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An Error Analysis in Division Problems in Fractions Posed by Pre-Service Elementary Mathematics Teachers

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Abstract

The present study aimed to make an error analysis in the problems posed by pre-service elementary mathematics teachers about fractional division operation. It was carried out with 64 pre-service teachers studying in their final year in the Department of Mathematics Teaching in an eastern university during the spring semester of academic year 2010-2011. The data collection instrument used in the study was the four-item Problem Posing Test (PPT) developed about fractional division operation. Semi-structured interviews were conducted with 16 pre-service teachers so as to confirm the errors observed in the responses to the PPT and to reveal the reasons behind these errors in detail. Seven types of errors were identified in the problems posed by the pre-service teachers about fractional division. In addition, it was also found that the pre-service teachers overlooked the conceptual aspect of division in the problems they posed about fractional division.

Key Words

Division in Fractions, Problem Posing, Pre-Service Elementary Mathematics Teachers.

Fraction concept and mathematical operations with fractions are the main mathematical subjects hard to be understood by students (Alacaci, 2009; Dorgan, 1994; Işıksal, 2006; Kocaoğlu & Yenilmez, 2010; Ma, 1999; Smith, 2002; Tirosh, 2000; Yim, 2010; Zembat, 2007). Various methods are used to teach fractions; however, it is very common among teachers to have students start making calculations without setting up an adequate substructure to be used in mathematical operations with fractions (Işıksal; Mack, 1990; Mok, Cai, & Fung, 2008; Rule & Hallagan, 2006; Utley & Redmond, 2008). Students develop limited understanding on fractions as teachers do not include adequate activities in their curriculums, which are likely to ensure students to understand the fractions and mathematical operations with fractions conceptually (Borko et al., 1992; Redmond, 2009; Sharp & Adams, 2002).

Researches show that teachers and pre-service teachers, exactly like students, have some difficulties on fraction concept and division in fractions (Ball, 1990; Borko et al., 1992; Carraher, 1996; Ma, 1999; Mok et al., 2008; Redmond, 2009; Sharp & Adams, 2002; Tirosh, 2000; Toluk-Uçar, 2009; Yim, 2010; Zembat, 2007). In their study conducted with 14 mathematics teacher; Post, Harel, Behr, and Lesh (1991) detected that teachers are not able to make any logical pedagogical explanations on mathematical operations with fractions, and that they have superficial knowledge on fraction-related concepts. Işıksal (2006) stated that even though the pre-service teachers are able to solve division problems related to fractions, their levels of reasoning for

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explaining the meanings of such operations are low. Several researchers (Fishbein, Deri, Nello, & Marino, 1985; Graeber, Tirosh, & Glover, 1989) stated that equal division dominant model causes limitations in solving verbal problems bearing fraction-related points.

In the studies conducted to avoid difficulties in fractions and division in fractions, importance of associating fractions and fraction-related operations with tangible situations is emphasized (Arcavi, 2003; Bezuk & Armstrong, 1993; Hasemann, 1981; Keijzer & Terwel, 2003; Li, 2008; Rule & Hallagan, 2006; Steffe, 2002). Linking fractions and fraction-related operations with the situations faced in real world, makes contributions to the development of reasoning and problem solving skills of students (Cianca, 2006). Problem posing has a very important place in linking fractions and fraction-related operations with the situations faced in real world. Abu-Elwan (2002) states that problem posing makes contributions to establish a relationship between the content of mathematics and daily circumstances, and this is an effective method for developing the students' mathematical thinking.

Researches indicate that when teachers and pre-service teachers are asked to pose a problem covering division operation in fractions; they, in general, tend to pose multiplication problems or are not able to pose a correct problem (Tirosh, 2000; Utley & Redmond, 2008). Ball (1990) stated that all pre-service teachers had been successful at calculating the result of $1\frac{3}{4} \div \frac{1}{2}$; however, they had not been able to decide on the verbal problem in consistence with this operation. Ma (1999) detected that mathematics teachers are able to solve problems containing division in fraction; however their skills of posing problems related to the situations faced in real world is low. Similarly, Toluk-Uçar (2009) stated that the verbal problem posing skills of elementary teachers are low. It was also stated that pre-service teachers expressed their opinions arguing that it is impossible to link division operation in fractions with the situations faced in real world. Zembat (2007) determined that pre-service elementary teachers approach fractions as "a part of the whole divided into several pieces", they pose problems converting division operation to multiplication operation, they have difficulties with the units within the problem sentences, they ignore the relevancy of the problems they pose with the situations faced in real world, and that they verbalize the grouped division in problem sentences inconsistently. Işık (2011) stated that pre-service elementary mathematics teachers generally ignore measurement meaning of division and as a result of this ignorance conceptual structure of division in fractions cannot be generated in problems.

Problem posing-related acquisitions regarding the mathematical operations with fractions were included in Mathematics Course Curriculum for 1^{th}-5^{th} and 6^{th}, 7^{th} and 8^{th} Grades in Elementary Schools (Milli Eğitim Bakanlığı [MEB], 2009a, 2009b). This curriculum also refers to use of mathematical concepts in everyday life and in other learning fields, and to the importance of developing communication, creativity, problem solving and critical thinking skills of students. According to the researchers, problem posing makes great contributions to students in developing these skills (Akay, 2006; Crespo & Sinclair, 2008; Cunningham, 2004; Knott, 2010; Mestre, 2002; National Council of Teachers of Mathematics [NCTM], 2000; Silver & Cai, 1996; Stickles, 2006; Toluk-Uçar, 2009; Yuan & Sriraman 2011).

According to Lowrie (2002), students are able to pose more preferable questions when they face with a meaningful content, and students should be given opportunities to achieve such condition. The results indicating that problem posing skills of pre-service teachers are likely to affect the mathematical performances and mathematic perception successes of their students, during in-service period, are also included in literature (Abu-Elwan, 1999; Crespo & Sinclair, 2008; Stickles, 2006; Stoyanova, 2003).

Purpose

From this point of view, detection of possible errors in problems posed for division in fractions is considered to bear importance. In this context, the aim of this study is to analyze the errors in the problems posed for division in fractions by pre-service elementary mathematics teachers.

Method

Research Design

This study is a case study. Case study means deeply discovering a system with limited borderlines via several data collection tools (Creswell, 2007; McMillan & Schumacher, 2010).

Participants

Study was conducted with 64 pre-service teachers (38 females and 26 males), studying at the final year of Elementary School Mathematics Teaching Department in a University located in the Eastern Region of Turkey, in 2010-2011 Spring Semester. Prior to the research, 95 pre-service teachers in total, studying in the final year, were informed about the study. 64 pre-service teachers accepted to participate in this study voluntarily. In the study, semi-constructed interviews with 16 pre-service teachers were conducted.

Data Collecting and Analysis

Problem Posing Test (PPT), prepared for the division in fractions and composed of four items, was used as data collection tool. Items, in one of which dividend is not known, in one of which divisor is not known and in one of which division is not known, are included in PPT for the division in fractions. Dividend fractions are greater than the divisor fractions in these three problem posing items included in PPT. However in the last item included in the PPT, the dividend fraction is smaller than the divisor fraction. Items included in PPT are (i) $\blacksquare \div \frac{1}{4} = \frac{16}{5}$, (ii) $\frac{7}{8} \div \blacksquare = 3\frac{1}{2}$, (iii) $\frac{3}{4} \div \frac{2}{5} = \blacksquare$ ve (iv) $\frac{1}{8} \div \frac{1}{2} = \blacksquare$, respectively.

Pre-service teachers were asked to pose a problem for each item in PPT within a course hour. And answers given by pre-service teachers for each item in PPT were analyzed in detail. Semi-structured interviews were conducted with 16 pre-service teachers to confirm the errors and to determine the reasons behind these errors.

Seven errors categories were detected in the analysis of the answers given by pre-service teachers to PPT. These categories are: *confusion in units [Mistake 1(M1)], assign natural number meanings to fractional numbers (M2), posing problem using ratio-proportion(M3), not being able to establish part-whole relationships (M4), dividing to the denominator of the divisor (M5), using multiplication operation instead of division operation (M6),* and *posing problem through inverting and multiplying the divisor fraction (M7).*

M1, M5, M6 and M7, covered in the above-mentioned error categories, had been experienced in different research results. Zembat (2007) stated that meanings assigned by pre-service elementary teachers to the fractional numbers, in terms of unit, were not clear. E1 error included in this study covers not only the case stated by Zembat but also the case indicating that units in the problem sentences are inconsistent with each other. Furthermore, studies on E5 category (Ma, 1999; Ball, 1990; Toluk-Uçar, 2009), E6 category (Ma; Toluk-Uçar) and E7 category (Ma; Toluk-Uçar; Zembat) are present in literature. In the problems posed by pre-service teachers, more than one error type can be observed together. Studies on error categories are included in findings section in detail. Answers given for the items in the PPT were analyzed by two researchers. Errors were detected individually by each two researchers in the wake of these analyses, and then the analyses were compared and cohesiveness was ensured for the classification of the error types detected.

Findings

Findings on the Errors Types Observed in the Answers Given in PPT

Explanations for the error types observed in the problems posed by pre-service teachers for division operations in the PPT are:

Confusion in Units (E1): This error type includes the cases where a unit appropriate for the fractional numbers is not used or where the units used for the fractional numbers are not consistent with each other. Problem sentence created by one of the pre-service teachers for $\blacksquare \div \frac{1}{4} = \frac{16}{5}$ operation is:

If we have $\frac{16}{5}$ units of trees, when we cut each tree into $\frac{1}{4}$ parts, how many tree parts will we have?

Statements like *when we cut each tree in* $\frac{1}{4}$ *parts* and *if we have* $\frac{16}{5}$ *units of trees* in the problem sentence are the examples of the confusion in units. If cutting trees into $\frac{1}{4}$ parts means dividing the tree in parts equivalent to one four of its length, then four equal parts will be ensured. As a result, remaining part of the problem will be meaningless. Furthermore, whether the statement tried to be pointed out with $\frac{16}{5}$ units is the length of the parts or the number of parts is not clear.

Assign Natural Number Meanings to Fractional Numbers (E2): This type of error covers the problem situations in which natural number meanings are assigned to the fractional numbers. Problem sentence created by one of the pre-service teachers for $\frac{7}{8} \div \blacksquare = 3\frac{1}{2}$ operation is:

To how many people we should share a $\frac{7}{8}$ meter ribbon to ensure that each person gets $3\frac{1}{2}$ meter ribbon?

In the exemplary operation, the answer to replace ■ sign is $\frac{1}{4}$. However, in the problem, meaning related to the number of people was assigned to the fractional number. As a result, it was observed in the problem sentence created by the pre-service teacher that the pre-service teacher ignored the fact that the divisor is a fractional number and represented the number of people with a divisor fraction.

Posing Problem Using Ratio-Proportion (E3): This error type includes the cases where problems are posed comparing different units and comparing two fractional numbers provided that the units are same. Problem sentence created by one of the pre-service teachers for $\frac{3}{4} \div \frac{2}{5} = \blacksquare$ operation is:

Cobblestones will be paved on a pedestrian road. As $\frac{3}{4}$ of this road was paved within $\frac{2}{5}$ hour, how much of this road would be paved in an hour?

Even though the problem solving method is convenient for the exemplary operation stylistically, it does not cover the division of two fractional numbers conceptually. Because the dividend $\frac{3}{4}$ refers to the area on which cobblestones are paved while divisor $\frac{2}{5}$ refers to time. As a result, we can say that engagement of time change with the change in the area, on which cobblestones are to be paved, through proportioning is in question.

Not being able to Establish Part-Whole Relationships (E4): This type of error covers the problem situations which, at the end of the division operation, requires a number greater than the one indicated in the dividend fraction. Problem sentence created by one of the pre-service teachers for $\frac{3}{4} \div \frac{2}{5} = \blacksquare$ operation is:

Seda separated $\frac{3}{4}$ of a cake and gave $\frac{2}{5}$ of this separated cake to Ayşe. How many slices did Ayşe get?

In the problem sentence, it is stated that Seda separated $\frac{3}{4}$ of a cake and gave $\frac{2}{5}$ of this separated part of cake to Ayşe. To solve this problem, $\frac{3}{4}x\frac{5}{2}$ operation is required to be performed. And the result of this problem, $\frac{15}{8}$, refers to a greater proportion than the existing cake.

Dividing to the Denominator of the Divisor (E5): This type of error covers the problems in which dividing to the natural number in the denominator of the fractional number is performed instead of dividing to the divisor fraction. Problem sentence created by one of the pre-service teachers for $\frac{1}{8} \div \frac{1}{2} = \blacksquare$ operation is:

I divided $\frac{1}{8}$ of a basket of apples between two friends of mine, equally. How many apples did I share with one of my friends?

Division to $\frac{1}{2}$ was tried to be covered with the statement of *between two friends of mine, equally.* However, it is clear in this statement that division operation will be performed with the denominator of this fraction, 2; instead of $\frac{1}{2}$.

Using Multiplication Operation instead of Division Operation (E6): This error type covers the problems in which dividend fraction is multiplied with the divisor fraction. Problem sentence created by one of the pre-service teachers for $\frac{3}{4} \div \frac{2}{5} = \blacksquare$ operation is:

Ali bought trail mix from the market. $\frac{3}{4}$ of the weight of the trail mix is hazel nuts, and $\frac{2}{5}$ of the weight of the hazel nuts is hull. What is the proportion of hulls to the trail mix?

In the problem sentence it is stated that $\frac{3}{4}$ of the weight of the trail mix is hazel nuts while $\frac{2}{5}$ of the weight of the hazel nuts is hull. The result of $\frac{3}{4}x\frac{5}{2}$ operation will refer to the proportion of the weight of the hulls to the weight of the trail mix.

Posing Problem through Inverting and Multiplying the Divisor Fraction (E7): This type of error includes the problems in which divisor fraction is inverted and multiplied with the dividend fraction. Problem sentence created by one of the pre-service teachers for $\frac{1}{8} \div \frac{1}{2} = \blacksquare$ operation is:

Ali was going to pay the price of his new sweater by 8 installments. He paid double installment the first month. What is the proportion of the amount he paid in the first month to the price of the sweater?

In the problem sentence, each installment price equals to $\frac{1}{8}$ of the price of sweater. Pre-service teacher stated that in the first month double installment was paid. The proportion of the amount to be paid in first month to the price of the sweater can be calculated using $\frac{1}{8} \times 2$ operation.

Findings on the Distribution of the Error Types in the Problems Posed

62 pre-service teachers made total 79 errors in posing problems as an answer to the first item of the PPT. Fewest errors were made under E6 category in the problems posed for this operation. Pre-service teachers could not be able to create units appropriate for the fractional numbers in the problems they posed for E1 category, the first category in the error types. 62 teachers made total 75 errors in posing problems as an answer to the second item of the PPT. Fewest errors were made under E6 category in the problems posed for this operation, while the most of the errors was observed in E2 category. Total 69 errors were made by 63 pre-service teachers who answered the third item of the PPT. Fewest errors were observed in E7 category while the most of the errors were experienced in E6 category. Total 88 errors were made by 64 pre-service teachers who answered the fourth item of the PPT. Fewest errors were observed in E1 category while the most of the errors were experienced in E3 category.

Discussion

Seven error types were identified in the problems posed by the pre-service teachers for division in fractions. 64 prospective teachers made total 311 errors in the four items included in the PPT. Most of the errors (88 errors) were observed in the problems posed for $\frac{1}{8} \div \frac{1}{2} = \blacksquare$ operation and the fewest errors (69 errors) were experienced in the problems posed for $\frac{3}{4} \div \frac{2}{5} = \blacksquare$ operation. When the number of the types of errors faced in the study is taken into consideration, it can be said that problem posing skills of the pre-service teachers are low regarding division in fractions. This result also bears resemblance to the other studies (Ball, 1990; Toluk-Uçar, 2009; Utley & Redmond, 2008; Zembat, 2007) arguing that problem posing skills of the pre-service teachers are low regarding division operation in fractions.

In $\blacksquare \div \frac{1}{4} = \frac{16}{5}$ operation where dividend is not known, E1 and E2 errors categories came into prominence. It can be said that the reason why pre-service teachers made E2 errors is that they focused on the equivalent division meaning of the division operation. In the problems posed for $\frac{7}{8} \div \blacksquare = 3\frac{1}{2}$ operation, in which divisor is not known, E2 and E4 error categories came into prominence. Pre-service teachers focused on the equivalent division meaning of the division operation as they did in the first item of the PPT. Equivalent division meaning of the division operation is convenient for the cases where dividend is a fraction and divisor is a natural number; however, measurement meaning (sequential subtraction) of division is convenient for the cases where both dividend and divisor are fractions (Olkun & Toluk, 2003; Van de Walle, 2004).

In the problems posed for $\frac{3}{4} \div \frac{2}{5} = \blacksquare$ operation, where division is not known, pre-service teachers made more errors in E2, E3 and E6 categories. Pre-service teachers tried to create measurement meaning (sequential subtraction) in the problems they pose, but they did not succeed in verbalizing this sense. Furthermore the fact that pre-teachers realized in the interviews that they had posed problems requiring multiplication operation instead of division operation indicates that they have some linguistic problems in making transitions from symbolical to verbal representation. According to

Rudnitsky, Etheredge, Freeman, and Gilbert (1995), language to be used in line with the objective of the problem occupies an important place for students to understand the concepts and to code the mathematical values.

In the problems posed for $\frac{1}{8} \div \frac{1}{2} = \blacksquare$ operation where dividend is smaller than the divisor, E2, E3 and E5 error categories came into prominence. As the pre-service teachers had some difficulties in verbally conveying the fact that dividend is smaller than the divisor in the problem, directed them to pose problems using ratio-proportion method. Even when the proportion is represented as $\frac{a}{b}$, it may not indicate a part-whole relationship. In the problems posed using ratio-proportion method, pre-service teachers tended to proportion comparing two quantities. Even though the operation appeared to be met stylistically, division in fractions was not ensured conceptually. Another striking error type in the problems posed for this operation is to divide to the denominator of divisor fraction. Other researches (Ball, 1990; Ma, 1999; Toluk-Uçar, 2009) indicate that error type surfaced as dividing to the denominator of the divisor fraction is observed in posing verbal problems corresponding to the mathematical statement given.

When the results of this study are taken into consideration, pre-service teachers were detected to ignore the conceptual dimension of the division operation in general in the problems they posed for division in fractions. Hence the results of this study reveal the requirement for improving the teachers' problem posing skills regarding the division in fractions. In other studies (Akay & Boz, 2010; Crespo & Sinclair, 2008; Dickerson, 1999; Stickles, 2006; Toluk-Uçar, 2009) it was stated that when teachers are offered an opportunity to pose their own problems, this opportunity makes contributions not only to the improvement of their problem posing skills but also to their conception comprehension. To this end, problem posing based learning environments should be ensured for the teachers to eliminate the error types regarding division in fractions. Conducting qualitative and quantitative studies together after increasing the number of items will make contributions to presenting other error types and to the generalisability of this study.

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