Understanding of the Equal Sign: A Case of Chinese Grade 5 Students

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An understanding of the equal sign is a fundamental concept for early algebra. While literature claimed that Chinese students commonly master the relational understanding of the equal sign in the elementary school, these claims are under-researched. This study used the Mathematics Equivalence Assessment with 237 Chinese Grade 5 students. The results showed that the majority of students possess a relational understanding of the equal sign, with some able to confidently apply the concept of structural equivalence. To complement the test results, six Grade 5 teachers were also interviewed to explore teaching approaches and contexts used to foster an understanding of the equal sign.

Students' misconceptions of the equal sign still remain widespread in many western countries nowadays (Stephens et al., 2022). Many students possess an operational understanding of the equal sign, considering the equal sign as a symbol of indicating calculation results, instead of perceiving the equal sign as an indication of an equivalent relationship of both sides (i.e., a relational understanding, McNeil et al., 2015). A relational understanding of the equal sign is fundamental during students' progression from arithmetic to algebra (e.g., Carpenter et al., 2003). A narrow operational conception of the equal sign can cause students' difficulties in algebra (Stephens et al., 2022). For instance, students without the relational understanding of the equal sign struggle to understand the number sentences such as '3 = 3', and '3 + 2 = 4 + 1', and these students will further experience difficulties in making sense of solving equations with unknowns on both sides (e.g., $3x + 2 = 2x + 1$) when learning algebra. There has been extensive research contributing to pedagogical approaches to develop students' relational understanding of the equal sign, such as the number sentence evaluation activity (Carpenter et al., 2003) and exposing students to non-conventional forms of arithmetic expressions, such as '__ = 1 + 2' and '3 = 1 + __'. (McNeil et al., 2015). These pedagogies aim to interrupt students' one-directional (left to right) conception of the equal sign and provide students with an opportunity to attend to the equivalence of both sides of the equal sign.

While the misconception of the equal sign is common in western countries, it is not universal. Researchers (e.g., Li et al., 2008; Jones et al., 2012) reported that by the end of primary school, students in China generally have developed a relational conception of the equal sign. For instance, a pioneer work by Li et al. (2008) revealed that the majority (98%) of Grade 6 tested students ($n = 145$) in China possess a robust relational understanding of the equal sign. Since Li et al. (2008), a growing body of literature is exploring Chinese students' conceptions of the equal sign, indicating their process of gaining the solid relational understanding is not without setbacks (e.g., Sun et al., 2023; Yang et al., 2014). This leaves a space for a further investigation on how Chinese students develop a relational understanding of the equal sign. The purpose of this report is to contribute further insights to this topic by examining Chinese Grade 5 students' understanding of the equal sign and exploring the possible factors influencing the development of their understanding.

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Literature Review

As mentioned, Li et al. (2008) reported students in China generally hold a relational understanding of the equal sign. Through a text-book analysis, Li et al. (2008) found that in China the equal sign was introduced to students before their exposure to addition and subtraction. Students started learning the formal symbol of the equal sign in a context of comparing and describing quantities relationship (e.g., ‘more than’, ‘the same as’, ‘less than’) of concrete objects, with the formal symbol of the equal sign being introduced afterwards. (Sun & Gu, 2023) conducted a close examination of the pedagogical approach to introduce equal sign in China. Echoing Li et al. (2008), Sun and Gu (2023) showed that the concept of the equality and the formal symbol of the equal sign is first introduced to children in kindergarten (for the children aged 5) in a quantity comparison context before they begin learning arithmetic operations (addition and subtraction), so the interference of misconception that the equal sign is a symbol of displaying results of calculation possibly brought by traditional forms of arithmetic expressions (e.g., 1 + 2 = 3, 7 − 5 = 2) can be reduced. Furthermore, they showed that children in kindergarten were introduced to an activity of drawing an equivalent number of any objects they like to match the other side of the equal sign, and the objects to be matched were allocated on either right side or left side of the equal sign (see Figure 1 below).

Figure 1

Match the Quantity of Objects Activity (Sun & Gu, 2023)

This activity supported students in building a bi-directional view of the equal sign. Finally, Sun and Gu (2023) mentioned introducing the formal symbol of the equal sign, the official curriculum document requires teachers to highlight the way of drawing the equal sign: ‘two short horizontal lines with the same length’. This step appeared to reinforce students’ conception that the equal sign refers to the ‘sameness’. Overall, Sun and Gu (2023) concluded that children’s experiences with the equal sign in kindergarten provided them with a foundation of a relational view towards the equal sign before beginning primary school. However, Sun et al. (2023) reported that only about half of Grade 3 tested Chinese students (n = 501) held the robust relational understanding of the equal sign. This percentage is even lower for Grade 1 students tested (approximately 36%, n = 497). Similarly, Yang et al. (2014) showed about 30% of tested Grade 3 students (n = 110) considered the equal sign as ‘show results’. These results tended to demonstrate that while Chinese students may have a foundation of relational understanding of the equal sign at pre-school level, their emerging relational view can revert to operational view after they enter early grades in primary school. Sun et al. (2023) speculated the extensive arithmetic operation drill in early primary grades could be a major factor leading to this failure. On the other hand, as mentioned above, others (e.g., Jones et al., 2012; Li et al., 2008) showed by Grade 6, students in China generally possessed a more robust relational understanding of the equal sign. While these studies were conducted in the different contexts (e.g., schools, regions) in China, the high degree of uniformity of national curriculum, nationally approved textbooks, and the consistency of teaching approaches in Chinese schools suggest that these findings from different studies are likely to be well-grounded to portray a landscape of students' conception of the equal sign in China. That is, while students in China generally have developed a relational understanding of the equal sign by end of primary school, the process that they gain this understanding is not without obstacles.

In this sense, there is a space to further explore how Chinese students' development of conception of the equal sign, in particular, to understand how their conceptions are further supported in progressing to relational level since Grade 3. This study contributes to this research gap by focusing on students who were at the start of Grade 5. The main part of this study reports on the use of the Mathematical Equivalence Assessment (MEA) to investigate students relational understanding of the equal sign. In addition, to complement the MEA test results, exploratory interviews with the students' teachers were conducted to identify possible factors that may help explain students' developing conceptions of the equal sign.

Methodology

The research was conducted in a primary school in Changchun, Jilin Province, China. The context of the participating school and students is similar to those in Sun et al. (2023), in terms of similar region, SES background and academic rankings. A total of 237 Grade 5 (aged 11–12) students participated in this study, 110 boys and 127 girls. Students took a diagnostic test that examined their understanding of the equal sign (elaborated below). Their responses were coded against three categories of understanding of the equal sign suggested by Stephens et al. (2013). Afterward, interviews were conducted with six mathematics teachers to gather their insights on the factors enhancing or impeding students' relational conception of the equal sign (four were the teachers of participating students when they were in Grade 4, the other two were not, but they also taught Grade 4). The study was conducted in September, when students had just started Grade 5.

Instrument

The Mathematics Equivalence Assessment (MEA) instrument is a well-established tool designed to measure students' understanding of the equal sign, and it has been proved effective in cross-cultural contexts, including its application in China (Simsek et al., 2021).

Table 1

Example Test Items for Number Sentence Evaluation and Solving

Number sentence type	Elaboration and example
$a + b = c$	Students evaluate true or false for number sentences such as 31 + 12 = 43 Students fill the missing number to the form __ + 35 = 91
$c = a + b$	Students evaluate true or false for number sentences such as 25 = 16 + 9 Students fill the missing number to the form 52 = 13 + __
$a = a$	Students evaluate true or false for number sentences such as 41 = 41 Students fill the missing number for the form 23 = __
$a + b = c + d$	Students evaluate true or false for number sentences such as 41 + 23 = 31 + 33 Students fill the missing number for the form 53 + 31= __ + 21

MEA consists of three types of problems: (1) structure evaluation, students to determine whether number sentences such as 10 = 3 + 7, 7 = 7 and 3 + 7 = 4 + 6, are true or false; (2) structure solving, students to fill the missing number in a number sentence such as 12 + 23 = __ + 25; and (3) definition of the equal sign, students to explain the meaning of the equal sign. Sun et al. (2023) further modified the items in MEA to the context in China (e.g., modifying the numbers to better align with students' grade levels) and applied them to measure students' understanding of the equal sign in the first three grades. Considering the similarity in school contexts between this research and Sun et al. (2023), this study used the same test items as Sun et al. (2023). In the test, students evaluated true/false for number sentences given first. They then filled the missing numbers in the number sentences. For number sentence evaluation and solving items, students were required to explain how they arrived at the answers. There are four types of number sentences for evaluation and solving, as shown in Table 1, which are conventional and non-conventional forms of number sentences that are commonly applied to examine students' understanding of equal sign. Finally, students wrote the definition of the equal sign.

Coding Process

Students' responses were coded according to three levels of understanding of the equal sign in Stephens et al. (2013). The first level is 'operational': students perceive the equal sign as 'show answer' symbol displaying the results of calculation carried out from the left side. The second level is 'relational-computational': students recognise the equal sign as indicating an equivalence of both sides, but they use full calculations to demonstrate this equivalence. The third level is 'relational-structural': students can apply relationships among the quantities to show the equivalence of both sides, with a minimum calculation. Both the second and third levels are evidence that students possess the relational understanding of the equal sign (Stephens et al., 2013). In this study, for the definition of equal sign, if a student stated that the equal sign meant "adding numbers", "answers", "results" or "totals", this response was coded as 'operational'; if a student explained equal sign means equivalence of both sides with a specific calculation example, it was coded as 'relational-computational'; if a student expressed equal sign meant equivalent quantities of both sides in general, it was coded as 'relational-structural'. For number sentence solving, for instance, when solving "$7 + 3 = \_\_ + 4$", if a student filled in 10 or 14 for the missing number, it was coded as 'operational'; if a student calculated $7 + 3 = 10$ and then $10 - 4 = 6$ for the missing number, it was coded as relational-computational; if a student recognised that 4 is 1 more than 3 and so the missing number should be 1 less than 7 so it is 6, it was coded as 'relational-structural'. The coding procedure for number sentence evaluation items was similar.

Interview

After the test was coded, six mathematics teachers were interviewed. The experiences of these teachers range from five to twenty years. All teachers had taught lower and middle primary levels. In the interviews, they shared their experiences in developing students' conceptions of the equal sign and provided their opinion about factors that supported and/or impeded the relational understanding of the equal sign. The interviews were open-ended and prompted by questions such as "*In your everyday teaching, would you emphasise the conception of the equal sign? If so, how?*", and "*What do you think helps or hinders students to understand the equal sign as a symbol indicating the equivalent relationship of both sides?*"

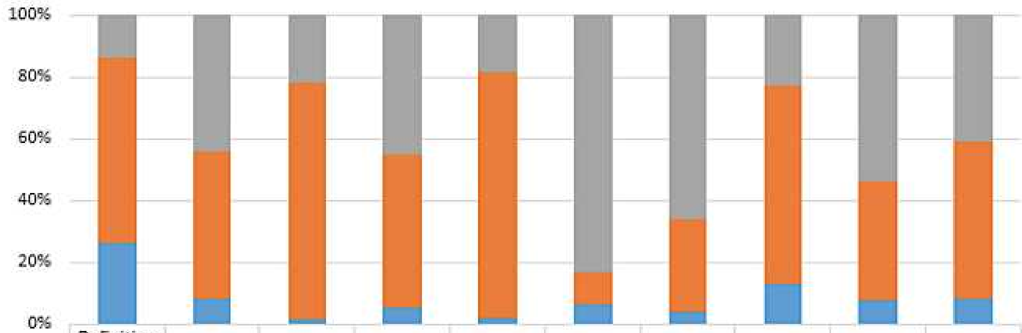
Results and Discussion

MEA Test

Figure 2 below shows the coded test results. There are ten result columns, with the first one for the definition of the equal sign. Subsequently, there are two columns for each type of number sentence (one for evaluation and one for solving). The final column shows the average results of the previous nine columns. For each column, the percentages of students' responses at three levels of understanding of the equal sign are categorised.

Figure 2

Distribution of Students' Responses to Test Items Against Level of Understanding (%)



	Definition of equal sign	"a+b=c" True/False	"a+b=c" Solving	"c=a+b" True/False	"c=a+b" Solving	"a=a" True/False	"a=a" Solving	"a+b=c+d" True/False	"a+b=c+d" Solving	average
relational-structural	13.5	44.3	21.9	45.1	18.6	83.1	66.2	22.8	53.8	41.0
relational-computational	60.3	47.3	76.4	49.4	79.3	10.5	30.0	64.4	38.2	50.6
operational	26.2	8.4	1.7	5.5	2.1	6.3	3.8	12.9	8.0	8.3

Figure 2 indicates that by the start of Grade 5, the majority of tested students possessed relational understanding of the equal sign. Across all number sentence items, at least 87% of responses fall at either relational-computational or relational-structural levels. Furthermore, the results shows that a substantial number of students used a strategy of applying quantity relations to fill the missing number in number sentences, without full calculations. For instance, 53.8% of answers to solving the number sentence type $a + b = c + d$ are classified as relational-structural level. For example, many students explained their approach to answer 53 + 31 = <u>63</u> + 21 as "since 21 is 10 smaller than 31, to keep two sides the same, the missing number needs to be 10 greater than 53, so it must be 63". Students with this structural view towards a number sentence have departed from focusing on calculating the results of operations and have started attending to the general structure and relations among terms in a number sentence, which is a hallmark of the early algebraic thinking (Molina & Ambrose, 2008). In this sense, it could be argued that a significant portion of tested students demonstrated their emergence of early algebraic thinking. On the other hand, it is noted that compared to number sentence items, fewer responses to the definition item are classified as relational understanding levels (74% compared to 87%). This result appears to suggest some students, who can apply their relational understanding of the equal sign to solve the problems, described the meaning of the equal sign as being coded as 'operational'. The possible explanation is that when expressing the meaning of the equal sign, some students stated, "the equal sign is to connect between the answer and the calculation". According to the coding scheme, this kind of responses was categorised as 'operational'. However, the use of words like "connection" and "between" implied that these students may hold the relational understanding, so they can apply it in solving number sentence items. There was an ambiguity in their explanation of their conceptions. This finding tended to suggest that students who have developed certain mathematical thinking possibly have difficulties in articulating this thinking verbally or in written form due to a lack of appropriate mathematical vocabulary. This was echoed by Kranda (2008), who showed many students had difficulties in updating their mathematical language to adapt to the new situations.

While an earlier study (Sun et al., 2023) documented that approximately half of the participating Chinese Grade 3 students still possessed an operational understanding of the equal sign, in this study, the majority of tested students, who had recently completed Grade 4, demonstrated their conception of the equal sign reached the relational level. Given that participating schools in both studies had the similar context (e.g., region, SES, academic ranking), it seems that by the start of Grade 5, Chinese students' relational understanding of the equal sign has been enhanced, compared to those in Grade 3. As will been seen later, teachers' comments in the interview also endorsed this claim. However, this claim may need more careful longitudinal evaluation (see Conclusion and Limitation section).

Teacher Interviews

In interviews, six teachers shared their experiences in facilitating students' development of the relational understanding of the equal sign. These experiences enabled this study to have an explorative understanding of factors promoting students' relational understanding of the equal sign. All teachers commented they would not formally highlight the definition of the equal sign in everyday teaching. However, teachers recognised that some learning activities, introduced not specifically for fostering students' relational understanding of the equal sign, actually promoted it. For instance, teachers revealed that students were exposed to number sentences such as '2 + 4 = 5 + __' and '__ + 3 = 5 − 1' sometimes during classroom practice, homework or tests. For completing these tasks, students needed to notice the equivalence of both sides. Three teachers used the word "embedding" to describe this experience, saying "embedding the conception of the equal sign into daily teaching practice." These words are concurred with by Cai et al. (2013), who showed that in Chinese primary school, the emergence of early algebraic thinking was immersed in everyday teaching and learning on arithmetic.

Furthermore, this study considers that an interesting finding from the interview is that five out of six teachers mentioned the introduction of solving simple formal equations (e.g., $2x + 1 = 5$, $5 = x - 2$) in Grade 4 contributed to reinforcing students' relational conception of the equal sign, and the revealed how they believed learning equations enhanced relational understanding. First, using a balance model to demonstrate the equation solving is frequently mentioned by teachers. For instance, Teacher A commented:

> When introducing equations, we used the balance model. Students visualised the similarity between the abstract equations and the concrete balance. So, they easily comprehend the equal sign as indicating equivalent relationship of both sides, like the balance beam.

Similarly, Teacher C said, "the use of balance model can help them to visualise the meaning of the equation and understand equal sign as … just like a balance, indicating two sides have the same weight".

Teachers' responses tended to suggest that the resemblance between the concrete representation (balance beam) and formal equation visually supports a conception that the equal sign refers to the equivalence of both sides. This supporting process is further elaborated by some teachers. For instance, Teacher B mentioned when learning equations, students engaged in a hands-on play with the balance, they saw the beam tilting if one side was heavier, and they observed it balancing again when they adjusted the weights to make both sides weight equally. This dynamic process led students to attend to the simultaneous changes of both sides of an equation, so as to further reinforce the conception of the equal sign that is to represent the equivalence of two sides. This kind of teachers' comment appeared to reveal that that students' hands-on interactions with the balance served as means to promote students' relational understanding of the equal sign. Teacher B stated, "students had learnt the equal sign in the context of comparing quantities at earlier age, but this comparison was static. In contrast, the dynamic play of balance model could press the meaning of the equal sign more". These words tended to highlight that compared to simply comparing quantities on two sides, the interactive play with the balance model is richer, enabling students to grasp a clearer understanding.

Other than the balance model, teachers commonly commented that, when learning equations, compared to earlier grades, students are exposed to more non-conventional form of arithmetic expressions, helping them depart from the operational mindset. As Teacher D stated:

> Before learning equations, students experienced traditional arithmetic expressions for years, so they tend to have a mindset that the equal sign connects a calculated result. When learning equations,

> they see many operators and the unknowns can appear on either side of the equal sign, this breaks this mindset.

Likewise, Teacher C and E said:

> Students normally do lots of the arithmetic operation questions with traditional form of number sentences, like the number to be answered on the right side. When they are learning to solve equations, they start experiencing many operations with unknown number on either side. They need to understand equal sign as 'two sides are equivalent' to make sense to these equations. (Teacher C)

> Before learning equations, students predominately see and think about calculating results, but when they were learning equations, they were pushed to consider the equivalent relations of quantities more than calculating results. (Teacher E)

The teachers' responses demonstrated that they recognised that overcoming a focus on calculating results (left to right) is an important premise for developing the relational understanding of the equal sign. They consider when learning formal equations in Grade 4, students were exposed to many non-conventional arithmetic expressions with operators and unknowns in different positions, providing students with ample opportunities to be aware of the equivalence of quantities on both sides. This finding concurs with other researchers (e.g., McNeil et al., 2015) who proposed non-conventional form of arithmetic expressions are effective to support students' relational understanding of the equal sign. Furthermore, teachers' comments suggested that during equation learning, students were facilitated to depart from focusing on calculating results, and they started noticing the relationships between the quantities, this focus on relationship might explain why a substantial number of students had started applying a structural strategy to evaluate or solve number sentence problems.

Taken together, teacher interview tended to show that the introduction of the formal equation could: (1) allow students to visualise the connection between the formal equation and the balance model through the hands-on experiences; (2) expose students to many non-conventional arithmetic expressions with operators and unknowns in different positions, whereby it promoted students' relational conception of the equal sign. This finding echoes an early study (Cai et al., 2013), which put forth that learning formal equations supported students in attending to quantity relationships in number sentences in Chinese primary school. Notably, recently researchers in western countries also start paying attention to use equations with the balance model to support students' relational understanding of the equal sign. For instance, Stephens et al. (2022) showed that equation solving activity with the assistance of a pan balance can promote US Grade 2 students' relational understanding of the equal sign. Therefore, it could be understood that the literature on utilising equation learning activity to promote students' relational understanding of the equal sign is emerging and is worth further exploration.

Finally, Sun et al. (2023) showed that many Chinese students' conceptions of the equal sign possibly revert to an operational level during the first three grades, and they speculated that this reversion might be to the excessive arithmetic operation drill practice. Teachers' comments in this research confirmed this speculation. Many teachers reported that students need to drill an extensive amount arithmetic expressions that are in the traditional 'left to right' form in early grades, this can press a 'show result' conception of the equal sign to students. However, all teachers agreed that most students are able to view equal sign as indicating the equivalence of two sides after equation learning in Grade 4. This aligns with the claim made earlier: at the start of Grade 5, students' relational understanding of the equal sign has been strengthened.

Conclusion and Limitations

In this study, the MEA test result suggested that by the start of Grade 5, the majority of tested Chinese students demonstrated a relational understanding of the equal sign, based on understanding levels categorised by Stephens et al. (2013), with some of the students exhibiting structural thinking that is a hallmark of early algebraic thinking. Teacher interview revealed that students' exposure to simple formal equations with the balance model in Grade 4 can strengthen their relational understanding of the equal sign. When learning formal equations, the concrete representation (balance model) visually aided students to perceive equal sign as indicating the equivalence of both sides. Also, students encountered many non-conventional arithmetic expressions during equation learning, which pushed them to notice the equivalence relationship instead of focusing on calculating results. With these findings, this study contributes to the research gap in literature on Chinese students' understanding of the equal sign, exploratively explaining how students were supported to step towards a robust relational understanding of the equal sign since Grade 3.

The claim that the relational understanding of Grade 5 is enhanced compared to Grade 3 students is obtained by comparing the test results of Grade 5 students in this report with the results of Grade 3 students in a similar research context (Sun et al., 2023), and this claim was echoed by teacher interview. However, in the future, a longitudinal study that tracks the same cohort of students from Grade 3 to Grade 5 will be highly desirable to make a more rigorous conclusion. Also, as stressed early, the findings of this study are explorative, therefore, a further investigation is worth conducting to understand more about how exposure to simple formal equations can enhance students' relational conception of equal sign in details, possibly with some in-depth classroom research.

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