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Preservice Mathematics Teachers' Personal Figural Concepts and Classifications About Quadrilaterals

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Abstract: The aim of this study was to determine preservice mathematics teachers' personal figural concepts and hierarchical classifications about quadrilaterals and to investigate the relationships between them. The participants were 57 preservice primary mathematics teachers in their senior year at a state university in Turkey. The preservice mathematics teachers were administered a questionnaire that consisted of 13 questions extracted from studies on the descriptions and images of quadrilaterals, identification of quadrilateral families among given images, and identification and classification of the relationships between quadrilaterals. The results showed that the preservice mathematics teachers' knowledge of quadrilaterals learnt at primary-secondary school level and prototypical images were dominant in their personal figural concepts. Also, the teachers didn't use the hierarchical definitions of quadrilaterals and were not able to establish relationships among quadrilaterals due to the effect of prototypical images in choosing a family category among the given images. On the other hand, the majority of the participants gave correct answers to the questions about the dual relationships among quadrilaterals. The study concluded that although the preservice teachers possessed formal definitions of quadrilaterals, their prototypical images affected their personal figural concepts.

Introduction

Geometry equips individuals with skills such as problem-solving, critical thinking, reasoning and higher-order thinking skills (NCTM, 2000). For this reason, the instruction of geometric concepts is of great importance in teaching mathematics. The instruction of geometric concepts helps individuals develop reasoning skills by recognising geometric shapes and exploring their attributes, comparing these attributes and developing certain shape classifications, and linking their attributes and making deductive inferences. Especially, the subject of quadrilaterals is a very rich source for research on these skills (van de Walle, 2012). Studies on quadrilaterals in the teaching of geometry focus on the identification and classification of quadrilaterals. An analysis of these studies shows that students experience difficulties in identifying quadrilaterals (Vinner, 1991, de Villers, 1998; Currie & Pegg, 1998; Pratt & Davison, 2003; Zaslavsky & Shir, 2005) and hierarchical classification (Monaghan, 2000; Erez & Yerushalmy, 2006; Pickreign, 2007; Fujita & Jones, 2007; Okazaki & Fujita, 2007; Fujita, 2012).

Research suggested that prototypical images are considered to be more important than the definitions and attributes of geometric figures (Hershkowitz, 1990), these prototypical examples of quadrilaterals are often identified correctly, but quadrilaterals in different orientations are not recognized (Fujita and Jones, 2007; Okazaki and Fujita, 2007; Fujita, 2012; Monaghan, 2000). These prototypical examples of concepts may sometimes lead to misconceptions and a conflict between the definition and family relations of a figure (Fujita and Jones, 2006; Fujita, 2012; Hershkowitz, 1990; Pratt and Davison, 2003). For example, although the standard definition of the parallelogram is given as "a quadrilateral with opposite sides parallel", the rectangle, the square and the rhombus are not thought to be

parallelograms because the concept image of the parallelogram does not allow all the angles and sides to be equal (Vinner, 1991).

Hierarchical classification and comprehension of quadrilaterals play a key role in establishing relations among quadrilaterals, solving problems, geometric proof studies and developing geometric reasoning skills (Fujita & Jones, 2007; Turnuklu et al., 2012; NCTM, 2000; van Hiele, 1999). For example, the solutions, proofs and properties of any quadrilateral in the family of parallelograms (e.g. the parallelogram) apply to other quadrilaterals (e.g. the square) as well. Research showed that students are affected by the prototypical images in their minds and cannot see the hierarchical relationships among quadrilaterals and, therefore, have difficulty in hierarchical classification (Fujita & Jones, 2007; Fujita, 2012; Monaghan, 2000). Many students cannot recognize the relationship among geometric properties (Fuys et al., 1988). For instance, Okazaki and Fujita (2007) stated that although most students think of a rhombus as a parallelogram, they do not realize that a square is a rectangle and a rhombus. Fujita and Jones (2007) suggested that images of quadrilaterals should be presented in connection with their properties when considering the relationships among quadrilaterals. Aktas and Aktas (2012a) found that, although the students in their study were able to recognize special quadrilaterals by using diagonal properties and making appropriate drawings, they were not able to identify the hierarchical relationships among quadrilaterals on their own.

Mathematics teachers play a key role in the perception of classifying and establishing a relationship between quadrilaterals (Turnuklu et al., 2012). For this reason, teachers' or preservice teachers' perceptions about this subject has been a popular research topic. Research revealed that teachers have difficulties similar to those of students. Most of the preservice teachers in some studies defined quadrilaterals under the effect of the images that they possessed (Kawasaki, 1992; Pickreign, 2007). In a study conducted with preservice teachers (aged 18 and 19-20), the majority of the preservice teachers were able to come up with correct drawings of quadrilaterals (except for the trapezoid), but they were not able to correctly define quadrilaterals (Fujita and Jones, 2007). Similarly, in another study conducted with preservice teachers (aged 19), although most of the preservice tea chers knew the correct definitions of quadrilaterals, they stated that they recognised quadrilaterals through their prototypical examples, and this situation made it difficult for them to understand the relationships among quadrilaterals (Fujita, 2012). Turnuklu et al. (2012) found that the quadrilaterals whose attributes were known most by the teachers were the square and rectangle, the teachers were able to correctly define quadrilaterals' angular and lateral properties, but they had problems about their diagonal properties. Turnuklu et al. (ibid) also reported that some of the teachers were not able to classify quadrilaterals and tried to configure the hierarchical classification without establishing any family relationships among quadrilaterals. This study first introduces the theoretical framework and presents the research objective. The method and instruments of data collection are then described and the findings presented and discussed.

Theoretical Framework
Formal Figural Concept and Personal Figural Concept

Mathematical definitions are an essential component of mathematics education. For this reason, student perceptions of mathematical definitions have been a popular research topic. The terms *concept definition* and *concept image* were proposed by Vinner and Hershkowitz (1980) and later developed by Tall and Vinner (1981). They were actually proposed to describe how students make sense of mathematical concepts. *A concept definition* is defined as a set of words that consists of terms explaining a concept. *A concept image* is defined as a set of cognitive structures involving the properties, mental images and

operations related to a concept. When students meet a concept that they already learnt in a new, unfamiliar context, they tend to employ the concept image rather than the concept definition as a result of their past experiences about that concept (ibid). A concept image should be formed by students themselves through activities designed to reveal what lies under a concept rather than making students just memorize a concept definition (Tall et al., 2000).

Definitions are particularly important in determining conceptual understanding of geometrical concepts (Silfverberg and Matsuo, 2008; Zazkis and Leikin, 2008; Usiskin et al., 2008). In addition to concept and image, which are defined as two separate mental categories of cognitive psychology, Fiscbein identified geometrical figures as a third category of mental representation (Fischbein, Nachlieli, 1998):

"A concept is usually defined as an abstract, general representation (an idea) of a category of objects or events. On the other hand, an image (especially a visual image) is a sensorial representation of an object or event. Visual images are sometimes described as 'pictures in the mind' because they possess spatial properties like extension, shape, location, magnitude. These two categories, images and concepts, though usually interacting in the course of mental activity, seem to be basically incompatible. A concept does not possess spatial properties, it is ideal and abstract, and an image is not reducible to an idea because of its sensorial properties. Nevertheless, one may identify a third category of mental representations which possess simultaneously both categories of properties. These are the geometrical figures."(p. 1193)

In this regard, geometrical concepts have a double nature characterised by two aspects: the figural and the conceptual (Mariotti, Fischbein, 1997; Fischbein 1993). While the figural aspect involves spatial properties (e.g. shape, position, and magnitude), the conceptual component involves abstract and theoretical nature (e.g. ideality, abstractness, generality and perfection) that geometrical concepts share with all other concepts. Fischbein (ibid.) called them *figural concepts*. For example, a circle is a figural concept. At the same time, it is a figure, a spatial (sensorial) representation and a concept (abstract, general, ideal). While the figural aspect of a figural concept facilitates mental operations with practical meaning such as modifying, cutting, and superposing, the conceptual aspect ensures the logical meaning and conceptual control of these operations. There is a harmony between the two aspects of a figural concept only in an ideal situation (Fischbein, Nachlieli, 1998). Fischbein (1993) suggested that the figural aspect is generally dominant and the conceptual aspect is not effective. A square, for instance, does not look figurally as a parallelogram. They have different views, but they are both formally parallelograms according to the definitions. Many mistakes made by students in geometric reasoning are actually caused by the gap between the two aspects of a figural concept (ibid). Fischbein (ibid) suggested that the development of figural concept into the ideal form is not a natural process. This process needs to be supplemented with didactic situations that will keep both the figural and conceptual aspect active.

Based on the definitions of *concept definition* and *concept image* (Tall and Vinner, 1981), Fujita and Jones (2007) reinterpreted the definition of *figural concept* (see Figure1). While Fischbein regarded figural concept as a process in which the harmony between the figural and conceptual aspect develops into the ideal form, he did not address the development of this process in individuals. Fujita and Jones, on the other hand, claimed that individuals have their own figural concept images and definitions that they construct through their own experiences of learning geometry, which they called *personal figural concept*. The notion of "ideal figural concept" that was proposed by Fischbein was considered as concept definition by Tall and Vinner. Fujita and Jones referred to the definitions discussed in Euclidean geometry including formal concept images and concept definitions as *formal figural concept*. The diagram below illustrates these concepts and the relationships among them:
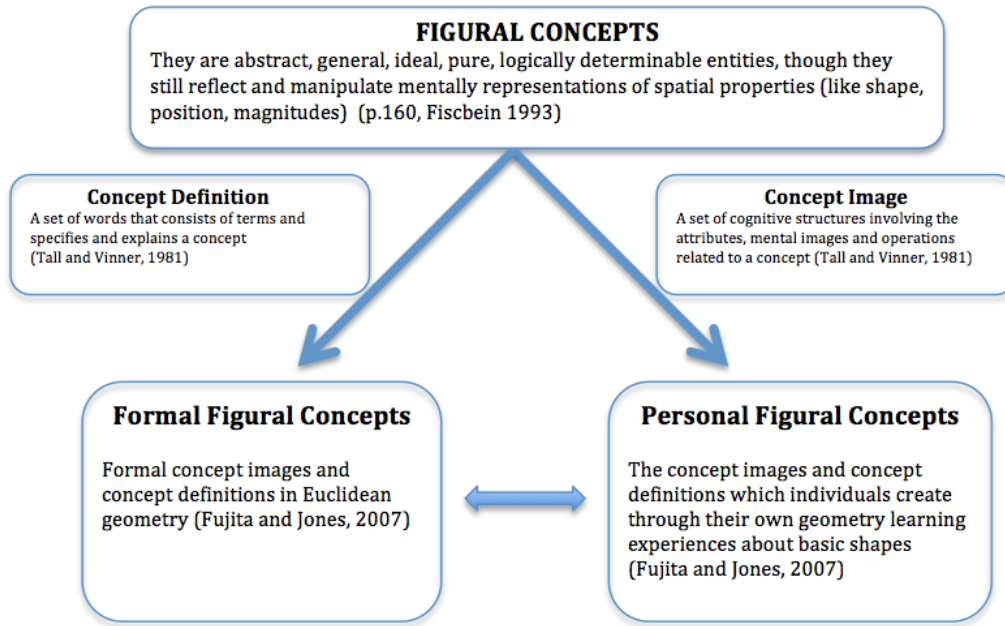
FIGURAL CONCEPTS
They are abstract, general, ideal, pure, logically determinable entities, though they still reflect and manipulate mentally representations of spatial properties (like shape, position, magnitudes) (p.160, Fiscbein 1993)

Concept Definition
A set of words that consists of terms and specifies and explains a concept (Tall and Vinner, 1981)

Concept Image
A set of cognitive structures involving the attributes, mental images and operations related to a concept (Tall and Vinner, 1981)

Formal Figural Concepts

Formal concept images and concept definitions in Euclidean geometry (Fujita and Jones, 2007)

Personal Figural Concepts

The concept images and concept definitions which individuals create through their own geometry learning experiences about basic shapes (Fujita and Jones, 2007)

Figure 1. Figural concepts

This diagram can be explained with an example: a student's (Ahmet) perception of "a rectangle". A rectangle has an image and a definition. In other words, a rectangle is a figural concept. In Euclidean geometry that is presented in textbooks and course syllabuses, a rectangle is defined as "a parallelogram with right angles" and it's shown with the image

(van De Walle, 2012, MEB, 2009). This is the formal figural concept definition of a rectangle. On the other hand, for Ahmet, a rectangle is "a quadrilateral with only opposite sides congruent and four 90° angles" and it has the image   . The figural concept about the rectangle in Ahmet's mind that consists of concept definition and concept image is considered to be Ahmet's personal figural concept (see Figure 2).
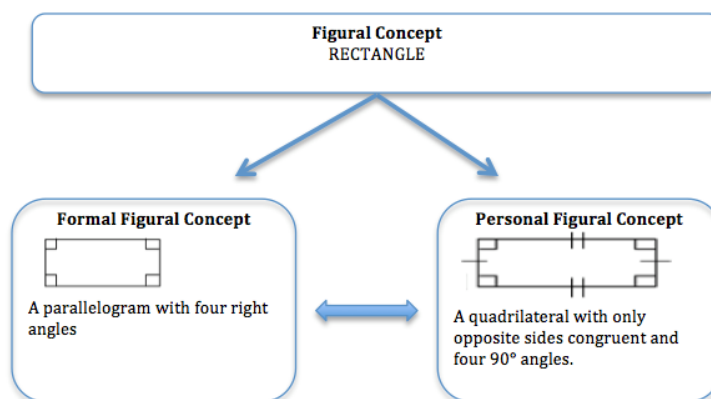
Figural Concept
RECTANGLE

Formal Figural Concept

A parallelogram with four right angles

Personal Figural Concept

A quadrilateral with only opposite sides congruent and four 90° angles.

Figure 2. Ahmet's perception of a rectangle

The instruction of geometric concepts requires that students have consistent knowledge of personal figural concepts and formal figural concepts. On the other hand, Fujita and Jones (2007) found that 80% of the participating students could define a parallelogram correctly and draw a correct image of it, but just 20% could identify all correct images of parallelograms (43% could only chose prototypical images). In other words, about half of the

students considered parallelograms in terms of their prototypical images and had difficulty in understanding the concept definition. This implies that these students' formal figural concepts and personal figural concepts of parallelograms were not consistent with each other and there was a gap between their formal and personal figural concepts.

Geometric Definition and Classification

It is often said that mathematics is a universal language and, therefore, mathematical concepts are the same in every context (Usiskin et al., 2008). However a mathematical concept is defined differently based on the logical relationship between different mathematical statements related to the concept (Winicki Landman and Leikin, 2000). In terms of mathematical activity, different perspectives are possible (Mariotti, Fischbein, 1997). The individual is free to make or choose statements about a mathematical concept within the perspective that he or she assumes. This freedom has been the starting point of many researches on mathematical definitions and how definitions should be made (De Villiers, 1994, 1998; Winicki Landman and Leikin, 2000; Leikin and Winicki Landman, 2000) An analysis of geometric definitions gives two different definition structures: partitional and hierarchical classification (de Villiers, 1994). The definitions that contain sufficient information to exclude non-examples are called partitional definitions. The definitions that contain all objects including all of the properties and that are more economical and shorter than partitional definitions are called hierarchical definitions. A property mentioned in a hierarchical definition applies to specific situations about the related concept. For example, as can be seen in both of the definitions of a parallelogram given below, the hierarchical definition is more economical than the partitional definition (ibid, p. 12)

Hierarchical definition: a quadrilateral with opposite sides parallel
Partitional definition: a quadrilateral with opposite sides equal and parallel, opposite angles equal, diagonals of different length halving each other, but not perpendicularly

While each concept is defined to be disjoint from one another in partitional classification, definitions are made by taking the relationships among concepts and inclusions into consideration in hierarchical classification. Usiskin et al. (2008) called the first situation as an exclusive definition and the other as an inclusive definition. Therefore, different relationships among figural concepts can be obtained based on the type of definition to be chosen. For example, a trapezoid is defined as "a quadrilateral with at least one pair of parallel sides" and, therefore, a parallelogram is a trapezoid. For this reason, the definition of a trapezoid includes the definition of a parallelogram and this is an inclusive definition. If a trapezoid is defined as "a quadrilateral having only one pair of parallel sides", a parallelogram will be excluded from the definition of a trapezoid and this is an exclusive definition.

The attributes that are necessary to define a concept can be considered as critical attributes whereas specific and irrelevant attributes can be considered as non-critical attributes (Hershkowitz, 1990). According to Erez & Yerushalmy (2006) and Markman (1991), the difficulties students have about quadrilaterals are caused by insufficient comprehension of the distinction between critical and non-critical attributes of quadrilaterals. The examples that include the subsets of the longest list of attributes containing all of the critical and non-critical attributes of a concept are called prototypical examples. These prototypical examples have an effect on concept image and these images are emphasized more than the concept itself (Hershkowitz, 1990; Fischbein, 1993). The perception of the prototypical example of a concept may prevent correct comprehension of that concept and lead to incorrect generalizations about the concept (Hershkowitz, 1990; Fujita and Jones, 2006; Fujita, 2012).

The classification of geometric shapes is also important, as well as identifying them in geometrical thinking. Both partitional classification and hierarchical classification, which are obtained depending on the type of the definition to be chosen, are equally valid in mathematics (de Villier, 1994). De Villier stated, "Since a classification and its corresponding definitions are arbitrary and not absolute, we should acknowledge that the choice between a hierarchical and a partition classification is often a matter of personal choice and convenience" (p.13, ibid). Therefore, it would be more appropriate to evaluate mathematics and the instruction of mathematics separately. According to the van Hiele model of the development of geometric thought, defining geometrical figures based on visualization and exploration of their properties are considered as Level 1 and Level 2, respectively, and making inferences by determining the relationships between figures is considered to be a higher level (Level 3). For example, while the expression "a square is a square" visually defines a square for a student at van Hiele Level 1, a square is a geometrical figure with four equal sides and four right angles for a student at Level 2. However, at Level 3, a student is expected to realize that a square is a special case of a rectangle. For this reason, in teaching mathematics, while partitional definition and classification can be used with younger students at van Hiele Levels 1 and 2, students at van Hiele Level 3 should be expected to understand and practice hierarchical definition and classification (ibid). Schwartz and Hershkowitz (1999) stated that, by its nature, partitional definition creates an environment that leads to the formation of prototypical example. In this regard, de Villier (1994) suggested that the concept of hierarchical inclusion could be developed in students at Level 1 and Level 2 by using dynamic geometry software. The ability to classify figures hierarchically is an indication of the development of the level of geometric thinking in students. According to de Villier (ibid), some of the most important functions of hierarchical classification are: "it leads to more economical definitions of concepts and formulation of theorems; it simplifies the deductive systematization and derivation of the properties of more special concepts; it often provides a useful conceptual schema during problem solving; it sometimes suggests alternative definitions and new propositions; it provides a useful global perspective "(p. 15, ibid). A number of classifications have been made throughout the history according to the accepted definitions and different properties of quadrilaterals (e.g. Euclid's, Posidonius' and Heron's, Ramus', Graumann's) (Athanasopoulou, 2008). For example, the classification proposed by Euclid in "Elements", which can be regarded as the first classification of quadrilaterals, does not include parallelogram (Figure 3). This is because "Euclid gave these definitions concerning rectangles, rhombuses, and squares as quadrilaterals independent of parallelograms. In the "Elements" Euclid defines the concept of parallel lines right after the definitions of the quadrilaterals. Therefore, he could not use the concept of parallel lines in his definitions of quadrilaterals" (p. 43, ibid).
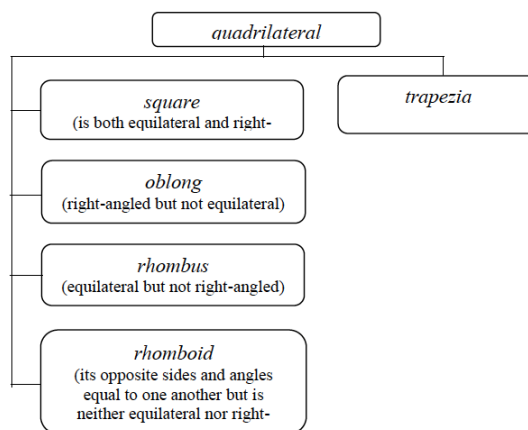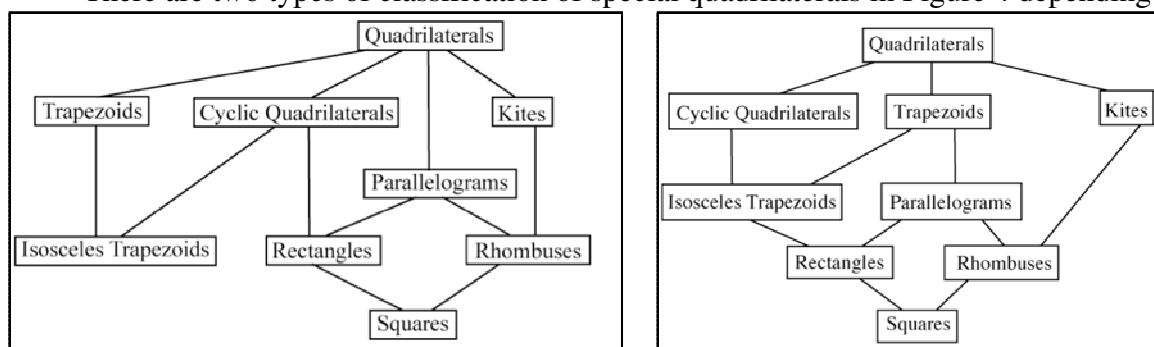

Figure 3. Euclid's classification of quadrilaterals (p. 42, Athanasopoulou, 2008)

Usiskin et al. (2008) examined a total of 100 textbooks published since 1838 in the US and concluded that modern classification of quadrilaterals consists of two types of definitions:

There are two types of classification of special quadrilaterals in Figure 4 depending

on the exclusive and inclusive definition of a trapezoid mentioned above.

Figure 4a. Exclusive definition   **Figure 4b. Inclusive definition**

Figure 4. Hierarchical classification (Usiskin et al., 2008)

Figure and Drawing

Describing, figure and drawing, the two concepts affecting concept image, and explaining the distinction between them is another important consideration. According to Laborde (1993) and Parzysz (1988), perception of geometric concepts (i.e. figural concept) is based on the concepts of figure and drawing. The distinction between figure and drawing can be useful in explaining how students interpret the geometric concepts that they study and the reasons for their interpretations. The term figure is defined as a mathematical object that depends on geometric attributes and is formed by the combination of these attributes. On the other hand, drawing is defined as the material representation of that object which consists of the object's trace on the screen or paper (ibid; Laborde, 1993). For example, the concept of the parallelogram as a figure depends on the geometric attributes. These attributes include parallelism and equality opposite sides, and their combination makes the concept. It is possible to determine an infinite number of parallelogram drawings related to this figure. When solving a problem, sometimes drawings might conflict with the geometric attributes used and prevent the perception of these attributes.

Dynamic geometry software facilitates students' understanding of the distinction between drawing and figure. If the image of a geometric shape on the screen changes without losing any of its properties when the dragging feature of this type of software is used and the image of the geometric shape is grabbed at a point and dragged, it is called "figure". However, if there is a change in the image on the screen as a result of dragging, then it is called drawing.

Aims of the Study

Research on the definitions of quadrilaterals and hierarchical classification, particularly studies about classifying the personal figural concepts mentioned above, is usually conducted with a wide range of participants – from high school students to prospective primary school teachers. However, none of these studies are about the personal figural concepts of primary mathematics teachers (preservice – inservice). On the other hand, secondary school mathematics curriculum coincides with a critical period for learning quadrilaterals and the relationships among them in the instruction of geometric concepts, and

transition from Level 2 to Level 3 of van Hiele levels of geometric thinking can occur in this period. For this reason, determining the perceptions of mathematics teachers and preservice mathematics teachers to teach at secondary schools about this subject is considered to be important. In this regard, the aim of this study is to determine preservice primary mathematics teachers' personal figural concepts about quadrilaterals.

A review of the literature suggested that findings about the definitions and classifications of quadrilaterals come from different study samples. Also, at the time of the study, there were no studies carried out with the same study sample in terms of both definitions and classifications. The existing relevant studies were limited only to some special quadrilaterals like parallelogram. In this study, on the other hand, a comprehensive questionnaire prepared based on the literature was administered to the same study sample. This study also aims to determine how preservice mathematics teachers make hierarchical classifications of quadrilaterals and to examine the relationship between their classifications and their personal figural concepts.

Method
Participants

This study was carried out with a total of 57 preservice primary mathematics teachers attending a state university in Turkey in 2011-2012 spring term. The Council of Higher Education regulates teacher-training departments of faculties of education in Turkey (for further information, see Altun, 2013; Yildirim & Ates, 2012; Uysal, 2012; Topkaya & Yavuz, 2011; Haciomeroglu, 2013; Kildan et al., 2013). In accordance with the regulations of this institution, the department of primary mathematics education that the participants were attending offered an eight-term (4 years) program. The participants took Geometry course and Analytic Geometry course in their 2nd and 5th terms, respectively. They also took Special Teaching Methods 1 and 2 in their 5th and 6th terms. Special Teaching Methods 1 and 2 syllabuses included teaching methods related to the instruction of mathematical concepts covered in secondary school mathematics curriculum. In addition, at the time of the study, they were doing their internship at a state school.

The participants were informed about this study, which would be carried out at the end of a required course, and they were explained that participation in the study was voluntary and they could see their knowledge of quadrilaterals by means of the study. Then they were asked to respond to the questionnaire items. Some students who were not present in the lesson or some of those who did not fill in the questionnaire that day visited the researchers in their office later and told they wanted to fill in the questionnaire. However, 14 preservice teachers did not participate in the study.

Data Collection Tools

Data were collected with a questionnaire consisting of some of the questions used by Fujita and Jones (2006; 2007), Okazaki and Fujita (2007) and Fujita (2012). The questionnaire consisted of three parts (see Annexe 1).
Part 1: The first question was about the relationships between some quadrilaterals. The second question asked the participants to define five special quadrilaterals and draw their figures. Also, in this part, the researchers added a question asking the participants to define and draw a rhombus.
Part 2: This part included questions about identification of quadrilateral families among given images and testing some postulates about parallelograms, rectangles and rhombuses.

Part 3: In this part, the participants were first asked to determine if a quadrilateral was a special case of another one by means of drawings between the images of quadrilaterals given. They were then asked to choose the images of a parallelogram, a rectangle and a rhombus among the images of quadrilaterals given in order to identify their personal images of quadrilaterals. After that, the participants were asked to test some postulates about parallelograms, rectangles and rhombuses. Finally, they were asked questions about the dual relationships between a rhombus and a parallelogram, a rectangle and a parallelogram, a square and a rhombus, and a square and a rectangle. These latter questions, which the students were asked to answer after the schema given at the beginning of Part 3, are considered to be important in that they support the relationships of prototypical images and definitions with classification.

Data Analysis

The quadrilateral definitions required in Part 1 were primarily evaluated according to the critical properties mentioned by the preservice teachers (e.g. either side properties or angular properties or both side properties and angular properties). The participants' partitional or hierarchical definitions about quadrilaterals were regarded as correct, but other definitions were regarded as incorrect. For example, a definition about trapezoid expressed as "a quadrilateral with no correlation between the sides (e.g. equality, parallelism, etc.)" was regarded as incorrect. The correct definitions were expressed in percentages.

The distinction between drawing and figure was taken into consideration while evaluating the participants' quadrilateral drawings. In this study, only the definitions and figures of quadrilaterals were required in paper-and-pencil environment, but solution of problems or configuration of geometric figures was not included. For some researchers, drawing transforms into figure by means of an explanation about geometric properties. This approach was adopted in this study while evaluating the preservice teachers' drawings about quadrilaterals. For this reason, the figures that were just simple images and were not drawn elaborately were regarded as "level of drawing" (see Figure 5) whereas those drawings showing the properties of the corresponding quadrilaterals on the images were regarded as "level of figure" (see Figure 5). As a result, the drawings at level of figure were regarded as correct.
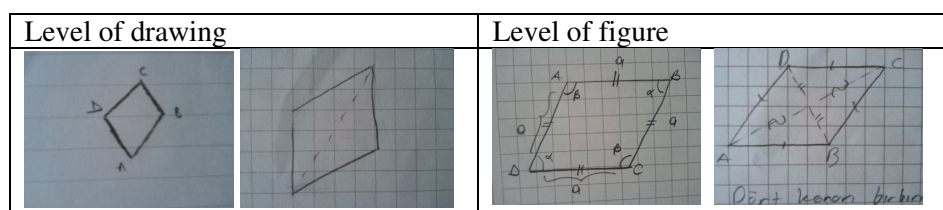

Figure 5. Sample drawings of the preservice mathematics teachers

The participants' responses to the other items in the questionnaire were evaluated with the marking criteria used in the studies where the questionnaire items in this study were taken from (Fujita and Jones 2006, 2007; Okazaki and Fujita, 2007; Fujita, 2012) (see Annexe 2). According the marking criteria, each question is measured by points between '0' and '3'. The preservice teachers who could identify all quadrilaterals correctly received '3' points whereas the teachers who could identify only prototypical images received '1' point. Similarly, the students who chose all the correct options among the given postulates received '3' points whereas the teachers who chose only the most obvious / the simplest options were awarded '1' point.

The participants' responses about the schema were evaluated with the marking criteria used in the study where the questionnaire items in this study were taken from. According to the criteria, the percentages of those making correct relationships were shown next to the arrows required to be drawn between the quadrilaterals.

In addition to these criteria, the responses given by the participants to the 1st question in Part 1 and to the schema question and the 5th question in Part 3 were analysed qualitatively and their internal validity was tested by the researchers. These qualitative analyses were used in both determining the preservice teachers' personal figural concepts and configuring the hierarchical classification schemas.

Results

Annexe 3 shows the overall evaluation of the preservice mathematics teachers' scores in the questionnaire. In addition to this evaluation, the results obtained in the qualitative analyses are presented in detail below.

Quadrilateral Definitions, Drawing Shapes and Family Identification

Table 1 below shows the responses given by the participants to the questions about definition and images. Among all the quadrilateral definitions made by the preservice mathematics teachers, the statements expressing either side properties or angular properties were evaluated under the 'side' category or 'angle' category whereas the statements expressing both of the properties simultaneously were evaluated under the 'side-angle' category. The percentages of the correct definitions made by the preservice mathematics teachers and the percentage of levels of drawing and figure in their quadrilateral drawings are shown in the table below.

	Definition					Shape		
	Side	Angle	Side-Angle	Correct Definition	Unanswered	Drawing	Figure	Unanswered
Parallelogram	46(81%)	-	11(19%)	55(96%)	-	11(19%)	41(72%)	5(9%)
Rhombus	29 (51%)	-	24(42%)	19(33%)	4(7%)	40(70%)	17(30%)	-
Rectangle	-	2(4%)	55(96%)	55(96%)	-	14(25%)	39(68%)	4(7%)
Square	4(7%)	-	51(89%)	54(95%)	2(6%)	11(19%)	41(72%)	5(9%)
Trapezoid	47 (82%)	1(2%)	5(9%)	26(46%)	4(7%)	23(40%)	27(48%)	7(12%)

Table 1. The definition and shape results about quadrilaterals

Parallelogram

All of the preservice mathematics teachers in the study were able to proffer a definition of a parallelogram. As can be seen in Table 1, almost 81% of the preservice mathematics teachers used only critical attributes including side lengths and parallelism such as "opposite sides parallel and equal". The rest of the preservice mathematics teachers (19%)

added non-critical angle attributes such as "opposite angles equal and consecutive angles supplementary". Out of the 57 preservice mathematics teachers, 4% stated that the angles were different from 90º while defining a parallelogram. It could be suggested that these preservice mathematics teachers made image-dependent definitions (visual definitions). According to the preservice teachers' responses to the items regarding shapes about parallelograms, approximately 91% were able to give an answer. Among them, 19% defined a parallelogram as a drawing and 72% gave it as a figure in their answers.

Out of the preservice mathematics teachers, 51% (29) were able to correctly identify the parallelogram family among the quadrilateral images given whereas 18% (10) chose only the prototypical examples among the shapes. However, some of the students did not include some special quadrilaterals in the parallelogram family. For example, 9% (5) of them did not think of the horizontal diamond-shaped rhombus (see Image 6 in Part 2, Annexe1) as a parallelogram and 5.26% (3) did not think of the rectangle standing vertically on the short side (see Image 7 in Part 2, Annexe1) as a parallelogram (see the Images in Part 2, Annex 1). Also, 4% of the participants did not think of the square in Image 4 and rectangles in Image 2 and 7 as a parallelogram.

Rhombus

About half of the preservice mathematics teachers (49.1%) did not mention parallelism (a critical attribute of a rhombus), but instead they defined it as "a quadrilateral with all sides equal". Only 33.3% (19) of the preservice teachers were able to proffer the correct definition of a rhombus. Unlike the other preservice mathematics teachers, one of them described it as a quadrilateral formed by combining the bases of two identical isosceles triangles. On the other hand, 7% of the preservice mathematics teachers were not able to come up with any definitions of a rhombus. According to the preservice mathematics teachers' answers about the shape of a rhombus, 70% of them regarded a rhombus as a drawing and 30% of them regarded it as just a figure. The preservice mathematics teachers' achievement level concerning the definitions and shape of a rhombus were similar.

The preservice mathematics teachers did better in identifying geometric families for a rhombus than in proffering definitions because 49% (28) of the teachers were able to identify the rhombus family correctly. However, 9% (5) of the teachers just marked the prototypical examples of a rhombus (see Images 5, 11 and 15 in Part 2, Annexe 1). In addition, some of the preservice mathematics teachers (5.26% (3)) had a narrow perception of a rhombus' prototypical examples and they thought of only images 5-11 as a rhombus (see Images in Part 2, Annexe 1). Moreover, 11% (6) of the preservice mathematics teachers ignored squares while identifying the rhombus family. Also, one teacher included a rectangle looking like a diamond shape in the rhombus family.

Rectangle

Almost all of the preservice mathematics teachers (96.49%) were able proffer a correct definition of a rectangular. While defining a rectangle, 60% (34) of them stated that its opposite sides and all of its angles were equal, but they did not mention the parallelism of the opposite sides. This situation was observed in drawing a rectangle as well, and 63.15% (36) of them did not specify parallelism attribute in their rectangle figures.

Only one of the preservice teachers defined a rectangle as "a parallelogram with right angles (90º)". Table 1 shows that more than half of the teachers (61.4%) preferred to make partitional definitions while 39% of the teachers were able to proffer hierarchical definitions.

The rectangle shapes showed that around 68% of the teachers drew a figure whereas about one-fourth of them thought of a rectangle just as a drawing independent from its attributes.

When the teachers were asked to identify the rectangle family, 46% of the preservice mathematics teachers were able to identify it correctly among the quadrilateral images given, 7% (4) did not choose the rectangle standing vertically on the short side (see Image 7 in Part 2, Annexe 1), 14% (8) excluded squares from the rectangle family, and 9% (5) did not include the oblique square (Image 11) in the rectangle family (see Images in Part 2, Annexe 1).

Square

Among the preservice mathematics teachers, 95% (54) were able to define a square correctly; 67% (38) of the preservice teachers stated that all of its sides and angels are equal, but they did not mention parallelism attribution of the opposite sides; 4% (2) were unable to proffer a definition of a square but one preservice teacher described it in a wrong way; 7% (4) defined a square as "a rectangle with all sides equal"; 4% (2) defined it as "a parallelogram with all four sides and all four angles equal"; and approximately 72% of the preservice mathematics teachers drew a square correctly. About one fifth of the preservice teachers thought of a square as just a drawing independent from its attributes.

Out of the preservice teachers, 84% (48) identified the square family correctly among the quadrilateral images given, 4% (2) of the teachers did not include an oblique square (Image 11, Annexe 1) in the square family, and 5% (3) of them thought of a rhombus as a square.

Trapezoid

As mentioned above, the literature presents two definitions of a trapezoid. In this part of the study, which deals with definitions of quadrilaterals, the responses of the preservice mathematics teachers who gave either of these two definitions precisely were regarded to be correct.

Among the preservice mathematics teachers, 46% (26) were able to define a trapezoid correctly. The vast majority of the preservice teachers who proffered correct definitions used the exclusive definition. Out of these teachers, 40% (23) defined a trapezoid as a rectangle "with two opposite sides parallel" or "with only two sides parallel". While defining a trapezoid, 30% (17) of the preservice teachers thought of it as a quadrilateral "with top and bottom sides parallel". Only 5% of the preservice teachers defined it as a quadrilateral "with at least two opposite sides parallel", 7% (4) of the teachers did not proffer any definitions of a trapezoid and 18% (10) proffered incorrect definitions of a trapezoid. These results revealed that the preservice teachers in this study had difficulty in defining a trapezoid. The definitions were based just on drawing without considering attributes such as "a quadrilateral with no congruent sides (e.g. equal, parallel, etc.), "a quadrilateral whose sides and angles are unknown but the sum of whose internal angles is 360 degrees", "a shape whose opposite sides are not 90⁰", "a quadrilateral with at least two angles not congruent", "a quadrilateral with sides and interior angles not congruent", "a quadrilateral with two opposite sides equal and the other two not equal" or "a quadrilateral with at least two sides not parallel".

According to the preservice mathematics teachers' responses about the shape of a trapezoid, 47% (27) of the teachers thought of a trapezoid as a figure and made a correct drawing, 40% (23) of the teachers thought of it as just a drawing, and 12% (7) of the teachers were not able to give any answer to this question.
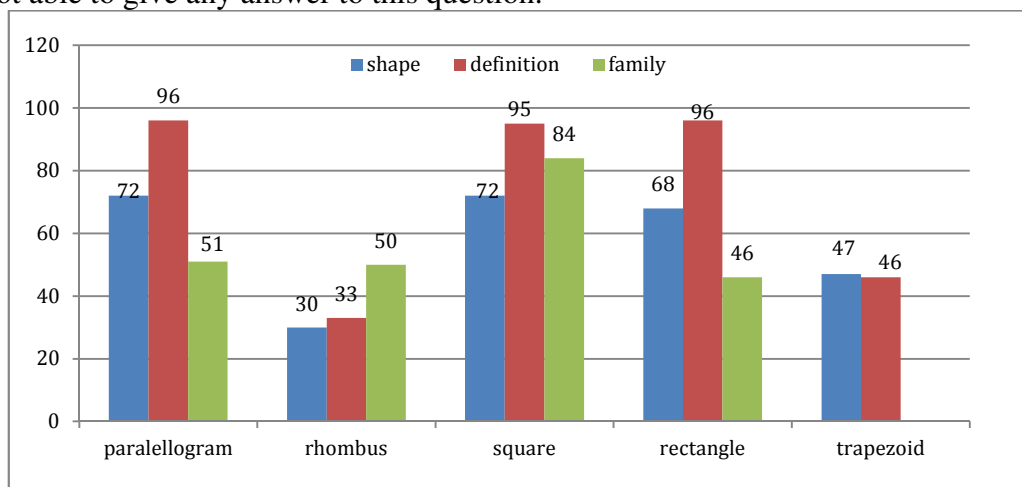

Table 2. Results about quadrilateral definitions, drawing shapes and family identification

The graph above shows the percentages of the correct definitions and figures proffered by the preservice mathematics teachers for each quadrilateral in Part 1 and the percentages of their correct responses about family identification in Part 2.

The percentages of the preservice teachers' achievement level concerning the definitions and drawings of a parallelogram, a square and a rectangle were similar (Table 2) whereas the percentage of achievement level for a trapezoid was different from the others. Although the majority of the teachers proffered correct definitions of a parallelogram, a square and a rectangle, those who thought of the shapes of quadrilaterals as figures had a lower percentage of achievement. This result showed that the preservice mathematics teachers were not able to distinguish between drawing and figure. About half of the teachers were able to identify the parallelogram family correctly, which is a significant result

compared to the results of other similar studies (Fujita & Jones, 2007; Aktas & Aktas, 2012b). Finally, only 26.3% (15) of the preservice teachers were able to correctly identify all of the parallelogram, rhombus, rectangle and square families.

Classification of Quadrilaterals

There were six types of classification among the responses given by the preservice teachers about the hierarchical classification of quadrilaterals. These classifications were made by the researchers based on the analysis of the hierarchical classification schema required in Part 3 and the responses given by the preservice mathematics teachers to the other questions.
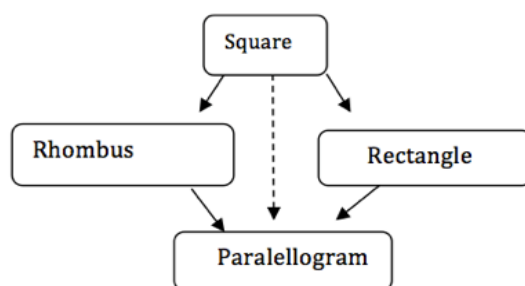
Figure 6. Preservice mathematics teachers' classification of quadrilaterals- Type 1

Among the preservice mathematics teachers, 53% (30) classified quadrilaterals as in Figure 6. While classifying quadrilaterals, 33.3% (19) of the preservice mathematics teachers stated that a square is a special case of a rhombus and a rectangle and they thought a rhombus and a rectangle are special cases of a parallelogram. Thus, the preservice mathematics teachers were able to make a hierarchical classification from the square to the parallelogram. In addition to this classification, 19.29% (11) of the preservice mathematics teachers again emphasized that a square is a special case of a parallelogram by drawing an arrow from the square to the parallelogram.
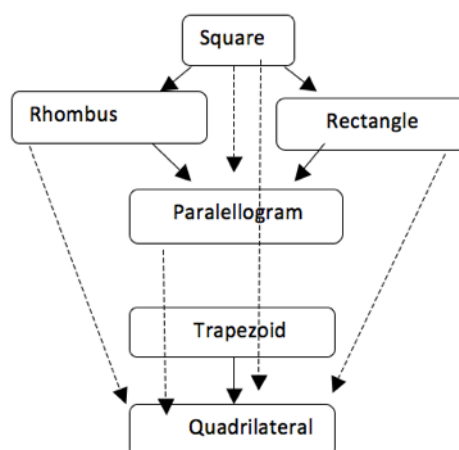
Figure 7. Preservice mathematics teachers' classification of quadrilaterals- Type 2

Out of the preservice teachers, 30% (17) classified quadrilaterals as in Figure 7. Unlike Figure 6, trapezoid and quadrilateral was included in classification here. In terms of hierarchical classification, no relationship was established between the parallelogram and the

trapezoid, a trapezoid was regarded as a special case of a quadrilateral and it was considered separately. In this classification, like in Figure 6, 16% (9) of the preservice mathematics teachers established a relationship between the square and the parallelogram. On the other hand, 4% (2) of the preservice mathematics teachers established a connection from each special quadrilateral to the quadrilateral and thought of each of the special cases as a special case of the quadrilateral.

It could be suggested that those preservice mathematics teachers establishing a relationship in Figures 6 and 7 with dashed arrows might have ignored the hierarchy among quadrilaterals and they might have remained at the level of pair correlation.
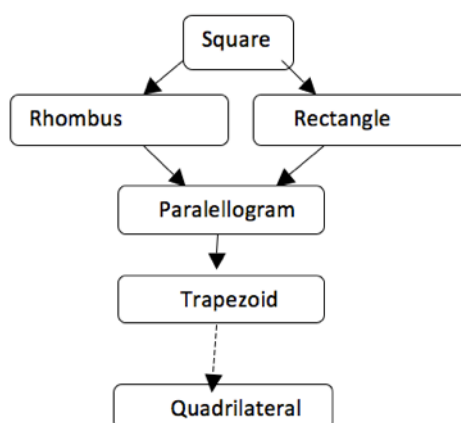

Figure 8. Preservice mathematics teachers' classification of quadrilaterals- Type 3

Among the preservice mathematics teachers, 7% (4) classified quadrilaterals as in Figure 8. Unlike the classification in Figure 6, a parallelogram was shown as a special case of a trapezoid. These preservice teachers established a connection between the parallelogram and the trapezoid and thought of a parallelogram as a trapezoid. On the other hand, one of the teachers was able to make a hierarchical classification from the square to the quadrilateral. This student thought of a parallelogram as a trapezoid.
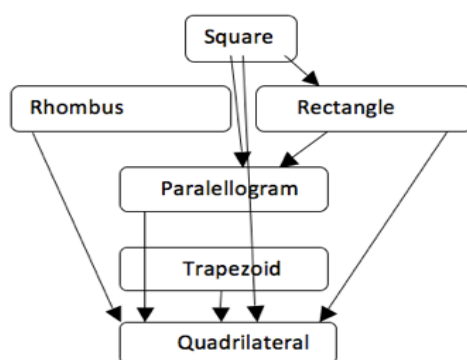

Figure 9. Preservice mathematics teachers' classification of quadrilaterals- Type 4

Among the preservice mathematics teachers, 2% (1) classified quadrilaterals as in Figure 9. This teacher connected quadrilaterals in pairs like the classification in Figure 7, but couldn't establish a relationship between the square and the rhombus and between the rhombus and the parallelogram.

Out of the preservice mathematics teachers, three teachers prepared the classification shown in Figure 10. Figure 10 shows that hierarchy was ignored in classification because the connections between the quadrilaterals were formed with two-way arrows. While thinking of a square as a special case of a rhombus, for example, these teachers also thought of a rhombus as a special case of a square.
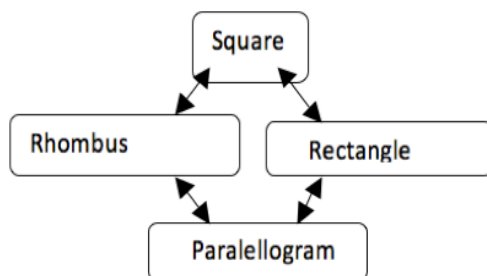

Figure 10. Preservice mathematics teachers' classification of quadrilaterals- Type 5

Another preservice teacher prepared the classification shown in Figure 11. This student established relationships between the square and the rectangle and between the quadrilateral and the parallelogram, but thought of a parallelogram as a special case of a rectangle in the relationship between the rectangle and the parallelogram. In addition, this teacher did not see a relationship between the rhombus and the square. In this hierarchical classification, this preservice teacher thought of the rectangle as the most common quadrilateral and placed it on the top.
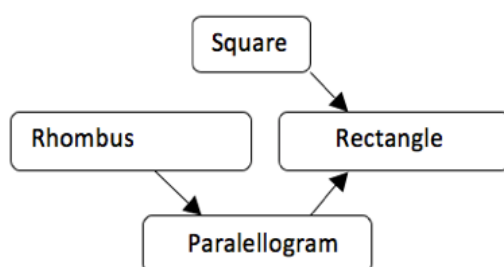

Figure 11. Preservice mathematics teachers' classification of quadrilaterals- Type 6

Discussion

The majority of the preservice mathematics teachers defined a parallelogram as "a quadrilateral with the opposite sides parallel". Also, they defined a trapezoid as "a quadrilateral with two opposite sides parallel", "a quadrilateral with only two sides parallel" or "a quadrilateral with bottom and top bases parallel". In addition, they defined a rhombus as "a quadrilateral with all of the sides equal", a rectangle as "a quadrilateral with the opposite sides and all of the angles equal", a square as "a quadrilateral with all the sides and angles equal". According to these definitions, the preservice mathematics teachers emphasised parallelism in the definitions of a parallelogram and a trapezoid whereas they did not mention it in the definitions of a rhombus, a square and a rectangle. Similar results were reported by Fujita (2012), Fujita & Jones (2007), Okazaki & Fujita (2007), Heinze & Ossietzky (2002) & Turnuklu et al. (2012). The term parallelism is related to the term parallelogram (Fujita & Jones, 2007) and it turned out to be the most common attribute of a prototypical example of a

trapezoid in this study. This may be the reason why the preservice mathematics teachers emphasized parallelism attribute in the definitions of these quadrilaterals. The quadrilaterals that the preservice mathematics teachers in this study had most difficulty in defining were the rhombus and trapezoid. One fourth of the preservice teachers either defined a trapezoid incorrectly or they didn't proffer any definition at all. This result is similar to the finding reported by Turnuklu et al. (2012), who found that trapezoid was the least known quadrilateral by their participants. It is also comparable to the finding reported by Fujita (2012), who found that the participants had insufficient knowledge of a rhombus.

According to the preservice mathematics teachers' definitions, partitional definitions were used for a square, a rhombus and a rectangle; there were very few hierarchical definitions; and exclusive definitions were made for a trapezoid. It could be suggested that what the preservice teachers understood by the term 'definition' was to write down all of the attributes of a shape (Herbst et al., 2005). The secondary school and high school curricula and textbooks adopt the exclusive definition of a trapezoid stated as "a quadrilateral with only one pair of opposite sides parallel" and partitional definitions of the other quadrilaterals. The reason why the majority of the preservice teachers adopted this attitude could be their own previous learning experiences at secondary school, high school and university.

According to the definition and shape drawing results concerning quadrilaterals, the achievement level of the participants for shape drawing, particularly for parallelogram, was lower than the achievement level for definitions. This result is different from those of Fujita and Jones (2007). This difference might have been caused by the fact that, among the preservice teachers' drawings in this study, only figures were regarded to be correct. On the other hand, the preservice teachers' failure to think of shapes as figures could be interpreted to mean that they did not know about the distinction between figure and drawing (Paryzsz, 1988; Laborde, 1993).

By means of the questions about identification of quadrilateral families, the preservice mathematics teachers' prototypical images were determined. In family identification, the teachers couldn't do as well as they did in parallelogram definition and shape drawing. The oblique prototype of a parallelogram (see Image 1 in Part 2, Annexe 1) was so dominant in their personal figural concepts that they did not think of a square, a rectangle and a rhombus as a parallelogram and they did not include Image 5 or 7 in the parallelogram family because they were affected particularly by images (see Images in Annexe 1).

The preservice teachers included Image 5 and Image 11 in the rhombus family but they excluded image 15, which showed that the teachers had a narrow perception of a rhombus' prototypical examples and they tended to consider only the most common examples (Monaghan, 2000). It could be suggested that those teachers who did not think of a square as a special case of a quadrilateral and did not include Image 4 and Image 11 in the family and those who included Image 13, which looks like a typical rhombus although it is actually a rectangle, in the family had very dominant typical image perceptions in their personal figural concepts.

Some of the preservice teachers didn't include Image 7, which is one of the prototypical examples of a rectangle, in the rectangle family. This might have been caused by the fact that Image 2, which dominates students' personal figural concepts and is often covered in textbooks in this way, was perceived as the most common shape of a rectangle, which was also reported by Monaghan (ibid). In addition, some of the preservice teachers didn't include Image 11 in the rectangle family probably because they were affected by the orientation and appearance of the shape and, therefore, they thought the shape could be a rhombus rather than a square. Although the majority of the preservice mathematics teachers did well in identifying the square family, some of the teachers did not include Image 11 in the family probably because Image 4 was the dominant typical image of a square especially in these teachers' personal figural concepts.

The results revealed that the concept images that made the preservice mathematics teachers' personal figural concepts consisted of dominant prototypical images. These results are similar to those reported in studies conducted with different samples (e.g. students, preservice primary school teachers, etc.) (Fujita & Jones, 2007; Okazaki & Fujita, 2007; Fujita, 2012; Clements et al., 1999; Monaghan, 2000). The preservice mathematics teachers' definitions of a quadrilateral showed that the preservice teachers actually defined prototypical images. For example, in their definitions of a parallelogram, those preservice teachers who stated that the angles were not equal to 90° made their definitions clearly based on the most common prototypical image of a parallelogram. This finding shows that these preservice teachers' personal figural concepts did not include a square and a rectangle as a parallelogram, and therefore, the hierarchy among the members of the parallelogram family could be limited. If these preservice teachers had taken formal figural concept into consideration, they could have realized that a square and a rectangle were also a parallelogram and angle couldn't be limited to 90°. As Okazaki (1995) also stated, however, with the effect of images, the teachers realized that a rhombus is a parallelogram more easily than they realized the same situation for a square and a rectangle (Okazaki & Fujita, 2007). Vinner (1991) stated that just proffering a correct definition of a mathematical concept is not sufficient to have an accurate understanding of that concept. The preservice mathematics teachers' identification of quadrilateral families among the images given actually supports this suggestion. In family identification, the teachers couldn't do as well as they did in parallelogram definition and shape drawing. The vast majority of the preservice mathematics teachers were not able to identify all of the families of parallelogram, rhombus, rectangle and square. This suggests that they had difficulty in establishing relations among quadrilaterals based on images and hierarchical classification (Fujita, 2012; Fujita & Jones, 2007; Okazaki & Fujita, 2007).

The preservice mathematics teachers' hierarchical classifications showed that the vast majority of the preservice teachers were able to establish a relationship between a square, a rhombus, a rectangle and a parallelogram. They were able to recognize a square as a special case of a rhombus and a rectangle. They were also able to recognize a rhombus and a rectangle as special cases of a parallelogram. In addition, they were able to make a hierarchical classification from a square to a parallelogram. About one third of these preservice teachers included trapezoid and quadrilateral in this classification and came up with a larger classification. This result is different from the result reported by Okazaki and Fujita (ibid),

Although not many, six of the preservice teachers tried to establish separate relations between special quadrilaterals and the common quadrilateral probably because they ignored the hierarchy among quadrilaterals and they remained at the level of pair correlation among quadrilaterals. The preservice teachers' failure to establish relations between a square and a rhombus and between a rhombus and a parallelogram could be an indication of limited images in their personal figural concepts about these quadrilaterals. In addition, it could be suggested that these students and those thinking that two-way relations could be established among quadrilaterals had lower levels of reasoning in a geometric environment.

Okazaki and Fujita (ibid), Fujita and Jones (2006) found that the difficulties experienced in the hierarchical classification of quadrilaterals were caused by the gaps between preservice teachers' formal figural concepts and personal figural concepts, and the images in their personal figural concepts affected their definitions and hierarchical classifications of quadrilaterals. Similarly, this study found that there were gaps between the preservice teachers' personal figural concepts of a parallelogram, a rhombus, a rectangle, a trapezoid and formal figural concepts, but their personal figural concept of a square was similar to the formal figural concept of it. Like the studies mentioned above, this study also found that the preservice teachers did not use the hierarchical definition to define quadrilaterals and, therefore, they were not able to establish a relationship between

quadrilaterals in identifying families and they were affected by prototypical images. However, there were different results about the hierarchical classification. When the preservice mathematics teachers were asked questions about the relations among quadrilaterals, the vast majority of the teachers were able to establish hierarchical relations. The preservice mathematics teachers in this study possessed formal definitions of quadrilaterals, but they were affected by the concept image-prototypes forming their personal figural concepts when they were working on images. Therefore, it could be suggested that the preservice mathematics teachers did not use the concept definitions.

Conclusion

Considering the fact that the definition and image are the components of figural concept, it is evident that, even if definition was consistent with formal definition, this was not sufficient in the preservice mathematics teachers' personal figural concepts and the existing prototypical images affected their personal figural concepts. Despite the preservice mathematics teachers attended university for four years, their learning experiences about quadrilaterals at primary and secondary schools were dominant in their personal figural concepts. The preservice mathematics teachers' definitions of quadrilaterals (partitional definitions) turned out to be the ones presented in secondary school and high-school textbooks and they dominated their personal figural concepts. These results suggest that preservice teachers need training to ensure that their personal figural concepts about quadrilaterals are similar to their formal figural concepts. For this reason, universities' pedagogical curricula regarding mathematics education should be revised. Also, preservice mathematics teachers should be provided with various teaching environments aided by contemporary popular dynamic geometry software to examine hierarchical definition and classification, relations among quadrilaterals and the cases of quadrilaterals apart from their prototypical images presented in textbooks and courses.

In addition, although both research and course syllabuses have recently put more emphasis on hierarchical definition and classification, preservice mathematics teachers need to realize that there can be different definitions of quadrilaterals, there can be different relationships between concepts, and it is essential that different definitions should be preferred according to students' levels of geometric knowledge and needs.

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Appendix One
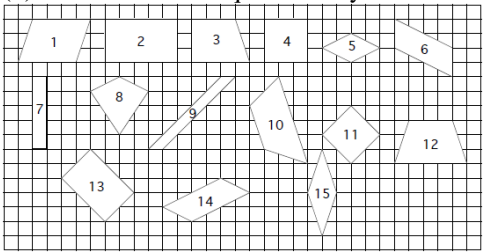**Questionnaire**

Part1
1. Answer the following questions, and state your reason briefly. • Is a square a trapezium? • Is a square a rectangle? • Is a parallelogram a trapezium? 2. A kite is defined as 'a quadrilateral, which has both pairs of adjacent sides equal'. Define the following quadrilaterals, and draw an image of each • A parallelogram • A square • A rectangle • A trapezium • A rhombus

Part 2	
1) Which of the quadrilaterals 1-15 above are … (a) members of the Parallelogram family (b) members of the Rhombus family (c) members of the Rectangle family (d) members of the Square family 2) Read the following sentences carefully, and circle the Statements which you think are correct. (a) There is a type of parallelogram which has right angles. (b) The lengths of the opposite sides of parallelograms are equal. (c) The opposite angles of parallelograms are equal. (d) There is a type of parallelogram which has 4 sides of equal length.	3) Read the following sentences carefully, and circle the statements which you think are correct. Also describe a rectangle in words. (a) The lengths of the opposite sides of rectangles are equal. (b) The opposite angles of rectangles are equal. (c) There is a type of rectangle which has 4 sides of equal length. 4). Read the following sentences carefully, and circle the statements which you think are correct. (a) The lengths of the opposite sides of rhombuses are equal. (b) The opposite angles of rhombuses are equal. (c) There is a rhombus which has right angles. 5) Answer whether the following statements are true or false a. There is no relationship between a rhombus and a parallelogram. True/False b. It is possible to say that a rhombus is a parallelogram. True/False c. It is possible to say that a parallelogram is a rhombus. True/False

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Part 3.1

Part 3

If a quadrilateral is a special case of another, show this by drawing arrows between them. An example drawing is given below.

Rhombus ➤ Kite: Rhombus is a special case of a kite

Part 3.2

1) In the following quadrilaterals (the shapes with the thick black lines), next to each one, put (/) for those you think are in the parallelogram family, (X) for those you think do not belong to the parallelogram family, or if you are not sure, put (?)

2) In the following quadrilaterals (the shapes with the thick black lines), put (/) for those you think are in the rectangle family, (X) for those you think do not belong to the rectangle family , or if you are not sure, put (?)

3) In the following quadrilaterals (thick lines), put (/) for those you think are in the rhombus family, (X) for those not in the rhombus family, or if you are not sure, put (?)

4) Read the following sentences carefully, and put (/) for those you think are correct, (X) for those that are incorrect, and if you are not sure, put (?)

a-Questions about Parallelograms

(a) ( ) The lengths of the opposite sides of parallelograms are equal.

(b) ( ) There are no parallelograms which have equal adjacent sides.

(c) ( ) The opposite angles of parallelograms are equal.

(d) ( ) There are no parallelograms which have equal adjacent angles.

(e) ( ) There is a parallelogram which has all its sides equal.

(f) ( ) There is a parallelogram which has all equal angles.
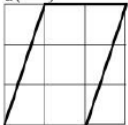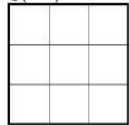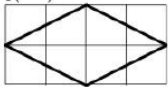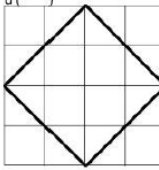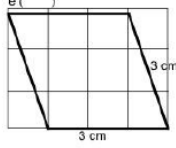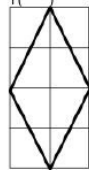
b-Questions about Rectangles

(a) ( ) The lengths of the opposite sides of rectangles are equal.

(b) ( ) There are no rectangles which have equal adjacent sides.

(c) ( ) The adjacent angles of rectangles are equal.

(d) ( ) The opposite angles of rectangles are equal.

(e) ( ) There is a rectangle which has all equal sides.

c-Questions about Rhombuses

(a) ( ) The lengths of the opposite sides of rhombuses are equal.

(b) ( ) The adjacent sides of rhombuses are equal.

(c) ( ) There are no rhombuses which have equal adjacent angles.

(d) ( ) The opposite angles of rhombuses are equal.

(e) ( ) There is a rhombus which has all equal angles.

5) Read the following sentences carefully, and put (/) for those you think are correct, (X) for those which are incorrect, or if you are not sure, put (?).

1. About parallelograms and rhombuses

(a) ( ) It is possible to say that parallelograms are special types of rhombuses.

(b) ( ) It is possible to say that rhombuses are special

a( ) b( ) c( ) d( ) e( ) 3 cm 3 cm f( )	types of parallelograms. 2. About parallelograms and rectangles (a) ( ) It is possible to say that parallelograms are special types of rectangles. (b) ( ) It is possible to say that rectangles are special types of parallelograms. 3. About squares and rhombuses (a) ( ) It is possible to say that squares are special types of rhombuses. (b) ( ) It is possible to say that rhombuses are special types of squares. 4. About squares and rectangles (a) ( ) It is possible to say that rectangles are special types of squares. (b) ( ) It is possible to say that squares are special types of rectangles.

Appendix Two
Marking criteria for evaluation of questionnaire

	Question	3 pt	2 pt	1 pt	0 pt
Part 2	1a	At least ten of the following: 1,2,4,5,6,7,9,11,13,14,15	At least six of 1,5,6,9,11,14,15 or eight of 1,2,4,6,7,9,11,13,14,15	At least three of the following: 1,6,9,14	others
	1b	4, 5,11,15	At least three of the following: 4,5,11,15	At least two of the following: 4,5,11,15	others
	1c	2,4,7,11,13	At least three of the following: 2,4,7,11,13	At least two of the following: 2,4,7,11,13	others
	1d	4,11	At least one of 4,11	At least one of 4,11 and others	others
	2	Correct for at least three of a,b,c,d	Correct for b and c, correct for at least one of a , d	Correct for at least one of b,c	others
	3	Correct for at least three of a,b,c,d	Correct for a and b, correct for at least one of c , d	Correct for at least one of a,b	others
	4	Correct for a,b,c	Correct for a and b	Correct for at least one of a,b	others
	5	Correct for b	Correct for a and b (if 2 or 3 pt in Q1)	Correct for c (if 2 or 3 pt in Q1)	others
Part 3	1	Correct for at least seven of a,b,c,d,e,f,g,h	Correct for at least three of b,d,f,g or Correct for at least six of a,b,c,d,e,f,g,h	Correct for b, g	others
	2	Correct for all of a,b,c,d,e,f	Correct for at least one of c,e and correct for a,b,d,f	Correct for at least four of a,b,c,d,e,f	others
	3	Correct for all of a,b,c,d,e,f	Correct for b and correct for at least four of a,c,d,e,f	Correct for at least four of a,c,d,e,f	others
	4a	Correct for at least five of a,b,c,d,e,f	Correct for a, c and correct for at least two of b,d,e,f	Correct for at least one of a, c and correct for at least two of b,d,e,f	others
	4b	Correct for all of a,b,c,d,e	Correct for a, c and correct for at least two of b,d,e	Correct for at least one of a, c and correct for at least two of b,d,e	others
	4c	Correct for all of a,b,c,d,e	Correct for a,b,d and correct for at least one of c,e	Correct for at least two of a,b,d and correct for at least one of c,e	others

Table 3. Marking criteria for Part 2 and Part 3 of questionnaire

Appendix Three
Preservice mathematics teachers' scores

Partitions	Questions		3pt	2pt	1pt	0pt
PART 2	Q1	a	39(68%)	5(9%)	10(18%)	3(5%)
		b	34(60%)	4(7%)	7(12%)	12(21%)
		c	27(47%)	12(21%)	9(16) %	9(16%)
		d	48(86%)	1(2%)	1(2%)	6(11%)
	Q2		42(74%)	4(7%)	9(16%)	2(4%)
	Q3		37(65%)	18(32%)	2(4%)	-
	Q4		48(84%)	2(4%)	7(12%)	-
	Q5		47(82%)	5(%)	1(2%)	4(7%)
PART 3	Q1		51(89%)	2(4%)	3(5%)	1(2%)
	Q2		51(89%)	1(2%)	5(9%)	-
	Q3		33(56%)	19(33%)	3(5%)	2(4%)
	Q4	a	53(93%)	1(2%)	2(4%)	1(2%)
		b	50(88%)	5(9%)	2(4%)	-
		c	44(77%)	3(5%)	10(18%)	-

Table 4. Preservice teachers' results of Part 2 and Part 3