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Pre-Service Mathematic Teachers’ Knowledge of Students about the Algebraic Concepts

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Abstract: The aim of this study was to evaluate preservice primary mathematics teachers' ability to discuss and investigate students' thinking process about the concepts of variable, equality and equation, to analyse their ability to predict student difficulties and misconceptions and, in this respect, to present their subject-matter knowledge and possible misconceptions on the related topics. The participants were 130 preservice primary mathematics teachers in their fourth year of university education. The data were collected through a questionnaire consisting of open-ended questions and clinical interviews and analysed qualitatively. The results showed that, in general, the preservice teachers were inadequate in terms of knowledge of students about algebraic concepts, they had insufficient subject-matter knowledge and they had misconceptions.

Introduction

Teachers are one of the key components of the reform of teaching and learning mathematics. Various structures of knowledge that teachers possess reveal what kind of teachers they might be. This fact raises the question "What should teachers know?" The answer can be explained by the concept of pedagogical content knowledge.

Shulman (1986) proposed the concept of pedagogical content knowledge that has a different meaning from content knowledge, which must be possessed by teachers. Shulman stated that pedagogical content knowledge is a specific mixture of subject-matter knowledge and pedagogical knowledge and defined it as "the most useful form of [content] representation, the most powerful analogies, illustrations, examples, explanation, and demonstrations—in a word, the ways of representing and formulating the subject that makes it comprehensible to others". The pedagogical content knowledge is a kind of knowledge that reveals a teacher's meaningful and effective ways of teaching. In other words, it is a special kind of information generated from the transformation of the subject-matter knowledge that teachers have and it consists of several components.

Shulman (1987) identified seven components of pedagogical content knowledge: subject-matter knowledge, pedagogical content knowledge, general pedagogical knowledge, knowledge of students, knowledge of curriculum, knowledge of educational contexts and knowledge of educational purposes. After Shulman's study, some researchers who focused on the pedagogical content knowledge demonstrated these components in different ways. Tamir (1988), for example, categorized pedagogical content knowledge into four components: knowledge of understanding students; knowledge of teaching methods, strategies and techniques; knowledge of measurement and evaluation; and knowledge of curriculum. Also, Grossman (1990) identified four components of pedagogical content knowledge: knowledge of strategies and representations for teaching particular topics;

knowledge of students' understanding, conceptions and misconceptions of these topics; knowledge and beliefs about the purposes of teaching particular topics; and knowledge of curriculum materials available for teaching. Similarly, Marks (1990) examined pedagogical content knowledge under four components: knowledge of understanding students, knowledge of teaching methods, strategies and techniques; subject-matter knowledge; and knowledge of the media. Dividing the knowledge areas of mathematics into two groups as knowledge of mathematics and pedagogical content knowledge, Ball (1990) defined pedagogical content knowledge based on student and content knowledge, teaching and content knowledge, and curriculum and content knowledge. Similarly, Fennema and Franke (1992) examined mathematics teachers' knowledge under four categories, one of which was knowledge of students. An, Kulm and Wu (2004) suggested three components of pedagogical content knowledge: subject-matter knowledge, knowledge of curriculum, and knowledge of teaching and they strongly emphasized that knowledge of teaching is the basic component of the pedagogical content knowledge and knowledge of students is the gist of it. In the same way, Park and Oliver (2008) suggested the following components: subject-matter knowledge, pedagogical knowledge, and pedagogical content knowledge and content knowledge, which are formed by combining the former two components. Considering the results of these studies on the topic, it can be concluded that subject-matter knowledge, pedagogical knowledge and knowledge of students are the emphasized parts of pedagogical content knowledge (Baker & Chick, 2006). In fact, some studies place knowledge of students in the centre of pedagogical content knowledge and it is considered as one of the important components (Shulman, 1986; Park & Oliver, 2008; An et al., 2004).

In general, knowledge of students is defined as a teacher's knowledge of students' operational and conceptual knowledge, students' thinking processes, learning styles, difficulties and misconceptions in the process of learning a subject (Shulman, 1987; Fennema & Franke, 1992; Even & Tirosh, 1995; Tirosh, 2000; An, Kulm & Wu, 2004). The literature presents several studies about inservice teachers' and preservice teachers' knowledge of students in various subject areas (Carpenter, Fennema, Peterson & Carey, 1988; Even, 1993; Stump, 2001; An, Kulm & Wu, 2004; Son, 2006; Chick, Baker, Pham & Cheng, 2006; Baker & Chick, 2006; Bayazit & Gray, 2006; Türnüklü & Ye ildere, 2007). Unfortunately, these studies reported that mathematics teachers and preservice mathematics teachers have incomplete or inadequate knowledge of students in general. However, teachers' knowledge of students is of great importance in performing an effective teaching and organizing teaching activities (Carpenter, Fennema, Peterson & Carey, 1988; Leinhardt & Smith, 1985). It is of no doubt that preservice teachers' knowledge of students is as important as that of inservice teachers in questioning teacher education.

This study focused on knowledge of students, which is considered to be one of the important components of pedagogical content knowledge, and examined preservice mathematics teachers' competence on knowledge of students in this respect. Exploring the concepts of variable, equality and equation, which students have problems and misconceptions about (e.g. Küchemann, 1978; Wagner, 1983; Philipp, 1992; Herscovics & Linchevski, 1994; MacGregor & Stacey, 1997; Dede, 2004; McNeil & Alibali, 2005; Soylu, 2008), the study investigated the participating preservice teachers' ability to discuss, question and predict students' thinking processes, difficulties and misconceptions relating to these concepts. The quality of the questions asked by inservice teachers or preservice teachers also plays a key role in gaining knowledge of students. This is because teachers or preservice teachers who are able pose qualified questions can better analyse the depth of students' thoughts (Moyer & Milewiez, 2002). In this respect, this study tried to determine to what extent the preservice teachers were able to ask qualified and effective questions in order to identify students' errors. Therefore, preservice teachers and their knowledge of students were the focus of this research. Preservice teachers are supposed to identify difficulties that students might encounter students' misconceptions and reasons for their misconceptions about the topics and concepts and to ask questions efficiently so that they can help their students and perform effective teaching in the future. Preservice teachers' subject-matter knowledge with respect to knowledge of students and identification of possible misconceptions are other important aspects of this study because preservice teachers' limited subject-matter knowledge and possible misconceptions are also important dimensions that should be discussed considering the achievement of their future students. In fact, research suggests that there is a relationship between teachers' subject-matter knowledge and student achievement in learning and understanding mathematics (Ma, 1999).

Conceptual Framework

This section presents research about mathematics teachers' and preservice mathematics teachers' knowledge of students with respect to learning algebra in order to provide a theoretical background to this study. Research indicates that students' previous knowledge, the subjects/concepts which they have difficulty understanding and their misconceptions are different from teachers' predictions and expectations about them. For example, in some studies about mathematics teachers' and preservice mathematics teachers' knowledge of students about the concepts of equals sign and variable, the participating inservice teachers and preservice teachers had difficulty in identifying students' misconceptions and the actual responses of the students were different from the teachers' predictions about possible student errors/difficulties (Asquith, Stephens, Knuth & Alibali, 2007; Stephens, 2006). Some other studies about preservice teachers' ability to predict the errors and misconceptions of primary school students in relation to algebraic expressions and manipulations showed that the surveyed preservice mathematics teachers generally made predictions about only one kind of errors and misconceptions and they predicted errors and misconceptions which students didn't have (Dede & Peker, 2007; Dobrynina & Tsankova, 2005). On the other hand, it is interesting that the predictions made by the inservice teachers and preservice teachers in some studies turned out to be the exact opposite of the actual situation. For example, some studies about teachers' predictions and expectations about students' difficulties and misconceptions in solving algebraic/mathematical problems revealed that the participating teachers predicted and expected the exact opposite of students' actual difficulties (Nathan & Koedinger, 2000a, 200b; Nathan, Koedinger & Tabachneck, 1996). In fact, the students in these studies had difficulty solving symbolically expressed algebra problems more than verbally expressed algebra problems although the teachers' predictions and expectations were the exact opposite of this result. Similarly, in Bergqvist's (2005) study, there were differences between the teachers' expectations about students' performance in confirming or refuting algebraic/mathematical hypotheses and students' actual performance. The literature also presents some studies about teachers' or preservice teachers' analysis and interpretation of students' structures of thinking and the reflection of these on their teaching (Stephens, 2008; Boz, 2002; Boz, 2004). In general, the preservice teachers in these studies were shown to need improvement in analysing and interpreting students' thoughts, to fail to identify the ideas and errors behind students' answers, to fail to explain the sources of students' errors, and to tend to consider students' errors as calculation or reading errors. These studies also revealed that preservice teachers could not come up with effective solution recommendations to eliminate students' errors. These studies generally highlighted knowledge of students and subject-matter knowledge and that teachers' or preservice teachers' inadequate knowledge of subject-matter affected their knowledge of students. For example, a study about preservice primary mathematics teachers' knowledge of algebraic concepts, their descriptions of algebra in general and their ability to analyse students' relational thinking skills or their understanding of the equal sign based the on students' studies in particular revealed that the preservice teachers had limited knowledge of algebra concepts as a part of subject-matter knowledge (Stephens, 2008). Another study on preservice mathematics teachers' subject-matter knowledge and pedagogical content knowledge on the concept of variable through their responses to a questionnaire that consisted of student responses and interviews analysed the preservice mathematics teachers' subject-matter knowledge in two categories as "knowledge of what" and "knowledge of why" and found that the preservice teachers in the study knew the rules for the letter symbols but could not demonstrate the same success in explaining the reasons for these rules (Boz, 2002). Another similar study about the relations between preservice teachers' subject-matter knowledge and their content-specific pedagogical knowledge on the subject of variables in terms of identifying students' errors and the reasons involved found that the preservice teachers confused simplification of algebraic expressions with solving equations and, as a result, this situation prevented them from identifying student errors (Boz, 2004).

Inservice teachers' or preservice teachers' questioning skills are significant for understanding what students already know and what they further need and for analysing and interpreting what they think. This is because the quality of the questions asked by teachers or their questioning skills play a key role in identifying students' difficulties, testing knowledge and ensuring control. Teachers who are able to ask qualified questions can also analyse the depth of their students' thoughts better (Moyer&Milewiez, 2002). The literature presents some studies about teachers' questioning skills and the type and frequency of questions used by them in classroom (Boaler&Brodie, 2004; Bonne&Pritchard, 2007; Buschman, 2001; Craig&Caino, 2005; Haydar, 2003; Mewborn&Huberty, 1999; Sahin&Kulm, 2008) but there's limited research on evaluating preservice teachers' questioning skills through interviews (Moyer&Milewicz, 2002; Tanı lı, Manuscript submitted for publication). These studies reported that the questions asked by the participating inservice teachers or preservice teachers represented a low level of achievement and the preservice teachers in general needed improvement in using effective questioning techniques. For example, a study investigating preservice primary school mathematics teachers' ability to question students' understanding of the concept of equality and their relational thinking skills through clinical interviews and to analyse and interpret the questioned students' thoughts found that, in general, the participating preservice teachers' questioning skills could be defined as "novice" and, as a result, they failed to expand on the questioned students' thoughts and to analyse the students' responses appropriately (Tanı lı, Manuscript submitted for publication).

A great deal of research generally reported on inservice mathematics teachers' and preservice mathematics teachers' knowledge of students with respect to learning algebra and mainly focused on predicting students' errors and providing solution recommendations (Nathan, Koedinger &Tabachneck, 1996; Nathan &Koedinger, 2000a, 200b; Boz, 2002; Boz, 2004; Bergqvist, 2005; Dobrynina & Tsankova, 2005; Asquith, Stephens, Knuth & Alibali, 2007; Stephens, 2006, Dede & Peker, 2007; Stephens, 2008). The most important feature that distinguishes this study from the others in the literature is that this study focuses on preservice teachers' skills to ask questions and investigate as well as discussing students' thoughts and predicting students' errors. Therefore, the aim of this study was to investigate primary school preservice teachers' ability to discuss and question students' thinking processes with respect to the concepts of variable, equality and equation; to predict students' difficulties and misconceptions; and, in this regard, to explore their subject-matter knowledge and possible misconceptions.

Method

Participants

The participants were 130 preservice teachers studying Primary School Mathematics Education in their fourth-year in two state-funded universities in Turkey. A criterion sampling method was employed to choose the possible participants in the study. With criterion sampling, all of the cases or individuals are required to meet a certain criterion (Yıldırım & im ek, 2005). In this respect, the main criterion used in this study was that the participants were required to have taken the courses "Mathematics Teaching I and II". These courses deal with theories of learning and teaching, teaching methods and techniques, presenting mathematics curricula, how mathematical concepts in the curricula could be taught, and discussion of possible misconceptions/difficulties related to these concepts and presenting micro-teaching sessions of these concepts. Therefore, taking these courses was determined as a sampling criterion because a considerable part of the qualifications which a mathematics teacher should possess as a part of pedagogical content knowledge are presented by these courses. The preservice teachers who failed these courses were not included in the study. In this way, a total of 130 participants were chosen - 60 participants from one of the universities and 70 participants from the other university.

Data Collection

The research data were collected in two stages through a questionnaire with open-ended questions and clinical interviews.

Questionnaire: The questionnaire was prepared in order to find out knowledge of students, which is one the most important components of pedagogical content knowledge, considering three main components: discussing students' thinking process, asking questions to identify students' errors, and predicting students' incorrect answers. The questionnaire contained eight open-ended questions to determine the preservice teachers' knowledge of students with respect to the concepts of variable, equality and equation. Before preparing the questionnaire, the literature was reviewed to determine student errors on the concepts of variable, equality, and equation and then student responses containing errors related to these concepts were used in preparing the open-ended questions (Kieran, 1992; Soylu, 2006, 2008; Vlassis, 2001; Hall, 2002).

Question 1: The question "*Ayse is 4 cm. taller than Seda. If Seda is n cm. tall, how tall is Ayse?*" is being discussed in class. The dialogue among three 6th grade students is given below:

Aral : Ayse's height is 4n,
Sena : No. Ayse's height is 104 cm.,
Ali : I think Ayse's height is x +4.

What kind of questions may be asked to each of these students to help them understand their errors?

Question 2: In the question "*In the expression 4n +7, what does the symbol n represent?*" 6th grader Ömer gives the following answer "*n does not mean anything here because there is no symbol "=" in the expression. For example, in an expression such as 4n +7 = 11, n = 1".*

Discuss the student's idea.

Question 3:

"*a) 4x-1=0* *b) x+10=47* *c)* $\frac{x}{2} + 3 = 5$ *d) -3x+6=2x+16"*

What kind of incorrect answers may be given to the questions above by your students? Try to predict.

Figure 1: Teaching Mathematics Survey

Two mathematics education experts reviewed the questionnaire and then, in line with the opinions of these experts, the number of questions was reduced by combining some questions. For the pilot study, the questionnaire was administered to 10 preservice teachers studying mathematics education in their fourth year. Answers from the preservice teachers, the comments, and the time required to answer the questionnaire were taken into account and sub-questions of some of the questions were removed and some of the questions were changed. Finally, the questionnaire's final version was prepared with a total of three open-ended questions -Figure 1. The questionnaire was administered to the selected participants in both of the universities. When administering the questionnaire, the participants were asked to answer the questions in detail.

Clinical Interview. After administering the questionnaire about mathematics teaching, the clinical interviews were carried out based on the participants' responses to the open-ended questions. Clinical interview is a technique that was pioneered by Piaget. It is used to deeply analyse students' thinking process and it includes interviews with students (Clement, 2000). Before starting the clinical interviews, the preservice teachers' answers to the open-ended questionnaire questions were analysed and the preservice teachers with misconceptions were identified. Because some of the preservice teachers didn't volunteer for the interview, five volunteers from each of the two universities, a total of 10 preservice teachers, were interviewed. The clinical interviews were recorded with a video camera and held in the preservice teachers' university campuses, where they could express themselves comfortably. The interviews lasted 15-35 minutes.

Data Analysis and Interpretation

The data obtained were analysed qualitatively. First of all, the answers to the questionnaire were examined separately by the two researchers and the categories and sub-categories were identified on the basis of each question. In accordance with the relevant literature, the following categories were organized as components of knowledge of students: asking questions to identify students' errors, discussing students' thinking process, and predicting students' errors. Each of these categories included three sub-categories. Asking instructional, investigative and inadequate/not-competent questions are the sub-categories under the category of asking questions to identify students' errors; understanding students' thinking process, understanding and explaining students' thinking process, not understanding/discussing students' thinking process are the sub-categories under the category of discussing students' thinking process. In addition, preservice teachers' misconceptions and difficulties as well as the language of mathematics subject-matter were identified as the last category. This category included two sub-categories: preservice teachers' misconceptions about the concepts of variable, equality, and equation and the language of mathematics subject-matter that they use. The relationship between this category and its sub-categories can be summarized as follows:

- **Category1:** Asking questions to identify students' errors
  - Instructional questions
    - Leading Question
    - Concept teaching questions
  - Investigative questions
    - Only questions about the incorrect response
    - Competent questions
  - Inadequate/not-competent questions
- **Category 2:** Discussing students' thinking process
  - Understanding students' thinking process
  - Understanding and explaining students' thinking process
  - Not understanding/discussing students' thinking process
- **Category 3:** Predicting students' errors
- **Category 4:** Preservice teachers' misconceptions, difficulties and the language of mathematics subject-matter which they use.
  - Preservice teachers' misconceptions about the concepts of variable, equality, and equation
  - The language of mathematics subject-matter

In addition, the sub-categories were tested by the two researchers in terms of reliability. The percentage of goodness-of-fit suggested by Miles and Huberman (1994) was used to calculate the reliability. The numbers of "consensus" and "disagreement" for the categories and sub-categories suggested by the field experts were determined and, as a result of the calculations (Reliability=Consensus/(Consensus+ Disagreement)), the percentage of goodness of fit was found to be 88%. The frequencies and percentages of the categories and sub-categories were calculated and interpreted and then the data were illustrated using figures. The categories, sub-categories, frequency and percentage distributions of the categories are included in the Figures. The data obtained from the clinical interviews are presented under the category of preservice teachers' misconceptions about the concepts of variable, equality, and equation in order to describe this category in greater detail.

Findings and Results

The preservice primary school mathematics teachers' knowledge of students about the concepts of variable, equality, and equation is presented according to categories determined under each concept.

Knowledge of Students about the Concept of Variable

It is very important that preservice teachers be able to discuss students' thinking process about the concept of variable, identify their difficulties or misconceptions and ask their students questions to help them recognize their misconceptions. In this respect, the preservice teachers in this study were asked two questions to determine their knowledge of students about the concept of variable.

In the first question, the preservice teachers were given a problem situation (*"Ayse is 4 cm. taller than Seda. If Seda is n cm. tall, how tall is Ayse?")*. Examples of incorrect student responses to this problem are presented (*Aral: Ayse's height is 4n, Sena: No. Ayse's height is 104 cm., Ali: I think Ayse's height is x +4.*). The preservice teachers were asked to figure out what kind of questions they can ask to each student to help them understand their errors.

Types of the questions asked by the preservice teachers and the percentages of the selected questions are presented in Figure 2. As shown in Figure 2, the preservice teachers asked three different types of questions to reveal the errors of three students (Aral, Sena, and Ali) given in the problem. These types of questions were inspired by the variety of questions asked by the preservice teachers in Moyer and Milewicz's (2002) clinical interviews and the preservice teachers' questions were classified as Instructional questions, investigative questions, and inadequate/not-competent questions.

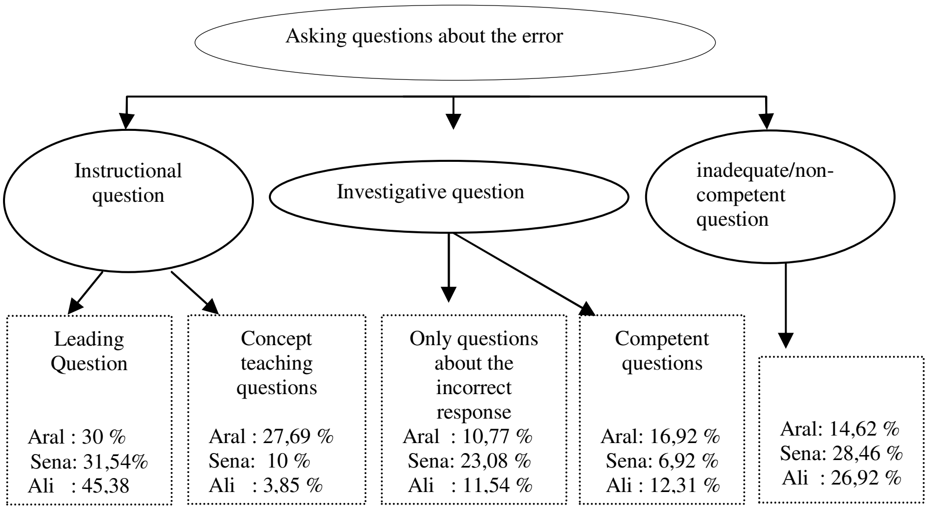

Figure 2: Preservice Teachers' Types of Questions to Identify Errors.

The study revealed that the majority of the preservice teachers asked mainly instructional questions to each of the three students. An instructional question can be defined as teaching a student instead of assessing the student's knowledge about concepts. Under this main category, the preservice teachers asked two different questions. The first type of questions is defined as leading questions, which the preservice teachers asked by giving students hints for the correct answer or by directly telling the correct answer. Examples of leading questions often asked by the preservice teachers include the following: the questions they asked to Aral, who thought that Ayse's height was 4n – "*The expression 4 cm. taller requires adding, not multiplying in mathematics*" and "*Does the question state that Ayse is four times taller than Seda, or 4 cm. taller than Seda? If Seda's height is n, and Ayse is 4 cm. taller than Seda, aren't we required to add 4 to Seda's height?*" – and the questions they asked to Ali who said that Ayse's height was x+4 – "*Is Seda's height given as x or n?*" In this type of questions, the preservice teachers emphasized their own thinking process and revealed the answer instead of taking student's thinking process on the concept of variable into account.

Another type of question that the preservice teachers asked under the category of instructional question was concept teaching questions. Although concept teaching is expressed as a type of question, it can be defined as the preservice teachers' explanation of errors through a sample situation, without asking questions about a concept, or teaching in a more leading and explanatory way. The statements of one of the preservice teachers to identify the errors of all of the three students can be presented as an example:

> *"Seda's height is n cm. and Ayse is 4 cm. taller than Seda, we can make a t table about Ayse and Seda's heights… Seda's height 100 cm. How tall is Ayse? Assume that Seda's height 101 cm. How tall is Ayse? The student can be asked to fill in the table by answering questions like that. After the completion of t table, we can ask them to find the relation between their heights by asking the question 'What is the relation between Ayse and Seda's heights?"*

Investigative questions were another type of question that the preservice teachers asked to identify student errors. Investigative questions are classified under two sub-categories: questions about the incorrect responses and competent questions. Questions about the incorrect responses can be defined as the questions which the preservice teachers asked about the students' incorrect responses. Examples of these questions, which were not used by most of the preservice teachers, include *"Is Ayse's height 4 times more than that of Seda, in your opinion?*", "*Why 104?*", and "*Why x+4?*" As can be seen in these examples, the preservice teachers were unable to ask proper and in-depth questions to identify students' errors. Another category of investigative questions is competent questions. Competent questions are more comprehensive questions requiring more information and, in this study, this type of questions were asked by the preservice teachers to help students recognize their own errors. In other words, competent questions have a guiding function for students to understand their errors. These questions were unfortunately asked by only a small number of the preservice teachers in this study. The following are some examples:

> "*What does 4 cm more mean? What does 4 times more mean? Does 4 more than Ayse's height equal to 4 times Ayse's height?"*
>
> *"Why 104 cm? How come did you come up with this answer? Do you know how tall Seda is? Then how can you say that?"*

The last type of questions used by the preservice teachers was inadequate/not-competent questions, which were used by 23% of the preservice teachers. Examples of these questions asked by the preservice teachers include "*What do the expressions in the equation represent?*" and "*The difference between a single letter symbol and a variable can be asked and the categories of x and n can be asked*".

In the scope of the preservice teachers' knowledge of students, another question asked in order to assess the preservice teachers' ability to discuss students' thinking process was about questioning the variable in a given algebraic expression. The research question "*What does n represent in the expression of 4n+7*", which was used by Soylu (2006), and the student answer to it were used in this section. Also, the preservice teachers were asked to assess the student's thinking process. These assessments presented in Figure 3 were classified into three sub-categories: understanding students' thinking process, understanding students' thinking process and explaining it, not understanding/not being able to discuss students' thinking process. The sub-category of understanding students' thinking process is defined as preservice teachers' understanding of the main misconception in students' answer to any given question. The category of understanding students' thinking process and explaining it refers to preservice teachers' ability to make correct inferences about the reason of the misconception as well as understanding the main misconception. The last sub-category is not-understanding and not-discussing students' thinking process.

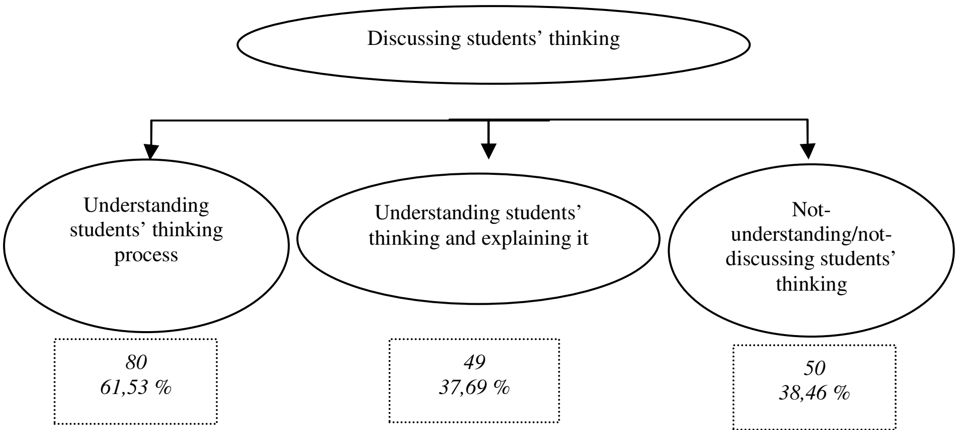

Figure 3: Discussing Students' Thinking process about the Concept of Variable

More than half of the preservice teachers in the study recognized students' incorrect thinking process about the question in which the variable *n* in the expression 4*n*+7 was asked (see Figure 3). Examples of the preservice teachers' statements about students' thinking process such as "*He thinks that n in the expression '4n+7' is meaningless because there is no equality*", "*He can't realize that n is a variable. He is conditioned to accept letters as unknown.*", and "*Without knowing the meaning of 'n', he is focused on solving equation in the expressions of 4n+7 and 4n+7=11*" indicate that the preservice teachers identified that students misinterpreted the variable *n* in the algebraic expression of 4*n*+7 and focused on the equals sign. It is remarkable that although almost 62% of the preservice teachers understood students' thinking process, only 37% of them figured out the reasons involved as well. The statements of the preservice teachers like "*they couldn't completely understand the concept of variable*", "*they can't differentiate the concepts of variable and unknown.*" and "*they didn't understand that n can represent more than one number*" show that the preservice teachers determined that different meanings of variable and students' not completely understanding the concept of algebraic expression were the reasons of students' misconceptions. However, the difference between the preservice teachers' responses to the first two sub-categories is important. It gives the impression that they had difficulty in analysing students' thinking process and determining the reasons involved. It is particularly remarkable that almost 40% of the preservice teachers did not understand students' thinking process or could not discuss it. The answers given under this sub-category also revealed that some of the preservice teachers made unnecessary, mostly irrelevant explanations to avoid giving answers about the topic. Examples of some of the preservice teachers' responses include the following:

"Since the expression 4n+7 equals nothing, there is no value of n."

"Ömer might be right. He might have considered n as a natural number."

"There is = in operations with unknown and operation is based on this equality. There is logic of balance, scales."

"If students give responses like these, questions are asked again using clearer expressions."

"He used his imagination since he did not see the equals sign in this expression."

Regarding this question, it can be suggested that in general the preservice teachers could not appropriately analyse students' thinking process.

Knowledge of Students about the Concepts of Equality and Equation

Under the topic of the preservice teachers' knowledge of students about the concepts of equality and equation, they were asked one question to predict the incorrect answers of the students. The equations "*4x-1=0, x+10=47*, *x/2+3=5, -3x+6=2x+16*", which are used in the literature to identify student errors, were used in the last question to evaluate the preservice teachers' ability to predict students' incorrect responses (Kieran, 1992; Vlassis, 2001; Hall, 2002). The results of these studies in the literature suggest that there are various types of student errors and they are classified under different categories. Utilizing these categories, this study aimed to evaluate whether the preservice teachers were able to predict incorrect answers or misconceptions in the literature. The preservice teachers' predictions are presented in Table 1.

Equations	Common errors and misconception in the literature	Number of the preservice teachers who predicted	Other predicted errors and misconceptions.	Number of the preservice teachers who predicted
$4x-1=0$	*if* 4*x*=1, *x*+4=1, *x*=1-4, *x*= -3	20 (15,38%)	*if* 4*x*= -1, *x*= -1/4	50 (38,46%)
			if 4*x*=0, *x*=0 or not exist	44 (33,85%)
			if 4*x*=1, *x*=1	35 (26,92%)
			if 4*x*-1=0, *x*=4	23 (17,69%)
			if 4*x*-1=0, 4*x*-1=3*x*	3 (2,31%)
			if 4(*x*-1)=0, 4*x*-4=0, *x*=1	3 (2,31%)
			-1=0	2 (1,54%)
$x+10=47$	*if* *x*=47+10, *x*=57	87 (66,92%)	*x*= 47/10	16 (12,31%)
			x= 47	8 (6,15%)
			x= -37	8 (6,15%)
			if *x*+10=47, 11*x*=47, *x*=47/11	4 (3,07%)
			x=36, *x*=40, *x*=27	3 (2,31%)
			1+10=47 (*x*=1)	2 (1,54%)
			x=10	1 (0,77%)
			if 10*x*-47=0, -37*x*=0	1 (0,77%)
$\frac{x}{2}$+3=5	*if* *x*+3=10, *x*=7	27 (20,77%)	*if* *x*/2=8, *x*=16 or *x*=4	48 (36,92%)
			if *x*/2=5-3, *x*=2or *x*=8	32 (24,61%)
			if *x*/2=2, *x*=1	27 (20,77%)
			if *x*+6=5, *x*= -1 or *x*=11	13 (10%)
			if *x*/2=5, *x*=10	5 (3,85%)
			if 3*x*/2=5, *x*=10/3	4 (3,07%)
			½+3+5=0 or ½+3=5 or 3/2-5=0	3 (2,31%)
			if 2*x*+6=10, 2*x*=4, *x*=2	1 (0,77%)
			if *x*/2=2, *x*/2-1/2=2-1/2, *x*=3/2	1 (0,77%)
$-3x+6=2x+16$	*if* *x*+6=16, *x*=10	47 (36,15%)	-*x*=22	31 (23,85%)
			5*x*=10, *x*=2	25 (19,23%)
			-5*x*=22 *x*= -22/5 or *x*=22/5	19 (14,61%)
			-3+6=2+16, 3=18	6 (4,61%)
			3*x*=18, *x*=6	3(2,31%)
			if -3*x*+6=18, *x*=-4 or *if* 3=2*x*+16, *x*=-13/2	2 (1,54%)
			-9*x*=18*x* or 3*x*=18*x*	2 (1,54%)
			3*x*=2*x*+16, *x*=16	2 (1,54%)
			-3(*x*+6)=2(*x*+16)	2 (1,54%)
			-*x*+6=*x*+16	1 (0,77%)
			-3*x*/2*x*=16/6	1 (0,77%)
			-3*x*.6=2*x*.18 , -18=36	1 (0,77%)
			5*x*=0	1 (0,77%)

Table 1: Errors or Misconceptions Predicted by the Pre-service Teachers.

The first equation was *4x-1=0*. Hall (2002) states that, solving this equation, students generally make the error of *4x=1, x=1-4*, which is called "The Other Inverse Error", and they focus on the reverse of the operation of addition instead of the reverse of the operation of multiplication. Only 15% of the preservice teachers in this study predicted that their students might make this error. On the other hand, 38% of the preservice teachers predicted their students' "Switching Addends Error: *4x-1=0, 4x= -1* and *x= -1/4*" which is defined by Kieran (1992). Also, 33% of the preservice teachers predicted their students' "*4x-1=0, x=0*" error and 26% of them predicted their students' "*4x=1, x=1*"error. These errors are the examples of "Omission Error: $4x\text{-}1\text{+}1\text{=}0$, $4x\text{=}0$, $x\text{=}0$" and "Absence of Structure Error", which were mentioned in Hall's study. In addition, 17% of the preservice teachers predicted the error "Mx=N, x=M", which results from limited usage of reverse operation and was presented as a student error related to linear equations by Erba , Çetinkaya and Ersoy (2009), in the form of "$4x=1$, $x=4$", and 2% of them predicted the error $4x-1=3x$, which is considered as a grammatical error by Kieran (1992).

Another equation asked to the preservice teachers was the equation $x + 10 = 47$. Kieran (1992) states that students mainly make two types of errors in this equation: "Redistribution Error: $x+10=47$, $x+10-10=47+10$" and "Switching Addends Error: $x+10=47$, $x=47+10$". The majority of the preservice teachers (66%) predicted this error correctly. When the preservice teachers' other predictions were analysed, it was found that in general they predicted that their students might make calculation errors and they might find the result of the equation $x+10=47$ *as* $x=36$, $x=40$, $x=27$ by making an error in subtracting 10 from 47. Also, 3% of the preservice teachers predicted that the students might find the results of "$11x=47$" and "$10x-47=0$, $-37x=0$" by making grammatical errors.

In the solution of the equation $x/2+3=5$, another equation given to the preservice teachers, Kieran states that most students tend to reach x+3=10 because they do not take the symmetry of the equation into consideration when they multiply both sides of the equation by two. This error, which is called "Transposing Error", was predicted by 20% of the preservice teachers in the study. In addition, 10% of the preservice teachers predicted that students might think this error as $x+6=5$, $x= -1$ and then they might change the position of the added items during the solution process, so they might reach the result of x=11. The preservice teachers also made predictions about their students' limited applications of equation's reverse operations during the beginning of the solution process or after a certain stage. For example, the preservice teachers predicted that students might reach $x/2=2/2$ and $x=1$ after the stage of $x/2=2$ or with an incorrect start they might reach the results $x=16$ or $x/2=8/2$, $x=4$ after $x/2=8$. Also, 3% of the preservice teachers estimated that their students might reach the result $x/2+3-3=5$, $x/2=5$ and $x=10$ through "The Omission Error" in the literature.

The last equation asked to the preservice teachers was $3x + 6 = 2x + 16$ and 36% of the preservice teachers predicted the error that students made in Vlassis' (2001) research, where students simplified the equation $-3x+6=2x+16$ as $–x+6=16$ by removing -2x from both sides of the equation and they found the result x=10 by ignoring the negativity of the unknown. In addition, the error "$-3x+2x=16+6$, $-x=22$" was predicted by 23% of the preservice teachers. The error was similar to the error $Mx+Px=N+Q$ which is one of the mal-rules errors made by the students in the solution of equation $Mx\pm N=Px\pm Q$ which was used in the study of Erba et al. (2009). Results such as "5x=10, x=2" and "-5x=22 x= -22/5 or x=22/5" are similar to the error $–Mx\text{-}Px= \pm N\pm Q$ which students made in solution of equation $Mx\pm N=Px\pm Q$ in the study of Erba et al (2009). The preservice teachers also made various error predictions for all four of the equations, none of which were mentioned in the literature.

The Preservice Teachers' Misconceptions, Difficulties and Language of Mathematics

Analysis of the preservice teachers' responses to the questionnaire with open-ended questions revealed that some of the preservice teachers did not use correct/proper language of mathematics subject-matter in their explanations; also some of the preservice teachers had serious misconceptions and difficulties. The clinical interviews were conducted with the preservice teachers who had misconceptions in order to examine this situation in detail. The data collected through open-ended questions and clinical interviews were classified under two categories: the preservice teachers' misconceptions about the concepts of variable, equality, and equation and the language of mathematics subject-matter which they used.

Analysis of the preservice teachers' misconceptions shows that the first important misconception is confusing the concepts of identity with algebraic expressions. Some of the preservice teachers who described the algebraic expression "*4n+7*" given in the open-ended questionnaire as an identity continued to use this description in the clinical interviews. For instance, one of the preservice teachers wrote "*Ömer knows the concept of equation, but he doesn't know the concept of identity.*" The clinical interview with this preservice teacher showed that the preservice teacher identified the concept of identity with an algebraic expression. For example;

R (Researcher) : What is identity? Can you explain it?
T (Preservice teacher) : It is something that is valid for every value of n
R : You described 4n+7 as an identity. Why is it an identity?
T : n takes every value in 4n+7

Moreover, some of the preservice teachers had misconceptions about the concepts of equation and identity and they had difficulty in defining the concept of identity. The following is an example from a clinical interview:

R : What is equation? Can you explain it?
T : Well, I am going to describe it in an easy way… Err, the unknown, Err, I am going to say it is a mechanism made of known values and unknowns but I can't. …What I say about equations is that… Well, I would explain it in that way now but I couldn't. You give unknowns, you know, you give known values…
R : Alright. What is $(x+y)^2=x^2+2xy+y^2$ *then?*
T : It is an equation, too.

Preservice teachers' use of mathematical rules, concepts or knowledge with correct content and correct terminology is important for their teaching mathematics in an effective way. However, some of the preservice teachers in this study had difficulty in using the language of mathematics subject-matter. It is particularly remarkable that some of the preservice teachers used the expression "equation system" instead of equation and "parity", "algebraic expression", "equation", "problem" or "question" instead of equality. Finally, one of the preservice teachers used the concepts of number and numeral improperly: "*Was an expression in Arabic numerals given about Seda's height?*"

Discussion

This part of the study presents a discussion of the preservice teachers' analysis of students' thinking process concerning the concepts of variable, equality and equation and their ability to ask appropriate questions to identify students' errors and predict their errors. Also, this part will discuss the preservice teachers' need for improvement in knowledge of these concepts and present some recommendations concerning teacher training.

An important result of this study was that the preservice primary school mathematics teachers in the study succeeded in understanding students' thinking process with respect to knowledge of the variable, equality and equation but they had difficulty in explaining the causes of their thoughts. National or international studies on the issue report similar results (Boz, 2002; Boz, 2004; Stephens, 2006; Asquith et al., 2007). On the other hand, the fact that 35% of the preservice teachers were unable to discuss students' thoughts is another important finding. It's worth noting that these preservice teachers came up with irrelevant or insignificant explanations when they couldn't understand students' thoughts or explain the reasons for those thoughts. This could be attributed to the inadequacy of their subject-matter knowledge and misconceptions on these concepts (Boz, 2004; Stephens, 2008). This situation is particularly interesting considering the relationship between teachers' subject-matter knowledge and student achievement (Ma, 1999) as well as the importance of knowledge of students' thinking process in guiding teaching.

In mathematics teaching, asking effective questions is an important tool for better identifying the depth of students' ideas (Moyer & Milewicz, 2002). One of the striking results obtained from this study in this sense is the preservice teachers' primary use of instructional question types to guide students instead of identifying students' errors, which is similar to the findings from Moyer and Milewicz's (2002) and Tanı lı's (Manuscript submitted for publication) studies. On the other hand, it is undesirable that approximately 22% of the preservice teachers in this study used insufficient or insignificant questions because use of effective and varied question types is an important factor for teachers to analyse their students' thoughts and evaluate their learning process.

The study also found that, in general, the preservice teachers were able to predict primary school students' common errors and misconceptions about the concept of equation referred in the literature. Moreover, some of the preservice teachers were able to predict other errors and misconceptions of primary school students about the concept of equation referred in the literature. In addition, in parallel to the findings of a study by Dede and Peker (2004), some of the preservice teachers were able to predict the errors and misunderstandings that were not reported in the literature before.

Another important finding of the study was that some of the preservice teachers themselves had misconceptions and difficulties about the concepts of variable, equality and equation. For example, regarding the item asking about the role of n in $4n+7$ in the questionnaire, some of the preservice teachers supported Ömer by saying, *"Ömer's statement is correct"* or *"Ömer might have had a point. He must have taken n as a natural number"* and this shows that they matched the variable n with only particular number and, therefore, they had improper or inadequate knowledge about the different uses of the variable concept (Soylu, 2006; Dede & Argün, 2003). On the other hand, the fact that some of the preservice teachers failed to study the variables in the given algebraic expression without the equal sign and their responses such as *"the symbol n does not mean anything since the expression 4n+7 is not equal to anything"* or *"it does not represent anything unless there is an equality"* indicate that the preservice teachers regarded the equal sign as "total" or "answer". And this reveals the preservice teachers' need for improvement in their subject-matter knowledge of equality and the equal sign. Similar to the findings from Stephen's (2006) study, this need seems to have caused them to have difficulty in identifying students' misconceptions concerning the equals sign. Another misconception of the preservice teachers was that they were confused about the concepts of equality, identity, or algebraic expression. The preservice teachers' misconceptions about these concepts prevented them from determining the students' errors, which was also reported by Boz (2004).

Finally, this study found that some of the preservice teachers had problems in using the language of mathematics subject-matter, which was reported by Ye ildere (2007) as well. Teachers' appropriate use of mathematics language is an important factor in the fulfilment of their effective teaching of mathematics. Considering the connection between the use of an appropriate language of mathematics and having adequate knowledge of mathematics subject-matter, it could be suggested that this problem might have been caused by their need for improvement in subject-matter knowledge.

Conclusion and Recommendations

This study investigated preservice primary mathematics teachers' knowledge of students regarding the concepts of variable, equality and equation and their own misconceptions and difficulties in this respect. The study showed that, in general, the preservice teachers needed improvement in their knowledge of students about algebraic concepts, which actually revealed the relationship between preservice teachers' knowledge of students and subject-matter knowledge and misconceptions. The preservice teachers' need for improvement in their subject-matter knowledge and their misconceptions prevented them from identifying knowledge of student thinking process and students' misconceptions. The preservice teachers' need for improvement in these areas is likely to have a negative impact on their teaching in their future professional lives. Moreover, their misconceptions will be reflected by student misconceptions and, therefore, this will lead to a vicious circle. This situation highlights the relationship between subject-matter knowledge and pedagogical content knowledge and raises the importance of teacher training.

Teacher training programs are of great importance for teacher professional development. Despite the 2005 revisions in teacher training curricula in Turkey in line with the renewed primary and secondary education curricula, there is still a need for further improvement in the courses designed for teaching a specific area. The reason for this situation is that pedagogical content knowledge, which a teacher is supposed to possess, consists of many components. With respect to the Primary Mathematics Education curriculum, it seems difficult to equip preservice teachers with these components through the existing must courses offered. Therefore, in order to minimize the difficulties experienced in teacher training, the number of these courses could be increased or they could be supplemented by means of elective courses. In addition to revising teacher training curricula, another recommendation could be investigating preservice teachers' pedagogical content knowledge in terms of various components, identifying the existing deficiencies and taking necessary measures in this regard. It could therefore be ensured that preservice teachers are more professionally qualified when they graduate and they can support the development of children's conceptual understanding.

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