Educational Sciences: Theory & Practice - 13(3) • 1920-1927

www.edam.com.tr/estp
DOI: 10.12738/estp.2013.3.1419

An Investigation of Problem Solving Approaches, Strategies, and Models Used by the 7th and 8th Grade Students when Solving Real-World Problems

İbrahim BAYAZİT[a]
Erciyes University

Abstract

This study scrutinises approaches and thinking processes displayed by the elementary school students when solving real-world problems. It employed a qualitative inquiry to produce rich and realistic data about the case at hand. The research sample included 116 students. The data were obtained from written exam and semi-structured interviews, and they were analysed using content and discourse analysis methods. The results indicated that most of the students disregarded the real-life situations that the problems are related to. They displayed non-realistic approaches that included application of rules, procedures, and factual knowledge in a straightforward and uncritical way. Most of the students lacked the ability to use alternative approaches and appropriate strategies. Some constructed models of the problem situations, yet most of these students were not able to utilise these instruments as a conceptual tool to gain insight into the situations. Rather, models were used as part of the routine – rules, procedures and factual knowledge – that the students applied uncritically.

Key Words

Real-World Problems, Problem Solving Approaches, Student Thinking, Problem Solving Strategies, Mathematical Models.

Problem solving is considered the most significant cognitive activity in professional and everyday life (Jonassen, 2000). International documents suggest that problem solving should be used as an integrating theme in the mathematics curricula assuming that this could increase students' achievements in mathematics (Cockroft, 1982; National Council of Teachers of Mathematics [NCTM], 1989). It is indicated that students who followed problem-based mathematics curricula outperformed their counterparts both in mathematics achievement and in problem solving (Cai, 2003; Cai & Nie, 2007; Clarke, Breed, & Fraser, 2004). Problem refers to a situation in which the desired goal has to be attained but direct path towards the goal is blocked. It refers to a situation that requires resolution, but an individual sees no apparent path to the solution (Krulik & Rudnick, 1985; Orton & Wain, 1994). Briefly, if a situation causes cognitive conflicts in the minds of individuals it can be considered as a problem (Baki, 2006). Problem solving is a dynamic process in which students try to understand the situation, make a plan for the solution, select or develop methods and strategies, apply all these heuristics to get the solutions; and finally they check out the answers obtained (Barnet, Sowder, & Vos, 1980; Mayer, 1985; Polya, 1973; Schoenfeld, 1992; Suydam, 1980). In this process one may use various strategies (e.g., making a list, looking for patterns, working backwards) and different kinds

a **İbrahim BAYAZİT, Ph.D.,** is currently an associated professor at the mathematics education. His research interests include qualitative relationships between teaching and learning, cognitive processing of algebraic concepts, teacher's pedagogical content knowledge, analogy use in teaching and learning mathematics, mathematical models and modelling, problem solving and problem posing. *Correspondence*: Assoc. Prof. İbrahim BAYAZİT, Erciyes University, Faculty of Education, Mathematics Education Department, Kayseri, Turkey. Email: ibayazit@erciyes.edu.tr, bayindiroglu@yahoo.com Phone: +90 352 207 6666/37110

of models (see Bayazit & Aksoy, 2008; Baykul, 2000; LeBlanc, 1977; Posamentier & Krulik, 1998). Mathematical problems are classified into two major categories: *routine* and *non-routine* problems. Routine problems can be resolved by the application of rules, procedures and basic operations that the problem solvers already know (Arslan & Altun, 2007; Mahlios, 1988). Non-routine problems causes great difficulties for students (Elia, Heuvel-Panhuizen, & Kolovou, 2009); this is because they do not have a straightforward solution; rather they request creative and critical thinking, employing alternative approaches and using various strategies and mathematical models (Altun, 2005; Inoue, 2005). In order to solve such problems one may need to use metacognitive skills including self-monitoring and self-regulations (Hartman, 1998; Mayer, Sims, & Tajika, 1995; Nancarrow, 2004).

One type of non-routine problem includes real-world problems (Verschaffel, De Corte, & Vierstraete, 1999). Their solution requests giving particular attention to real-life contexts that the problems are related to. Students need to utilise their intuitive knowledge and daily-life experiences to resolve real-world problems (Nesher & Hershkovitz, 1997). They provide an opportunity to apply mathematical knowledge to real-world situations (Brown, 2001; NCTM, 1991); it is for this reason educators suggest that real-world problems should be incorporated into the teaching of mathematics. Nevertheless, many researchers (Chacko, 2004; Greer, 1993; Reusser & Stebler, 1997; Verschaffel, De Corte, & Lasure, 1994; Yoshida, Verschaffel, & De Corte, 1997) have reported that students from primary school to university level display strong tendency to act non-realistically and exclude real-world knowledge from their solutions. They do not consider the factual relationships between real-world situations evoked by the problem statements and the mathematical operations they carry out. Three problems of this kind are given below (derived from Verschaffel et al., 1994).

1. 450 solders must be bussed to their training site. Each army bus can hold 36 solders. How many buses are needed?
2. Steve has bought 4 planks each 2.5 meters long. How many planks 1 meter long can he saw from these planks?
3. Runner Item: John's best time to run 100 meters is 17 second. How long will it take him to run 1 kilometer?

These problems have in common the potential for the students' responses to include some realistic considerations. A realistic answer is taken to mean one which pays some attention to just those sorts of realistic considerations that might characterise problem solving outside of the classroom. From this perspective, a realistic answer for the first problem would be 13 rather than 12.5. A realistic answer to the second problem is 8; because in reality one can saw only 2 planks of 1 meter from a plank 2.5 meters long. The third problem does not have a single correct answer; a wide range of answers considerably larger than 170 would be a realistic response to this problem. Yet, Verschaffel et al. (1994) reported that 49% of the students – the research sample included 75 students at the ages of 10-11 years – displayed realistic considerations when responding to the first problem. This figure declined to 13% for the second problem and further to 3% for the third one. These findings were replicated by many researchers who reported that not only elementary school students (Chacko, 2004; Greer, 1993; Yoshida et al., 1997) but also prospective teachers (Verschaffel, De Corte, & Borghart, 1997) try to solve real-world problems without apparent concern for what would be realistically meaningful outside of the classroom. These authors argue that this is a consequence of the way mathematics is traditionally taught in schools.

Purpose and Research Questions

A review of literature indicated that in Turkey a considerable body of research has been conducted to investigate primary school students' proficiencies at solving routine problems (see Altun, 1995; Özsoy, 2002; Sulak, 2005) and non-routine problems (Altun & Memnun, 2008; Dönmez, 2002). Yet, there is almost no research that examines the students' performances in solving real-world problems. It is for this reason the present study takes the interest further and investigates elementary school students' proficiencies at solving real-world problems. It attempts to scrutinise their capability at using realistic considerations, problem solving strategies and mathematical models. The study addresses the following research questions:

1. How successful 7^{th} and 8^{th} grade students are at solving real-world problems?
2. What sort of approaches and thinking do they display when solving real-world problems?
3. Do they use problem solving strategies and mathematical models; if they do so, how successful they are at using these instruments?

Method

Research Design, Sampling and Data Collection Instruments

The study employed a qualitative inquiry (Yin, 2003) to produce rich and realistic data about the case at hand. The research sample included 116 students from four different schools located in an urban setting in Kayseri. Of them 32 students were 7th graders and 84 were 8th graders. Data were collected through written exam and semi-structured interviews. First, a written exam that included six real-world problems was administered to the students (Table 1). To ensure validity and reliability issues all the problems were checked and revised in terms of content and language through a pilot study and in accord with the experts' opinions. The written exam was completed within 45-60 minutes. Students were encouraged to provide reasons for their answers. Clarification interviews were conducted with four students after the written exam. The interviewees were selected based on the diversity of approaches and the quality of thinking that they revealed in the exam; accordingly one from low achievers, two from medium achievers and one from high achievers were involved in the interviews. In the interviews students were invited to resolve the problems one by one. Then, the line of inquiry developed in accord with the students' responses and the aspects of clinical interview (Gingsburg, 1981) were considered to gain insight into the students' thinking processes. Interviews were audio-taped and the annotated field notes were taken for consideration.

Theoretical Framework and Data Analysis

Overall, literature about real-world problems (Chacko, 2004; Reusser & Stebler, 1997; Verschaffel et al., 1994; Yoshida et al., 1997) and using strategies and mathematical models in problem solving activities (Lesh & Harel, 2003; Verschaffel et al., 1999) provided a theoretical basis for the data analysis. Several methods and techniques were used. Content and discourse analysis (Miles & Huberman, 1994; Philips & Hardy, 2002) were utilised to discern meaning embedded in the students' written and spoken expressions. Data analysis was an iterative process that evolved gradually. First, students' exam papers were examined and a summary of their answers to each question was written up. In-depth examination of the students' responses continued and codes were established to identify approaches, strategies and models that they used when solving the problems. In the last phase of analysis a pattern coding was applied and the codes were collected under more general categories. Also, interviews were fully transcribed and considered line by line. A summary of students' answers to each question was written up. These documents were read thoroughly a couple of times and, then, codes were established to distinguish sort of approaches, strategies and models that the students used. Repeated on different copies of the text this eventually led to the creation of three major categories: Realistic answer (R-A), non-realistic answer (N-R-A) and other answers (O-A).

Table 1.
The Six Problems Used in the Study

Short Name	Problem
Field	Ahmet owns a rectangular field with the lengths of its sides 1000 and 1500 meters. He split the land among his children in a way each child gets a square field. In this case, what is the least number of children that Ahmet has?
Tablet	Alican has to take a pill every three hours and there are 10 pills in a box. How long does it take for him to take all the pills?
Course	Mustafa goes 3 days to art and 2 days to guitar courses in week. How many days he does not have a course (Adapted from Verschaffel et al., 1997).
Queue	Nihat and Aykut stand in a queue. Nihat is the 8th person from the start and Aykut is the 12th person from the end. There are also three people between them. How many people are there in the queue?
Picnic	A bus can carry 37 students and 350 students are to be transported for a picnic. How many busses are needed? (Adapted from Verschaffel et al., 1997).
Laundry	A housewife hangs out the laundry on a sunny day. It takes 25 minutes for 3 kg laundry to dry out. How long does it take for the same kind of 9 kg laundry to dry out under the same weather condition? (Adapted from Verschaffel et al., 1997).

Realistic answers reflected rational contemplation and included an understanding of the real-world knowledge even though problem solvers could not obtain all the alternative solutions. Non-realistic answers displayed no understanding of the real-world situations elicited by the problem statements at all. Such responses resulted from using rules, procedures and arithmetical operations in a straightforward and uncritically way. For instance, the *course problem* (Table 1) has three different solutions. If a student obtained only one solution through arithmetical operations evoked by the problem statement (such that, 3+2=5, so Mustafa does go to course 7-5=2 days in a week) his/her answer was classified as N-R-A. If a student illustrated two or three solutions his/her answer was collected under the category of R-A. If the students' responses included serious mistakes concerning problem solving approaches, methods, strategies, models and basic operations they were collected under the category of O-A. In addition, a fourth category was added, namely *no answer* (N-A).

Results

The research findings indicated that most of the students did not display realistic considerations when responding to the problems. They applied rules, procedures and factual knowledge without reflecting upon whether or not these were appropriate for the solution of the problems. The majority of them lacked the ability to use alternative approaches, authentic strategies and appropriate mathematical models. A summary of students' responses to six problems in the written exam can be seen in table 2.

Table 2.
A Summary of the Written Exam Results

Problem Name	%R-A	%N-R-A	%O-A	%N-A
Field	10,3	45,7	17,2	26,7
Tablet	20,8	69,8	9,5	---
Course	19,0	70,7	---	10,3
Queue	---	65,5	27,6	6,9
Picnic	44,8	30,2	12,9	12,1
Laundry	19,8	71,6	2,6	6,0

Abbreviations: R-A: Realistic answers, N-R-A: Non-realistic answers, O-A: Other answers, N-A: No answer.

The table shows that less than half of the participants gave realistic answers to *picnic problem*. Nearly one third of the students demonstrated non-realistic approaches and identified the number of buses with the decimals like 9.5. The percentages of realistic answers declined to 19.8% for the *laundry* problem. More than two third of the students applied the idea of direct proportion uncritically and claimed that if 3 kg laundry dries out in 25 minutes 9 kg laundry needs 75 minutes to dry out in the same weather condition. Concerning the *tablet problem* the overwhelming majority (69,8%) ignored the real-life condition that there is no need to three hours time to take the first pill and claimed that one takes all the pills in 30 hours. This indicated again students' inclination to apply proportional reasoning in a straightforward way. The remaining four items also required realistic considerations. Various approaches, several strategies and different kinds of mathematical models could be used to tackle these tasks; and two of them, *course* and *queue problems*, have more than one solution. Yet, again the vast majority of the students lacked the ability to consider realities of the daily-life situations that the problems are related to. Only one tenth of the participants gave R-A to the field problem. Working on a rectangular model these students partitioned the field into three square lands – one in 1000x1000 m^2 area and two in 500x500 m^2 area. Nearly half of the participants produced N-R-A and the majority of them (37% of the participants) employed the idea of *greatest common divisor (GCD)*. 19% of the students displayed realistic considerations in responding to the course problem. Three quarter of the participants gave N-R-A and these all followed strictly a sequence of operations elicited by the problem statement and argued that 'since Mustafa goes 3 days to art and 2 days to guitar courses he takes two days off in a week with the accompanying operations such that 3+2=5, 7-5=2'. *Queue* problem yielded no realistic answer. Those who produced N-R-A found out one of the solutions following strictly the way suggested by the problem statement such that 8+3+12=23.

The research findings also showed that the majority of the students had not attained an ability to use strategies and models properly. In responding to the field problem 55.5% of the students worked on a rectangular field model. However, less than one fifth of these students (10.3%) utilised these instruments as a conceptual tool for their realistic considerations and obtained three plots of lands – one in 1000x1000 m^2 and two in 500x500 m^2. The rest of them manipulated the models as part of the routine, which included an application of the idea of *greatest common divisor* in a straightforward way, and partitioned the field into six equal parts. 20.7% of the students constructed models for the tablet problem. These models included a series of 10 drawings in square or circle shapes each of which standing for a pill; so they were appropriate to represent the problem situation. Yet, again two third of these students produced N-R-A. These students focused upon the number of pills, not upon the time span between two successive pills, and claimed that if one takes a pill in three hours he/she would take 10 pills in 30 hours. For the queue problem, 44.8% of the students worked on appropriate models; nevertheless, none of them was able to use these instruments in a way that increased their realistic considerations. Making a table or systematic list could be an effective strategy to resolve *course* and *tablet* problems. However, only 8% of the students incorporated making a list strategy into their solution of the tablet problem and gave R-A. The remaining did no use strategy at all.

Interview with four students produced results that complemented the findings obtained from the written exam (Table 3). Students acted more realistically when the problems included simple ideas and basic operations.

Table 3.
A Summary of the Interviews Results

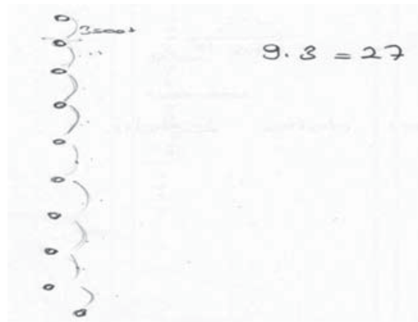
Problem Name	Gökçe[3]	Onur	Ayça	Alper
Field	R-A	N-R-A	N-R-A	N-R-A
Tablet	R-A	N-R-A	N-R-A	N-R-A
Course	R-A	R-A	N-R-A	N-R-A
Queue	N-R-A	N-R-A	N-R-A	N-A
Picnic	R-A	R-A	N-R-A	R-A
Laundry	R-A	R-A	R-A	N-R-A

Three students gave R-A to each of the picnic and laundry problems. Employing the idea of direct proportion Ayça claimed that if 3 kg laundry dries out in 25 minutes, 9 kg laundry needs 75 minutes to dry out under the same weather condition. On further probing she said: "*... Ohh, I see [laughing], nothing changes, no matter how much laundry we hang out; it could be 3 kg or 9 kg, it does not matter, the time does not change...*". This exchange suggests that students are likely to revise their thinking when the problem contexts are very familiar to them. However, they had difficulties at producing R-A if the problems were cognitively more demanding. This resulted from their lack of flexibility at thinking and using strategies and models properly. None of the interviewees used problem solving strategies. Gökçe was the only student who gave R-A to the field and tablet problems. In both cases she worked on models and utilised these instruments as a conceptual to gain insight into the situations. This can be seen in the following exchange (*Dialogue 1*):

> Int: There are ten pills here, so did not you multiply the time by 10?
>
> Gökçe: It is not like that; actually we can see it here [on the model, see Figure 1]. ... ...there are ten pills; yet, here the important thing is the time between the pills; that is why I multiplied it by 9... this is the tricky point in this question... We do not need to wait three hours to take the first pill...

As to the course problem, Gökçe and Onur displayed realistic considerations while the other two claimed that Mustafa takes two days off, because he goes 5 days to courses in a week. Queue problem yielded no R-A. Gökçe and Onur worked on models while Ayça made arithmetical calculations. Following a sequence of operations evoked by the problem statement they claimed that there are 23 people in the queue. The following exchange indicates students' lack of flexibility at interpreting the situation from different perspectives (*Dialogue 2*):

Figure 1.
A Model Constructed by Gökçe for the Tablet Problem



> Int: Cannot you think that Aykut is standing somewhere on the left side of Nihat. Is not that possible?
>
> Onur: How he [Aykut] comes before Nihat? Nihat is the 8th person from the start.
>
> Int: Imagine that you are the 19th person from the end in a queue including 20 people. In this case, you stand at the second place from the beginning and many people come after you...
>
> Onur: I cannot understand what you mean…[silence]… Do not you think my solution is more plausible… Here it is [pointing at the model, see Figure 2] ... Nihat is the 8th person from the beginning and Aykut is the 12th person from the end... and there are three mode people in between...

Although Onur is given a suggestion as to how he could interpret the model from different perspective to obtain a realistic solution he is not able to do so. He continues to use it in a limited way that is actually suggested, implicitly though, in the problem statement.

Figure 2.
A Model Constructed by Onur for the Queue Problem

Discussion and Conclusion

The purpose of this study was to examine problem solving approaches, strategies and models used by the 7th and 8th grade students when solving real-world problems. Our findings complement the results of previous studies (Chacko, 2004; Reusser & Stebler, 1997; Verschaffel & De Corte, 1997; Verschaffel et al., 1994; Yoshida et al., 1997). Overall, the research findings indicated that the students excluded real-world knowledge and experiences from their solutions. They did not consider the factual relations between the real-life contexts elicited by the problem statements and the operations they carry out. Most students acted uncritically and applied their prior knowledge in a way evoked by the story of the problems. In the written exam less than half of the students displayed realistic considerations in responding to the picnic problem; however, this figure declined dramatically and only one fifth of the students gave realistic answers to each of the tablet and laundry problems. More than two third of the students misused their knowledge of direct proportion and produced non-realistic answers. Likewise, nearly one fifth of the students recognised that the course problem has more than one solution. The situation got worse when the problem requested more cognitive demands and flexibility in thinking. Only one tenth of the students displayed realistic considerations in responding to the field problem. Nearly half of the students produced N-R-A and with the overwhelming majority being inclined to use uncritically their knowledge of greatest common divisor. The queue problem yielded no realistic answer. The interviews with four students produced results that complemented the students' performances in the written exam. Only Gökçe was consistent in her provision of realistic answers to most of the problems. The remaining three students took into account the realities of the problem context when the problem included simple ideas and operations. In other cases, they totally ignored realities of the problem contexts or produced no answer.

Students' ignorance of the realities of the problem contexts appears to have some cognitive and pedagogical reasons. Our evidences suggest that most students display result-oriented, not process-oriented, problem solving approaches. The priority for them is getting an answer. They do not pay attention to the underlying meaning of the rules and procedures that they use. None of the students checked out plausibility of the answers they obtained, and this indicate their inclination to skip one crucial step of Polya's (1973) acclaimed problem solving model – the so called *looking back*. Also, their written answers included no indication that they made attempts to incorporate critical and creative thinking into their solutions. Students' concentration on the result but not on the process of solution suggests, indirectly though, that they possess a sort of misconception that every problem has only one solution (De Corte, 2000).

The research findings also provide us with some insight into the strategy and model use. It is illustrated throughout the result section that students were dependent upon rules, procedures and some sort of arithmetical operations. The majority of them did not use strategies at all. Making a table or systematic list could be an efficient strategy to produce realistic answers to the course problem; yet, none of the students utilised these instruments. In the written exam, less than one tenth of the students used making a list strategy in responding to the tablet problem, and these all produced realistic answers. This confirms the results of previous studies in that there appears to be positive relations between students' capability at using strategies and their proficiency in solving non-routine problems (Cai, 2003; Kantowski, 1977). Mathematical models would enable students to identify key aspect of the real life situations that the problems are related to; thus they could enhance students' realistic considerations (Blum, 1993; Blum & Ferri, 2009; Fischbein, 2001; Zbiek, 1998). In this study some students constructed models of the problem situations; however, a few of them were able to utilise these instruments as a conceptual tool to gain insight into the situations. The remaining encountered difficulties. Most of these students constructed inappropriate models – models were inappropriate because they lacked the content validity to represent the problem situations (Bayazit & Uğur, 2011) -and manipulated them as part of the routine. An analysis of the data sets indicated that construction of inappropriate models was resulting from the students' lack of understanding of the problem situations, the so called *cognitive models* (Greca & Moreira, 2002). On the other hand, although some students produced appropriate conceptual models (Norman, 1983)- conceptual models included some kind of drawing that are visually capable of representing the problem situation – by intuition they could not use them in conjunction with the information given in the problem statements. These students were unable to revise their thinking by reflecting upon the models that they constructed (Dialogue 2). It can be concluded from these evidences that construction of appropriate models would be essential but not sufficient to raise realistic considerations. One needs to be capable of using such instruments in cooperation with the information in the problem statements so that she/he could develop more realistic conceptions for the solution of real-world problems.

In conclusion, the new Turkish mathematics curriculum (Talim Terbiye Kurulu Başkanlığı [TTKB], 2008) delineates problem solving as a process-oriented activity and suggests that teachers should appreciate what the students do in this process rather than the results that they obtain. It recommends that students should be allowed to work on the problems the solution of which requests being flexible in thinking, adopting various approaches, using appropriate strategies and models, and incorporating creative and critical thinking into the solution. Also, the new curriculum reflects Freudenthal's (1991) idea of mathematisation in that it greatly appreciates the transfer of knowledge from mathematics to daily life. However, the majority of the students in the present research have not attained learning objectives and standards set down in the new curriculum (TTKB). This might be resulting from the limitations associated with the textbooks and the classroom practices. Thus, the relationships between students' performances at solving real-world problems and the way such problems are presented in the textbooks and classroom teachings need to be explored. This is the issue for further research.

References/Kaynakça

Altun, M. (1995). *İlkokul 3., 4. ve 5. sınıf öğrencilerinin problem çözme davranışları üzerine bir çalışma*. Yayımlanmamış doktora tezi, Hacettepe Üniversitesi, Sosyal Bilimler Enstitüsü, Ankara, Türkiye.

Altun, M. (2005). *Eğitim fakülteleri ve ilköğretim matematik öğretmenleri için matematik öğretimi*. Bursa: Aktüel Yayınları.

Altun, M. ve Memnun, D. S. (2008). Matematik öğretmeni adaylarının rutin olmayan matematiksel problemleri çözme becerileri ve bu konudaki düşünceleri. *Eğitimde Kuram ve Uygulama*, *4* (2), 213-238.

Arslan, Ç., & Altun, M. (2007). Learning to solve non-routine problems. *Elementary Education Online*, *6* (1), 50-61.

Baki, A. (2006). *Kuramdan uygulamaya matematik eğitimi*. İstanbul: Bilge Matbaacılık.

Barnet, J. C., Sowder, L., & Vos, K. E. (1980). Textbook problems: supplementing and understanding them. In Krulik, S. (Ed.), *Problem solving in school mathematics: NCTM yearbook 1980* (pp. 334-371). Reston VA: NCTM Inc.

Bayazit, İ. ve Aksoy, Y. (2008). Matematiksel problemlerin öğrenim ve öğretimi. E. Bingölbali ve M. F. Özmantar (Ed.), *İlköğretimde karşılaşılan matematiksel zorluklar ve çözüm önerileri* içinde (s. 287-312). Ankara: Pegem Akademi.

Bayazit, İ. ve Uğur, D. (2011). Öğretmen adaylarının ürettiği matematik modellerinin bilişsel ve kavramsal boyutları itibariyle incelenmesi. *Ahmet Keleşoğlu Eğitim Fakültesi Dergisi*, *32,* 49-67.

Baykul, Y. (2000). *İlköğretimde matematik öğretimi: 1-5. sınıflar için*. Ankara: Pegema Yayınevi.

Blum, W. (1993). Mathematical modelling in mathematics education and instruction. In T. Breiteig, I. Huntley, & G. Kaiser-Messmer (Eds.), *Teaching and learning mathematics in context* (pp. 3-14). New York: Ellis Horwood Limited.

Blum, W., & Ferri, B. (2009). Mathematical modelling: Can it be taught and learned? *Journal of Mathematical Modelling and Application*, *1* (1), 45-58.

Brown, S. (2001). *Reconstructing school mathematics: Problems with problems and the real world*. New York: Lang.

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: An exploratory study. *International Journal of Mathematical Education in Science and Technology*, *34* (5), 719-737.

Cai, J., & Nie, B. (2007). Problem solving in Chinese mathematics education: Research and practice. *ZDM Mathematics Education*, *39*, 459-473.

Chacko, I. (2004). Solution of real-world and standard problems by primary and secondary school students: A Zimbabwean example. *African Journal of Research in SMT Education*, *8* (2), 91-103.

Clarke, D., Breed, M., & Fraser, S. (2004). The consequences of a problem-based mathematics curriculum. *The Mathematics Educator*, *14* (2), 7-16.

Cockroft, W. H. (1982). *Mathematics count*. London: Her Majesty's Stationary Office.

De Corte, E. (2000). New perspectives for research and practice in mathematics education. *Pythagoras*, *52*, 33-46.

Dönmez, N. (2002). *İlköğretim 2. ve 3. sınıf öğrencilerinin problem çözme stratejilerini kullanabilme düzeyleri üzerine bir çalışma*. Yayımlanmamış yüksek lisans tezi, Uludağ Üniversitesi, Sosyal Bilimler Enstitüsü, Bursa, Türkiye.

Elia, I., Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics. *ZDM Mathematics Education*, *41*, 605-618.

Fischbein, E. (2001). Tacit models and infinity. *Educational Studies in Mathematics*, *48*, 309-329.

Freudenthal, H. (1991). *Revisiting mathematics education*. Dordrecht, The Netherlands: Kluwer.

Ginsburg, H. (1981). The clinical interview in psychological research on mathematical thinking: Aims, rationales, techniques. *For the Learning of Mathematics*, *1* (3), 57-64.

Greca, I. M., & Moreira, M. A. (2002). Mental, physical, and mathematical models in the teaching and learning of physics. *Science Education*, *1*, 106-121.

Greer, B. (1993). The mathematical modelling perspective on world problems. *Journal of Mathematical Behaviour*, *12*, 239-250.

Hartman, H. J. (1998). Metacognition in teaching and learning: An introduction. *Instructional Science*, *26*, 1-3.

Inoue, N. (2005). The realistic reasons behind unrealistic solutions: The role of interpretive activity in word problem solving. *Learning and Instruction*, *15*, 69-83.

Jonassen, J. (2000). Toward a design theory of problem solving. *Educational Technology Research and Development*, *48* (4), 63–85.

Kantowski, M. G. (1977). Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education*, *8*, 163–180.

Krulik, S., & Rudnick, J. A. (1985). Developing problem solving skills. *Mathematics Teacher*, *79* (9), 685-692.

LeBlanc, J. F. (1977). You can teach problem solving. *Arithmetic Teacher*, *25* (2), 16-20.

Lesh, R., & Harel, G. (2003). Problem solving, modelling and conceptual development. *Mathematical Thinking and Learning*, *5* (2), 157-189.

Mahlios, J. (1988). Word problems: Do I add or subtract. *Arithmetic Teacher*, *36* (3), 48-52.

Mayer, R. E. (1985). Implications of cognitive psychology for instruction in mathematical problem solving. In E. A. Silver, (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 123-138). London: Lawrance Erlbaum Associates.

Mayer, R. E., Sims, V., & Tajika, H. (1995). A comparison of how textbooks teaching mathematical problem solving in Japan and the United States. *American Educational Research Journal*, *35*, 443-459.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis-An expanded sourcebook*. London: Sage.

Nancarrow, M. (2004). *Exploration of metacognition and non-routine problem based mathematics instruction on undergraduate student problem solving success*. Unpublished doctoral dissertation, The Florida State University, Florida.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics*. Virginia, Reston: Author.

National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Nesher, P., & Hershkovitz, S. (1997). Real-world knowledge and mathematical knowledge. *Proceedings of 21st Conference on the International Group for the Psychology of Mathematics Education*, *3*, 280-287.

Norman, D. A. (1983). Some observations on mental models. In D. Gentner & A. L. Stevens (Eds.), *Mental models* (pp. 7-14). Hillsdale, New Jersey: Lawrence Erlbaum.

Orton, A., & Wain, G. (1994). Problems, investigations and an investigative approach. In, A. Orton & G. Wain (Eds.), *Issues in teaching mathematics* (pp. 150-173). London: Cassel.

Özsoy, G. (2002). *İlköğretim 5. sınıfta matematik dersi genel başarısı ile problem çözme becerisi arasındaki ilişki*. Yayımlanmamış yüksek lisans tezi, Gazi Üniversitesi, Eğitim Bilimleri Enstitüsü, Ankara, Türkiye.

Phillips, N., & Hardy, C. (2002). *Discourse analysis: Investigating processes of social construction*. United Kingdom: Sage Publications Ltd.

Posamentier, A. S., & Krulik, S. (1998). *Problem solving strategies for efficient and elegant solutions – A resource for the mathematics teacher*. California: Sage Publications Ltd.

Poyla, G. (1973). *How to solve it*. Princeton, USA: Princeton University Press.

Reusser, K., & Stebler, R. (1997). Every word problem has a solution: the social rationality of mathematical modelling in schools. *Learning and Instruction*, *7* (4), 309-327.

Schoenfeld, A. H. (1992). Learning to think mathematically: problem solving, metacognition, and sense making in mathematics. In D. Grouvs (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334-370). New York: MacMillan.

Sulak, S. (2005). *İlköğretim matematik dersinde problem çözme stratejilerinin problem çözme başarısına etkisi*. Yayımlanmamış yüksek lisans tezi, Selçuk Üniversitesi, Sosyal Bilimler Enstitüsü, Konya, Türkiye.

Suydam, M. N. (1980). Untangling clues from research on problem solving. In *NCTM 1980 Yearbook, Problem solving in school mathematics* (pp. 34-50). Virginia: NCTM Inc.

Talim Terbiye Kurulu Başkanlığı (TTKB). (2008). *İlköğretim matematik dersi 6-8 sınıflar öğretim programı ve kılavuzu*. Ankara: Milli Eğitim Bakanlığı.

Verchaffel, L., De Corte, E., & Borghart, I. (1997). Pre-service teachers' conceptions and beliefs about the real-world knowledge in mathematical modelling of school word problems. *Learning and Instruction*, *7*, 339-359.

Verschaffel, L., & De Corte, E. (1997). Teaching realistic mathematical modelling in the elementary school: A teaching experiment with fifth graders. *Journal for Research in Mathematics Education*, *28*, 577-601.

Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modelling of school arithmetic word problems. *Learning and Instruction*, *4*, 273-294.

Verschaffel, L., De Corte, E., & Vierstraete, H. (1999). Upper elementary school pupils' difficulties in modeling and solving nonstandard additive word problems involving ordinal numbers. *Journal for Research in Mathematics Education*, *30* (3), 265-285.

Yin, R. K. (2003). *Case study research: Design and methods*. United Kingdom: Sage Publications Ltd.

Yoshida, H., Verschaffel, L., & De Corte, E. (1997). Realistic considerations in solving problematic word problems: Do Japanese and Belgian children have the same difficulties? *Learning and Instruction*, *7*, 329-338.

Zbiek, R. M. (1998). Prospective teaches' use of computing tools to develop and validate functions as mathematical models. *Journal for Research in Mathematics Education*, *29* (2), 184-201.

(Footnotes)

1 Yazılı sınava katılan 6 numaralı öğrenciyi temsil etmektedir; yazılı sınav kâğıtlarından yapılan alıntılarda bu sunum şekli kullanılacaktır.

2 Öğrencilerin gerçek isimleri yerine kod adları kullanılmıştır.

3 Students' names are altered.