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Last Name, First Name	Academic/Organizational Affiliation	ORCID ID
Park, Yunji	Stanford Medicine, Stanford University	https://orcid.org/0000-0003-2904-9505
Viegut, Alexandria A.	University of Wisconsin–Eau Claire	https://orcid.org/0000-0003-2608-3441
Matthews, Percival G.	University of Wisconsin–Eau Claire	https://orcid.org/0000-0001-6265-813X

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More than the Sum of its Parts:
Exploring the Development of Ratio Magnitude vs. Simple Magnitude Perception

Yunji Park, Alexandria A. Viegut, Percival G. Matthews

Department of Educational Psychology, University of Wisconsin – Madison

Correspondence of this manuscript should be addressed to Yunji Park, Department of Educational Psychology, 1025 W. Johnson Street, Madison, WI, 53706. Phone: (708)-374-5461; Email: ypark246@wisc.edu

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Research Highlights

- We examined developmental trajectories of simple and ratio magnitude discrimination in multiple formats among preschoolers, 2^{nd}- and 5^{th}-graders, and adults.
- Simple and ratio magnitude acuities showed similar group-level format effects and developmental trajectories.
- Subject-level acuities for ratio magnitudes were better predicted by cross-format ratio acuities than by within-format simple magnitude acuity.
- Findings demonstrate ratio magnitude processing bears several similarities to simple magnitude processing, but is also a substantially different process.

Abstract

Humans perceptually extract quantity information from our environments, be it from simple stimuli in isolation, or from relational magnitudes formed by taking ratios of pairs of simple stimuli. Some have proposed that these two types of magnitude are processed by a common system, whereas others have proposed separate systems. To test these competing possibilities, the present study examined the developmental trajectories of simple and relational magnitude discrimination and relations among these abilities for preschoolers (n = 42), 2nd-graders (n = 31), 5th-graders (n = 29), and adults (n = 32). Participants completed simple magnitude and ratio discrimination tasks in four different nonsymbolic formats, using dots, lines, circles, and irregular blobs. All age cohorts accurately discriminated both simple and ratio magnitudes. Discriminability differed by format such that performance was highest with line and lowest with dot stimuli. Moreover, developmental trajectories calculated for each format were similar across simple and ratio discriminations. Although some characteristics were similar for both types of discrimination, ratio acuity in a given format was more closely related with ratio acuities in alternate formats than to within-format simple magnitude acuity. Results demonstrate ratio magnitude processing several similarities to simple magnitude processing, but is also substantially different.

Key words: Magnitude discrimination, relational magnitude, development, acuity, ratio processing system, nonsymbolic ratio.

1 Introduction

We encounter various forms of quantity in our daily lives. These not only include symbolic labels like numbers of coins or sizes of clothes, but also include naturally occurring continuous quantities like sizes of fruit or sweetness of drinks. Indeed, a wealth of research in psychophysics has demonstrated that we humans perceptually extract many different forms of quantity information from the environment (e.g., Brannon, Lutz, & Cordes, 2006; Cantlon, Platt, & Brannon, 2009; Odic, 2017; Xu & Spelke, 2000). Many quantities can be classified as *simple magnitudes* – also known as *extensive* magnitudes – be they discrete (e.g., numerosity) or continuous (e.g., line-lengths or volumes). Simple magnitudes are defined by the raw amount of a substance (Newcombe, Levine, & Mix, 2015; Strauss, Piaget, Gattegno, & Hodgson, 1953).

Other quantities featured in our environments are relational in nature. In contrast to simple magnitudes that can be measured in solitude, these *relational magnitudes* – also known as *intensive* magnitudes – emerge from the comparative relations among simple magnitudes. Their magnitudes correspond not to the raw amount of material measured, but instead emerge from the composition of simple magnitudes (e.g., density reflects a relation between the simple magnitudes mass and volume, and a line's slope reflects the relations between the relative vertical change for every unit change in horizontal distance). Relational magnitudes include quantities such as nonsymbolic ratio magnitudes composed of two simple magnitudes (e.g., ratios instantiated by juxtaposing two line-segments, Figure 1) (Matthews, Lewis, & Hubbard, 2016; McCrink & Wynn, 2007).

Although human abilities to perceptually represent magnitudes have been widely investigated, the overwhelming majority of these investigations have been limited to exploring representations of simple magnitudes. The acuity or precision of representations has been measured by paradigms comparing pairs of magnitudes. These discrimination paradigms have been used across various stimulus formats such as line segments, blob areas, or dot arrays (e.g., DeWind & Brannon, 2012; Feigenson, 2007; Odic, 2017; Park & Cho, 2017). Task performance varies depending on these formats. Multiple studies have charted the development of simple magnitude discrimination acuities for different formats using cross-sectional analyses of different age cohorts (e.g., Halberda & Feigenson, 2008; Odic, 2017). These studies have commonly reported that the simple magnitude acuities differ by format such that acuities for continuous magnitudes (e.g., line-lengths or areas) are sharper than those for discrete magnitudes (i.e., numerosities).

Comparatively little is known about relational magnitude processing and how discrimination acuity for this type of magnitude varies depending on format. Furthermore, developmental changes in relational magnitude representation remain almost entirely uncharted. Additionally, the extent to which processing of relational and simple magnitudes are related remains unclear. We investigated these outstanding questions in the current study.

1.1 Relations between ratio and simple magnitudes: Competing hypotheses

One major question we wanted to investigate concerns whether ratio processing is tightly tied to simple magnitude processing or if the two are processed significantly independently. The case for the two being yoked is straightforward: Because a ratio magnitude is composed of two simple magnitudes, it seems logical that we must first represent each simple magnitude component to extract ratio magnitude. Indeed, theory about the nature of simple magnitude discrimination can be read as suggesting there should be close relations between the two: The discrimination of simple magnitudes follows Weber’s law, whereby discriminability becomes worse as the *ratio* between compared magnitudes approaches one (Gibbon, 1977; Stevens, 1957). Thus, the difficulty of discriminating simple magnitudes is parameterized in terms of a ratio. Moreover, simple magnitude discrimination tasks are often characterized as differencing operations in a logarithmically transformed space, which is mathematically equivalent to taking a ratio (Birnbaum & Veit, 1973). On this logic, Chesney and Matthews (2018) hypothesized that magnitudes of both individual numerosities and ratios constructed from pairs of them might be processed by a shared system. If this hypothesis is true and applies across formats, representations of simple magnitudes should be more closely associated with within-format ratio magnitudes than they are with other simple magnitudes across formats.

On the other hand, there is reason to suspect representations of nonsymbolic ratios, and simple magnitudes might be more distinct. Some researchers have proposed that there is a ratio processing system (or RPS) specifically dedicated to processing nonsymbolic ratio magnitudes, which are intensive magnitudes as referenced above (e.g., Jacob, Vallentin, & Nieder, 2012; Lewis, Matthews, & Hubbard, 2016). Both adults (Matthews & Chesney, 2015) and children (Boyer, Levine, & Huttenlocher, 2008; Jeong, Levine, & Huttenlocher, 2007) have been found capable of discriminating nonsymbolic ratios. Indeed, even infants (McCrink & Wynn, 2007) and non-human animals such as monkeys (Drucker, Rossa, & Brannon, 2016; Vallentin & Nieder, 2008), parrots (Bastos & Taylor, 2020), and chicks (Rugani, Mccrink, Hevia, & Vallortigara, 2016) demonstrate this ability. For example, McCrink and Wynn (2007) found that infants who were habituated to multiple examples of a 2:1 ratio of different colored dots looked longer at novel 4:1 ratios, indicating that they could discriminate ratio magnitudes. Furthermore, Vallentin and Nieder (2008) found that nonsymbolic ratio is represented at the level of individual neurons tuned to specific ratio magnitudes

The theory extends further to hypothesize that the RPS may serve as a neurocognitive foundation to support acquisition of symbolic fractions knowledge. In fact, some behavioral evidence suggests that the ability to perceive nonsymbolic ratio magnitudes –an intuitive sense of proportion – better predicts symbolic fractions knowledge than does simple magnitude representation, consistent with the ratio-specific RPS hypothesis (Lewis et al., 2016; Matthews et al., 2016). The present study puts competing hypotheses regarding the relations between simple and ratio magnitude representations to the test. If ratio processing is dependent upon first processing simple magnitudes, comparison performance should depend more on format (e.g., lines, dots, etc.) than magnitude type (i.e., simple vs. ratio). If, instead, ratios are processed directly by the RPS, comparison performance should depend more on magnitude type than format.

1.2 Developmental trajectories of ratio and simple magnitudes

Two related questions concern (a) how nonsymbolic ratio representation abilities develop with age, and (b) how developmental trajectories for these abilities vary across formats. Previous studies investigating nonsymbolic ratios have not explicitly examined RPS acuity differences across age cohorts. In contrast, much more is known about how simple magnitude acuity develops. For simple magnitudes, sensitivity improves over development as perceptual abilities mature (Feigenson, 2007; Halberda & Feigenson, 2008; Odic et al., 2013). Numerosity acuity, for instance, grows from 1:2 among 6-month-olds, to 3:4 among preschoolers, to a mature 9:10 or better among adults (Brannon et al., 2006; Halberda & Feigenson, 2008; Odic et al., 2013; VanMarle & Wynn, 2006; Xu & Spelke, 2000).

However, simple magnitude acuity appears to differ depending on the nonsymbolic formats tested. Prior work has found that although acuities in multiple formats are similar for 6-

month-old infants (Brannon et al., 2006; Feigenson, 2007; Odic et al., 2013; Xu & Spelke, 2000), these acuities diverge across formats with age (DeWind & Brannon, 2012; Odic et al., 2013; Park & Cho, 2017). Odic (2017) investigated children aged 2 to 12 along with adults and found that overall acuity was highest for line-length, followed by area, numerosities, duration and density respectively. Moreover, using logistic growth modeling, Odic found that each format had a distinctive developmental trajectory (see also Odic et al., 2013).

If there is a substantial association between simple and ratio magnitude representations, we might expect RPS acuities in different formats to follow the same ordinal rank (i.e., to be most acute with line segments and least acute with dot arrays). We might also expect the developmental trajectory of discrimination acuity to depend more on format (e.g., dots versus lines) than on magnitude type (i.e., simple or ratio magnitudes). However, there are no extant data on the development of ratio magnitude acuities in different formats. It is true that nonsymbolic ratio magnitudes have been studied in various formats, including those composed of line segments (e.g., Boyer & Levine, 2015; Boyer, Levine, & Huttenlocher, 2008; Hansen et al., 2015; Jacob & Nieder, 2009; Kalra, Binzak, Matthews, & Hubbard, 2020; Matthews et al., 2016; Vallentin & Nieder, 2008), dot arrays (e.g., Chesney & Matthews, 2018; Fabbri, Caviola, Tang, Zorzi, & Butterworth, 2012), pie wedges (Mock et al., 2018), and circles (Matthews & Chesney, 2015). However, no prior research explicitly measured ratio acuity among multiple developmental cohorts. The current research explores these developmental relations. Based on prior findings regarding infants' (McCrink & Wynn, 2007) and adults' (e.g., Matthews et al., 2016) ratio discrimination performance, we hypothesized that ratio acuity, just as with simple magnitude acuity, would become sharper over the course of development and that the trajectory of this development would vary with magnitude format.

1.3 The present study

We investigated relations between simple and ratio magnitudes for different age cross sections, ranging from 4-5-year-olds to adults. We focused on four magnitude formats: line lengths, circle areas, dot arrays, and irregular blob areas. We chose lines, circles, and dot arrays because wanted to examine nonsymbolic ratio formats that have been previously been studied systematically (e.g., Matthews & Chesney, 2015; Matthews et al., 2016; Mock et al., 2018). We additionally included irregular blobs that (a) have previously been studied using simple magnitude tasks (Odic, 2017) and (b) preclude the possibility of using partitioning or simple (non-area) correlates of area magnitude such as diameter when making magnitude judgments.

We also measured inhibitory control as a covariate, consistent with prior studies using magnitude discrimination tasks (Fuhs & Mcneil, 2013; Roell, Viarouge, Houdé, & Borst, 2017). Furthermore, prior work with fractions and ratios has shown that participants often have to inhibit interference effects due to their inclinations to respond on the basis of the sizes of fraction or ratio components rather than overall magnitude (e.g., Boyer et al., 2008; Matthews & Lewis, 2017; Ni & Zhou, 2005).

We examined differences in ratio acuity across formats in four age ranges and compared each ratio acuity with simple magnitude acuities. If ratio and simple magnitude representation abilities are closely related as Chesney and Matthews (2018) hypothesized, within-format acuities for simple and ratio magnitudes should be strongly associated. For instance, individuals with high acuity for line ratio magnitudes should also have high acuity for corresponding simple magnitude comparisons of lines, but not necessarily for other ratio formats. However, if ratio magnitude is processed by a separate dedicated system, as hypothesized by Lewis et al. (2016), it may be that ratio acuities across formats correlate more strongly and that ratio and simple acuities within-format do not.

Our hypotheses were largely in line with those of Chesney and Matthews (2018). Specifically, we hypothesized 1) that ratio acuity would improve with age across all formats with similar growth trajectories for simple magnitude acuity, 2) that ratio acuity in each format would be better predicted by within-format simple magnitude acuity than by cross-format ratio acuity. A third prediction followed from the second: that acuities for ratios composed of line-lengths and areas would be sharper than those for numerosity ratios in all age ranges, as has been found for simple magnitudes(e.g., Odic, 2017). However, if simple and ratio magnitudes are processed by substantially divergent routes, contrary to our hypotheses, ratio acuities may be better predicted by ratio acuities in alternate formats than by simple comparisons in the same format.

2 Methods

2.1 Participants

We recruited 42 preschoolers (20 Females; M_{age} = 4.55, SD = .50), 31 2nd-grade children (15 Females, M_{age} = 6.87, SD = .34), and 29 5th-grade children (10 Females; M_{age} = 9.86, SD = .44) from a mid-sized Midwestern city via visits to local daycare centers, online postings to local parenting blogs, and mass emailings to employees at a major university in the city. We recruited more preschoolers than participants from other groups out of caution that they would yield noisier data – due to lack of attention and other factors – and thus be more likely to have their data excluded from the analytic sample. Children were individually tested in a quiet room with a trained experimenter. Each was given a book and a small amount of cash for their participation. Additionally, 32 undergraduate students from the same university participated for credit (26 Females, M_{age} = 19.74, SD = 1.24). Eight preschoolers, three 2nd-grade, and two 5th-grade children were excluded from the analytic sample for failing to complete all tasks. Also, one 5th-grader was excluded due to an ADHD diagnosis. The final sample included 34 preschoolers, 28 2nd-graders, 27 5th-graders, and 32 adults.

2.2 Overview

Participants completed both simple magnitude comparison tasks and ratio comparison tasks. Prior to the ratio comparison tasks, we introduced children to nonsymbolic ratio with a brief set of instructions (details described below). For all trials, two nonsymbolic stimuli were presented side-by-side on a 1920 X 1080 resolution screen using E-prime software (Figures 2 & 4). Stimuli are described in detail below. Task paradigms, stimuli, and data are available online via the Open Science Framework (https://osf.io/ky69x).

2.2.1 Simple Magnitude Comparison Tasks

On each trial, a fixation cross appeared for 200ms followed by comparison stimuli which remained visible briefly before disappearing, leaving a blank screen. Participants determined which of two stimuli had a 'larger' magnitude (i.e., longer line, larger area, or more dots). Stimulus display time was adjusted for each age group based on protocols from Halberda and Feigenson (2008): 2000ms for 4-5-year-olds, 1500ms for primary school children, and 750ms for adults. The computer did not proceed to the next trial until participants made a choice. Participants (except preschoolers) indicated their judgment of which stimulus was larger via keystroke – "j" for right and "f" for left. Keys were marked by colored stickers to remove any cognitive load due to letter identification. To avoid key press errors with preschoolers, they indicated their answer by pointing at the stimulus judged larger on the screen, and the experimenter pressed the corresponding key.

Task difficulty was determined by varying the *ratio distance* between each pair of magnitudes, defined as the ratio between compared magnitudes (M_1:M_2 where M_1 is the smaller of the two magnitudes). We varied difficulty using five ratio distance bins based on Odic (2017): 1:2, 2:3, 5:6, 7:8 and 15:16, with difficulty increasing as ratio distance approached 1. For example, the 15:16 bin (e.g., 30 vs. 32 dots) was expected to be more difficult than the 1:2 bin (e.g., 16 vs. 32 dots).

We also sought to control the *congruity* of relevant and irrelevant dimensions. Trials in which making judgments on irrelevant dimensions can lead participants to a correct decision are considered congruent, whereas trials in which relevant and irrelevant dimensions point in opposite directions are considered incongruent. For example, people are faster to judge 7 as greater than 5 when the font size and magnitude are congruent (e.g., 5 7) than when the font size and magnitude are incongruent (e.g., 5 7) (Henik & Tzelgov, 1982). We controlled congruency in the numerosity comparison task, as it featured both relevant (i.e., numerosity) and irrelevant (e.g., dot size, summed area of dots) dimensions (see description below for details). We did not control congruency in other formats of the simple magnitude task because there was no competing irrelevant dimension. For these formats, we included two instantiations of each ratio with different locations of stimuli (all lines, circles, and blobs were given random vertical jitter as described below) so that the trials were not identical.

Each stimulus pair was presented twice, once with the larger stimulus on the left, and once with the larger on the right. Participants completed a total of 60 trials in each format (5 distance bins x 3 pairs per bin x 2 instantiations of each pair, x 2 sides for presentation of the correct response, see description of stimuli below). Trials were presented in random order. Participants were prompted by the software to take a break every 20 trials.

2.2.1.1 Simple Dot Comparisons. Boxes of yellow and blue dot arrays were presented against black backgrounds on each side of the screen (Figure 2). Dots were randomly placed so that they evenly covered the area of each box. The number of dots in each array ranged from 16 to 118. Each distance bin contained three classes of arrays to be compared: small, medium, and large, which used three different numerosity ranges, spanning arrays as small as 16 and as large as 118. For example, for the 1:2 distance bin, 16 vs. 32, 37 vs. 74, and 59 vs. 118 represented small, medium, and large pairs, respectively.

We also controlled for the possibility that participants might choose based solely on the summed area or average size of dots throughout the task (Fazio, Bailey, Thompson, & Siegler, 2014; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011). In half of all dot trials, the summed area of dots was equivalent across the two arrays on any given trial, meaning average dot size decreased with numerosity. In the other half of trials, average dot size was the same across the two arrays compared, meaning that the summed area of an array increased with numerosity. The size of each dot in a given array was allowed to vary from 80% - 120% of the average size of dots in that array.

2.2.1.2 Simple Line Comparisons. Vertically oriented yellow and blue line segments appeared on each side of the screen (Figure 2) ranging from approximately 32 to 235 pixels in length. Both lines were given random vertical jitter to minimize the possibility that participants might compare lines by using the slope of the implied line connecting the tops of the stimuli. Each stimulus pair was presented twice with different levels of vertical jitter, to make the number of trials match those of dot comparisons, which had two congruity conditions.

2.2.1.3 Simple Circle Comparisons. Yellow and blue circles appeared on each side of the screen (Figure 2). Circle stimuli areas varied from approximately 2590 to 19104 square pixels. Both circles were given random vertical jitter as for simple line tasks. As with lines, each stimulus pair was presented twice with different levels of vertical jitter.

2.2.1.4 Simple Blob Comparisons. Irregular yellow and blue blobs appeared on each side of screen (Figure 2), varying in area from approximately 2395 to 17663 square pixels. Blobs were also given random vertical jitter to parallel the line and circle tasks. As with other continuous formats, each stimulus pair was presented twice with different levels of vertical jitter.

2.2.2 Ratio Instruction

Because many children in the sample had no prior formal instruction on symbolic ratios or fractions, and because the nonsymbolic ratio representations were novel, all children received a brief PowerPoint lesson introducing the concept of ratios in each nonsymbolic ratio format prior to performing the corresponding ratio comparison task. For example, prior to circle ratio comparison, children received instruction using circle figures. The instruction introduced the concept that ratios get larger as the two components become closer to the same value. To introduce the concept in a child-friendly manner, we used cartoon characters to depict how height/size/number comparisons can make a ratio. For instance, in our circle ratio instruction, we used cartoon cows, instructing children that "Larry is half as big as Bessy. When we think of Larry's and Bessy's sizes together, we can call it a RATIO." The ratio instruction slideshow also included a few examples of ratio comparison problems to make sure children understood what we meant by the 'larger' ratio. These examples included ratio pairs where the larger ratio had smaller absolute size (e.g., Figure 3b) to make sure that children chose the ratio based on the relation between the yellow and blue magnitudes, not based on the absolute size.

2.2.3 Ratio Comparison Tasks

Ratio comparisons paralleled the simple comparisons, but stimuli compared were pairs of ratios, each composed of two juxtaposed simple nonsymbolic magnitudes (Figure 3 & 4). As with simple comparisons, trials began with a fixation cross appearing for 200ms followed by brief presentation of stimuli which disappeared, leaving a blank screen. Participants were shown two stimuli and asked to "choose the larger ratio" of yellow:blue (see Figures 3 & 4). For instance, Figure 3 shows two pairs of ratios, both representing 1:4 vs. 8:9. In both cases, regardless of the different lengths of components, the right ratio representing 8:9 is larger than the left ratio representing 1:4. Because ratio magnitude is relationally defined, the larger ratio can have greater components (Figure 3a) or smaller components (Figure 3b). Considering that this irrelevant dimension (i.e., overall size of component lengths, areas, or numerosities) might influence participants' judgment of ratio magnitude, we set two *congruity* conditions based on the summed component sizes in each format. For congruent trials, the summed extent of the larger ratios components what larger than the summed extent of the smaller ratio's components. For incongruent trials, the summed extent of the larger ratio's components was smaller than that of the smaller ratio. Details on this manipulation are described below.

Stimulus display time was adjusted for each age group based on Matthews et al. (2016) and our own pilot studies to ensure that participants would have enough time to respond but not enough time to count or use explicit calculation. Stimuli remained visible for 6000ms for primary school children and 1500ms for adults. For preschoolers, stimuli remained visible until they made a decision. Participants indicated their choices with the same method used for simple comparisons.

Task difficulty was determined by varying the ratio distance between each pair of magnitudes – in this case indicating a ratio of ratios. Pairs fell into one of 5 distance bins: 3:1, 2:1, 2:3, 3:4, and 5:6, with difficulty increasing as the distance approached 1. Magnitudes of individual ratio stimuli ranged from 1:5 to 4:5 (i.e., 0.2 to 0.8) (Table 1). For adults, we used three sizes of ratio pairs – small, medium and large – just as with simple magnitude tasks. The small pair always included 1:5, the large pair always included 4:5, and the medium pair was always midway between the other two sizes. For example, for the 2:1 ratio bin, 1:2 vs. 2:5 (i.e. , 0.2 vs. 0.4), 3:10 vs. 3:5 (i.e., 0.3 vs. 0.6) , and 2:5 vs. 4:5 (i.e., 0.4 vs. 0.8) are representative comparisons for the small, medium, and large pairs, respectively (see Table 1). We excluded medium pairs for children to reduce the number of trials and maximize attention and engagement.

Trials were blocked by format. In every block, each pair was presented twice, once with the larger stimulus on the left, and once with the larger on the right. Children completed 40 trials in each format (5 ratio distance x 2 size x 2 congruity of summed length/area/numerosity of components x 2 left/right, see stimuli descriptions below). Adults completed 60 trials in each format (5 ratio distance x 3 size x 2 congruity x 2 left/right). Trials were randomized within each block. Children were prompted to take a short break after every 10 trials, and adults were prompted to take a short break after every 20 trials.

2.2.3.1 Dot-ratio Comparison. Dot-ratios were constructed from pairs of yellow and blue dot arrays (Figure 4). For each ratio, the yellow dot array was presented in the top box, and the blue array presented in the bottom. To control for the possibility that participants might make judgments based on the summed areas of dot array components in each ratio stimulus, we controlled the summed areas of dots in each array such that it was the same across stimuli. Dots were randomly and evenly spaced in each array.

The number of dots in the numerator ranged from 12 to 92, and the number dots in the denominator ranged from 20 to 120. The size of dots in an array was allowed to vary from 80% - 120% of the average size of dots in the array. To control for the possibility that judgments would be based on the total number of dots summed across components in each ratio, we constructed stimuli such that, in half of the trials, the larger ratio had more total dots (i.e., congruent), and on the other half, the larger ratio had fewer total dots (i.e., incongruent). The ratio between the summed numerosities in the compared stimuli was always 1.3 or 1.77.

2.2.3.2 Line-ratio Comparisons. Line-ratios were constructed by juxtaposing yellow and blue line-segments (Figure 4). The yellow line (numerator) always appeared on the left, and the blue line (denominator) on the right. The yellow line was given random vertical jitter relative to the blue line. The numerator segment ranged from approximately 35 to 228 pixels long, and the denominator ranged from 50 to 304 pixels. To control for the possibility that participants might make their judgments based on the summed lengths of components for each stimulus, the larger ratio had greater summed length (i.e., congruent) in half of all trials, and the larger ratio had lesser summed length (i.e., incongruent) in the other half. The ratio of summed lengths was always approximately 1.3.

2.2.3.3 Circle-ratio Comparisons. Circle-ratios were constructed of yellow circles in the numerator/top position and blue circles in the denominator/bottom (Figure 4). Yellow circle areas varied from approximately 1602 to 11540 square pixels, and blue circles varied from approximately 2289 to 15386 square pixels. To control for the possibility that participants would judge based on summed areas of each stimulus, the larger ratio had larger summed area (i.e., congruent) in half of all trials, and the larger ratio had smaller summed area (i.e., incongruent) in the other half. The ratio of summed areas was always approximately 1.3.

2.2.3.4 Blob-ratio Comparisons. Similar to the circle-ratio task stimuli, yellow and blue blobs corresponded to the numerators and denominators, respectively (Figure 4). Yellow blob areas varied from approximately 1467 to 3518 square pixels, and the blue blobs varied from approximately 4891 to 29348 pixels. We controlled congruity relations among summed areas of yellow and blue blobs just as we did for the circle tasks.

2.4 Inhibitory Control Tasks

2.4.1 Flanker Task

We administered a flanker task to measure individual differences in inhibitory control for all participants except preschoolers. Because young children show more variability in child-specific versions of the flanker (e.g., Rueda et al., 2004), children and adults completed different versions. Adults saw five arrows presented simultaneously in the middle of the screen, and decided which direction the center arrow was pointing. For children, arrows were replaced with fish facing either left or right. After a fixation cross appeared for 500ms, five evenly spaced stimuli (i.e., fish or arrows) appeared. Second-grade and fifth-grade children were asked to respond within 1700ms (Rueda et al., 2004) and adults within 800ms before trials timed out. For children, testing began with 8 practice trials followed by 32 test trials. For adults, testing began with 12 practice trials followed by 80 test trials. For both adults and children, half of all experimental trials were congruent, in which all stimuli pointed the same direction, and the other half of the trials were incongruent, in which the center stimulus pointed the opposite direction from the others. We used congruity-based normalized differences in RTs ($RT_{incongruent}$ – $RT_{congruent}$)/(mean RT) as a measure of inhibitory control. We converted performance to z-scores to allow analysis alongside preschoolers' Day/Night Task (described below) as a measure of inhibitory control across age ranges.

2.4.2 Day/night Task

For preschoolers, we used the Day/Night task (Fuhs & Mcneil, 2013; Gerstadt, Hong, & Diamond, 1994) to measure inhibitory control. An experimenter presented a cartoon picture of the sun or the moon and asked children to respond "Day" when they saw a picture of the moon and "Night" when they saw a picture of the sun. There were 4 practice trials followed by 16 real trials. We used the number of correct responses out of 16 questions as the raw score for this measure and converted it to a standardized z-score for analytical purposes as described above.

2.5 Procedure

For children, the experiment was divided into two sessions, each on a different day. Adults performed all tasks on the same day. Over the course of the experiment, adult and primary school participants completed tasks in all four formats for both simple and ratio magnitude comparison tasks. However, preschoolers only completed tasks in three out of four formats to limit fatigue and lapses in attention. Thus, they either completed Lines, Circles, and Blobs or Lines, Circles, and Dot tasks in each session (Table 3)[1]. In session 1, children completed flanker and simple magnitude comparison tasks, and they completed ratio comparison tasks in session 2. All comparison tasks were blocked by format. We gave priority to circle and line formats for two reasons: First, prior research suggests that comparison tasks with dots yield noisier signals, and we wanted to maximize the amount of clean data we would receive prior to the onset of fatigue in children (e.g., Matthews & Chesney, 2015; Matthews et al., 2016; Odic, 2017). Second, less is known about blob tasks in general, so we opted to reduce the risk of

[1] Due to experimenter error, two children performed simple Blob comparisons in session 1, but performed Dot ratio comparisons in session 2.

fatigue effects among children for line and circle tasks. Thus, Line and Circle tasks were always conducted prior to Blob and Dot tasks. However, the order of Line/Circle and Blob/Dot was counterbalanced. Prior to test trials, participants saw 4 practice trials for the simple magnitude comparison tasks and 12 practice trials for the ratio comparison tasks. When children's accuracy was lower than chance in practice trials, we had them repeat the practice segment.

3 Results

We excluded an individual's data from a given task – though retained the individual's information from other tasks – if performance on that task was below chance (50% accuracy). However, in each session, if a preschool participant showed below chance level performance in two out of three tasks, or if a primary school child or adult showed below chance level performance on three out of four tasks, we excluded that individual's data for the entire session (1 preschooler). Additionally, trials with RTs shorter than 250ms or with RTs more than 3 SD from a participant's mean RT were trimmed prior to analysis. After data trimming, 2% of all ratio comparison data and 3% of simple magnitude comparison data were excluded. For the inhibition task, after taking into account the participants who couldn't finish the sessions (8) as stated above (section 2.1), five preschoolers' Day/Night task data (5/34 =11%) were additionally excluded from analysis due to lack of attention (3) and coding errors (2). Two 2nd-graders and two adults did not finish the flanker test. One 2nd-grader's and one adult's flanker data were additionally excluded due to lower than chance level performance. Taken together, 28 preschoolers, 25 2nd-graders, 27 5th-graders, and 31 adults were included for analyses where inhibitory control was entered as a regressor. As a supplementary analysis, we also conducted the same logistic regression without inhibitory control so that we could include participants without inhibitory control measures and found consistent results (see supplementary materials, Table S1).

3.1 Distance, Format, and Grade Effects.

3.1.1 Simple Magnitude Accuracy

Both children and adults were capable of quickly comparing simple magnitudes more accurately than chance. Mean performance for each age cohort is indicated in Table 2. We conducted mixed effects logistic regression to account for within-subject correlation among trials using the ‘gImer’ function of the lme4 package in R software. We examined fixed effects by regressing accuracy (0 or 1) against inhibitory control, format (4 levels, dot = 0, blob = 1, circle = 2, line = 3), age group (4 levels, preschoolers = 0, 2^{nd}-graders = 1, 5^{th}-graders = 2, adults = 3), and ratio distance. Since we had hypotheses regarding how accuracy would vary with age and format (i.e., preschoolers < 2^{nd}-graders < 5^{th}-graders < adults and Lines > Circles > Blobs >Dots), to facilitate analysis, we used a backward difference coding scheme (i.e., coded so that the mean of a given level is compared with the mean of the immediately prior level) to compare adjacent levels of variables. Figure 5 shows accuracy by format and age group.

Analysis revealed a distance effect; accuracy decreased precipitously as the distance between stimulus magnitudes decreased ($OR = 17.18$, $p < .001$) (Table 3). We also found that performance improved significantly with increasing age: 2^{nd}-graders were 1.31 times more likely to be correct than preschoolers ($p = .014$), 5^{th}-graders were 1.46 times more likely to be correct than 2^{nd}-graders ($p < .001$), and adults were 1.37 times more likely to be correct than 5^{th}-graders ($p = .004$).

We also found format effects largely consistent with our predictions. Accuracy in line and circle formats were not significantly different ($p = .054$), but participants were 1.54 times more likely to be correct with circles when compared to blobs ($p <.001$) and 1.65 times more likely to be correct with blobs than with dots (p <.001). There was no effect of inhibitory control (p = .888).

3.1.2 Ratio Comparison Accuracy

Children and adults were also capable of accurately and rapidly comparing ratio magnitudes. Mean performance for each age cohort is indicated in Table 2. Our analyses of ratio magnitude comparisons paralleled those for simple magnitude comparisons. We regressed accuracy against inhibitory control, format, age group, and ratio distance using backward contrast coding. As with simple comparisons, we found a significant distance effect with accuracy decreasing as distance between stimuli decreased (OR = 4.42, p < .001) (Table 3). Age effects also roughly paralleled those for simple magnitudes: 2^{nd}-graders were 1.33 times more likely to be correct than preschoolers (p = .026), and 5^{th}-graders were 1.39 times more likely to be correct than 2^{nd}-graders (p = .011). Adults and 5^{th}-graders were not significantly different in terms of accuracy (p = .074).

Although format effects for ratios bore similarities to those for simple magnitudes, there were some differences. First, ratio comparisons significantly more accurate for the line format than for circles, (OR = 1.71, p <.001). Second, there was no difference in performance between circle and blob ratio format (OR = .97, p = .648). Dots again proved to be the most difficult format, yielding significantly less accurate responses than blob ratios (OR = 1.62, p <.001). In contrast to simple magnitude trials, we did find an effect for inhibitory control on ratio comparison accuracy (OR = 1.10, p = .043).

3.2 Growth modeling

The above analyses clearly show that acuity improved with age cohort, both for simple magnitude and for ratio magnitude comparisons. To map developmental trajectories of acuities across age cross-sections, we employed logistic growth modeling following Odic (2017). Below, we present results for the growth modeling analysis conducted using the entire analytic sample (34 preschoolers, 28 2nd-graders, 27 5th-graders, and 34 adults) for maximum power, but we ran supplementary analyses using the trimmed sample (as the logistic regressions) and found similar results (see SI Table S2). Equation 1 describes the model, which assumes that early development progresses with a constant speed until performance asymptotes at maturity. The model uses the mean accuracy at each time to estimate three parameters: 1) peak accuracy at the point of maturity, *a;* 2) onset age of development, *1/-b*; and 3) developmental speed *c*. When it comes to *c*, lower values indicate faster development. Note, however, that age of onset was expected to be overestimated due to limits of our sample, since the ages of the youngest participants in the sample only went as low as four. Thus, below we only report growth rate and approximate age of maturity (Table 4; Figure 6). We used the mle2 function of the ‘bbmle’ package in R software to estimate the best-fit parameters. Parameters were estimated by minimizing the negative log-likelihood. The age of maturity was estimated based on the mean of adults’ performance.

$$Accuracy = \frac{a}{1 + e^{(-(b + Age * c))}}$$

Eq. 1. Logistic growth model.

We used a z-test to compare model parameters between formats. Contrary to Odic’s prior findings (Odic et al., 2013; Odic, 2017), growth rate estimates were not significantly different among formats. This was true both for and simple and for ratio magnitudes. Furthermore, estimated ages of maturity were similar across all tasks, generally in the 16-20 years of age range. This result, however, must be qualified by the fact that adult participants in our sample were generally 18-20 years old. Thus, current estimates are likely limited by a restriction of range. It is quite possible that these perceptual abilities continue to develop after 20 years of age as has been found for the ANS (e.g., Halberda, Ly, Wilmer, Naiman, & Germine, 2012)

3.3 Relations between magnitude acuity and ratio acuity

We performed linear regressions for each format to determine whether ratio acuity in a given format was better predicted by within-format simple magnitude acuity, simple acuity in alternate formats, or by ratio acuity in alternate formats. Because preschoolers were only tested in 3 out of 4 possible formats (line/circle/blob or line/circle/dot), their performance could not be included in analyses that included all 4 levels of format. Thus, we present regressions excluding preschoolers. Three 2nd-graders and one 5th-grader were excluded due to below chance performance in one or two tasks out of eight. Thus, regression data were from 79 participants from three different age cohorts (22 2nd-graders, 26 5th-graders and 31 adults). Table 5 shows the bivariate correlations among variables.

For each format of the ratio comparison task, we regressed ratio accuracy against age and flanker performance as control measures, along with all simple magnitude acuities and remaining ratio acuities (Table 6). We report semi-partial correlations (sr^2) in order to compare unique variance that each regressor explains. We also calculated variance inflation factors (VIF) to screen for any potential issues of multicolinearity. All VIF scores were under 3, indicating that there were no major collinearilty issues.

Contrary to our hypothesis, in every format, ratio acuity was better predicted by ratio acuity in one or more alternate formats than by the corresponding within-format simple acuity. Indeed, within-format simple acuity explained very little independent variance in ratio acuity in any of the four formats. For example, circle ratio acuity explained *twelve times* as much independent variance (as measured by sr^2) in line ratio acuity than simple line acuity did. Note that the one instance in which simple magnitude acuity significantly predicted ratio acuity, it was across formats; simple line acuity explained unique variance in dot ratio acuity (see Table 6).

4 General Discussion

We investigated the comparative processing of simple and ratio magnitudes – two different classes of magnitudes frequently encountered in the natural environment. Specifically, we tested two competing hypotheses about whether representation of simple and ratio magnitudes is achieved through common or independent mechanisms. We examined both simple and ratio magnitude representation and their direct relations among multiple developmental cohorts: preschoolers, 2nd-graders, 5th-graders, and adults. We found similarities between simple and ratio magnitude representations in terms of format effects at the group level. However, at the subject level we found that ratio acuities were often more related to each other across formats than they were to simple acuities within the same format.

4.1 Early development of nonsymbolic ratio magnitude acuity.

We found that children as young as 4 years old could effectively represent nonsymbolic ratios in multiple formats. These results are consistent with prior studies showing that children demonstrate capacity for nonsymbolic proportional reasoning (e.g., Boyer et al., 2008; Hansen et al., 2015; Jeong, Levine, & Huttenlocher, 2007; McCrink & Wynn, 2007; Möhring, Newcombe, Levine, & Frick, 2015; Sophian, 2000; Spinillo & Bryant, 1991). We extended prior research by demonstrating that all age groups' ratio representations followed Weber's law in all formats, showing significant ratio distance effects. This result is consistent with prior research on ratio processing with adults (e.g., Jacob et al., 2012; Matthews & Chesney, 2015; Mock et al., 2018) and primary school children (Kalra et al., 2020), and also consistent with prior research investigating simple magnitude representations with children and adults (e.g., Halberda &

Feigenson, 2008; Odic, 2017). Although a non-trivial number of preschoolers (8 of 42) were excluded from analysis due to lack of attention, our rate of exclusion was consistent with that of Odic (2017), which we used as a model. Our study stands as the first to provide estimates of nonsymbolic ratio acuities among children as young as 4- and 5-year-old and as the first to chart growth trajectories.

We found that developmental trajectories and ages of maturity were similar across formats for both ratio and simple magnitudes. These similarities in trajectory are consistent with the suggestion that something in the kernel of magnitude processing is the same across formats, as suggested by previous theories positing a generalized magnitude system (Bueti & Walsh, 2009; Feigenson, 2007; Lourenco & Longo, 2011; Walsh, 2003). We interpret the fact that young children in our study showed competence with nonsymbolic ratio magnitudes prior to receiving formal instruction on ratio concepts as indicative that there is a strong role for biological maturation in the development of ratio processing ability. This is also consistent with prior findings indicating that several animal species demonstrate competence representing nonsymbolic ratios (see Jacob et al., 2012). These conclusions remain speculative, as our design did not control for participants' environmental histories. Even if maturation is a major factor driving early development of ratio representation, it may still be that the early environment provides informal support for these abilities. Future longitudinal studies integrating measures of children's exposure to informal and formal ratio concepts are required to deliver a final verdict on this issue.

Unlike previous studies showing faster growth rates in continuous formats when compared to dots, we did not find significant differences in developmental trajectories between formats. This suggests that all formats might develop similarly, potentially driven by the same fundamental underlying process, even though acuity at any one point in time is sharper for some formats (e.g., lines) than others (e.g., dots). However, the age range of our sample may have limited our ability to detect subtle differences in growth trajectories. For instance, Odic (2017) included a wider range, spanning ages from 3–11 in five evenly spaced cohorts, whereas our design spanned ages 4 – 10 in three cohorts. To more accurately estimate developmental trajectories and to establish their generalizability, future investigations should extend to younger and to older participants.

4.2 Relationships between simple vs. ratio magnitudes

As we expected, there were substantial within-format similarities between simple and ratio magnitude representations. First, we observed that acuities of both simple magnitudes and ratios varied significantly by the format of magnitudes. In line with previous studies, child and adult participants demonstrated higher acuities for simple magnitude in all continuous formats (e.g., line-lengths and areas) than they did for discrete numerosities (Odic, 2017; Odic et al., 2013). Acuity was sharpest for lines, as has been found elsewhere (Matthews et al., 2016). This ordinal pattern for simple magnitude acuity by format was similar to what we found for ratios. Acuity for ratio discrimination tasks was also best for lines and worst with dots, although the rank of circles and blobs differed from that of simple magnitudes. These similarities indicate that both simple and ratio magnitude representations are constrained in similar ways by format.

However, our analyses also uncovered one key result that contradicted our hypotheses: Overall acuity of ratio magnitude within a given format was better predicted by ratio acuity in alternate formats than by within-format simple magnitude acuity. This result implies that although the format of simple magnitudes helps determine acuity of ratio discrimination, its influence is limited. There are at least two plausible ways to account for this. First, it may be that simple and ratio magnitudes are represented by different metric systems. That is, extraction of ratio information might begin with representation of two simple magnitudes, but judgment of the *relative value* of the two may occur via a substantially different mechanism, employing the RPS. Thus, it may be that how efficiently one can extract this relational information plays a large role above and beyond simple magnitude acuity in determining one's ratio acuity. Another possibility is that there is indeed a deeply shared mechanism for processing simple and ratio magnitude differences, but that the different tasks require substantially different demands on domain general attentional and/or working memory capacity. Thus, the close relations between ratio tasks across different formats may be attributable to these domain-general abilities rather than format-general RPS processing. For example, comparison of two line ratio stimuli involves four separate lines grouped into two pairs. It may be that the complexity of the task imposes substantial working memory or executive function demands that drive differences between simple and ratio tasks. Indeed, we found that inhibitory control was a significant – if minor – predictor for ratio comparisons, but not for simple comparisons.

Future studies are necessary to adjudicate between these possibilities. One path forward is to include more varied and robust measures of executive function. Although we included flanker measures, they showed limited effects here and null effects in prior research on nonsymbolic ratio processing (Matthews et al., 2016). Future studies could include other measures of attentional control and various aspects of working memory.

4.3 Beyond magnitude: Possible implications for numerical cognition

So far, we have framed this project primarily in terms of basic magnitude perception and psychophysics. However, implications of the work may extend beyond this realm and inform theories concerning human numerical cognition. A substantial body of research has posited that numerical intuitions find their deepest roots in simple magnitudes (e.g., Dehaene, 2011; Feigenson, 2007; Leibovich et al., 2017; Lourenco & Longo, 2011; Newcombe et al., 2015; Piazza, 2011; Walsh, 2003).

However, recently some have posited that human sensitivity to ratio may serve as a basis for generalized magnitude representation shared across perceptual continua (Bonn & Cantlon, 2017), and others have suggested that ratio may prove to be a bridge from generalized magnitude representation to number (Meng, Matthews, & Toomarian, 2019). Regarding the link to number specifically, several researchers have suggested that ratio perception may serve as a cognitive primitive that undergirds the formation of numerical concepts (e.g., Jacob et al., 2012; Leibovich, Kallai, & Itamar, 2016; Lewis et al., 2016; Sidney et al., 2017). In contrast to the ANS, which is limited to processing discrete analogs of whole numbers, nonsymbolic ratio magnitudes can be mapped to fractions and to real numbers more generally. For example, there is no discrete nonsymbolic analog of π or $\sqrt{2}$, but these magnitudes can be respectively be expressed by the ratio of a circle's diameter to its circumference or a square's side length to its diagonal.

If ratio magnitude processing is to support the development of symbolic number processing, it should be present in young children. Until now, however, no studies have systematically documented nonsymbolic ratio processing acuity among young children in paradigms that allow comparison with adults. The current research furnishes these comparisons and confirms that child performance parallels that of adults, providing a critical link for cognitive primitive theories associating ratio perception with numerical cognition (e.g., Lewis et al., 2016; Jacob et al., 2012). In light of our findings, we echo Chesney and Matthews (2018) in calling for more research foregrounding ratio perception and its role in the development of the human number sense.

5 Conclusion

The present study examined relations between simple and ratio magnitude representations using parallel protocols with young children and adults. We found that simple and ratio magnitudes processing bore similarities at the group level, but yielded substantial differences at the individual level. Perhaps our most significant finding ran contrary to our prior hypotheses: Acuity for ratio magnitude in a given format was better predicted by ratio performance in other formats than by within-format simple magnitude acuity. This is consistent with the notion that the intuitive ability to access nonsymbolic ratio might represent a level of abstraction that is somewhat independent from simple magnitude representation – and that this is the case throughout development. That is, even for young children, perception of a nonsymbolic ratio may be about much more than the sum of its parts.

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Figure legends

Figure 1. Examples of nonsymbolic ratio magnitudes. From left to right, line-ratios, circle-ratios, and dot-ratios.

Figure 2. Example stimuli for each simple magnitude format: (a) lines, (b) circles, (c) blobs, and (d) dots. In each example, the correct response is “j”, with the blue stimulus on the right showing a larger simple magnitude.

Figure 3. Examples of congruent and incongruent nonsymbolic ratio comparison stimuli. Both panels represent 1:4 on the left and 8:9 on the right with different instantiations of each ratio. Ratio trials can be congruent, such that the larger ratio features larger components (panel a) or incongruent, such that the larger ratio features smaller components (panel b).

Figure 4. Example stimuli for each ratio comparison format: (A) line-ratio, (B) circle-ratio, (C) blob-ratio, and (D) dot-ratio tasks. In each example, the correct response is “j”, because the right stimulus shows a larger ratio magnitude.

Figure 5. Box plots with individual values for mean accuracy for all comparison tasks and all age cohorts.

Figure 6. Mean accuracy (%) for each participant in the ratio comparison task (upper) and the magnitude comparison task (lower) along with the curve for the best-fit logistic growth model. The age of maturity is the point at which the asymptote reaches peak accuracy.

Table 1

Ratio distance between stimulus pairs in the ratio comparison task.

	Ratio Bin				
Size	3:1	2:1	2:3	3:4	5:6
	Stimulus Values				
Small	0.20:0.60	0.20:0.40	0.20:0.30	0.20:0.27	0.20:0.24
Medium†	0.23:0.70	0.30:0.60	0.37:0.55	0.40:0.53	0.43:0.52
Large	0.27:0.80	0.40:0.80	0.53:0.80	0.60:0.80	0.67:0.80

†Medium pairs were presented only to adults.

Note, decimal notation is used to more clearly indicate the sizes of the presented ratio. For example, 0.2 indicates a ratio of 1:5.

Table 2

Mean (SD) accuracy and reaction times in ms by task, format, and age group

		Preschoolers	*2nd-Graders*	*5th-Graders*	*Adults*
Magnitude Comparison Task	*ACC*	.76(.01)	.79(.01)	.84(.01)	.88(.01)
	RT(ms)	†1953.84 (109.60)	1075.82 (47.88)	910.11 (42.63)	564.47 (12.24)
Ratio Comparison Task	*ACC*	.78(.02)	.81(.01)	.85(.01)	.87(.01)
	RT(ms)	†3009.56 (220.71)	1741.32 (81.22)	1464.15 (77.40)	850.84 (24.92)

†*Note*, reaction times of preschool children are slower in part because the experimenter pressed the key instead of the participant.

Table 3

Fixed effects logistic regression results for accuracy of magnitude and ratio comparison tasks

Magnitude Comparison Task				Ratio Comparison Task			
	β	Odds Ratio	*p*		β	Odds Ratio	*p*
Intercept	-2.08†	.12	<.001***	Intercept	-.74	.48	<.001***
Inhibition	.00	.99	.888	Inhibition	.09	1.10	.043*
Ratio distance	2.84	17.18	<.001***	Ratio distance	1.48	4.42	<.001***
Blob-Dot	.50	1.65	<.001***	Blob-Dot	.48	1.62	<.001***
Circle-Blob	.43	1.54	<.001***	Circle-Blob	-.03	.97	.648
Line-Circle	.10	1.11	.054	Line-Circle	.54	1.71	<.001***
2nd-graders-preschooler	.27	1.31	.014*	2nd-graders-preschooler	.28	1.33	.026*
5th-graders–2nd-graders	.38	1.46	<.001***	5th-graders–2nd-graders	.33	1.39	.011*
Adults–5th-graders	.31	1.37	.004**	Adults–5th-graders	.22	1.24	.074

Note, estimates are backwards difference coded to compare one level to the next. For example Blob-Dot refers to how participants fared on blob stimuli compared to dot stimuli (previous level). A positive β indicates that participants were more likely to be correct compared to the previous level, and a negative β indicates that participants were less likely to be correct compared to the previous level, as reflected in an odds ratio less than 1.

$^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$

Table 4

Parameters of the best-fit logistic growth model for each format by task

Task	Format	Growth rate(SE)	Approximate age of maturity(SE)
Magnitude	Line	0.29(0.08)	19.28(5.56)
	Circle	0.32(0.09)	15.20(3.26)
	Blob	0.19(0.06)	19.90(3.16)
	Dot	0.21(0.06)	19.45(3.01)
Ratio	Line	0.29(0.12)	16.20(3.64)
	Circle	0.15(0.08)	20.63(3.71)
	Blob	0.42(0.18)	16.83(6.37)
	Dot	0.31(0.11)	19.12(6.18)

Table 5

Bivariate correlations between variables

			Ratio Comparison				Magnitude Comparison			
		Inhibition	Line	Circle	Blob	Dot	Line	Circle	Blob	Dot
	Age	-.06	.34**	.46**	.29**	.40**	.50**	.46**	.63**	.54**
	Inhibition		.16	.05	.15	.09	.11	-.02	-.09	-.16
Ratio	Line			.52**	.38**	.35**	.45**	.38**	.37**	.39**
	Circle				.38**	.50**	.49**	.41**	.39**	.41**
	Blob					.39**	.40**	.36**	.37**	.42**
	Dot						.41**	.28**	.28**	.30**
Magnitude	Line							.59**	.60**	.51**
	Circle								.65**	.58**
	Blob									.62**

** p <.01

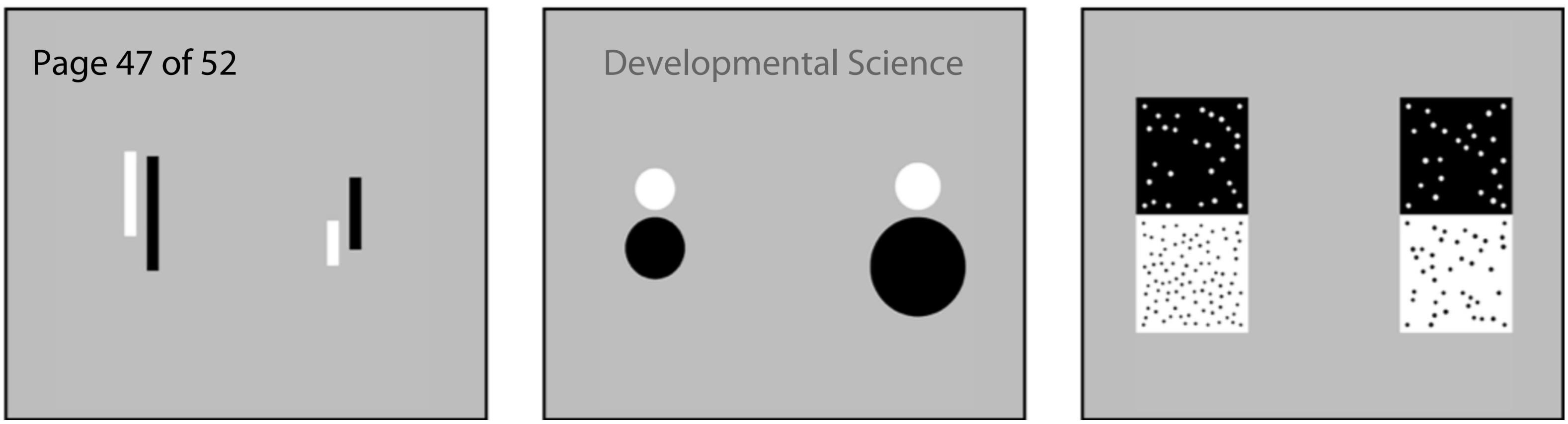
Table 6

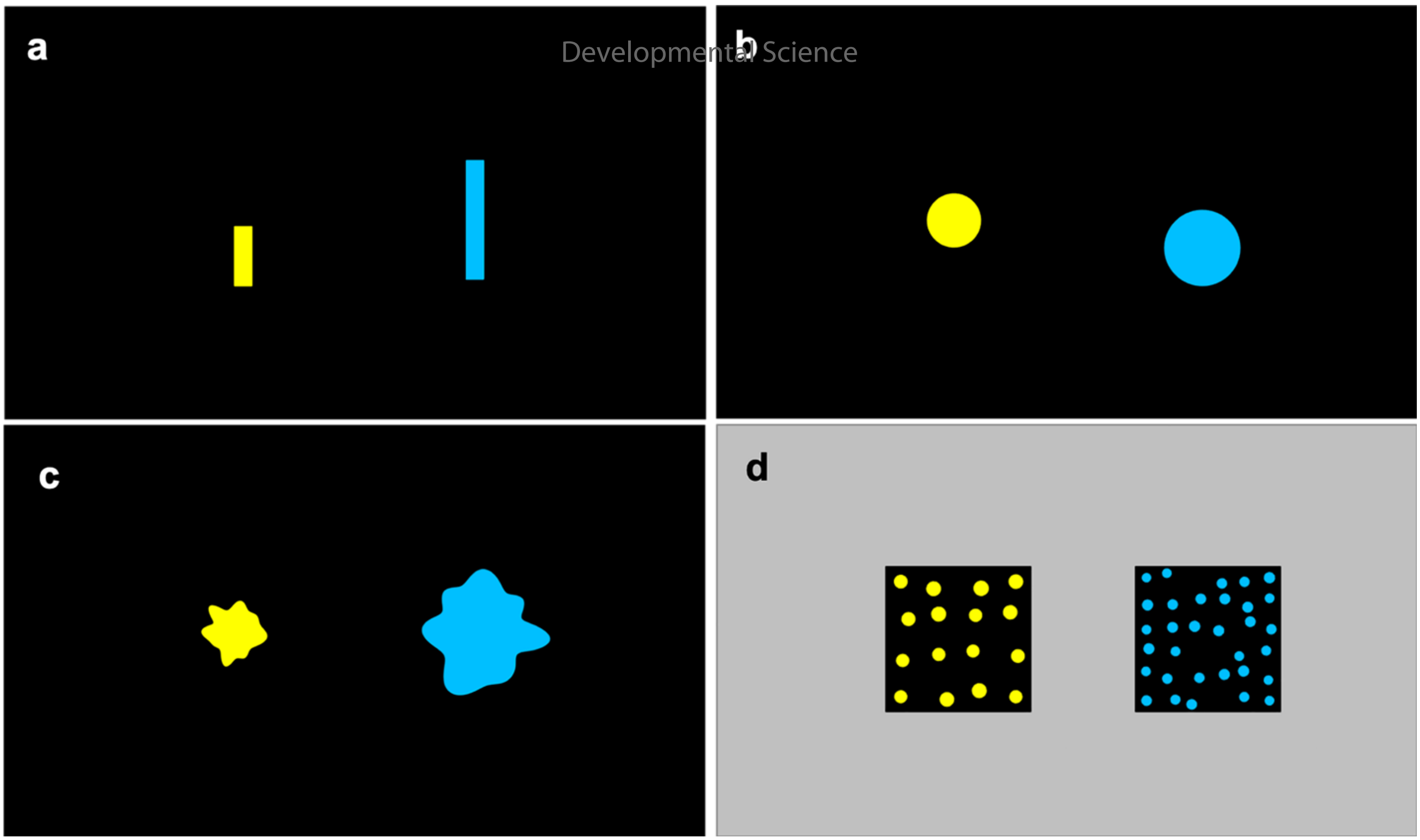
Results from the linear regressions predicting accuracy for each format of ratio comparison tasks.

	Line Ratio $R^2_{adjusted} = .26$			Circle Ratio $R^2_{adjusted} = .36$			Blob Ratio $R^2_{adjusted} = .26$			Dot Ratio $R^2_{adjusted} = .30$		
	β	*p*	$sr^{2‡}$	β	*p*	sr^2	β	*p*	sr^2	β	*p*	sr^2
Inhibition	.02	.032*	.055	-.01	.138	.018	.002	.810	.001	.014	.124	.022
Age	-.01<	.586	.009	<.01	.197	.014	-.001	.330	.009	.002	.274	.011
Line	-.11	.435	.006	.18	.267	.010	-.27	.121	.023	.41	.018*	.053
Circle	.07	.638	.002	.01	.937	<.001	.16	.354	.008	-.16	.356	.008
Blob	.19	.191	.016	-.12	.492	.004	.21	.257	.012	-.06	.755	<.001
Dot	.16	.104	.026	.02	.828	<.001	.20	.083	.029	-.07	.573	.003
Line Ratio				.36	.007**	.062	.02	.903	<.001	.05	.752	<.001
Circle Ratio	.28	.007**	.072				.19	.139	.021	.29	.029*	.045
Blob Ratio	.01	.903	<.001	.16	.139	.018				.30	.013*	.058
Dot Ratio	.03	.752	.001	.23	.029*	.041	.28	.013*	.061			

‡sr^2 indicates the square of the semipartial correlation coefficient.

** $p < .01$, * $p < .05$

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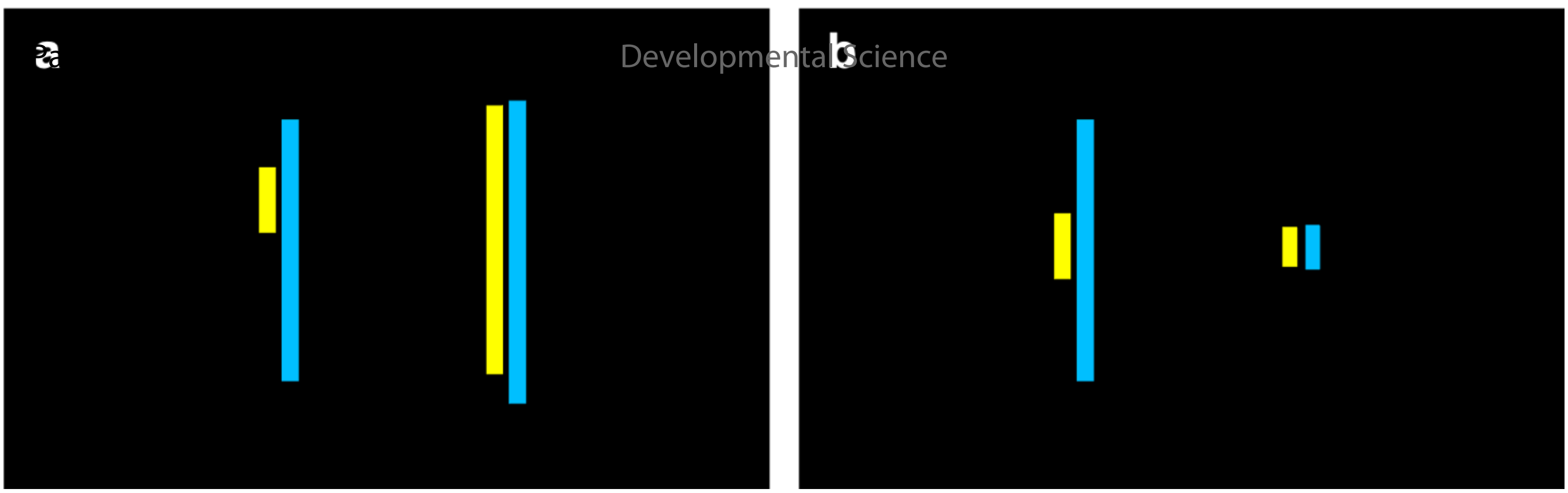
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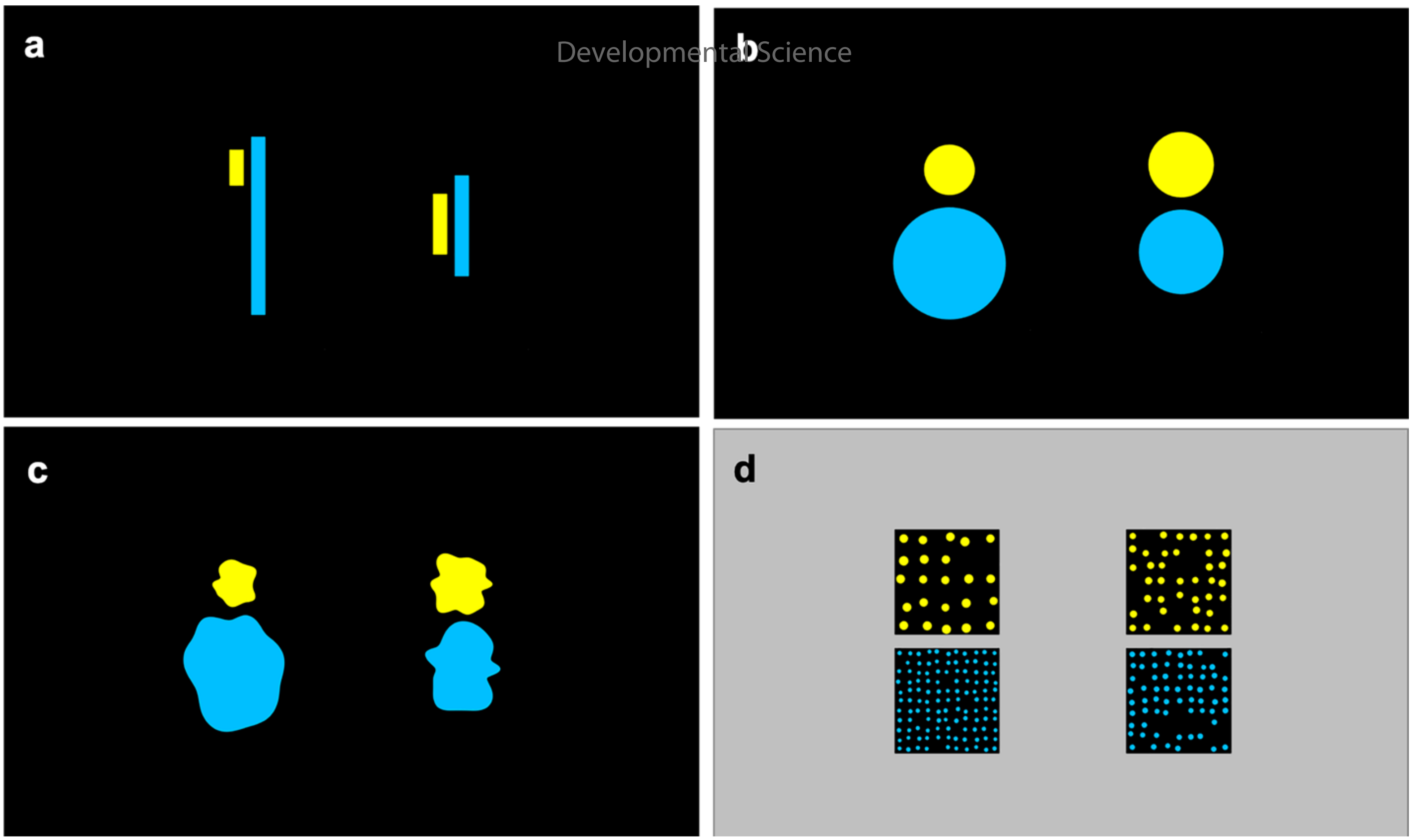
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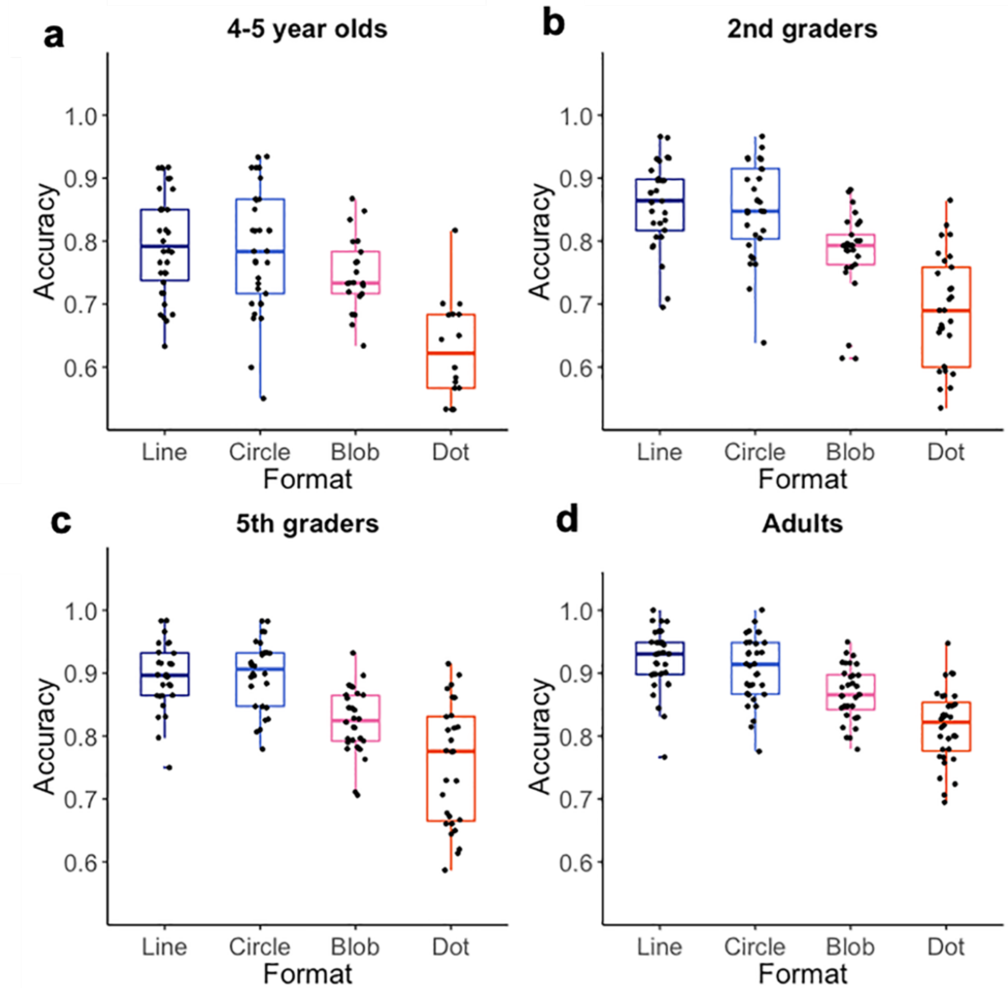
Magnitude Comparison



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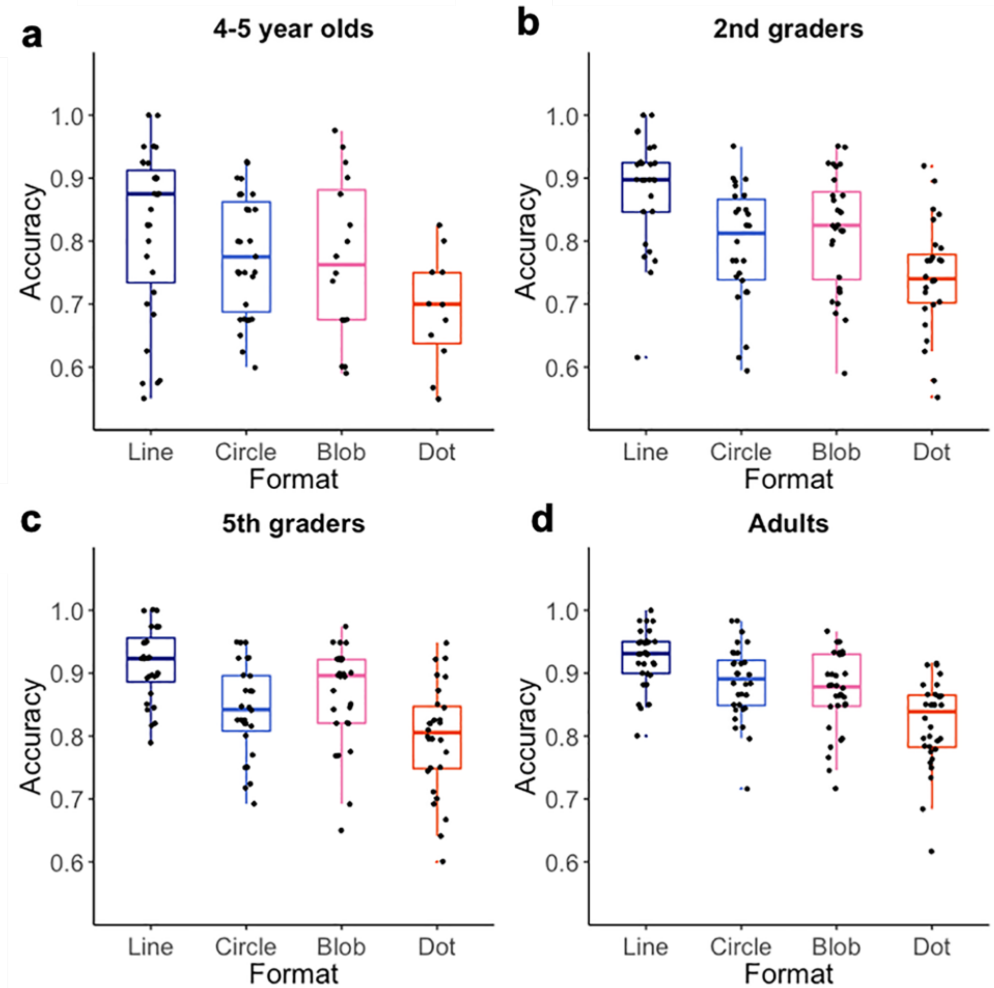

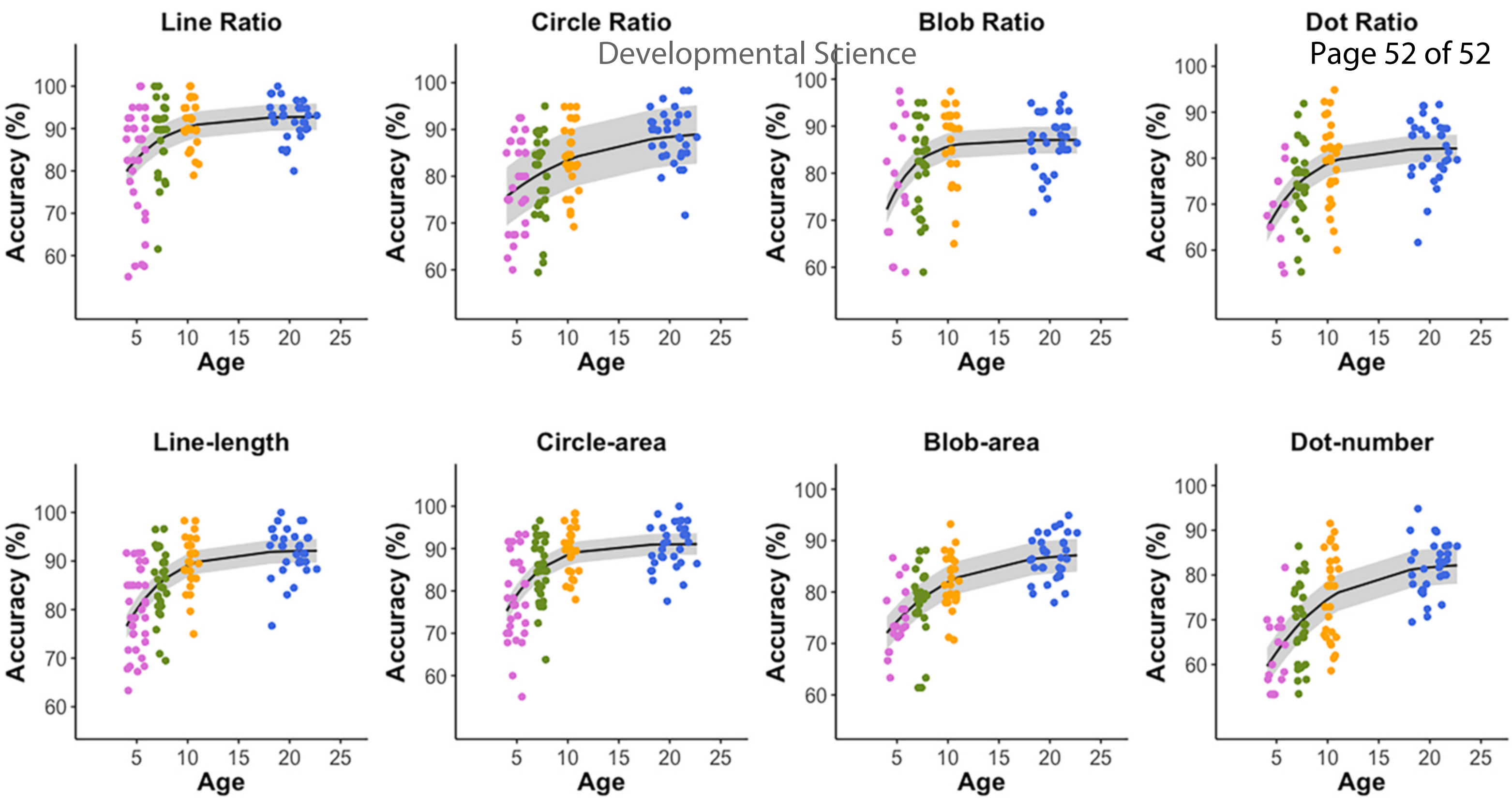
Line Ratio
Circle Ratio
Blob Ratio
Dot Ratio
Line-length
Circle-area
Blob-area
Dot-number
Accuracy (%)
Age
5 10 15 20 25
60 70 80 90 100