RELATIONSHIP BETWEEN MATHEMATICAL PROBLEM-SOLVING SKILLS AND ASYNCHRONOUS COLLABORATION IN DIGITAL LEARNING ENVIRONMENTS

Alice Barana, Marina Marchisio Conte and Sara Omegna
Department of Molecular Biotechnology and Health Sciences, University of Turin, Piazza Nizza 44, 10153 Torino, Italy

ABSTRACT

This study investigates the relationship between students' problem-solving skills in solving mathematical problems and asynchronous collaboration in digital learning environments. The research focuses on the Digital Math Training (DMT) project, designed by the DELTA research group at the University of Turin, which aims to enhance students' digital mathematics skills and problem-solving abilities. Through a detailed analysis of student participation and performance data, including responses to a final questionnaire on the perceived benefits of asynchronous collaboration, the study identifies significant relationships between students' problem-solving abilities and the effectiveness of their collaborative activities. The findings highlight the benefits of asynchronous forums in fostering a sense of community, collaboration and reflective thinking among students, ultimately contributing to improved mathematical competence. The study's results underline the importance of integrating asynchronous collaborative tools into digital learning environments to support students' academic growth and prepare them for future challenges.

KEYWORDS

Asynchronous Collaboration, Digital Learning Environment, Problem Solving, Mathematics Education, Student Performance, Collaborative Learning

1. INTRODUCTION

Collaborative Learning is defined as a group of learners working together by sharing ideas, solving problems, or pursuing common goals (Lahann & Lambdin 2014). Although traditionally developed in the classroom through peer-to-peer discussions, technological advances have allowed for innovations such as access to transcripts of previous discussions, expanding opportunities for collaboration (Hammond 2017). Technology-enabled asynchronous discussions encourage the articulation of ideas, feedback, problem solving, and reflection on the opinions of others. This has led to talking about Computer Supported Collaborative Learning, a pedagogical approach that uses computing devices to enhance learning through social interaction and by offering new tools such as simulations and visualisations to improve students' conceptual understanding (Ludvigsen & Arnseth 2017). Technological innovations in education have also had a significant impact on how problem-solving skills in Mathematics are developed. For example, asynchronous online discussions, such as forums, allow students to engage in reflective thinking and knowledge construction. These platforms allow students to articulate their ideas and receive feedback, thus fostering a deeper understanding of mathematical concepts and the development of critical thinking. The aim of this research is to examine the relationship between achievement of problem-solving skills in mathematics and asynchronous collaboration in digital learning environments.

2. THEORETICAL FRAMEWORK

In the literature, problem solving is defined by (Martinez 1998) as "the process of moving toward a goal when the path to that goal is uncertain." It elaborates that problem solving occurs every time we achieve something without knowing beforehand how to do it. This encompasses both simple, everyday challenges and more complex, ill-defined problems that require navigating uncertain paths and making decisions step by step (Martinez 1998). It is the basis for creative thinking, new inventions, evolution, continuous improvement, communication, and learning. These skills are therefore essential for every citizen of the world, not only for their careers but also for their everyday life outside work. In this context, mathematics plays a crucial role in enhancing problem-solving skills, particularly in higher education (Fissore et al. 2021).

With the advent of technology in education, we started to talk about Digital Learning Environment (DLE), which is composed by a human component (the learning community), the technological component and the interrelations among the two (Barana & Marchisio 2022). The technological component of a DLE is based on a Learning Management System, through which it is possible to evaluate learning objectives, keep track of the progress of the students involved, and collect data to supervise the learning process (Barana, Marchisio & Miori 2019). Furthermore, within a DLE, online and asynchronous learning modes can be developed; for example, online discussion forums stimulate critical evaluation and thinking skills, as well as reflection on the ideas presented by other participants (Thomas 2002). This leads to the construction of personal meaning, which is not individual but a product of interaction between learners. This tool is asynchronous, non-hierarchical and reciprocal: instead of a mere a transfer of declarative knowledge, the construction of complex knowledge structures is now promoted. Indeed, each participant has the opportunity to be exposed to a variety of different perspectives, to engage in critical reflection, and to change his or her own perspective (Thomas 2002).

Asynchronous discussion groups are one of the best tools to promote interaction and learning, especially when facilitated by new technologies (Lucas, Gunawardena & Moreira, 2014). Indeed, they allow the automatic recording of discussions or messages, which can be consulted by students for reflection or by teachers to assess the level of knowledge achieved by students. This mode of interaction is often used because it has several advantages, such as allowing time to think, reflect, and search for information before participating in the discussion, thus avoiding off-topic messages or misunderstandings, equal opportunities for interaction and participation, and the development of learning communities and networks, where knowledge is the result of discussions, shared practices and collaboration (Lucas, Gunawardena & Moreira 2014).

Before going into the details of this research, we would like to focus on three studies in the literature.

The first study (Jacob & Sam 2008) is a pilot study exploring the promotion of critical thinking among first-year university mathematics students through asynchronous discussions. The problem-solving sessions activated through online discussion forums in these classes were examined. 46 participants were involved with the aim of adapting a model to assess critical thinking at the individual level in mathematical problem-solving sessions in online discussion forums; examining the relationship between mathematical achievement, as measured by final exam grades, and critical thinking in online discussion forums; testing whether there was a progression in critical thinking skills based on discussion forum posts in week 3 and week 11. The results showed an increase in the number of posts in the second forum compared to the first one, but the lower stage of critical thinking (clarification and evaluation) was prevalent in most posts, while the higher stages (inference and strategies) occurred for only a few students.

The second study (Jacob 2012) examines the relationship between critical thinking (CT) skills and student performance in an engineering mathematics course. Specifically, participation in asynchronous discussion forums (ADFs) was measured to assess CT skills using the CAIS model, which assigns a weighted CT score to each student. The ADFs were integrated into the course to encourage collaborative problem solving and critical thinking among students. Mathematical performance was measured by final exam scores, while initial mathematical ability was assessed by an initial test. This analysis showed that CT skills, when properly encouraged through structured and moderated ADF sessions, can improve students' mathematical success. For instance, students engaged in ADFs exhibited a notable improvement in their CT scores between the two problem-solving sessions, reflecting their enhanced ability to analyse, infer, and strategize effectively. This highlights the importance of encouraging CT skills in engineering students to improve their mathematical success. It also suggests that further research into the use of online discussion forums to facilitate CT could be useful for educators and researchers both in mathematics and other disciplines.

The third study (Alshaye, Tasir & Jumaat, 2023) involved 120 eleventh grade students from two secondary schools in Riyadh, equally divided into a control group and an experimental group. The research aimed to investigate the effectiveness of online problem-based learning (PBL) tasks in improving students' problem-solving ability (PSA) and programming skills. In particular, the experimental group engaged in complex programming tasks through asynchronous collaboration in Facebook groups, which allowed them to reflect and contribute to problem solving at their own pace, fostering a more critical and in-depth approach to learning. The study concluded that online PBL significantly improved both students' problem-solving and programming skills, promoting more autonomous and effective learning.

3. RESEARCH METHOD

3.1 The Digital Math Training Project

The context of this study is the Digital Math Training (DMT) project, designed and managed by the DELTA - Digital Education for Learning and Teaching Advances research group at the University of Turin. The DMT project aims to improve the mathematical, digital, and problem-solving skills of the participating students in grades 10, 11 and 12 (when students are between 16 and 18 years old), in order to prepare them to understand the processes of social transformation of which they must be an active and conscious part (Barana & Marchisio 2016). The DMT project offers optional extra-curricular activities, open to different schools each year, which can be recognised as part of the PCTO (Pathways for Transversal Skills and Orientation), which combines theoretical and practical learning to prepare students for the labour market. The aim is to use the mathematical skills learnt at school to solve real-world problems, thereby reducing the gap between theory and practice and between school and work. Specifically, students work in a DLE based on a dedicated Moodle platform and solve problems every ten days, submitting their solutions for assessment. These problems, open to different solution strategies, encourage asynchronous collaboration through forums; for each relevant intervention, points can be earned that contribute to a final ranking, thus motivating active participation (Barana et al. 2023; Floris et al. 2024). This platform is integrated with an Advanced Computing Environment (ACE), a system that allows students to do mathematics by performing numerical calculations, symbolic computations, graphical visualisations in two or three dimensions, and programming embedded components such as sliders, buttons, and boxes through a simple programming language (Barana et al. 2020). During the DMT project, the difficulty of the proposed problems gradually increases. Initial problems are simpler and allow students to familiarise with the use of the ACE, while subsequent problems become progressively more complex, focusing on developing problem-solving strategies. The problems are related to school topics to enhance the understanding of the subject outside school. The problems are divided into sequential and open-ended prompts: the former ones help to understand, explore, and identify a solution model, while the latter ones require generalisation using the interactive components of ACE, which can be programmed to perform calculations and provide numerical or graphical results (Barana et al. 2020).

Each problem is evaluated by the tutors according to a specifically designed assessment rubric containing five indicators: 1. understanding of the problem, 2. identification of a solution strategy, 3. development of the solution process, 4. argumentation, 5. use of Maple. For each of these, the tutor must indicate the level of competence achieved (from one to four), identified by descriptors, and the corresponding score, which can vary from 1 to 100 for each problem. In particular, the first four indicators are taken from the (Polya 1945) model and refer to the four phases of solving a problem, while the fifth indicator refers to the use of ACE.

3.2 Data Collection

The sample considered in this study is composed of 172 grade 11 participants (i.e. 17 years old) in the school year 2021/22. The data analysed to understand if there is a relationship between the achievement of problem-solving skills and asynchronous collaboration are the following:

- The number of messages sent by each student in the forums throughout the project to measure their level of active engagement in asynchronous collaboration;
- The total score, i.e., the sum of the scores obtained by the students in solving the eight problems of the online training, which we considered an index of the development of problem-solving skills achieved by each of them through the project (Barana et al. 2023);
- The students’ answers to specific questions concerning their perception of the usefulness of asynchronous collaboration included in the final questionnaire given to all the participants at the end of the project via the platform.

The questions, asked on a Likert scale from 1 (not at all) to 5 (very much), are the following:

1. I felt part of a community;
2. The forums allowed me to collaborate with other students to solve problems;
3. The atmosphere on the platform was positive;
4. The interventions in the forums were helpful in solving problems;
5. I felt competitive with other participants;
6. I felt helped;
7. I posted on the forums to get points;
8. This kind of online collaboration could be useful for my future work;
9. I found answers to my questions on the forums.

3.3 The Research Process

Before carrying out the analyses, all participants who did not take part in the survey were eliminated from the data set, leaving 131 participants, or approximately 76% of the participating students. For each student, the score obtained from the sum of the results for each delivery and in all project activities was calculated, and the number of messages posted in the discussion forums for each proposed problem was identified.

The first step was to divide the students into classes according to the final score obtained. A histogram was created using R-studio to describe qualitatively the frequency distribution of the total scores and to calculate the empirical density, approximating the theoretical density. In this way, it was possible to divide the students into two classes using a classification based on a mixture of normals. Next, the correlation between the total score and the number of posts was examined to see if there is a relationship between problem-solving skills and participation in asynchronous discussions in discussion forums. Lastly, a comparison was then made between the score means for each class into which we divided the students, using the bootstrap technique, analysing the relative p-values to understand which questions were influenced by the total scores. In particular, we expect the mean of the class characterised by having achieved higher problem-solving skills to be greater than or equal to the mean of the other class.

4. RESULTS

The first step in the analysis was to create a histogram (see Figure 1a), representing the distribution of scores, with the total score on the abscissa and the frequency of occurrence of these scores on the ordinate.

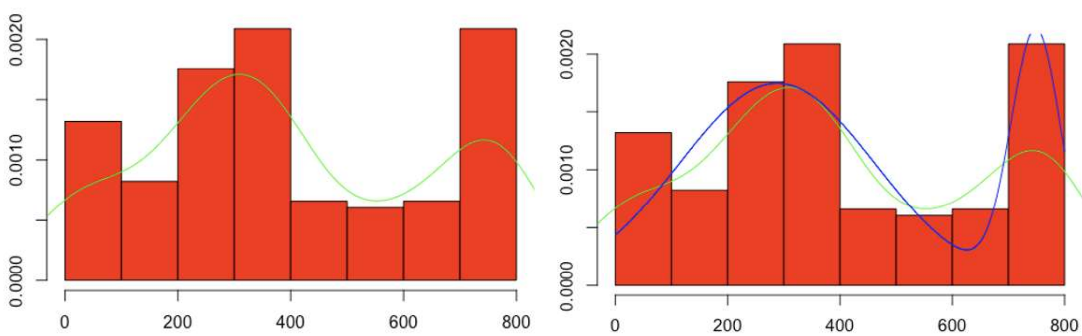

Figure 1a. Histogram of scores; Figure 1b. Histogram with BIC method estimation

The green curve of Figure 1a is the non-parametric estimate of the density function and it allowed us to assume that the whole group of students could be divided into two classes, represented by the two 'peaks', based on the score obtained; an attempt has therefore been made to estimate this curve. In particular, looking at the shape of the empirical density, it could be assumed that the two groups, taken individually, are distributed as normals. Therefore, we used an algorithm included in Mclust, which employs an iterative method, known as the E-M algorithm. It aims to find the mean variance and probability values that maximised the maximum likelihood function. In addition, in order to find the ideal number of classes, we used the Bayesian Information

Criterion (BIC), which removed values from the maximum likelihood function as the number of classes increased. This is because, as the number of classes increased, it became easier to describe the data due to the increased degrees of freedom. In this way, a balance was struck between the number of classes used and the quality of data modelling. As we expected, the most likely situation was one with two classes, where the blue curve in Figure 1b represents the estimate resulting from the BIC method.

Conclusions about the two classes could be drawn from the graph shown in Figure 2, where the dots represent the students, positioned alphabetically along the x-axis and according to their scores on the y-axis.

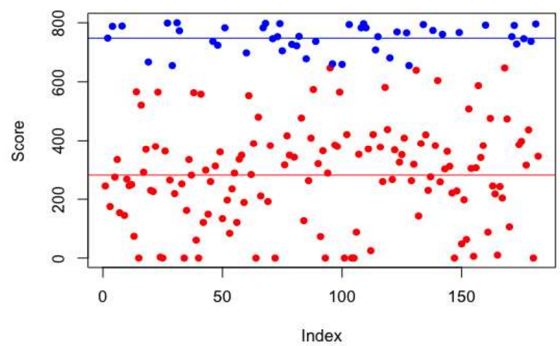

Figure 2. Scatterplot on the division of students into classes according to score

The graph shows that those belonging to the first normal, represented by the red dots, were the students with the lowest scores and were clearly separated from those in the other class, represented by the blue colour. In the following, these classes have been referred to as “lower achieving” and “higher achieving” respectively. Figure 3 shows a scatterplot where each dot represents a student, positioned alphabetically on the x-axis and according to their scores on the y-axis, with colours indicating their group: red for "lower achieving" and blue for "higher achieving."

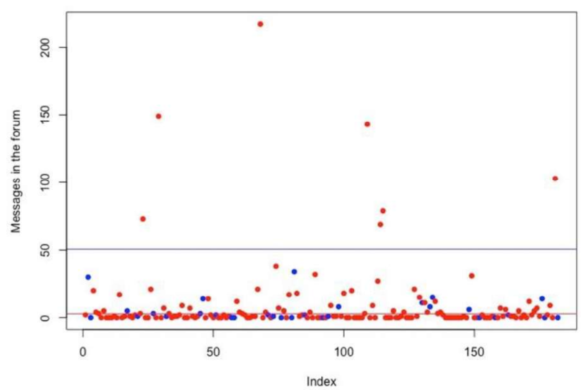

Figure 3. Scatterplot of the messages sent by each participant, where the colour indicates whether the participant belongs to the "lower achieving" (red) or "higher achieving" (blue) class

From the graph, it can be observed that students with higher scores generally sent, on average, a greater number of messages. The mean of messages sent by each class was represented by a line, red for the former and blue for the latter. In addition, the correlation index between the total score and the number of messages sent is 0.369, which is slightly above the significance level commonly accepted in educational research. This allows us to state that there is a relationship between the two variables considered.

At this point, we investigated whether this division into classes was related to the answers to the questions. The graphs in Figure 4 display each student's answers, with their indices on the x-axis and responses on the y-axis, color-coded according to the previous classification. Specifically, red represents students in the first class, while blue represents students in the second class, i.e. those who scored higher by solving the problems. In particular, Table 1 shows the means for each class and the relative p-value for each question.

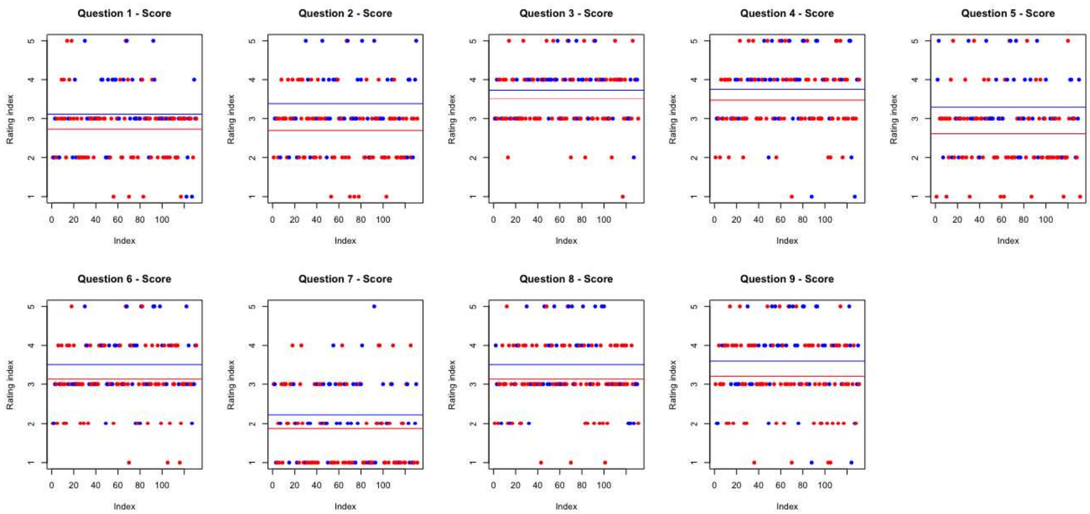

Figure 4. Analysis of the answers according to the classification given by the scores obtained by solving the problems

Table 1. Mean of the answers divided into the two classes obtained from the scores obtained by solving the problems and study of the p-Value

Question	“Lower achieving” class	“Higher achieving” class	p-value
I felt part of a community.	2.732558	3.111111	0.0210
The forums allowed me to collaborate with other students to solve problems.	2.697674	3.377778	0.0000
The atmosphere on the platform was positive.	3.511628	3.733333	0.0582
The interventions in the forur were helpful in solving problems.	3.465116	3.755556	0.0812
I felt competitive with other participants.	2.616279	3.288889	0.0000
I felt helped.	3.127907	3.511111	0.0100
I posted on the forums to get points.	1.860465	2.222222	0.0260
This kind of online collaboration could be useful for my future work.	3.127907	3.511111	0.0172
I found answers to my questions on the forums.	3.197674	3.600000	0.0326

We can reject the assumption that the means of the two groups are equal if the p-value is less than 0.05 using a one-tailed test (since we expect the mean of the class “Higher achieving” to be greater than or equal to the mean of the class “Lower achieving”). In this case, our initial hypothesis is correct: the mean of the class “Higher achieving” (shown in blue in Figure 4) is greater than the mean of the class “Lower achieving” (shown in red in Figure 4). However, the results vary according to the question. Specifically, we reject the hypothesis in the following cases: *I felt part of a community*; *the forums allowed me to collaborate with other students to solve problems*; *I felt competitive with other participants; I felt helped*; *I posted on the forums to get points*; *this kind of online collaboration could be useful for my future work* and *I found answers to my questions on the forums*. However, the hypothesis was not rejected in the following cases: *the atmosphere on the platform was positive* and *the interventions in the forums were helpful in solving problems*.

5. DISCUSSION AND CONCLUSION

From the analysis of the correlation between the total score and the number of messages sent by each student, we were able to observe a relationship between the perception of asynchronous collaboration and the problem-solving skills developed: in fact, greater collaboration is associated with higher levels of problem-solving ability. Furthermore, by examining the p-values, we observed a relationship between the problem-solving skills achieved by each student and the answers to the self-assessment questions on the perceived usefulness of asynchronous collaboration. When the p-value was less than 0.05, it indicated that belonging to a particular class, i.e. possessing certain problem-solving abilities, influenced the perception of specific aspects. In particular, this relationship was evident in the following statements:

- I felt competitive with other participants;
- I posted on the forums to get points.

These elements highlight the more competitive side of the project: each student earns points for each forum intervention, and accumulating a lot of points is essential to win. This competitive feature undoubtedly encourages students to engage more actively in the forum; however, it may serve as an initial incentive for some students to view the forum differently, i.e. as an opportunity to solve problems in a better way through interaction with other participants. In addition, influences were observed in the following questions:

- I felt part of a community;
- The forums allowed me to collaborate with other students to solve problems;
- I felt helped;
- This kind of online collaboration could be useful for my future work;
- I found answers to my questions on the forums.

Students who achieved higher levels of problem-solving skills at the end of the project also exhibited greater engagement with asynchronous collaboration, actively using the forum to post questions and answers. These students perceived the DLE as collaborative and supportive, suggesting that asynchronous collaboration facilitated the formation of a learning community. These results indicate that the competitive side of the project did not undermine the development of a community where participants, by sharing their opinions, both assist others and open themselves to feedback and validation.

Notably, no significant difference was found between the two classes in the answers to the statements "The atmosphere on the platform was positive" and "The interventions in the forums were helpful in solving problems". These are also the items that received the highest scores. This indicates that both the positive climate and the perceived usefulness of asynchronous collaboration were recognised by students with lower problem-solving skills as well as those with higher skills.

Our research has proven the existence of a relationship between students' final grades, reflecting their problem-solving skills in mathematics and their engagement with asynchronous collaboration, including their perceptions of its usefulness. Notably, we found a significant connection between the students' sense of competition and their motivation to contribute to forums for points. Additionally, students' views on asynchronous collaboration highlighted a strong sense of community and the value of working together to solve problems. Many students, regardless of their final point totals, reported feeling supported and believed that this form of collaboration would benefit their future work. This suggests that, beyond its cognitive advantages, asynchronous collaboration can positively impact community building and socio-emotional learning by fostering a supportive environment that promotes active participation and knowledge sharing. When conducting this discussion, however, it is important to keep in mind the research context in which the results were obtained: the project aims to stimulate and enhance mathematical and computational skills. At the same time, participants are in a competitive atmosphere due to the final ranking, feature that has undoubtedly influenced their behaviour.

In conclusion, the studies reviewed, together with our empirical observations, indicate the existence of a relationship between students' mathematical problem-solving skills and the perception of asynchronous collaboration in digital learning environments. These results are significant, as online learning continues to grow in popularity: it is therefore necessary to continue researching and optimising these tools to maximise their educational potential. In this analysis, we only took into account grade 11 students (when they are 17 years old); future research could extend the sample to include other classes participating in the DMT project to determine if similar results are observed across different age groups. Additionally, it could be interesting to differentiate between students participating for the first time and those with prior experience in the project, examining how their perceptions of asynchronous collaboration evolve over time and to extend this study to other skills, such as programming, to explore the full potential of asynchronous collaboration in improving different academic skills.

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