FracSi:
Testing Efficacy of Language Arts Approach to Teaching Fractions

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ABSTRACT

Urban school districts have seen low scores on NWEA Map math scores (Education Recovery Scoreboard, 2024). To foster and promote urban youths' capabilities to learn mathematics, a new method to teach, enhance, and remediate understanding of fractions was developed, Fractions and Signed Numbers Workshop (FracSi). It utilizes the approach of an early literacy classroom to teaching fractions. The implementation of this methodology yielded a substantial improvement in NWEA MAP math scores for one urban high school.

INTRODUCTION

Newark Public Schools of Newark, NJ, (NPS) has seen a decrease of 1.13 points in NWEA MAP math test scores between 2019 and 2023 (Education Recovery Scoreboard, 2024). While there can be many different causes to the decline, researchers have linked the importance of learning fractions to math achievement (Stigler et al., 2010; Barbieri et al., 2021).

To foster and promote urban youths' capabilities to learn mathematics, a new method to teach, enhance, and remediate understanding of fractions adapted the blueprints set by the late Susan Glazer (1996) for early literacy classrooms. The Fractions and Signed Numbers Workshop (FracSi) falls in line with the advice and recommendations of researchers who promote using language literacy for mathematics (Pitkethly, 1996; LeFevre et al., 2010; Molina, 2012; Bingham et al., 2019; Moleko, 2021). The implementation of this methodology yielded a substantial improvement in the student test scores.

BACKGROUND

Schwartz et al. (2011) found students who invented their own index for density transferred ratios for density to speed. The success was attributed to finding deep structure through guided discovery. The researchers noted that their method "…is one among many possible ways to support students in learning deep structure (p. 772)."

The researcher of this paper theorized that deep structure can be gained through a language arts approach. Researchers have advocated using language to teach math in general, but special emphasis has been given to fractions. Zhang et al. (2015) argue that there is an overemphasis on pictorial representations of fractions. Pitkethly (1996) stressed developing language, seeing fractions as requiring action, and recognizing images will foster more complex and sophisticated understanding of fractions. Bingham et al. (2019) recommended using vocabulary, much in the sense of English Language Arts programs, to teach fractions.

Adapting the practices and approaches set by Glazer (1996) to early literacy, FracSi developed new methods to teach fractions. Of particular interest, the depictions of unit fractions commonly found in textbooks to introduce young students to fractions (Zhang et al., 2015) were replaced by photographs of people or objects in situations that needed resolution to a problem. The goal of the study is to demonstrate a method to teach fractions in a manner advocated by researchers like Pitkethly and Bingham do in fact yield positive results.

METHOD

Bard High School Early College, a magnet school, selects students based upon writing samples, not standardized tests or other testing criteria. FracSi was a quasi-experimental study that taught fractions using reading and writing program components. It was introduced during Summer Bridge 2023, which is a voluntary two-week summer school program open to all incoming 9th grade students.

Summer Bridge 2023 took place at Bard High School from August 7 through August 18, 2023. In August 2023, Bard High School was expecting 118 incoming freshmen.

Incoming freshmen of academic 2023-2024 were placed into three groups:

- Control Group - Students voluntarily declined to participate in Summer Bridge 2023.
- Subject Group - Students voluntarily enrolled in Summer Bridge 2023 and underwent 2 hours of FracSi instruction. Their day ended at 1pm.
- Full Participants - Students voluntarily agreed to undergo ~17 hours of FracSi instruction. These students stayed in school until 2:45pm.

The students participated Monday through Friday. Instruction started each day at 8:30am. The schedule for Summer Bridge 2023 allocated a portion of the time to math. On the first day, a PreTest was administered. A PostTest identical to the PreTest was administered on the last day. All students received a blue book which was used for note taking and solving situation-based activities. Ap-proximately two hours of instructions spread across three days focused on teaching fractions as verbs, nouns, and prepositions.

Each lesson had four sections: Reading, Writing, Composition and Comprehension. Each lesson began with Reading and ended with Composition. The Comprehension section utilized Glazer's "Picture Talk" and replaced the unit fraction depictions commonly found in math textbooks (Zhang et al, 2015) with pictures that told stories. Students were tasked to discuss as a group what is happening in the picture and what needs to be done so that an equitable situation can arise. The study purposely chose photos of real people that met a certain level of aesthetic value set by the researcher. Also, the photos were selected because they reflected the demographic diversity of the classroom. See Figure 1

Figure 1-One of the slides for Division lesson contains a photo of ten soccer players of various ethnic and racial backgrounds and four pizzas (Creative Commons license)

To illustrate the methodology, one lesson introduced fractions as a verb, "to divide" or "to share". The Reading section opened the lesson by asking the students to read the improper fraction 6/2. Some students responded, six halves. The others remained silent. The lesson introduced the idea that the vinculum is read as "divided by" or "shared by." The students were presented different improper and proper fractions and required students to write how you would say those fractions in English. This meets conceptual framing and integration of content and language of WIDA ELD Standard 3 which emphasizes different parts of speech in mathematics (University of Wisconsin-Madison , 2020). Also abiding by the natural development of multilingual language learners, students can describe picture-based stories in their native languages or a mixture (Olshansky, 2018).

For the Writing section, students were tasked to write fractional equivalents to different English sentences. For example, "Divide into six parts, take three parts" would be written as 3/6.

The Comprehension portion presented situational scenarios that display both discrete and continuous partitioning (Pitkethly, 1996), and each scenario required students to reach an equitable solution to sharing. The first scenario showed discrete partitioning, such as six cookies and three siblings. See figure 2. Then a continuous partitioning scenario followed. For example, one cookie is shared between three siblings. See Figure 3.

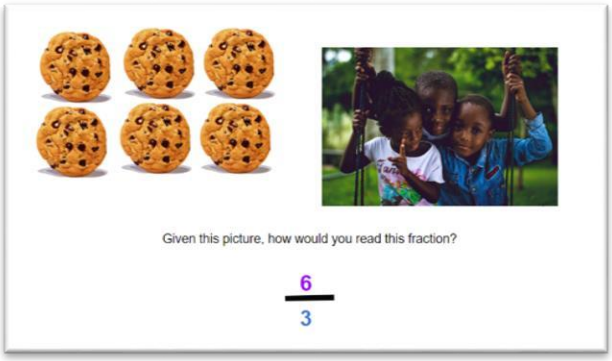

Figure 2 – This is a sample scenario for discrete partitioning.

Figure 3 – This is a sample scenario for continuous partitioning.

The Composition section required students to write their own scenarios for a partner to solve, write and read. The methodology for FracSi has since been expounded upon and improved [1].

At the end of Summer Bridge 2023, the blue books were collected. The contents of the blue books gave insights to possible reasons why certain students improved between PreTest and PostTest. For evaluation of the overall efficacy of FracSi, NWEA MAP scores were used.

DATA

The Newark Board of Education granted the rights to access NWEA MAP data, and the Newark Public School Research Department provided all raw data. The researcher requested NWEA MAP data for the end of 8th grade test and throughout 9th grade. The test at the end of 8th grade took place in Spring of 2023, and it was administered at all NPS middle schools in the May 2023 time frame. The Fall 2023 NWEA MAP, which was administered on October 3 and October 4, 2023, at Bard High School, marked the beginning of 9th grade. Midway through their 9th grade, the Winter NWEA MAP was administered on February 7 and 8, 2024. At the end of the year, the Spring NWEA MAP was administered on June 3 and 4, 2024. See Table 1

[1] See https://nrs.harvard.edu/URN-3:HUL.INSTREPOS:37380084 for full thesis

NWEA MAP Test	Dates Administered	Testing Location
Spring 2023	In the May 2023 timeframe	middle schools within the *** district
August 7-18, 2023 Summer Bridge		
Fall 2023	October 3 and 4, 2024	*** High School
Winter 2023	February 7 and 8, 2024	*** High School
Spring 2023	June 3, 2024	*** High School

Table 1 – This table shows NWEA MAP Testing dates and times along with the Summer Bridge dates and times

Because Bard High School is a magnet school, a fair evaluation of efficacy requires comparing test results amongst the Bard student population. The researcher then reviewed the raw data and transformed it into meaningful comparison criteria. End of 8th grade NWEA MAP results became the baseline. Only students who were in NPS middle school would have that data available, and out of district students' middle school data could not be obtained. A student from outside of NPS district could mean a student from another country, another state, and another town. It is important to note that private and charter schools within Newark, NJ, are also out of district. As a result of having this data selection criterion, the number of potential data points decreased.

The Control Group had 16 students' data, and the Subject Group had 36 students' data. For the Full Participants, there were only five students' test results. Therefore, the study did not analyze Full Participants due to insufficient data.

The final sample size was 52 students which represented 91.2% of Bard High School freshmen students who attended a NPS middle school. Due to the small sample size, the data as a discrete set, and therefore many statistical analysis techniques for continuous data like regression continuity were not used.

The results focused on the difference between Spring and Fall 2023 NWEA MAP test results. The mean difference between the two tests of the Control Group will be denoted as μ_0. The mean difference between the two tests for the Subject Group will be denoted as μ_1. The absolute error provided by NWEA MAP varied

between 3.1 and 3.8%. When reviewing the difference of Spring and Fall test results, the relative error varied between 0.1% and 0.8% which is negligible for the study.

RESULTS

NULL HYPOTHESIS

The null hypothesis says FracSi did not improve students' NWEA MAP math scores. The alternative hypothesis says FracSi improved student scores.

There are five identified cases that would make the null hypothesis true. The first case, Subject Group and Control Group had comparable score increases between Spring and Fall testing. Even though the relative error is negligible, the study set a 5% error to the Control Group's mean score ($1.05\mu_0 \geq \mu_1$). The second case, the Subject Group's mean difference is greater than the first, but the increase in scores was due to chance. The third case, students who enroll in Summer Bridge have a stronger work ethic than those who do not. Worth ethics caused an increase in scores, not FracSi. The fourth case, the time interval between the end of Summer Bridge 2023 and the actual date of the Fall NWEA MAP test allowed enough instructional time that test scores improved as a result of regular academic year instruction and not FracSi. The fifth case, the teacher's teaching style caused the increase in scores and not FracSi.

CASE 1: CONTROL GROUP AND SUBJECT GROUP HAD COMPARABLE INCREASE IN TEST SCORES.

Although both groups showed an increased mean score, the Subject Group's results were approximately nineteen times greater than the Control Group's results. FracSi had a significant impact on the test results of students. This is shown by the fact that $\mu_0 = 0.19$ and $\mu_1 = 4.25$. ($1.05\mu_0 \leq \mu_1$). This rejects the first case of the null hypothesis.

Furthermore, the increase in raw scores is clearly reflected in the percentiles. The Control Group increased 2.87 percentiles from 52.56 to 55.44. For the Subject Group, the percentile increased almost 10 percentiles from 38.86 to 48.58. This indicates FracSi substantially improved Subject Group students' performance. See Figure 2

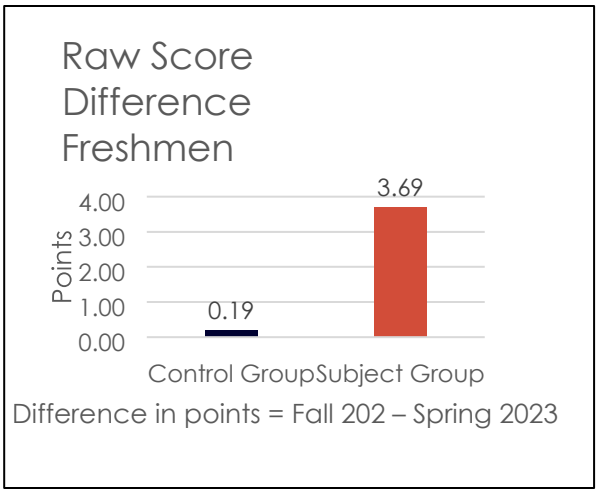

Figure 4 – The blue bar is the difference between Spring and Fall 2024 NWEA MAP Math Scores for the Control Group, and the orange graph is for the Subject Group.

CASE 2: THE SUBJECT GROUP'S INCREASE IN SCORES IS DUE TO RANDOM CHANCE.

Results for a paired one-tailed independent T-test show the results were not by chance. The 36 participants in the Subject Group (M= 4.25, SD = 12.41) demonstrated significantly better scores, $t(50) = 2.78$, $p = .004$. At the specified level of significance ($a = 0.05$), the calculated p-value rejects the second case of the null hypothesis and supports the alternative hypothesis.

CASE 3: SUMMER BRIDGE STUDENTS HAVE A STRONGER WORK ETHIC

It is possible that the improvements in the freshmen's scores are due to summer school students possessing stronger work habits than those who do not go to summer programs, and FracSi played no factor in the results. If the difference of scores between end of 8th grade and beginning of 9th for all other cohorts were comparable to that of the freshmen, then work ethics lead to increased test scores and not FracSi. However, data of previous cohorts do not support that.

NWEA MAP math 8th grade and 9th grade results for sophomores and juniors of academic year 20232024 were reviewed. Seniors' data was not used for comparison purposes because they did not undergo the COVID pandemic in middle school to the same extent as their schoolmates. The freshmen, sophomores, and juniors were online learners for one full year during COVID. The freshmen were online learners in the last quarter of 5th grade and all of 6th grade, and the beginning of 7th grade. The sophomores were online in the last quarter of 6th grade and all of 7th grade, and the beginning of 8th grade. The juniors were online for last quarter 7th grade, all of 8th grade, and beginning of 9th grade. The seniors were affected last quarter of 8th grade. Their middle school experience was not as heavily impacted as the freshmen, sophomores and juniors.

End of 8th grade and beginning of 9th grade results for the three cohorts were collected. Because NWEA MAP tests are normalized, percentile results were not analyzed due to the complexity of analysis. Raw

scores are more convenient for comparing across cohorts. It is important to note that while the freshmen and sophomores had comparable proportions of Algebra 1 and Geometry students (no more than 25% were enrolled in Geometry), the juniors did not. Over 90% of the juniors were enrolled in Algebra 1 in their freshmen year. See Table 2

	Number students who took Algebra 1 during their 9th Grade	Number of students who took Geometry during their 9th Grade
Freshmen	81	27
Sophomores	79	21
Juniors	84	4

Table 2 – This table shows the number of freshmen by cohort who took the Algebra 1 NJSLA and the Geometry NJSLA.

The sophomores of academic year 2023-2024 had an average of 225.73 points at the end of their 8th grade. After Summer Bridge 2022, their raw score improved by 1.04 points. The juniors left 8th grade with an average of 234.62 raw points. After Summer Bridge 2021, their score increased by 0.89 raw points. There were no comparisons of end of 9th Grade NWEA MAP results across cohorts, because in September 2023, NPS introduced a new curriculum for all high school students. Freshmen of academic year 2023-2024 learned Algebra 1 and Geometry with new textbooks and teaching approaches.

When comparing the gains of all three cohorts, the freshmen had the greatest increase in raw points. Freshmen who participated in FracSi as part of Summer Bridge had improvements in scores over three times that of the sophomores and juniors who participated in Summer Bridge without FracSi. See Figure 5

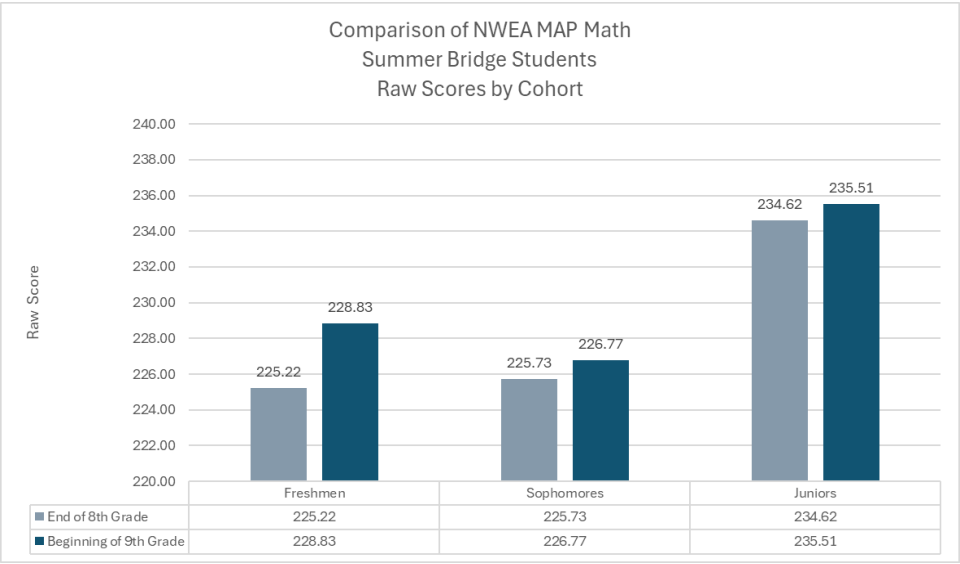

Figure 5 – Organized by cohort, this graph shows the end of 8th grade NWEA MAP Math results and the

beginning of 9th grade.

CASE 4: REGULAR CLASSROOM INSTRUCTION BETWEEN START OF ACADEMIC YEAR 2023-2024 AND FALL NWEA MAP TESTING INCREASED TEST SCORES, NOT FRACSI.

Another possible explanation as to why the freshmen had more gain than the sophomores or juniors, the freshmen had more instructional time between the start of school and when the Fall 2023 NWEA MAP was administered. However, the dates provided by NPS do not support this. According to the NPS school calendar, in 2023, school started the first day after Labor Day for all three years and the Fall NWEA MAP were administered in October for all three years. In the academic year 2023-2024, school started September 5, 2023, and NPS' Fall testing window was between September 28 through October 5, 2023. NWEA MAP took place at Bard High School on October 3 and 4, 2023. The freshmen had 31 calendar days between the start of school and testing. For the academic year 2023-2023, school started September 5, 2022. The FALL NWEA MAP on October 20 and 24, 2023. They had 45 calendar days between the start of school and testing. In the academic year 2021-2022, school started on September 7, 2021. Fall NWEA MAP testing took place in October 2021, but no specific date could be determined.

CASE 5: SUMMER BRIDGE 2023 MATH TEACHER'S TEACHING STYLE CAUSED THE INCREASE IN TEST SCORES, NOT FRACSI.

The existing data cannot show if null hypothesis is true or false for the fifth case. Currently, the only identified possible method to prove to disprove the fifth case is having different teachers from different schools implement FracSi. They can compare their results to a control group within their school or compare it to the general population of their school.

DISCUSSION

The study used a T distribution to represent the data due to the small sample size. Because T distribution is close to a normal distribution, all graphs are normal distributions. See Graph 6 and 7, and Table 3.

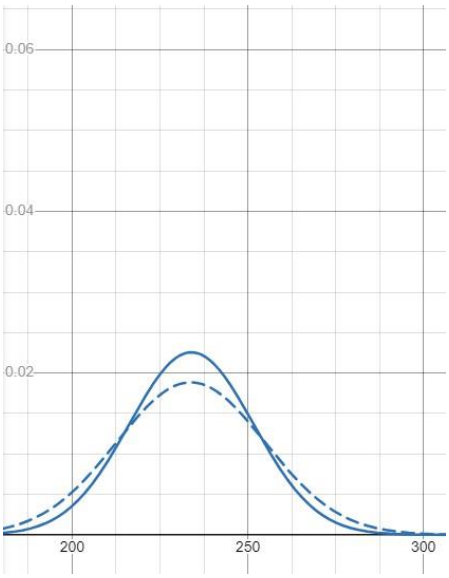

Figure 6 – This graph shows the mean and standard deviation of the Control Group

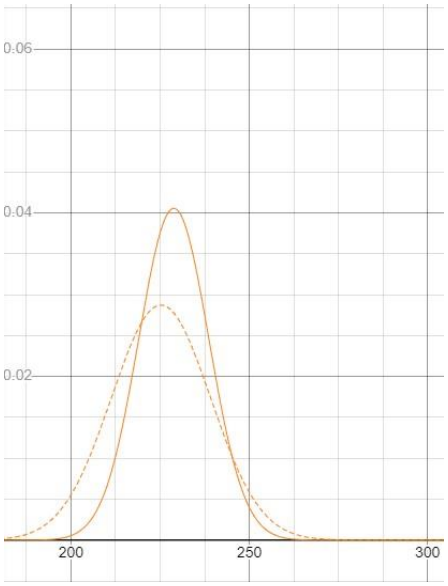

Figure 7 – This graph shows the mean and standard deviation of the Subject Group.

	End of 8th Grade (dotted line)	Beginning of 9th Grade (solid line)	End of 8th Grade (dotted line)	Beginning of 9th Grade (solid line)
Mean	233.75	233.94	225.22	228.83
STDV	21.16	17.68	13.89	9.84

Table 3 – The blue graphs compare the mean and standard deviation results of the Control Group's end of 8th grade and beginning of 9th grade NWEA MAP Math results. The orange graphs compare the mean and standard deviation results of the Subject Group's end of 8th grade and beginning of 9th grade NWEA MAP Math results

BLUE BOOK ANALYSIS

PreTest and PostTest scores were analyzed and cross compared to the NWEA MAP Fall 2023 math test results. Analysis focused on how students' drawings represented one specific situation that needed solving: If three people share two pizzas how much pizza does each person get? The students who scored over 245 raw points on the Fall 2023 NWEA MAP and had more than 8 points improvement between PreTest and PostTest

varied in their pictorial depictions. Some used circles to represent pizzas. Most used rectangles to represent the situation. Many did not draw anything and just solved the problem.

The students who scored below 225 and had less than 5 points increase in PreTest and PostTest, shared a general commonality. They used circles to depict pizzas and drew figures showing how many people shared the pizza. Their pictures were concrete in representation.

One possible explanation for the increase in NWEA MAP math scores amongst students in the Subject Group, the reading and writing approach to teach fractions helped students abstract.

RECOMMENDATION

While the NWEA MAP math results indicated that two hours of FracSi instruction increased student performance on the test, the sample data was small. It can be argued that the teacher's teaching style played a role. In addition, the data represents only Bard students who attended a middle school within the NPS district. It does not represent students from private or charter schools within NPS. While the PreTest and PostTests could potentially be correlated to NWEA MAP results, that would be a different study.

To understand FracSi's ability to improve students' NWEA MAP scores due to content and approach and not teaching style and to determine for efficacy outside of Bard High School or the NPS school system, FracSi would ideally be repeated by different teachers from different schools, and over 50 students participate at each school. The aggregate results from all schools can be evaluated and analyzed against control groups of previous years' students as well as students who voluntarily choose not to participate.

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