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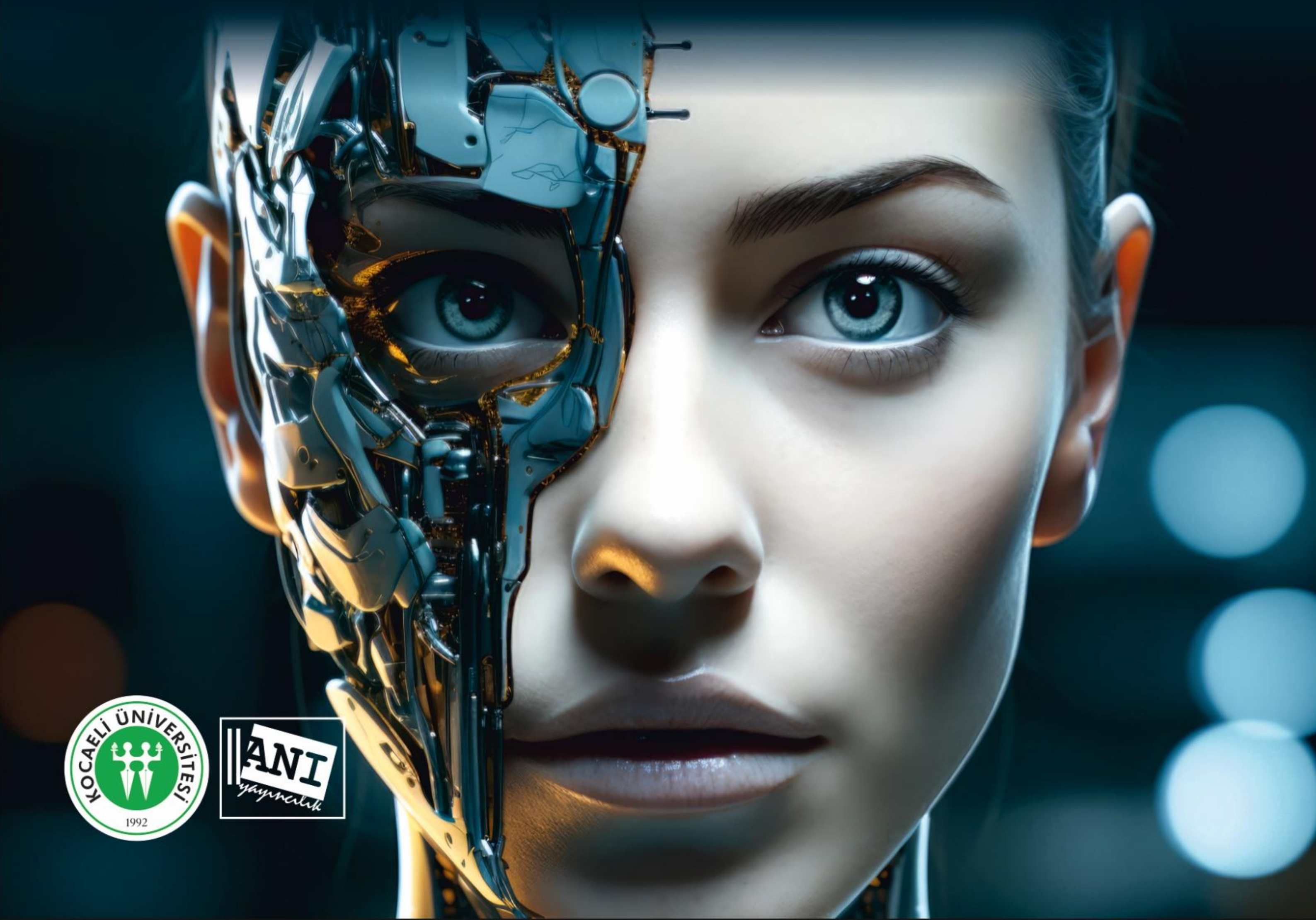

XI INTERNATIONAL EURASIAN

EDUCATIONAL RESEARCH CONGRESS

EJERCONGRESS 2024
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May 21-24, 2024/ Kocaeli University - Türkiye

Editor

Distinguished Professor Şenel POYRAZLI,

Penn State University, USA

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“Designing the Future: Changing Paradigms and Transhumanism with Artificial Intelligence in Education”

Sub-Themes

- Academic freedom, autonomy, and social responsibility in education
- Artificial intelligence and educational applications
- Augmented reality applications
- Barriers to learning
- Blended learning
- Computer-assisted measurement and evaluation
- Core skill sets for students and teachers
- Design of school buildings in the future
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- Earthquake and Effective Psychosocial Intervention Methods
- Earthquake and Trauma
- The Impact of Earthquakes on School Staff
- Education and society
- Education for healthy living and healthy communities
- Education for a sustainable life
- Education in the digital age: Primary, secondary, high school, higher education, and application examples
- Educational leadership in the digital age
- Effects of regional differences on education
- Equity, Diversity, and Inclusion Related to Marginalized Groups
- Emergency Management at Schools
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- Globalisation and Education
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- Vocational education
- Ways to overcome the digital divide

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Author Information

This book has been compiled with contributions from 61 authors representing 35 different universities in Turkiye, the United States, and Iran, as well as Turkiye's Ministry of National Education. Among the contributors, there are 51 authors from 31 universities 6 authors from education institutions in Turkey, 3 authors from 2 universities in the United States, and 1 author from a university in Iran.

CONTENTS

Sixth Grade Students' Construction Processes of Circle, Disc, and Their Basic Elements in a Dynamic Mathematics Software Supported Environment

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Abstract

Circle and disc are fundamental concepts introduced in middle school, forming the basis for many other mathematical ideas. Students often face challenges in comprehending circles, discs, and their basic elements. This research examines the processes of sixth-grade students in constructing circles, discs, and their essential components within a dynamic mathematics software-supported learning environment. Conducted in a sixth-grade class at a public middle school in the Black Sea region, this research was designed as a case study within the qualitative research paradigm. A readiness test was administered to assess students' preliminary knowledge related to these concepts, and participants were categorized into three levels: advanced, intermediate, and lower-intermediate. Data from observations, interviews, and individual worksheets were analyzed within the framework of APOS theory. Results indicate that participants reached the object stage by identifying points equidistant from a fixed point to denote a circle. The participants reached the radius and diameter objects. However the participant at the lower intermediate level had difficulty reaching the radius object, could not coordinate with the concept of set and reach the disc object. In the research, the abstraction process was described in depth and suggestions were made.

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Keywords: *Circle, disc, center, radius, diameter, APOS, concept formation, abstraction, dynamic mathematics software, GeoGebra*

Introduction

A circle is a geometric figure formed by points in a plane equidistant from a fixed point, while a disc represents the two-dimensional area enclosed by the circle. Euclid's *Elements*, written around 300 BCE, includes a third postulate stating that "a circle can be drawn with any center and any radius," laying a foundation for geometry. During the same era, Eratosthenes estimated the Earth's circumference by calculating the central angle and arc length via shadow measurements in Egypt. Aristarchus used solar eclipses to gauge the Moon's diameter and lunar eclipses to approximate the Earth's shadow size, deducing the Moon's approximate size. These foundational uses of circles illustrate their significance in both geometry and scientific fields, emphasizing their inclusion in mathematics education from elementary through university levels.

The National Council of Teachers of Mathematics (NCTM) outlines that students in grades 6 to 8 should learn and apply formulas for determining the circumference of a circle, and the areas of triangle, parallelogram, trapezoid, and disc. They are also encouraged to develop strategies for calculating areas of more complex figures (NCTM, 2000). In accordance with these standards, the mathematics curriculum incorporates the meaningful construction of circles and discs as part of the 6th-grade outcomes (MEB, 2018).

With the rapid evolution of information and communication technologies, integrating technology into mathematics curricula has become increasingly important (MEB, 2018), enhancing students' learning experiences (NCTM, 2000). Although there is existing research on the teaching and learning of certain geometric concepts using dynamic mathematics software, little is explored about students' construction and abstraction of circle and disc concepts. Most studies use quantitative methods to assess the effects of dynamic mathematics software on academic achievement, attitudes, geometric self-efficacy, and retention in topics such as circles, arc length, and area (e.g., Acar, 2017; Bayrambeğ, 2022; Topuz & Birgin, 2017; Şeker & Erdoğan, 2017; Uzun, 2014). Research also shows that students face epistemological obstacles in understanding these concepts (Diana et al., 2020), struggle to connect different concepts (Cantimer &Şengül, 2017), fail to relate them to other geometric figures (Özsoy & Kemankaşlı, 2004), have difficulties defining concepts (Kristianti et al., 2022), and harbor misconceptions (Kaygusuz, 2012). Addressing these challenges and misconceptions, it's essential to construct the fundamental concepts of circle and disc meaningfully. Investigating their abstraction in a dynamic software-supported learning environment can provide valuable insights for effective teaching and learning.

This study presents the APOS theory's framework model, providing a cognitive explanation for the abstraction process, along with computer-assisted mathematics teaching and dynamic mathematics software.

Computer-Assisted Mathematics Instruction and Dynamic Mathematics Software

In technology-supported contexts, designing student-centered learning environments is recommended. As students interact with computers, they experience a mathematician's perspective, grasping the mathematical foundations and reorganizing their knowledge (Kabaca, 2016). Heid's (1997) four principles serve as a guide for technology-enhanced learning environments: technology offers a robust alternative for student-centered learning, supports students in studying like mathematicians, enhances learning through deep thinking, and redefines epistemological authority with technology.

Research in technology use in geometry shows that well-designed tasks and pedagogical organization in technological environments enhance students' conceptual understanding. Technological tools allow students to establish mathematical relationships (Olive et al., 2009); dynamic software aids visualization (Presmeg, 2006), connects formal and informal mathematics (Mariotti, 2006), and mediates between students' informal and formal understanding of mathematical concepts (Balacheff & Kaput, 1996).

To explore learning and teaching geometry, various dynamic software applications like Cabri, Geometer's Sketchpad, Cinderella, and GeoGebra are used. GeoGebra, developed by Markus Hohenwarter, integrates algebra and geometry, providing free activities and is recommended in mathematics curricula due to its accessibility across all levels, from elementary to university (Hohenwarter & Fuchs, 2004; MEB, 2018). Literature suggests that using GeoGebra improves students' reasoning and visualization skills and positively impacts their academic performance (e.g., Acar, 2017; Ağaçdiken, 2021; Bayrambeğ, 2022; Bhagat & Chang, 2015; Çetin, 2018; Dikovic, 2009; Reisa, 2010; Zengin, 2019). In this research, GeoGebra was chosen as the dynamic mathematics software.

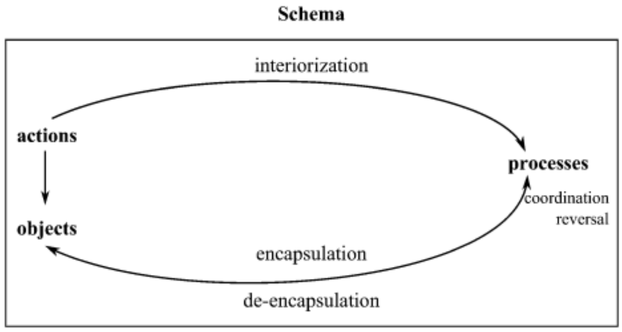
Abstraction and APOS Theory

Abstraction involves identifying relationships and common properties among mathematical objects and transforming these relationships into specific expressions by abstracting them from details (Chihara, 1963; Skiff, 1953; Von Glasersfeld, 1991). Through abstraction, individuals assimilate new experiences, leading to a cognitive understanding of concepts (Skemp, 1986).

The APOS Theory, an extension of Piaget's reflective abstraction, offers a framework for understanding mathematical concept construction (Arnon et al., 2014; Dubinsky, 1984; Dubinsky & McDonald, 2001). APOS Theory explains mental construction through structures of Action, Process, Object, and Schema, and mechanisms relating to these structures, such as coordination, reversal, generalization, and encapsulation (Dubinsky, 1991).

Figure 1

Mental Structures and Mechanisms in the Construction of a Mathematical Concept (Arnon et al., 2014)



The action starts the transformation of previously constructed objects using external stimuli. Considered external, Action is explicit and guided by external instructions. Though the most basic structure, Action is crucial in APOS Theory. Actions are internalized into processes; when an action is repeated and reflected upon, it becomes a Process. At this stage, individuals understand the action, think deeply, and combine it with other actions. The Process can be encapsulated into objects, reversed, or coordinated with other processes to form new processes. Encapsulating a process into a cognitive object allows individuals to view the process as a whole, enabling further transformations. De-encapsulation and coordination among processes and objects facilitate cognitive development (Asiala et al., 1996; Dubinsky, 1991; Dubinsky et al., 2005; Yilmaz, 2023).

Genetic decomposition in APOS theory identifies the mental structures and mechanisms necessary for constructing a mathematical concept. This model remains hypothetical until experimentally validated (Arnon et al., 2014).

Research using the APOS theoretical framework has explored the construction processes of various geometric concepts, such as regular polygons at the university level (Asiala et al., 1998), the concept of slope at both universities (Nagle et al. 2019; Tabaghi et al., Sinclair, 2009) and middle school levels (Deniz & Kabael, 2017), transformation concepts in high school (Hollebrands, 2003) and university settings (Yilmaz, 2011), angle concepts at the university level (Yiğit, 2014), area concepts at the middle school level (Ağaçdiken, 2021), and volume concepts at the middle school level (Dündar, 2019). However, research on the abstraction of geometric concepts concerning circles, discs, and their fundamental elements is limited.

This study aims to address the research question: "How do 6th-grade students construct the concepts of circle, disc, and their fundamental elements in a dynamic mathematics software-supported environment?"

Method

Research Design

This research utilized a qualitative research design to explore how 6th-grade students construct their understanding of circle, disc, and their fundamental elements within a dynamic mathematics software environment. Given the focus on in-depth examination of students' abstraction processes, the study was structured as a case study (Yıldırım & Şimşek, 2021).

Participants

The research involved 6th-grade students from a public school in a major Black Sea city. School and class were selected through criterion sampling, considering factors such as internet access, interactive boards, class academic diversity, student interaction, and self-expression. Three participants were chosen based on readiness test results and teacher input: advanced (P1), intermediate (P2), and lower-intermediate (P3). The readiness test results are summarized below:

Table 1

Readiness Test Results of Participants Selected for Individual Interviews

Participants	Correct Answers	Incorrect Answers	Level
P1	15	3	Advanced
P2	12	6	Intermediate
P3	9	9	Lower-intermediate

Data Collection Tools

Data collection involved several tools: a readiness test, observations, and individual interviews, along with activity sheets and individual worksheets. An 18-question test covering prerequisites for circle, disc, and their elements was developed and validated by mathematics teachers and experts. It included 14 open-ended questions, 3 multiple-choice questions, and 1 fill-in-the-blank question on topics such as polygon, length calculation, circumference, exponential expression, line segment, transformation, and ratio.

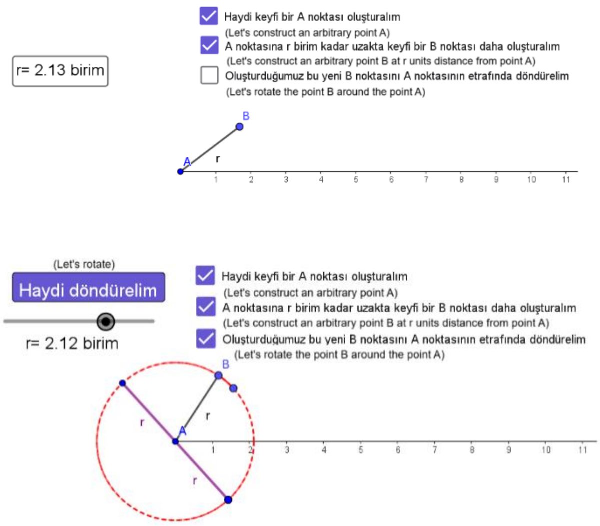
Observations were conducted with the researcher as a participant observer, using audio recordings and video cameras to capture classroom interactions and discussions. A semi-structured interview form was developed to delve into concept formation, with feedback from experts ensuring its effectiveness. Interviews were conducted in an empty classroom and recorded, with individual worksheets used to document participants' bakıyresponses and thought processes.

Teaching Environment

Activity 1, supported by GeoGebra, aimed to introduce concepts of circle, center, radius, and diameter (Figure 2 as a and b). Students were to recognize that rotating point B around point A (the center) forms a circle and that the length of line segments from the center to any point on the circle is equal, termed the radius. Line segments passing through the center and joining two points on the circle were identified as the diameter.

Figure 2

GeoGebra Supported Activity 1 Visual for the Concepts of Circle, Center, Radius, and Diameter



(a)

(b)

Activity 2 focused on developing the concept of a disc, including the circle's interior and exterior regions. The concepts of circle and disc were compared using GeoGebra (Figure 3).

Figure 3

GeoGebra-Supported Activity 2 Visual for the Concepts of Circle, Interior Region, and Exterior Region

Data Analysis

Data were analyzed using content analysis, which involved deep examination to identify themes and dimensions (Yıldırım & Şimşek, 2021). The APOS theoretical framework guided the coding of data, examining how students form concepts. The preliminary genetic decomposition informed the design and planning of activities.

Based on the preliminary genetic decomposition, the study begins with a random point. In the action stage, students draw points equidistant from this fixed point to define a circle. Progression to the process stage occurs when students can visualize all such points and identify the fixed point as the center of the figure. In the process stage, students coordinate concepts of point, line segment, length, and distance, leading to the encapsulation of the circle concept as 'the set of points equidistant from a fixed point (center)'. This encapsulation advances to the object stage, where the circle is fully conceptualized.

De-encapsulation of the circle object involves understanding that 'any point on the circle and the center form a line segment with a constant length,' which defines the radius. Further encapsulation of the concept reveals that 'a line segment passing through the center and joining two points on the circle is twice the length of the radius,' leading to the diameter object. The progression continues to the disc object by recognizing that 'the set of points on the circle and its interior forms an area.' Through these stages of encapsulation and coordination, students achieved a comprehensive understanding of circle, radius, diameter, and disc.

Findings

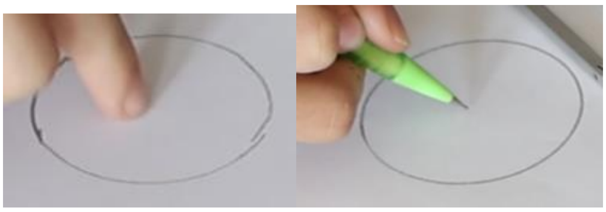
Interviews revealed that participants identified the (initial) object as the center when drawing points equidistant from it. For example, during an interview with P3, who recalled Activity 1 in the class, he recognized point A as the center:

Researcher (R): *We had point A in the activity. What was this point A?*
P3: *Our central position. Our central point.*

All participants were able to correctly identify the center when drawing circles (Figure 4).

Figure 4

Identification of the Centers of the Circles Represented by P1 and P2 During the Interview



In Activity 1, which used GeoGebra software to illustrate circles as sets of points equidistant from a fixed point, participants were asked about their understanding of the circle. P1 described it as *"We rotated point B around point A"* while P3 said, *"We rotated (point B) around the central point."* P2 explained as follows:

R: *What did we do with this point B?*

P2: *Teacher, that (circle)... we drew the thing of the circle there (pointing with his hand to the circle).*

R: *How did we draw it?*

P2*: Like this (rotating with his finger).*

P2 described the figure formed by rotating point B around a fixed point A as a circle. During the interview:

R: *Do you remember there was a 'Let's rotate' button? What happened when we clicked it?*

P2: *Point... it becomes a point (pointing to the dotted lines).*

R: *Do these dotted lines mean anything to you?*

P2: *Those are points... Points can go on infinitely.*

P2 recognized that the circle is made up of an infinite number of points and internalized this concept, indicating an understanding of the circle as a set of equidistant points. When asked about a sprinkler watering up to 3 meters, P2 said:

R: *What happens only within 3 meters?*

P2: *Doesn't it fill up within 3 meters?*

R: *But it only waters up to 3 meters.*

P2: Oh... it doesn't reach here at 2 meters.

R: *Yes.*

P2: Then it becomes a circle.

P2 demonstrated that he encapsulated the knowledge and reached the object stage by applying operations to it. Both P3 and P1, when asked the same question about the sprinkler, identified the watered area as a circle, confirming their understanding of the circle as a set of equidistant points and reaching the object stage.

Regarding the radius, participants recognized it as a line segment from the center to any point on the circle. P2 said, *"We drew a line between them (from the central point to the point on the circle),"* and P1 noted, *"They are all the same length,"* calling it the *"radius."* P3 initially referred to it as the diameter but corrected it with the researcher's help:

R: *Was there a relationship between the points on the circle and point A?*

P3: *Being equal.*

R: *Being equal, right? Equal to what?*

P3: *Between point A (central point) and point B (a point on the circle).*

R: *Do you remember what we called the line (segment) between the points on the circle and point A?*

P3: *Diameter...*

R: *Did we call it the diameter? What was the diameter?*

P3: *Diameter, I can't define it but... for example, if there is a point C (showing the circle) between A and B,… the diameter of C with A.*

R: *Did we call it the diameter or something else?*

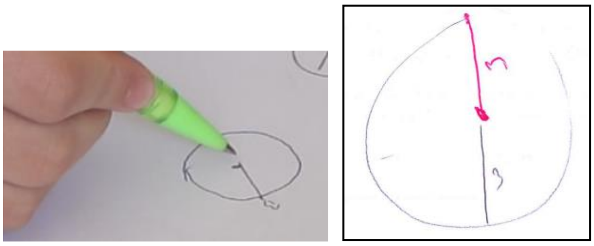
P3: *Radius...*

R: *We called it the radius.*

P1 and P2 effectively related the circle to the line segment, length, and center, reaching the object stage. P3 coordinated with the concept of distance rather than the concept of length. All participants accurately identified the radius and applied it in tasks (see Figure 5).

Figure 5

Representation of the Radius by P3 and the Value of the Radius Found by P2 During the Interview



When discussing the diameter, P3 said, *"It passes through the central point,"* while P2 mentioned *"2r (two radii)."* P1 explained:

R: *Can you explain the concept of the diameter?*

P1: *The diameter is twice the radius. It... divides the circle in half. so it is 180 degrees.*

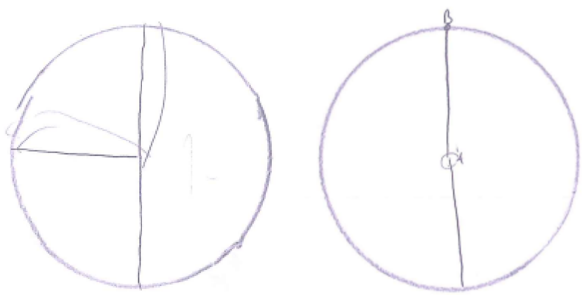
R: *Can you define the diameter again?*

P1: *The diameter is twice the radius. It... divides the circle in half and passes right through the middle.*

P1 demonstrated an understanding of the diameter by coordinating the concepts of center, angle, radius, and part-whole. Similarly, P3 and P2 also identified the diameter by linking it to the concepts of center and radius. All participants successfully drew the diameter of the circle and accurately identified the diameter when asked to perform operations on the object (Figure 6).

Figure 6

Representation of the Diameter by P1 and P2 During the Interview



For the disc, participants understood it as the area enclosed by the circle. P2 described it as *"It's in the figure of a circle but filled inside"* while P1 referred to it as *"the circle's interior with its ring filled."* P3 initially struggled but later defined the disc as *"filled inside."* P3 differentiated the disc from the circle:

R: *Do you think the circumference around the circle (pointing to the circle). Is included in the disc?*

P3: *It is not.*

R*: Why is it not?*

P3: *Well, this is the circle. (drawing the circumference) When we fill it in, it becomes a disc.*

P3's difficulty in connecting the disc concept with the circle and set concepts indicated a lack of progression. However, P1 and P2 correctly described the disc as including the circle and its interior, demonstrating a comprehensive understanding. P2 responded, "*It's a circle, but it's filled in, so it's a disc*", while P1 simply stated, *"Disc"* showing they had reached the object stage for the disc concept.

Discussion and Conclusion

This research aimed to investigate the concept formation processes of 6th-grade students regarding circle, disc, and their fundamental elements. To support these processes, the learning environment was designed with the aid of dynamic mathematics software. Results from the instructional process and individual interviews showed that participants were able to think about circle, disc, and their fundamental elements more easily and establish relationships between these concepts in a shorter period. It is believed that the support of dynamic mathematics software played a positive role in teaching these concepts, making it more feasible compared to compass and ruler, as supported by literature (e.g., Günhan & Açan, 2016; Öçal & Şimşek, 2016; Topuz & Birgin, 2022). The research found that the use of dynamic mathematics software and interactive boards increased students' interest in learning, which aligns with previous research findings (e.g., Lavicza & Papp-Varga, 2010; Wall et al., 2005). Moreover, it is thought that technology has a positive impact on the teaching of these concepts. This view is consistent with the results of studies on the effect of technology use in mathematics education on academic achievement (e.g., Zengin, 2019; Hutkemri & Zakaria, 2014; Thambi & Eu, 2013).

Participants were observed to coordinate the concepts of point and distance while forming the circle object. For the formation of the radius concept, it was observed that participants at advanced and intermediate levels coordinated it with the concepts of line segment, length, and center point. The participant at the lower-intermediate level also coordinated these concepts but struggled to associate them with the line segment concept. In the study by Çantimer and Şengül (2017), it was found that 7th-grade students had difficulties in defining concepts related to the circle, provided insufficient or incorrect definitions, and struggled with drawing and relating the concepts. Similarly, Kristianti et al.'s (2022) research revealed that many students had misconceptions when defining the radius concept. These findings differ from the general outcome of this research. The encapsulation to the object stage is discussed as a challenging mechanism (Asiala et al., 1996; Piaget & Garcia, 1989), and researches related to APOS theory emphasize this difficulty (Clark et al., 2007; Trigueros & Martinez-Plannel, 2010). Comparing participants who reached the object stage in the radius concept with those at the lower-intermediate level indicates difficulties in the concept formation, suggesting the possibility of a stage in the progress from process to object, supporting the idea of a mental stage of totality between process and object (Arnon et al., 2014; Dubinsky, Arnon & Weller, 2013; Oktaç & Çetin, 2016). For the concept of diameter, although participants coordinated the concepts of points and line segments, the participant at the advanced level also coordinated them with angles and whole-part relationships. For the disc concept, participants who reached the object stage coordinated it with the concept of set, while the lower-intermediate level participant did not associate it with the concept of set and therefore did not reach the object stage.

Recommendations

The APOS theory is an important model in examining the formation of concepts in students' minds through the stages of action, process, object, and schema. In this research, a genetic decomposition for circle, disc and their fundamental elements was established, and by examining the processes in depth, the stages to be focused on in the teaching process were identified. The concept formation processes of the mentioned concepts can be analyzed using different abstraction theories, and by identifying differences, recommendations for the teaching of these concepts can be made. Similarly, the concept formation of these concepts in different teaching environments can be researched and compared.

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