higher on conceptual assessments when compared to students who do not participate in directed learning groups at the Learning

Center?

2. Will students who participate in directed learning groups demonstrate long-term benefits from their participation in the directed learning groups at the Learning Center?
3. Are students satisfied with their experiences in the directed learning groups?

Method

Participants

Participants in the study included students enrolled in Applied Calculus (MAT 181) at a mid-sized comprehensive university located in the Mid-Atlantic region. MAT 181 is a terminal mathematics course that primarily serves first-year students in the College of Business. Most participants were first-year students between the ages of 18 and 20 and enrolled in their first spring semester at the university. Because most students who take this course are first-year students, the researchers decided to use this sample in order to introduce these students to the value of learning groups and learning center services early in their academic career. Since research has shown that tutors can help students improve their self-efficacy, confidence, and ability to do well in their studies (Tinto, 1999), first-year students could benefit from early exposure to services.

Two MAT 181 sections, which met for 15 weeks in three 50-minute periods per week, participated in two directed learning group activities completed at two different intervals during the semester. In Section A, 41 students participated in Directed Learning Group Activity 1 (DLGA1) during weeks two to four, while 36 students in section in Section B, Control Group 1, were not required to complete DLGA1. During weeks seven to nine, 32 students from Section B completed Directed Learning Group Activity 2 (DLGA2), and 38 students from Section A, Control Group 2, were not required to complete DLGA2. By alternating the directed learning group sessions, control and experimental groups were established for both groups. Students who did not complete the directed learning group activities, pretest, posttest, and surveys were removed from the final data set.

Students are admitted to MAT 181 based on one of three criteria: an acceptable score on a college entrance exam, a passing score on the university-administered mathematics placement test, or the successful completion of College Algebra with a grade of "C" or better. The prerequisites for entrance into the course ensure that students enrolled in MAT 181 possess similar mathematical ability.

Procedures

During the second week, students who agreed to participate in the study completed an IRB-approved consent form. DLGA1 was conducted during weeks two, three, and four, while the DLGA2 was conducted during weeks seven, eight, and nine.

Three experienced Learning Center tutors, who were all upper-class math majors, were cross-trained by the MAT 181 instructor and the Director of the Learning Center. The math instructor discussed conceptual learning goals for the class and presented tutors with a variety of scenarios designed to prepare them for implementing a scaffolding approach with the student groups. The Director of the Learning Center focused on procedures for coaching, technology usage, recording student visits, and reviewing best practices in mathematics coaching.

Prior to the start of both iterations of the study, students received instruction focused on two mathematical topics commonly taught in the standard Calculus curriculum: limits and derivatives. The instruction provided an introduction to both topics and incorporated procedural and conceptual approaches on a regular basis. The instructor taught both sections, and each section completed four distinct activities:

1. Following instruction on a new topic, students were administered a pretest (Appendices A and D) consisting of six questions. Students were not notified ahead of time that they would be taking a quiz during that class period, and this pretest was not calculated into their course grade. This pretest provided a baseline of students' understanding of the concepts and helped determine if there were significant differences between the groups.
2. After students completed their pretest and attended class, they were assigned a worksheet (Appendix B and E) con-

taining questions that required them to explore the curricular topic in real-world situations, which encouraged them to develop conceptual understanding.

3. Students in the experimental group formed small groups of three to five students and completed a one-hour group tutoring session with one of three trained Learning Center tutors prior to submitting the worksheet for a grade.
4. On the same day directly after students submitted the worksheet, they completed the posttest and survey. Students were not notified ahead of time that they would be taking a quiz (Appendices C and F) during that class period.

These procedures remained constant for both groups, although conceptual topics varied. For the first topic, Section A, the experimental group, was required to complete DLGA1 and Section B, the control group, was not required to complete DLGA1. For the second topic, Section B, the experimental group, completed DLGA2 while Section A, the control group, was not required to complete DLGA2 activities. Both groups completed worksheets for each unit.

During the implementation of the study, an unexpected turn of events occurred. Students who completed DLGA1 wanted to continue working in groups for the second unit. The researchers did not want to forbid the group meetings since students appeared to benefit from this learning activity, so students who voluntarily formed groups were instructed to identify their group members on their worksheet. Therefore, students from Section A, who engaged in learning groups when not required to do so, were identified.

Measurement instruments. This quasiexperimental study analyzed three types of collected data that included the following: (a) a comparison between students' achievement of learning outcomes before attending a directed learning group session and after attending a directed learning group session; (b) a comparison between students' achievement of learning outcomes with and without a directed learning group experience; and (c) student perceptions of the effectiveness of directed learning groups and the structure of this teaching model.

The assessment of learning outcomes in students' conceptual understanding of standard topics in calculus was completed by using multiple choice pretests and posttests that focused on subject matter presented in the classroom prior to the administration of the pretest. The first unit quizzes on limits (Appendices A and C) each contained three questions requiring students to use procedural knowledge and three questions testing students' conceptual knowledge of mathematical ideas. For example, a procedural question on the limit quiz presented students with the function, $x = \frac{x^2 - 16}{x - 4}$, and then asked them to find the limit of the function as x approaches the number four, which is written in mathematical notation as: $\lim_{x \to 4} f(x)$. On the limit quiz, the conceptual question paired with the previously described procedural example asked the following: "Suppose that the cost C of removing *p*% of pollutants from a chemical dumping site is given by $(p) = \frac{\$20,000p}{100 - p}$. Can a company afford to remove 100% of the pollutants? Explain."

In the second unit on derivatives, the two quizzes (Appendices D and F) each contained three questions requiring students to initially use a combination of conceptual knowledge and procedural knowledge of the derivative to help them solve the problems and three questions focused solely on testing students' conceptual knowledge of the derivative applied to a graph or function embedded in a word problem, with the second set of activities requiring a direct application of concepts.

Each group completed the same pretest and posttest for each module, resulting in a total of four quizzes for both modules. After both groups completed the pretest, the instructor discussed math concepts and then assigned a worksheet that required students to use applications on three multi-part, open-ended questions about real-world problems. These worksheets served as the focus point for discussion in directed learning group sessions.

The instructor collected data on students' perceptions of directed learning groups at the culmination of each topic through self-reported measures using the MAT 181 Student Learning Surveys, which were developed by the researchers. The experimental and control group surveys differed: the experimental group answered eight scaled questions while the control group responded to a seven-question survey with five scaled questions. The additional questions on the experimental group surveys were focused on identifying student experiences during the directed learning group sessions. Control group surveys focused queries on the assigned worksheet.

Analysis of data. After the pretests and posttests had been administered, the researchers collected all four sets of test data that included results from both groups for each directed learning group activity. Results of the MAT 181 Student Learning Surveys were also collected along with notes from the instructor and the student tutors. A quantitative analysis using independent and paired samples *t*-tests was conducted from both DLGA1 and DLGA2 test scores. The researchers conducted a descriptive analysis of student perceptions from the MAT 181 Student Learning Surveys. Only students who attended DLGA1 and DLGA2 sessions had their test and survey results included in the data set. Students who had not completed consent forms and both the pretest and posttest for a particular study were removed from the sample set. Descriptive statistics and paired samples *t*-tests were calculated to determine if significant differences in Conceptual Comprehension had occurred over the course of the study. Results from all measures were merged to determine common themes and student perceptions. Conclusions from these data were determined from the frequency of repeated themes and scores from the quantitative sections of the survey.

Results

Evaluations of Directed Learning Group Activity 1 (DLGA1)

The first research question sought to determine if there was a significant difference between students' achievement of learning outcomes before and after attending directed learning sessions in the Learning Center. For the limits unit, 41 students completed the pretest and posttest in the experimental group; 36 students completed both tests in the control group. An independent samples *t*-test, with an alpha level set at .05, was used to determine if a significant difference in students' mathematical abilities existed before instructional activities commenced. The results, $t(75) = .497$, $p<.05$, clearly indicate no significant differences existed between students' abilities in each class before DLGA1 began. Table 1 presents differences between the experimental (DLGA1) and control (CRTL1) groups' pretest and posttest results.

Results from the paired samples *t*-test reveal a significant difference for both groups on Question 1, which was a conceptu-

Table 1

Paired Samples t-test Comparing Pretest and Posttest Results for Unit 1

	DLGA1				CTRL1			
SLO	M(SD)	95% CI	*t*(40)	Sig.	M(SD)	95% CI	*t*(35)	Sig.
Q1 - CON	-.34(.57)	[-.52, -.16]	-3.80	.00	-.44(.50)	[-.61, -.27]	-5.29	0.00
Q2 - PRO	-.07(.57)	[-.25, .11]	-0.83	.41	-.03(.61)	[-.23, .18]	-0.27	0.79
Q3 - CON	-.27(.45)	[-.41, -.13]	-3.83	.00	-.03(.69)	[-.26, .21]	-0.24	0.81
Q4 - PRO	-.05(.50)	[-.21, -.11]	-0.63	.53	.00(.76)	[-.26, .26]	0.00	1.00
Q5 - PRO	-.12(.56)	[-.30, .05]	-1.40	.17	.00(.76)	[-.26, .26]	0.00	1.00
Q6- CON	-.29(.60)	[-.48, -.10]	-3.11	.00	-.03(.56)	[-.22, .16]	-0.03	0.77

Note. DLGA1= Students who completed Directed Learning Group Activities. CTRL1= Students in Control Group 1 who did not complete Directed Learning Group Activities. SLO = Student Learning Outcome; M = Mean; SD = Standard Deviation; CI = Confidence Interval that includes the lower and upper limits; *t*(40) = paired samples *t*-test with 40 degrees of freedom; *t*(35) = paired samples *t*-test with 35 degrees of freedom; Sig. = Significance (two-tailed); Q = Question; CON = Conceptual Mathematical Problem; PRO = Procedural Mathematical Problem.

ally-based question. However, there were no significant differences for other questions for the control group. Students who completed DLGA1 demonstrated statistically significant differences for the other two conceptual questions. Both groups did not demonstrate significant differences in procedural questions.

An independent samples *t*-test, with an alpha level set at .05, was used to determine if an overall significant difference in students' mathematical abilities existed between groups after all instructional activities had been completed. The results, $t(75) = 2.47$, $p<.05$ with a significance score of 0.016, clearly indicate an overall significant difference between the groups' abilities to apply mathematical concepts taught in the unit. DLGA1 students scored significantly higher in their overall understanding of concepts and procedures when compared to the control group.

Evaluations of Directed Learning Group Activity 2 (DLGA2)

A continuation of the first research question, DLGA2 on derivatives similarly sought to determine if there was a significant difference between students' achievement of learning outcomes before and after attending directed learning group sessions in the Learning Center. However, one major difference was that most of the students in the control group, while not required to meet in groups in the Learning Center, continued to independently meet in their groups. In the DLGA2 group, 32 students completed the pretests and posttests while 38 students completed the pretests and posttests in the control group. An independent samples *t*-test, with an alpha level set at .05, was used to determine if a significant difference between groups existed before instructional activities commenced for this second unit. The results, $t(68) =.523$, $p<.05$, clearly indicate no significant differences between groups' abilities before DLGA2 began. Table 2 presents student results on pretests and posttests for both groups.

Results for DLGA2 students on all conceptual questions reveal a significant difference between experimental students' pre-directed learning session and post-directed learning session. A statistically significant difference was not revealed in students' learning of conceptual/procedural material for either group.

The control group's results from the paired samples *t*-test

Table 2

Paired Samples t-test Comparing Pretest and Posttest Results for Unit 2

	DLGA2				CTRL2			
SLO	M(SD)	95% CI	$t(31)$	Sig.	M(SD)	95% CI	$t(37)$	Sig.
Q1 - CON	-.44(.56)	[-.64 -.23]	-4.39	.00	-.55(.50)	[-.72, -.39]	-6.76	0.00
Q2 - C/P	.09(.64)	[-.14, .32]	-0.83	.41	-.03(.59)	[-.22, .17]	-0.27	0.79
Q3 - C/P	-.16(.51)	[-.34, .03]	-1.72	.10	-.21(.66)	[-.43, -.01]	-1.90	0.06
Q4 - CON	-.31(.64)	[-.54, -.08]	-2.74	.01	-.24(.59)	[-.43, .04]	-2.48	0.02
Q5 - CON	-.34(.70)	[-.60, .09]	-2.78	.01	-.26(.60)	[-.46, -.07]	-2.70	0.01
Q6- C/P	-.06(.62)	[-.16, .29]	0.57	.57	.08(.63)	[-.13, .29]	0.77	0.45

Note. DLGA2 = Students who completed Directed Learning Group Activities. CTRL2 = Students in Control Group 2 who did not complete Directed Learning Group Activities. SLO = Student Learning Outcome; M = Mean; SD = Standard Deviation; CI = Confidence Interval that includes the lower and upper limits; $t(31)$ = paired samples t-test with 31 degrees of freedom; $t(37)$ = paired samples t-test with 37 degrees of freedom; Sig. = Significance (two-tailed); Q = Question; CON = Conceptual Mathematical Problem; C/P = Conceptual/Procedural Mathematical Problem in which a procedural step follows conceptualization of the situation.

reveal significant differences in all the conceptual activities, which are similar to their results of their first unit and the experimental group's results for the second unit. An independent samples *t*-test, with an alpha level set at .05, was used to determine if a significant difference in students' mathematical abilities existed between the groups after all instructional activities had been completed. The result, $t(68) = .755$, $p<.05$ with a significance score of 0.811, indicates no overall significant differences between the groups' abilities to apply mathematical concepts taught in the unit.

MAT 181 Group Learning Surveys

After students completed their posttest for both units, they received the MAT 181 Student Learning Survey. Students completed the survey during class to provide a 100% response rate for both groups. The quantitative portion of the survey asked students about their attitudes concerning course components by using a scale from 7 to 1. Students who completed the DLGA1 received an eight question-survey in which five of the questions were stated in a positive fashion while three were stated negatively. In Control Group 1, students received a different survey that contained only five of the Likert questions from the test group survey because three of the experimental group questions were not applicable for the control group. Of the five questions given to the control groups, three of the questions were stated positively while two were worded negatively. Positive and negative questions were given in order to measure reliability of student responses. For the second unit, students who completed DLGA2 completed the eight-question survey and Control Group 2 completed the five-question survey.

Tabulating results on the scale required weighting of the responses. For the positively stated items, numeric values ranged from 7 to 1, with the highest rating given to favorable responses and respectively decreasing to unfavorable ones. Thus, Strongly Agree would have a rating of 7 while Strongly Disagree would be rated as 1. On the negatively stated items, the weighting is reversed with the Strongly Agree weighted as 1 and Strongly Disagree weighted as 7. Table 3 lists the questions and the mean scores obtained from students.

Students in DLGA1 appeared to experience more confidence

Table 3

Mean Scores of Statements on MAT 181 Student Learning Surveys

	Mean Scores			
Survey Statement	DLGA1	CTRL1	DLGA2	CTRL2
1. I experienced an overall improvement in my understanding of mathematical concepts after completing the worksheet.	3.93	2.86	3.53	3.58
2. My Leaning Center meeting helped me understand and complete the assignment.	4.67	---	4.12	---
3. The feedback I received from my group was helpful.	5.18	---	4.51	---
4. I am disappointed in the lack of improvement in my calculation skills.	3.80	3.78	3.52	3.63
5. Discussing the worksheet with a Learning Center Tutor did little to improve my understanding of concepts.	4.34	---	4.03	---
6. I feel more confident in my ability to calculate problems.	4.12	3.83	3.94	4.53
7. I feel comfortable sharing ideas with members of my group.*	5.95	4.64	4.97	4.82
8. I will not work in a group on future homework assignments and/or projects.	5.12	4.68	4.68	4.73

Notes. CTRL1 = Control Group. CTRL2 = Control Group 2. *Questions for Control Group 1 and 2 substituted the words "with my classmates" for "members of my group" in question 7. CTRL1 and CTRL2 did not answer questions 2, 3, and 5.

than Control Group 1 in their ability to understand mathematical limits, and they credited their Learning Center meetings as helpful in understanding the assignment. Despite fairly positive reviews of group meetings, students in DLGA1 did not consistently credit their tutor with helping them understand mathematical concepts. An item analysis of question five revealed that students who worked with two of the tutors ranked their tutors positively while students who worked with the third tutor rated this tutor's assistance less favorably. Finally, students in the DLGA1 felt more comfortable sharing ideas with group members than students who did not attend Learning Center meetings.

Students in DLGA2 did not demonstrate more confidence in understanding mathematical limits than students in the Control Group 2, which differed from the results from DLGA1 Survey. DLGA2 students rated their Learning Center meetings and feedback from their group favorably, but their ratings were less favorable than the ratings given by DLGA1. Surprisingly, DLGA2 students rated their confidence in their ability to calculate limits less positively than students in Control Group 2. Last of all, DLGA2 students positively rated their comfort in sharing ideas with their group, but even though their rating was higher with the tutors' facilitation of the group, their rating was not much higher than the rating from Control Group 2.

Discussion

Our results concur with conclusions by Gillard et al. (2011) that measuring effectiveness of math tutoring is very difficult. A simple look at the results may seem to reveal confounding effects; nevertheless, anecdotal records and observations provide insight into interpretation of these multiple measures. Sound assessment practices incorporate multiple measures to provide rich layers for interpretation, and results from this study illustrate the importance of following such practices.

This study sought to determine whether students who participate in directed learning groups score higher on conceptual assessments-- which require critical reasoning and application skills-- than students who do not participate in directed learning groups. Students in DLGA1 and DLGA2 demonstrated significant differences in growth in all areas of conceptual knowledge (see Tables 1 and 2), and students in DLGA1 revealed an overall significant difference in growth for both conceptual and procedural knowledge when compared to Control Group 1 [t (75) = 2.47, p<.05]. These results appear to support the premise that directed learning groups were effective in helping students grow significantly in their conceptual knowledge. However, when comparing Control Group 1 and Control Group 2, a simple analysis cannot explain the outcomes.

In Control Group 1, students only demonstrated significant growth in one out of three conceptual areas, but Control Group 2 demonstrated significant growth in all conceptual areas, which required higher-order, conceptual thinking; therefore, students who completed DLGA1 experienced the same level of growth as students who completed DLGA2. At first inspection, these results do not appear to corroborate; however, students in Control Group 2, who had experienced the benefits of working in their learning groups, continued to meet in their groups without a math tutor for the second learning activity even though they were not required to do so. One might think that Control Group 2 began the second unit with a stronger conceptual foundation, yet, this does not appear to be the case since the pretest scores were similar and the t-tests for independent samples [t (68) = .523, p<.05] did not indicate any significant differences between groups before the unit was taught. Apparently, students in Control Group 2 were empowered to transfer successful learning strategies they had learned during their time in DLGA1 to new concepts they were learning in the second unit. Thus, it appears that first-year students who participate in directed learning groups may continue to meet in groups and employ practices learned in groups that enable them to achieve success. These results strengthen the results of Gillard et al. (2011) that concluded that math support is a valuable resource for students' academic development.

In procedural problems, students did not make significant gains in either unit. Several reasons account for a lack of significant improvement in this area. First, tutor training focused on scaffolding the problems from a real-world perspective because tutors are often comfortable with procedural coaching and may not naturally connect problems to real-life situations. Thus, tutors focused sessions primarily on real-world problems, encouraging students to think more about concepts than procedures. Second, once students were empowered to conceptualize word problems, the word problems actually became easier because they understood the problems and could rule out false possibilities in the multiple choice quiz. Third, in solving conceptual problems, students were less likely to make calculation errors, and their responses were based more on reality and their understanding of the problem. Reasoning made it easier for students to select the correct answer while procedural exercises held more possibilities for error due to the calculation procedures students had to complete.

Student perceptions of their experience revealed valuable insights concerning the strategies and the importance of instituting directed learning groups early in the semester. Students who completed DLGA1 evaluated their ability to understand mathematical limits more positively than those who participated in DLGA2. Evaluations from students who participated in DLGA2 demonstrated their awareness of the effectiveness of directed learning group strategies that helped build their understanding, which corroborated with quantitative results shown in Table 1. Because students understood the value gained from participating in the groups, Control Group 2 continued to meet in their learning groups and maintained the positive momentum of active learning strategies, which helped them significantly improve their critical thinking and application skills for the second unit. Most likely, the early implementation of directed learning groups and students' continuance of meeting in groups made such an impact in students' development of conceptual understanding, that no significant difference in overall learning was determined between the two groups for the second unit. Instituting directed learning groups early in the semester appears to have long-term effects in students' ability to apply concepts to future problems, feel comfortable participating in groups, increase their awareness of their improvement in understanding real-world applications, and maintain their confidence in their ability to understand mathematical concepts.

Students' perception of the value of directed learning seemed to vary according to the composition of the group and the time in the semester when the group was formed. Those who participated in DLGA2 did not rate the groups as positively as students who participated in DLGA1, which may be a result of their initial decreased ability to understand concepts in the first unit. Since students in DLGA2 began meeting in groups later in the semester, they did not achieve the same level of early success as students in DLGA1. Even though students in DLGA1 met for just one hour for the first unit, the effectiveness of the group discussions helped students in DLGA1 experience enough success to build more confidence for future learning. By building students' confidence in their ability to solve more complex algorithms, their attitudes toward strategies may be more positive. As Houston and Lazenbatt (1996) found, group composition does determine the effectiveness of a directed learning group model, but in this study, problems were related more to the tutor leading the group rather than members of the group. Because the instructor formed the groups to avoid problems in social group compositions like those described by Houston and Lazenbatt, group effectiveness did not seem to be a result of student members. The results from this study support Webb's (1991) research that concluded that tutors leading the group set the tone and environment for learning.

Out of the three tutors leading the groups, one of the three tutors consistently received lower ratings than the other two tutors. The researchers noted that the tutor with the lowest ratings lacked essential interpersonal skills that hampered his ability to establish strong bonds with his groups. For directed learning groups to operate effectively, specialized tutor training should discuss strategies for building interpersonal communication and approaches for creating an environment conducive for active learning.

The directed learning groups provided a structure in which tutors were not only able to help students build conceptual knowledge, but, as Valkenburg suggests, the tutors also empowered students to independently apply knowledge to solve future problems. Tutors' ability to scaffold learning by directing language interactions appeared to help students understand, retain, and apply concepts to new situations. Given that students' greatest gains occurred with conceptual ideas that involve critical thinking skills and application of real-world problems, this strategy holds promise for instructors of mathematics courses. However, to be optimally effective, learning centers and mathematics instructors both need to actively train and prepare tutors for scaffolding content and leading the groups. Additionally, instructors should support the formation of groups to launch the initiative and provide structure for students so that groups can be started at the onset of the semester.

Formal measurements of conceptual growth and effectiveness of math tutoring are difficult to construct, yet holistic measures of assessment in students' gains in knowledge and their perceptions of strategies are useful for both learning centers and instructors. The quasi-experimental model in this study and qualitative analysis provided a useful model for understanding how students gain conceptual knowledge and view such strategies. Directed learning groups can help students improve their understanding of difficult concepts through interactive discussions led by a skilled tutor. When students achieve early success in critical thinking strategies, they may tend to employ them again to new situations and enjoy working in groups when they experience success and comfort in the group. Therefore, when learning center personnel and math instructors collaborate to design extended learning opportunities such as directed learning groups, students are introduced to valuable resources that can enhance their academic development.

Recommendations for Future Research

This study involved two classes at one institution and is limited in its ability to transfer to other institutions. Replication of these methods would help confirm the findings of this study and allow the results to be generalized to larger populations. In order to fully understand the effect of directed group learning on students' perceptions of curricular material, tutoring, and group work in a math class, further investigations should include administering a student learning survey before and after the directed learning activity to determine changes in student perceptions. Furthermore, because quantitative and qualitative results of the control group following DLGA2 suggest that those who find success with directed group learning may continue to study using these techniques, additional studies could include longitudinal surveys, interviews, and focus groups that seek to investigate study habits and learning center usage of participants throughout that semester and subsequent semesters.

References

Bamforth, S.E., Robinson, C.L., Croft, A.C., & Crawford, A.R. (2007). Retention and progression of engineering students with diverse mathematical backgrounds. *Teaching Math Applications*, 26, 156-166.

Biddlecomb, B. D. (2005). Uniting mathematical modeling and statistics: Data analysis in the college classroom. *Learning Assistance Review (TLAR)*, 10(2), 41-51.

Dowling, D. & Nolan, B. (2006). Measuring effectiveness of a math's learning support centre: The Dublin City experience. *CETL-MSOR Conference Proceedings*, 2006, 51-54.

Gillard, J., Robathan, K., & Wilson, R. (2011). Measuring the effectiveness of a mathematics support service: An email survey. *Teaching Mathematics & Its Applications, 30*(1), 43-52.

Hodges, R. (2001). Encouraging high-risk student participation in tutoring and supplemental instruction. *Journal of Developmental Education, 24*(3), 2-8.

Houston, K., & Lazenbatt, A. (1996). A peer-tutoring scheme to support independent learning and group project work in mathematics. *Assessment & Evaluation in Higher Education, 21*(3), 251-266.

Laskey, M. L. & Hetzel, C.J. (2011). Investigating factors related to retention of at-risk college students. *The Learning Assistance Review (TLAR), 16*(1), 31-43.

Rheinheimer, D. C., Grace-Odeleye, B., Francois, G. E., & Kusorgbor, C. (2010). Tutoring: A support strategy for at-risk students. *Learning Assistance Review (TLAR)*, 15(1), 23-33.

Solomon, Y., Croft, T. & Lawson, D.A. (2010). Safety in numbers: Mathematics support centres and their derivatives as social learning spaces. *Studies in Higher Education*, 35,421-431.

Tinto, V. (1999). Taking student retention seriously: Rethinking the first year of college. *National Academic Advising Association Journal, 19*(2), 5-9.

Valkenburg (2010). Joining the conversation: Scaffolding and tutoring mathematics. *The Learning Assistance Review (TLAR), 15*(2), 33-41.

Valkenburg, J. & Dzuback, C. (2009). *The engaged mind: Cognitive skills and learning*. Charleston, SC: CreateSpace.

Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

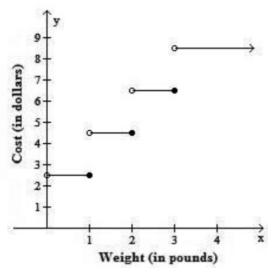
Webb, N.M. (1991). Task-related verbal interaction and mathematics learning in small groups, *Journal for Research in Mathematics Education, 22,* (5), 366-389.

Appendix A

REAL-WORLD LIMITS PRETEST

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1. An express mail service uses the following graph to determine how much to charge for overnight delivery of packages. You have to mail two packages; one weighing 1.97 pounds, the other weighing 3.02 pounds. How much will it cost to send both packages using the overnight service?



A) \$9.00
B) \$11.00
C) \$13.00
D) \$17.00
E) None of the above

Find the limit, if it exists.

2. Let $(x) = \frac{x^2 - 3x - 10}{x + 2}$. Find $\lim_{x \to -2} f(x)$.

A) -7
B) -2
C) 0
D) 5
E) Does not exist

Solve the problem.

3. Suppose the cost of removing p% of the pollutants from a

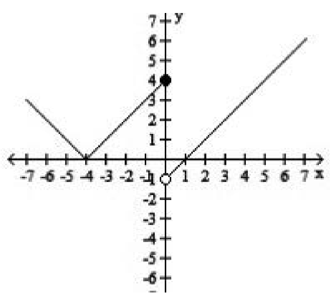
chemical dumping site is given by $C(p) = \frac{\$35{,}000p}{100 - p}$.

Can a company afford to remove 100% of the pollutants? Explain.

A) Yes, the cost of removing p% of the pollutants is $35,000, which is certainly affordable.
B) No, the cost of removing p% of the pollutants is $350, which is a prohibitive amount of money.
C) Yes, the cost of removing p% of the pollutants is $350, which is certainly affordable.
D) No, the cost of removing p% of the pollutants increases without bound as p approaches 100.
E) Yes, the cost of removing p% of the pollutants is $3,500, which is certainly affordable.

Use the graph to evaluate the indicated limit and function value or state that it does not exist.

4. Find $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$.



A) $\lim_{x\to 0^-} f(x) = 4$; $\lim_{x\to 0^+} f(x)$ does not exist

B) $\lim_{x\to 0^-} f(x) = 4$; $\lim_{x\to 0^+} f(x) = -1$

C) $\lim_{x\to 0^-} f(x) = -1$; $\lim_{x\to 0^+} f(x) = 4$

D) $\lim_{x\to 0^-} f(x)$ does not exist; $\lim_{x\to 0^+} f(x)$ does not exist

E) $\lim_{x\to 0^-} f(x) = 4$; $\lim_{x\to 0^+} f(x) = 4$

Provide an appropriate response.

5. If the limit at infinity exists, find the limit.

$$\lim_{x\to\infty}\frac{3x^3+5x}{4x^4+10x^3+2}$$

A) 3⁄4
B) ∞
C) 1
D) 0
E) None of the above

Solve the problem.

6. It has been determined that the value V of a certain product decreases, or depreciates, with time t in years, where

$$V(t)=100-\frac{60t^2}{(t+2)^2}$$

Find $\lim_{t\to\infty}V(t)$.

A) \$100
B) \$60
C) \$40
D) \$70
E) Does not exist

Appendix B

Real-World Limits Worksheet

This worksheet explores some possible applications of limits in real life. You are allowed to work in groups (< 5 people/group) to determine solutions to these problems; however, each individual must turn in a solution. If you do choose to work with others, you must write ALL the names of the members of your group on the paper you turn in.

1. Analyze the progression of men's and women's world record times in the marathon (Information can be found at this link: http://www.arrs.net/RecProg/RP_wwR.htm).

   a. When (if ever) will the men's world record drop below 2 hours? 1 hour and 45 minutes? Use the data to support your answer.
   b. Give an example of a function that models the progression of men's world record times in the marathon.
   c. Similarly use the data to determine when the women's world record will be within 5 minutes of the men's? Will the women's world record time ever surpass the men's?
2. The rates for two metropolitan parking ramps are given below:

   Mid City Parking Lot:
   - $4 per hour or fraction thereof
   - $36 maximum for 24 hours.

   Central Garage:
   - $5 per hour or fraction thereof
   - $21 maximum for 24 hours.

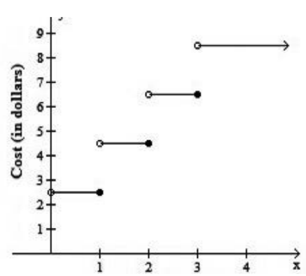
   a. Draw graphs to represent both parking situations (let t = time in hours from 0 to 24).
   b. You are in a line of cars waiting to exit the Mid City lot and notice that you have been in the parking ramp for 5 hours and 58 minutes. Do you want the cars in front of you to "hurry up, pay, and get out of the way"? Why? (Describe this situation using limits.)
   c. You are in a line of cars waiting to exit the Central Garage and notice that you have been in the parking ramp for 5 hours and 58 minutes. Do you want the cars in front of you to "hurry up, pay, and get out of the way"? Why? (Describe this situation using limits.)
   d. Which garage is more affordable?
3. Snow plows in Boatsville are working overtime this winter. Each time a plow makes a pass on a street (i.e. plows one side of the street), it removes 45% of the total snow on the road.

a. Assuming no melting due to sun or salt, how many passes will it take to remove 90% of the snow?
b. Boatsville's snow removal department has a contract with the borough that pays them based on the percentage of snow removed; specifically they get paid 400p/(100-p) dollars for removing p% of snow. How much do they get paid for removing 90% of the snow?
c. At a borough meeting, a Boatsville resident stands up and says "I pay taxes to this town, so I demand 100% of the snow is removed from my street!" Is this a reasonable request? Explain why or why not.

Appendix C

REAL-WORLD LIMITS POSTTEST
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1. An express mail service uses the following graph to determine how much to charge for overnight delivery of packages. You have to mail two packages; one weighing 1.03 pounds, the other weighing 2.98 pounds. How much will it cost to send both packages using the overnight service?



A) $9.00
B) $11.00
C) $13.00
D) $17.00
E) None of the above

Find the limit, if it exists.

2. Let $(x) = \frac{x^2 - 16}{x - 4}$. Find $\lim_{x \to 4} f(x)$.

A) 8
B) 2
C) 0
D) -8
E) Does not exist

Solve the problem.

3. Suppose the cost of removing p% of the pollutants from a chemical dumping site is given by

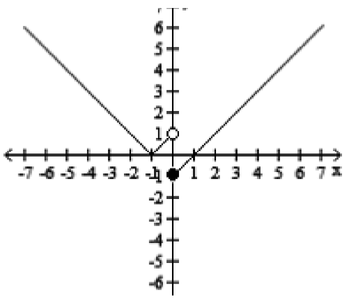
$$C(p) = \frac{\$20{,}000\,p}{100 - p}$$

Can a company afford to remove 100% of the pollutants? Explain.

A) Yes, the cost of removing p% of the pollutants is $200, which is certainly affordable.
B) No, the cost of removing p% of the pollutants is $200, a prohibitive amount of money.
C) No, the cost of removing p% of the pollutants increases without bound as p approaches 100.
D) Yes, the cost of removing p% of the pollutants is $2,000, which is certainly affordable.
E) Yes, the cost of removing p% of the pollutants is $20,000, which is certainly affordable.

Use the graph to evaluate the indicated limit and function value or state that it does not exist.

4. Find $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$.



A) $\lim_{x\to 0^-} f(x)$ does not exist; $\lim_{x\to 0^+} f(x) = -1$

B) $\lim_{x\to 0^-} f(x) = 1$; $\lim_{x\to 0^+} f(x) = -1$

C) $\lim_{x\to 0^-} f(x) = -1$; $\lim_{x\to 0^+} f(x) = 1$

D) $\lim_{x\to 0^-} f(x)$ does not exist; $\lim_{x\to 0^+} f(x)$ does not exist

E) $\lim_{x\to 0^-} f(x) = 1$; $\lim_{x\to 0^+} f(x) = 1$

Provide an appropriate response.

5. If the limit at infinity exists, find the limit.

$$\lim_{x\to\infty} \frac{16x^5 + 5x + 11}{12x^6 + 16x^5 + 32x^3 + 2}$$

A) 0
B) 1
C) 4⁄3
D) ∞
E) None of the above

Solve the problem.

6. It has been determined that the value V of a certain product decreases, or depreciates, with time t in years, where

$$V(t) = 100 - \frac{20t^2}{(t+2)^2}.$$

Find $\lim_{t\to\infty} V(t)$.

A) \$100
B) \$20
C) \$90
D) \$80

E) Does not exist

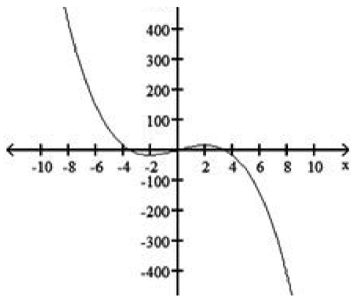
Appendix D

DERIVATIVE PRETEST

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the given graph of $f(x)$ to find the intervals on which $f'(x)>0$ or $f'(x)<0$ as stated below.

1.



A) $f'(x)$ is always < 0
B) $f'(x)>0$ on $(-4,4)$, $f'(x)<0$ on $(-\infty,-4)\cup(4,\infty)$
C) $f'(x)>0$ on $(-\infty,-2)\cup(2,\infty)$, $f'(x)<0$ on $(-2,2)$
D) $f'(x)>0$ on $(-\infty,2)$, $f'(x)<0$ on $(2,\infty)$
E) $f'(x)>0$ on $(-2,2)$, $f'(x)<0$ on $(-\infty,-2)\cup(2,\infty)$

Solve the problem.

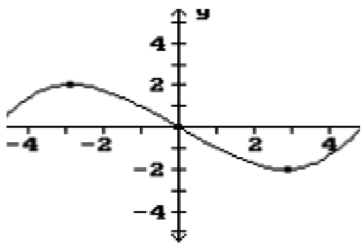
2. A company estimates that it will sell $N(x)$ pens after spending $\$x$ thousands on advertising as given by:

$$N(x) = -3x^3 + 450x^2 - 21,600x + 500,000 \quad \text{where } 40<x<70.$$

For which values of x is the rate of sales, $N'(x)$ increasing?
A) $50<x<60$
B) $x>40$
C) $40<x<50$
D) $40<x<60$
E) None of the above

Use the given graph of f(x) to find the intervals on which f''(x)<0 as indicated.

3.



A) (0,3)
B) (-3,∞)
C) (-3,3)
D) (0,∞)
E) (-∞,0)

Solve the problem.

4. The percent of concentration of a certain drug in the bloodstream x hours after the drug is administered is given by $K(x) = \frac{2x}{x^2 + 36}$. How long after the drug has been administered is the concentration a maximum? Round answer to the nearest tenth, if necessary.

A) 6 hours
B) 1.8 hours
C) 2 hours
D) 3.6 hours
E) 10 hours

Provide an appropriate response.

5. A drug that stimulates reproduction is introduced into a colony of bacteria. After t minutes, the number of bacteria is given approximately by:

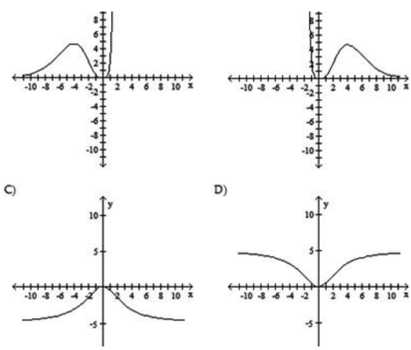
$N(t) = 1{,}000 + 36t^2 - t^3, 0 \le t \le 30$

At what value of t is the rate of growth maximum?

A) 24 minutes
B) 12 minutes
C) 6 minutes
D) 30 minutes
E) None of the above

6. Use the given information about the first and second derivatives of the function $f(x)$ in order to determine which of the following graphs (if any) represents $f(x)$.

$f'(x)>0$ on $(-\infty,-4)$ and $(0,\infty)$, $f'(x)<0$ on $(-4,0)$
$f''(x)>0$ on $(-\infty,-6)$ and $(-2,\infty)$, $f''(x)<(-6,-2)$, & $f''(x)=0$ at $x=-6$ and $x=-2$



E) None of the above

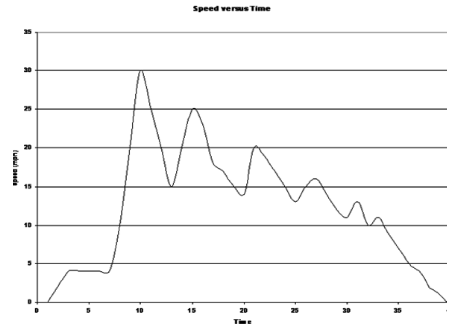
Appendix E

REAL-WORLD DERIVATIVES WORKSHEET

Real-World Rates of Change

This worksheet explores some possible applications of derivatives in real life. You are allowed to work in groups (3-5 people/group) to determine solutions to these problems; however, each individual must turn in a solution. If you do choose to work with others, you must write ALL the names of the members of your group on the paper you turn in.

1. Below is a graph showing the speed of a roller coaster at all times during one ride.



   a. What does the roller coaster look like? (i.e., draw a graph that shows the height of the roller coaster versus time during one ride.)
   b. When (approximately) is the roller coaster traveling the fastest?
   c. When (approximately) is the speed of the roller coaster increasing at the greatest rate?
   d. On what intervals (list all) is the speed of the roller coaster decreasing?

2. Almost all states in the U.S. increased in population from 2000 to 2010 (see results of the 2000 and 2010 census, below). In accordance with the U.S. Constitution, after the results of the 2010 census were declared official, changes were made to each state's apportionment in the U.S. House of Representatives (also listed on the chart):

	Results of 2000 Census	**Results of 2010 Census**	**Change in # of U.S. Reps.**
Arizona	5,140,683	6,412,700	+1
California	33,930,798	37,341,989	0
New York	19,004,973	19,421,055	-2
Pennsylvania	12,300,670	12,734,905	-1
South Carolina	4,025,061	4,645,975	+1
South Dakota	756,874	819,761	0

Utah	2,236,714	2,770,65	+1
Washington	5,908,684	6,753,369	+1
Total - USA	281,424,177	309,183,463	N/A

a. The population of California increased by more than the entire population of Utah, yet Utah gained a seat and California didn't. Explain why this makes sense.
b. How can a state gain population but lose a seat? (e.g. New York or Pennsylvania) Explain.
c. Using census results from 1950 – 2010, forecast the 2020 population of each of the states listed above. (A good place to find all this information is: http://www.census.gov/). Justify your reason for arriving at each number.
d. Using the census results from 1950 – 2010 (again) determine the growth rate over each 10-year period (there are six; 1960 vs. 1950, 1970 vs. 1960, and so on…) for each of the eight states. In which state(s) is the rate of growth currently increasing?

3. In the United States the consumer price index (CPI) measures changes in price levels of goods and services frequently purchased by U.S. consumers. The rate of change of the CPI is often used to represent decreases (or increases) in the purchasing power of the U.S. dollar; this figure is more commonly known as the inflation rate (when the inflation rate is negative it's referred to as deflation). In an attempt to compare "apples to apples" the monthly inflation rate is often calculated as the change in the CPI over one year (for example, February 2011 is compared with February 2010). Use data found here: (http://inflationdata.com/inflation/Inflation_Rate/HistoricalInflation.aspx) to investigate and answer the following questions. (Only use monthly data, not the yearly "AVE")

a. Graph monthly inflation rate versus time from January 1976 to February 2011. (use Excel!)
b. Using your graph, find the three consecutive 12 month periods during which the inflation rate decreased 11 out of 12 months (or 12 out of 12). When did each of these periods of continued deflation increasing inflation end? Can you find a contrasting 12 month period during which inflation

increased 11 out of 12 months (or 12 out of 12)? If so, when?

c. When did the greatest month-to-month inflation rate jump take place? What was the difference between the two consecutive months?

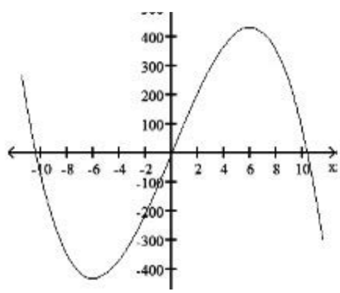
d. What is the greatest month-to-month inflation decrease? When did it occur?

Appendix F
Real-World Derivatives Posttest

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Use the given graph of $f(x)$ to find the intervals on which $f'(x)>0$ or $f'(x)<0$ as stated below.

1.



A) $f'(x)$ is always > 0
B) $f'(x)>0$ on $(-430,430)$, $f'(x)<0$ on $(-\infty,-430) \cup (430,\infty)$
C) $f'(x)>0$ on $(-6,6)$, $f'(x)<0$ on $(-\infty,-6) \cup (6,\infty)$
D) $f'(x)>0$ on $(-\infty,6)$, $f'(x)<0$ on $(6,\infty)$
E) $f'(x)>0$ on $(-6,6) \cup (6,\infty)$, $f'(x)<0$ on $(-6,6)$

Solve the problem.

2. A drug that stimulates reproduction is introduced into a colony of bacteria. After x minutes, the number of bacteria is given approximately by the following equation.

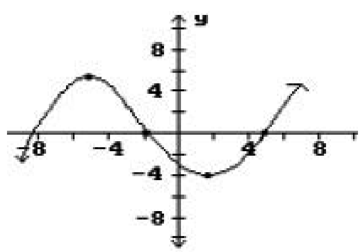
$$N(x) = 1,300 + 33x^2 - x^3 \quad \text{where} \quad 0 \le x \le 30$$

When is the rate of growth, $N'(x)$ increasing?

A) $11<x<22$
B) $11<x<30$
C) $0<x<11$
D) $0<x<22$
E) None of the above

Use the given graph of* f(x) *to find the intervals on which* f" (x)*<0 as indicated.

3.



A) (-5,5)
B) (-5,2)
C) (-∞,2)
D) (-2,∞)
E) (-∞,-2)

Solve the problem.

4. The percent of concentration of a certain drug in the bloodstream x hours after the drug is administered is given by $K(x)=\frac{3x}{x^2+36}$. How long after the drug has been administered is the concentration a maximum? Round answer to the nearest tenth, if necessary.

A) 1.8 hours
B) 3 hours
C) 3.6 hours
D) 6 hours

E) 10 hours

Provide an appropriate response.

5. A company estimates that it will sell $N(x)$ pens after spending \$$x$ thousands on advertising as given by:

$$N(x) = -2^3 + 318x^2 - 13,600x + 200,000 \text{ where } 10 \le x \le 90 \text{ .}$$

At what value of x do the rate of sales reach maximum?

A) 29.7
B) 53
C) 76.3
D) 90
E) None of the above

Use the given information about the first and second derivatives of the function $f(x)$ in order to determine which of the following graphs (if any) represents $f(x)$.
$f'(x)>0$ on $(-\infty,0)$, $f'(x)<0$ on $(0,\infty)$
$f''(x)>0$ on $(-\infty,-2)$ and $(2,\infty)$, $cf''(x)<(-2,2)$, c & $f''(x)=0$ at $x=-2$ and $x=-2$