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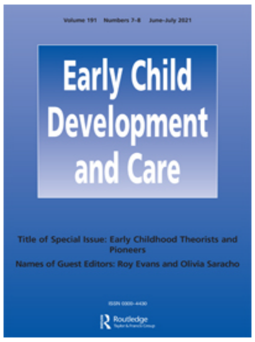

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Math achievement outcomes associated with Montessori education

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ABSTRACT

The math curriculum of the Montessori system of education for children ages 3–12 is distinctive, incorporating multiple manipulatives and educational practices which have theoretical and empirical support in research. However, studies investigating the math achievement and learning of Montessori students and alumni have not consistently found Montessori programmes to be more effective than conventional or other programmes. Through a detailed review of such studies, we find that a Montessori advantage in math is more likely when programmes adhere to important principles of Montessori education, when students have had longer immersion in Montessori programmes, and when assessments are more conceptual in nature. We suggest that future research should take into account programme fidelity and enrolment duration, and outline other directions for future research.

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In 1907, one of the first women physicians in Italy started a school in a tenement building in Rome based on principles of what she termed 'scientific pedagogy' (Montessori, 1967/2016). Her method would eventually travel to many regions of the world, be embraced by teachers and communities in several cultures, and as 'the Montessori method', become eponymous with its founder. Dr Montessori based her educational method on close observations of children's self-directed engagement and activity, and through practice and experimentation, she expanded and implemented her ideas for helping children's development and learning. Dr Montessori's practice-oriented perspective on child development stemmed from her training as a physician. This same training also led her to view children as self-organizing dynamical systems (see Lillard, 2020, this issue). Like Itard and Seguin, she worked with atypically developing children initially, and focused on the training of the senses. Eventually, those initial children performed far beyond expectations by passing the state exams for normal children, which led Montessori to question the common education system. In 1906, she was invited to set up schools for children of ages 2–7 years in the San Lorenzo quarter in Rome, where tenements were being erected to house an uneducated and poor working-class population. This first school opened in January of the new year and was named *Casa dei Bambini*, or Children's House.

Here, while building on the methods she had used with atypically developing children, Montessori was guided by observations of children's reactions to the environment. She was careful to distance these observations from contemporaneous methods in the domain of educational psychology (Montessori, 1967/2016). Her focus was not on precisely measuring children's learning of target content in the artificially created situations typical of school, but rather on interpreting observations of children's behaviour in more naturalistic environments within the holistic context of the child's development. She leveraged her insights to offer children an environment that boosted their

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learning and engagement in a manner consonant with their specific abilities and inclinations. In Montessori's view, a good teacher is foremost a keen observer who 'learn[s] from the child himself the ways and means to his own education' (Montessori, 1967/2016, p. 8). Her focus on allowing children to guide their own educations went hand in hand with her espousal of children's liberty in the classroom. Montessori came to believe that children should be free to move about in the classroom and engage in work of their choosing, without rewards or punishment which could undermine their agency. In this way, she believed, children could respond to personal developmental needs to which each individual child had privileged access.

Montessori recorded several examples of how she came across her insights and incorporated them into her pedagogy. For instance, she noticed that although children were initially clumsy in retrieving materials from a narrow shelf, their movements became better coordinated and more careful to accommodate this feature of the environment. This led her to design materials which, to be used appropriately, required concentration and careful action even from very young children. As another example, she noticed that children were often indifferent to or unwelcoming of rewards offered by their teachers or visitors to the school. Because of this, she eliminated external rewards from school. She was in several respects ahead of her time in recognizing children's intrinsic motivation to learn by interacting with their environment. Yet, another example of her insight is noting that children seek order and repetition. This is evident in the alterations she made while building on the work of predecessors like Seguin and Froebel. She noticed that young children were easily able to distinguish between Froebel's blocks and bricks by shape when blindfolded; moreover, they seemed to enjoy this activity, since they spontaneously began using the blindfold to identify by touch other materials in the Children's House. This made her reflect on the role of repetitive movement in learning, which she made a central part of her pedagogy. Similarly, noticing that children willingly shut their eyes when trying to discern differences in auditory or tactile stimuli, Montessori made educational materials as simple as possible, to highlight salient properties through the 'isolation of a single quality' (Montessori, 1967/2016, p. 104) such as colour or shape.

Such observations fed directly into several features of the Montessori method, including the math curriculum. Today, children in Montessori classrooms engage in activities involving repeated movements to affix the length, shape, or other mathematical properties of certain materials in their mind. As we will see, the design of Montessori math manipulatives involves isolation of qualities like shape and colour, with the expectation that isolation and consistency of these qualities will aid the learning of abstract concepts. The math manipulatives, a visually striking feature of the Montessori method, were a significant innovation of Montessori's. Froebel had earlier introduced the idea of using concrete materials for teaching abstract concepts to children, but Montessori developed an extensive system of manipulatives, including ones for mathematical concepts ranging from natural numbers to multinomial algebraic equations (Kim & Albert, 2014). Some of these representations have become common in mainstream practice, e.g. the base-10 representation used in many classrooms today is similar to the base-10 bead materials Montessori pioneered. Using manipulatives for math instruction has garnered much attention, but some complementary aspects of Montessori math instruction, such as coordinating actions to arrive at set goal states and creating optimal conditions for sustained engagement with these manipulatives, have been relatively overlooked.

In the next section, we discuss distinguishing features of the Montessori math curriculum, reviewing theoretical and empirical support for these features. Next, we critically review research comparing the math achievement of children enrolled in Montessori or conventional schools, and finally, we conclude with suggestions for future research.

Key features of the Montessori math curriculum

Beginning in preschool, children in Montessori experience a continuous progression of hands-on math manipulatives coupled with real-world problem-solving. Abstract concepts are introduced systematically, with an initial focus on math's perceptual (thus spatial) basis leading to formal instruction of concepts. The manipulatives bear minimal resemblance to objects encountered elsewhere and are used in a context that assumes and facilitates sustained engagement with freely chosen activities. The teaching of number provides a comprehensive example of this process.

Spatial basis for teaching math

Before being taught natural numbers, 3-year-olds in Montessori classrooms are introduced to the idea of sequential progression through two types of activities. Practical life activities, like washing tables, render repeated experiences of undertaking set sequences of separate actions in pursuit of specific goals. Math computation also involves repeated undertaking of set sequences of actions. Sensorial activities, like making an ordered tower of 10 cubes whose sides range from 1 to 10 cm, attune children to systematic change in dimensions like volume, length, width, height, and circumference (Montessori, 1967/2016). For instance, as a precursor to formal number activities, children use a material called the red rods, which is a set of 10 rods that successively increase in length by 10 cm, such that the smallest is 10 cm long and the longest is 100 cm long. As part of the work, these rods are mixed up and then arranged by children in the order of increasing length. This represents the spatial basis for introducing number with a later material. The concept is embodied, as children carry the rods one by one from shelf to rug (where they do the work), feeling the change in length as they spread their hands to hold the rods by their ends. Later the teacher will disperse the rods around the room and ask the children to bring the next rod in the sequence, requiring children to hold length as an abstraction in their mind as they seek that next rod.

Given that the primary purpose of mathematics is measuring aspects of the world, it is unsurprising that spatial and math skills are strongly related. Mix (2019) suggests that the relation between spatial and mathematical skills is stable at different ages, may involve automatic as well as strategic processing, and may be recruited especially for novel content. This last point suggests spatial representations might be especially helpful at younger ages when math concepts are first introduced (Mix, 2010), consistent with the Montessori materials.

Systematic variation in manipulatives

In Montessori classrooms, natural numbers are formally introduced using number rods, which incorporate one clear variation on an already familiar manipulative. Number rods are identical to the red rods except alternating 10 cm segments are blue, and now they are named, 'The 1 rod', 'the 2 rod', and so on. The *cardinal principle*, that the last number counted is the count of a set (Gelman & Gallistel, 1978), is implicated in the number rod lessons. In three periods, children are taught to count the segments on a rod, select the appropriate rod in response to its segment number (recognition), and then name the appropriate number for the rod (recall). Cardinality is reinforced by asking children for both the physical quantity and the abstract number. It is embodied spatially on rods increasing in length by uniform quantities, in accordance with positions of numbers on the count list.

The alternating colours on the number rods also demonstrate the *successor function,* or the idea that successive numbers on one's count list are generated by addition of exactly one unit to the previous unit (Sarnecka & Carey, 2008). By placing the one rod at the end of any other rod, and aligning against the very next rod, children see the precise unit quantity by which rods of different numbers measure up to each other. Exercises with number rods are meant to facilitate such insights about the structure of the number system, laying the ground for addition and subtraction. Children learn to place cards revealing the symbol for each number rod; children embody the symbols by tracing sandpaper number cut-outs. Embodiment is a known strategy for better learning (see below).

The Montessori classroom has multiple manipulatives representing natural numbers, providing redundancy. The spindle box has 10 compartments labelled 0–9 and 81 spindles; children place the appropriate number of spindles in each compartment, and if they do so correctly, no spindles are left over. The exercise requires counting and verification, reinforcing cardinality. The successor function is again reinforced since each successive compartment requires exactly one more spindle. Multiple representations encourage *abstraction,* understanding that numbers can be used for counting anything (Gelman & Gallistel, 1978), including discrete objects and portions of objects (Montessori, 1934/2016).

Manipulatives have generally been found useful in math instruction. A recent meta-analysis of studies comparing effects of instruction with concrete manipulatives or abstract symbols showed that manipulatives improved retention, problem-solving, transfer, and justification of solutions (Carbonneau, Marley, & Selig, 2013). However, the effect sizes for the more advanced skills were small, possibly because manipulatives are not always of high quality and can be confusing. The features of Montessori math manipulatives (including dimensionality, colour, and shape) were chosen to facilitate the representation of a mathematical concept, with no extraneous features to distract from the core concept (Montessori, 1934/2016). Further, Montessori math materials are used only for math work. These features are desirable according to the dual representation hypothesis (DeLoache, 2000): young children find it difficult to maintain two representations of the same object, as both a symbol for something else and something in its own right. Manipulatives are objects intended to represent abstract concepts. If their meaning as objects is salient to children, this interferes with children's ability to acquire the symbolic meanings. When children are familiar with manipulatives from another context, manipulatives are less effective (Petersen & McNeil, 2013).

Montessori materials also demonstrate *concreteness fading*, the quality of moving from more concrete materials to more abstract ones, such as gradually moving from concrete manipulatives for multi-digit numbers to just using numerals. Further, in all materials using multi-digit Arabic numerals, the same colours represent unit, ten, hundred, and thousand place values. Reinforcing regularities in this way may facilitate learning the decimal system. Taken together, these characteristics may make Montessori manipulatives especially suitable for learning math (Laski, J'ordan, Daoust, & Murray, 2015).

Coordination of action and perception

While learning multi-digit numbers and place value using golden beads, children work with individual beads, strips of 10 beads, squares of 100 beads (made of ten 10-bead strips), and cubes made of 1000 beads (10 stacked 100-bead squares) to construct and de-construct multi-digit numbers. The primary objective is abstract: 'to provide children with the construction of the decimal system itself' (Montessori, 1934/2016, p. 11), not merely to count or calculate. Negative numbers are introduced and practiced using manipulatives for the negative 'snake game', during which children calculate the cumulative value of a long chain composed of smaller bead strips representing different positive and negative numbers. The positive numbers are multicoloured, with one colour for each number from 1 to 9, and the negative numbers are black. The objective of the game is to arrive at the cumulative total of all the numbers composing the long bead chain, entailing quick and flexible additions and subtractions. Children also use long chains to do 'skip counting,' placing number tabs at say every sixth bead on a chain of 1000 beads; this would seem to prepare the child's mind for multiplication and division.

Encountering large quantities perceptually and motorically, rather than primarily abstractly, may facilitate deeper understanding, in concert with embodied cognition. According to Barsalou's (1999) theory of perceptual symbol systems, abstract concepts are rooted in perceptual experiences in the world, which result in a record of neural activation. When experience reinforces perceptual regularities, this neural activation becomes the basis for abstract symbols. Representations in one modality, such as vision, are coordinated with representations in other modalities, such as touch, which develops representations; multimodal experiences thereby produce abstract concepts. In support of multimodality, multisensory training protocols are more efficient than unisensory protocols (Shams & Seitz, 2008). In addition, neurological studies suggest that actions influence the connectivity of neural networks, which is important for cognitive and behavioural tasks (Byrge, Sporns, & Smith, 2014). Self-generated actions are influential for learning early in life – for example, infants who are given sticky mittens that let them lift objects early are more sensitive to other people's object-related goals (Sommerville, Woodward, & Needham, 2005). Toddlers' object recognition is associated with specific views of the objects that toddlers generate themselves by manipulating the objects (James, Jones, Smith, & Swain, 2014). Converging findings from several fields offer grounds for believing that coordination of perception with action on math manipulatives would be conducive to learning the relevant math concepts and skills.

General Montessori learning principles

Montessori has many characteristics pertinent to learning more generally. The first, already mentioned, is that all learning is embodied. Lessons incorporate precise movements and gestures by the teacher intended to enhance imitative fidelity, priming students' attention – —in this case, to the salient properties of manipulatives which vary numerically. Other general features include allowing student interest to guide learning, and the cultivation of sustained attention (Lillard, 2017). At all ages, children choose what material to work with, with whom to work, and for how long. This is crucial, since perceptual experience develops abstract concepts only through active, sustained, and selective attention on essential qualities of the concept (Barsalou, 1999). Sustained attention also correlates positively with cognitive tasks such as problem-solving among toddlers (Choudhury & Gorman, 2000), with math achievement among elementary school children (Anobile, Stievano, & Burr, 2013), and with academic achievement among high-school students (Steinmayr, Ziegler, & Träuble, 2010). These general principles of embodied cognition, free choice, peer learning, and sustained attention are not specific to math, but could enhance its learning.

In sum, the Montessori approach to teaching math is a coordinated amalgam of several features individually supported by theory and research. Taken holistically, this approach should be effective for math education, leading one to expect good math outcomes from Montessori schools. In the next section, we see whether this is the case, comparing math achievement of students enrolled in Montessori or other kinds of schools.

Review of research on math outcomes from Montessori schools

Several studies over the past few decades have assessed outcomes associated with Montessori schooling. However, study designs vary in their ability to isolate programme effects from parent or school-specific effects. In addition, studies vary in students' ages and sociodemographic characteristics, assessments used, and crucially, the fidelity of the Montessori programmes.

Lillard and McHugh (2019a, 2019b) mined Dr Montessori's writings to identify several features of high-fidelity Montessori programmes, including mixed-age groupings, a 2.5–3 h uninterrupted work period, a full set of Montessori materials, and high-quality Montessori teacher training. Departure from these features may hamper learning, and two studies suggest the importance of programme fidelity for math learning specifically. Lillard (2012) measured school-year math growth in high and low-fidelity Montessori 3–6 programmes, indexed by the percentage of children engaged with Montessori (as opposed to conventional preschool) materials. Students in high-fidelity classrooms advanced significantly more. In another study, non-Montessori materials were removed from two classrooms and left undisturbed in another classroom at a Montessori school. Children from the higher-fidelity classrooms advanced 0.19 standard deviations more in math, which was not significant given the small sample, but verges on a large effect size for education field research using standardized measures (Kraft, 2020), which is impressive given the study duration of 4 months (Lillard & Heise, 2016). Significantly greater advances were seen in executive function, which is related to math. These findings suggest Montessori implementation fidelity must be considered when evaluating studies.

Next, we discuss math outcomes for children ages 3–6 years and, building on that early childhood foundation, for ages 6–12 years, according to the Montessori system of mixed-age grouping. The final section synthesizes these findings.

Preschool

There are several studies examining math achievement in Montessori preschools. Two studies involved public Montessori schools with random lotteries, allowing causal inferences about programme effects by effectively controlling for unselected variables. Lillard and Else-Quest (2006) compared the math scores of 5-year-olds in a public Montessori school with those of children who were waitlisted and enrolled in non-Montessori schools. Significant differences favoured Montessori 5-year-olds on the Woodcock-Johnson® III Test Battery Applied Problems subtest (Woodcock, McGrew, & Mather, 2001), with a medium effect size. Yet, recruitment of Montessori students from only one school confounds curriculum and school effects. In the second lottery-controlled study, researchers longitudinally compared the academic achievement of students from two public Montessori preschools with waitlisted conventional preschool students (Lillard et al., 2017). Math achievement, which contributed to the academic achievement composite, was again measured with the WJ®-III. The academic achievement scores did not differ significantly in their first year in the programme, but children in the Montessori programmes had significantly higher scores by the end of preschool. Although these findings are compelling, they do not shed light specifically on math learning. Taken together, the lottery-control studies suggest but do not establish positive causal effects of the Montessori approach for teaching math.

Most other studies use correlational designs, comparing math achievement of children enrolled in Montessori preschools and other kinds of preschools. Lillard (2012) compared math gains on WJ®-III of students enrolled in high-fidelity and lower fidelity Primary Montessori classrooms, and found significantly greater gains in the high-fidelity classrooms. The students from the two types of classrooms were similar in terms of age and SES, and all parents had chosen Montessori, but unmeasured parent variables might have been operative. Another study compared Sudoku skills of 5- and 6-year-olds in Montessori preschools and Ministry of National Education (MoNE) preschools in Turkey (Guven, Gültekin, & Dedeoğlu, 2020), finding that Montessori preschoolers solved more Sudoku puzzles. However, children's Sudoku skills were positively related to parents' education, which was not accounted for in analysis. Programme implementation information was also not provided. Such studies open interesting areas for research, but are inconclusive.

Some correlational studies carry special relevance for children from disadvantaged backgrounds. Chisnall and Maher (2007) studied the numeracy skills of 5-year-olds in New Zealand, comparing those who had received at least 18 months of education in Montessori preschools and others who had received a similar duration of conventional or alternative preschool education. Students were tested with the New Zealand Numeracy Project Assessment, including knowledge and strategy use in addition, subtraction, forward number word sequence, backward number word sequence, number identification, and place value understanding. Montessori children outperformed the comparison children on backward number word sequence and place value understanding, two of the more advanced skills assessed, but no difference was found on other measures. Two-thirds of Montessori students as opposed to one-fourth of the comparison students were of low SES. The pattern of positive effects of Montessori programmes for children from disadvantaged settings has been found elsewhere as well (Ansari & Winsler, 2014, 2020; Lillard et al., 2017; Miller & Bizzell, 1983, 1984).

However, other studies have also reported mixed effects of the Montessori programme for minority and low-SES children. Ansari and Winsler (2014) compared the effects of 1 year of Montessori versus HighScope on the school readiness of 4-year-old low-income Black and Latinx children; math was assessed with the Learning Accomplishment Profile-Diagnostic (LAP-D™). The Montessori classrooms had neither mixed-age grouping nor the 3-year cycle, and in that sense were of low fidelity. Latinx students in the Montessori classrooms had significantly higher growth from pre-to-posttest than those in HighScope; Black students in Montessori classrooms showed no significant difference. Ansari and Winsler (2020) found that the 1 year of Montessori was also associated with better third grade math scores for Latinx students.

Findings from early Head Start programmes also suggest mixed effects of one-year Montessori-like programmes, in this case for gender rather than racial group. Miller and Bizzell (1983, 1984) examined children who had been randomly assigned to a brief intervention with four different preschool programmes, including low-fidelity Montessori; 90% of participants were Black. At the end of preschool, no significant differences had emerged. A Montessori advantage in math emerged through grades 7 and 8 on the Advanced Battery (Form B) of the Stanford Achievement test, but only for boys. At grades 9 and 10, too, a Montessori advantage for boys was found on Math Total scores of Comprehensive Test of Basic Skills (CTBS). Girls from DARCEE and the control group outperformed Montessori alumna (Miller & Bizzell, 1984). While noteworthy, such sex differences have not since been reported in other studies, and *n*s were quite small by high school.

Peng and Md-Yunus (2014) assessed the academic achievement of Taiwanese school children in grades 1–3, comparing Montessori preschool alumni with others. Math achievement was assessed using the Elementary School Math Ability Achievement Test (ESMAAT), including assessments of place value, base-10 number system, fractions, patterning (including sequences of shapes, sounds and numbers), describing two- and three-dimensional shapes, and mentally manipulating two- and three-dimensional shapes. Whereas first graders with Montessori preschool experience scored higher than first graders with other preschool experience, no differences were found at older ages, suggesting fade out. Little information was provided about the preschool programmes, and many Montessori alumni appeared to have had only 1 year of Montessori experience and thus failed to complete the normal 3-year cycle.

Questions about the effectiveness of Montessori programmes often invite a closer look not only at the Montessori programmes, but also at the programmes with which they are being compared. The absence of a difference in favour of Montessori alumni in Taiwan may not be as concerning as a lack of significant differences in other contexts, given the generally high math learning outcomes in Taiwan (Gonzales et al., 2009). Similarly, differences in favour of Montessori in the diverse preschool settings of Turkey, the U.S., New Zealand, and Taiwan may not imply the same thing. Even with this caveat, studies with younger children generally find positive effects from Montessori programmes, especially high-fidelity ones. Partially favourable findings are associated with lower fidelity implementations. We next examine math outcomes for older children.

Elementary

One lottery-control study assessed math achievement of 12-year-olds (Lillard & Else-Quest, 2006) and revealed no significant differences, in spite of the school implementing a high-fidelity programme. However, the participants belonged to one of the early cohorts at this school, so they may not have experienced the full implementation (which requires older peers who have themselves had the full implementation). Moreover, with only one Montessori school included in the study, the result may not be generalizable to the Montessori programme as a whole. Similar limitations negatively affect the generalizability of the findings of Lopata, Wallace, and Finn (2005), who compared the math achievement of fourth grade and eighth grade students from one of each type of four public schools, including Montessori, open magnet, structured magnet, and conventional. Scores on New York State Mathematics (NYSM) test and the Math portion of the TerraNova (TN) did not demonstrate any consistent math advantage for any type of school, since differences at fourth grade had washed out by eighth grade. The Montessori programme here appeared to be low fidelity, based on information on the school website.

Other studies using correlational designs have tried to maximize control by comparing academic outcomes of demographically similar students enrolled in different programmes. One compared high-school performance of students who had previously attended high-fidelity public Montessori and conventional programmes (Dohrmann, Nishida, Gartner, Lipsky, & Grimm, 2007). Participants were matched within their high schools on gender, race/ethnicity and free/reduced-price lunch status. Montessori alumni outperformed conventional schools' alumni in math by a medium effect size, as measured through composite and subtest scores for the standardized ACT and WKCE tests as well as the students' high-school math grade point averages.

The students from Dohrmann et al. (2007) had attended Montessori schools from preschool through fifth grade, suggesting that a Montessori advantage may reflect the contribution of the students' preschool education as well as their elementary education. Indeed, findings from other studies indicate that a Montessori foundation at preschool may be important for optimal math learning outcomes at higher ages. Mallett and Schroeder (2015) assessed achievement differences from first through fifth grade for Montessori and non-Montessori school students in Dallas public schools using the Iowa Test of Basic Skills (ITBS) in grades 1 and 2, and the Texas Assessment of Knowledge and Skills (TAKS) in grades 3 through 5. No significant difference was found between groups in first through third grades, but emerged in favour of Montessori in fourth and fifth grades. Most Montessori students in this study had not received Montessori education at younger ages, which might have hindered them from reaping benefits earlier. A similar finding was obtained by Brown and Lewis (2017), who compared students in three Montessori schools and three matched traditional magnet schools (including one STEM-focused school) on third grade test scores. No significant difference emerged for math, although both groups scored over a half standard deviation higher than the district average. The higher scores from the STEM-focused magnet may have elevated the magnet school scores to be on par with the Montessori schools, but this is difficult to ascertain due to limited information on comparison schools. Like the students in Mallett and Schroeder's (2015) study, the Montessori Elementary students may not have been in Montessori preschools.

Laski, Vasilyeva, and Schiffman (2016) present an alternative possibility: that Primary Montessori programmes, but not Elementary Montessori programmes, confer math advantages. They found an early Montessori advantage was nullified later. Children were tested first at ages 6–7 on single-digit addition and construction of multi-digit numbers using base-10 blocks, and at ages 8–9 on double-digit arithmetic problems and place value. Montessori 6-year-olds outperformed conventional ones on the base-10 block task, but no differences were seen at 7, 8, or 9. However, the study did not report on fidelity, and many Montessori schools do not have high-fidelity programmes (Daoust, 2004).

Exactly the opposite pattern of findings was obtained in another study on the same competency, i.e. children's understanding of base-10 concepts and place value, in high-fidelity Montessori programmes. Mix, Smith, Stockton, Cheng, and Barterian (2017) compared base-10 understanding of 5- and 8-year-olds in Montessori and conventional schools. Students were assessed using a place value test, a practical problem requiring the application of the base-10 structure to a base-5 setup, and a number line estimation task. Older Montessori students showed a better understanding of base-10 structure and place value than their peers in traditional elementary schools, but no difference was found on the number line estimation task, nor were differences seen at age 5 years. The base-10 task used by Mix et al. (2017) assessed children's understanding of the structure of the decimal system, including hierarchical relations between different place values, and place value understanding was assessed using a variety of questions with multi-digit numbers. These tasks may be considered conceptually more demanding than those used by Laski et al. (2016), which had asked first graders to decompose two-digit numbers using base-10 blocks, and second and third graders to identify the largest number among a set of multi-digit numbers.

In addition to programme quality, another clue hinting at a source of discrepant findings is seen in Reed (2008), which compared place value understanding among students in first through third grades in one Montessori and one traditional school. Students were tested on different aspects of place value understanding, including grouping, partitioning, horizontally represented addition problems (testing conceptual knowledge), and vertically represented addition problems (testing procedural knowledge). Montessori students outperformed conventional school students on conceptual understanding, but no differences were found on tasks measuring procedural understanding. The Montessori curriculum's orientation towards helping children understand and work with basic mathematical structures without emphasizing algorithms may explain the Montessori advantage on conceptual understanding. The distinction between procedural and conceptual knowledge may also help explain the discrepant findings of other studies, such as Laski et al. (2016) and Mix et al. (2017). While interesting, only one school of each type was included in Reed's (2008) study, thus conflating programme type and school.

Preschool and Elementary school studies raise many of the same concerns, including fidelity of programme implementation, relative benefits for students of different social groups, and difficulty of distinguishing between school-related and programme-related effects. Elementary school studies also raise other questions: Is a Primary Montessori education necessary or merely conducive for good math outcomes from Elementary Montessori programmes? Are some kinds of assessments better than others at identifying the mathematical knowledge and skills of Montessori students? In the next section, we summarize the regularities and discrepancies from presented findings and offer directions for future research.

Summary and conclusions

Straightforward conclusions about math learning in Montessori schools are difficult to draw: cross-study comparisons are complicated by different types of assessments; studies rarely report key programme fidelity information; and a bird's eye-view presents some contradictory findings. However, a closer look at the studies and the included schools, assessments and students reveals some consistencies. First, most studies show either a Montessori advantage or no difference between Montessori and conventional schools. Very few studies show the superiority of another approach, and those that do show it only at one time point or for a subgroup and with low-fidelity Montessori programmes (Miller & Bizzell, 1984; Lopata et al., 2005). This suggests that Montessori programmes are as good as or better than conventional programmes for math learning. Secondly, high-fidelity Montessori schools are likely to confer advantages in math relative to conventional schools (e.g. Lillard & Else-Quest, 2006; Mix et al., 2017; Dohrmann et al., 2007; Lillard et al., 2017; Mallett & Schroeder, 2015). Studies featuring Montessori schools with discernibly lower fidelity (Ansari & Winsler, 2014, 2020; Brown & Lewis, 2017; Miller & Bizzell, 1983, 1984; Peng & Md-Yunus, 2014), or reporting little programme information (Laski et al., 2016; Lopata et al., 2005) have usually simply not found a difference. Similarly, Montessori-favouring differences are likely with longer-term Montessori immersion (Dohrmann et al., 2007; Mallett & Schroeder, 2015; Mix et al., 2017), as might be predicted by theories of concept grounding on the basis of perceptual experience, which emphasize the importance of a large body of experience. Further in line with concept grounding theories, some studies also suggest a Montessori advantage is more likely when testing deeper conceptual knowledge (Mix et al., 2017; Reed, 2008). Standardized measurements and end-of-grade tests may not plumb the full extent of Montessori students' understanding, not only because Montessori students may not rely on algorithms to the extent assumed, but also because of their relative unfamiliarity with large-scale impersonal assessments.

Therefore, we suggest that sustaining positive effects of the Montessori approach are seen with high Montessori programme fidelity, long-term exposure to the Montessori environment, and on deeper conceptual assessments. Since conceptual understanding is arguably a worthy educational goal, and since an educational approach that confers long-term benefits on the strength of a solid foundation is a desirable one, we believe that key stakeholders such as parents and teachers would find these conditions for a positive Montessori effect reasonable.

For a fuller understanding of the math learning process in Montessori schools, future studies should investigate how the Primary (ages 3–6) Montessori experience influences math learning in Elementary Montessori schools. Assessing Montessori students' algorithm and strategy use, possibly with application and transfer oriented assessments, may yield insights about their learning process and conceptual understanding. The Montessori math advantage may trace its source to perceptual grounding, or to positive attitudes towards math fostered by autonomy support, or both. Future studies should try to probe these compatible possibilities.

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No potential conflict of interest was reported by the author(s).

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