Examining Students' Mathematical Thinking: The Case of Porridge Words

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Mathematical thinking is a complex, multi-faceted process that has been described as messy and difficult but can also show growth and insights into conceptual understanding and knowing. This paper explores the application of Edward de Bono's practical thinking, in particular, the occurrence of porridge words to examine the mathematical thinking of primary school students. The research employed qualitative research techniques using participant's drawings, their written descriptions, and interviews about their drawings. Employing discourse analysis uncovered patterns in how students used porridge words to communicate their mathematical thinking.

In the realm of mathematics education, understanding and unravelling the intricacies of students' mathematical thinking is an enduring challenge. Mathematical thinking underpins students' abilities to problem solve in mathematics (Monteleone et al., 2023), and can lead to a more thorough understanding of mathematics (Stein et al., 1996). Further, mathematical thinking is a means to "describe mathematical growth" (Rasmussen et al., 2005, p. 52) and is a function of mathematical processes and operations (Burton, 1984). To achieve a deep level of understanding of mathematics, researchers suggest that students need to engage in the process of mathematical thinking (Schoenfeld, 1989; Stein et al., 1996). However, the thinking process is complex, and multi-faceted (Quane & Booth, 2023). Liljedahl (2021) describes mathematical thinking as "messy" requiring risk-taking (p. 72) is "difficult" (p. 87) and "is a necessary precursor to learning" (p. 5).

Despite the increased attention and acknowledgment of the important role mathematical thinking plays in mathematical understanding, there is little research examining how primary-aged children communicate their mathematical thinking and make their thinking visible to others. Research has investigated professional learning for teachers and pedagogical practices in the areas of mathematical thinking (Liljedahl, 2021; Sfard, 2008; Stein et al., 1996). Few studies have explored mathematical thinking from a student's perspective. No studies as far as the author is aware, have examined students' mathematical thinking and the use of porridge words.

Different scholars have theorised the thinking process. Edward de Bono, a proponent of teaching thinking and renowned for theorising thinking eloquently describes thinking as "simply a matter of moving from one idea to another" (de Bono, 1971, p. 55). The transition between ideas can be challenging, resulting in students using filler words phrases or emergent language to describe the transition. de Bono (1971) refers to these words or phrases as "porridge words" (p. 60) and uses the phrase in generalised contexts and not subject-specific contexts such as mathematics. This research investigated the research question:

- How do students use porridge words to communicate their mathematical thinking?

Conceptualisation of Porridge Words in Mathematics Education

Generalised language can inhibit and also enable thinking with de Bono (1971) using the term "porridge words" which, "allow us to make definite statements or ask definite questions when we do not really know what we are talking about". These vague blurred porridge words

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have an extremely important part to play in thinking (p. 61).

As such, “porridge words” help us make the connections to keep thinking moving and progressing “from one idea to another” (de Bono, 1971, p. 67). Without “porridge words” de Bono (1971) warns that thinking will cease providing no way to keep thinking moving. Further, “porridge words” can be a means to ask “vague questions” when a person is not yet familiar with a topic to help develop their thinking and understanding of a topic (de Bono, 1971, p. 67). Giving a name to different aspects of thinking allows for the identification of the different processes used when engaging in thinking. While de Bono (1971) used “porridge words” in generalised contexts, it can be argued that the process of using “porridge words” be applied to specific contexts and topics including mathematics education. In mathematics, the word ‘sum’ could be considered a porridge word, especially for children who use the word to mean ‘perform a mathematical operation’ such as multiplication. Here the word ‘sum’ is vague and does not accurately describe the processes involved in or the concept of multiplication.

However, de Bono (1971) warns that we often dismiss porridge words, especially in the thinking process. Yet, it is through such devices as “porridge words” that we can make a commitment to thinking, take action on our thinking, develop ideas through thinking, have parallel and intersecting thoughts, and abstract our thinking. de Bono (1971) posits that often we start with very general, non-specific thinking to establish more specific ideas and that there is a general attitude that we need to start with rigid, constrained, and specific thinking. The caveat of this attitude is that we are at risk of being “completely trapped by existing ideas” (p. 68). Porridge words in mathematics education, therefore, could be defined as *words or phrases that have an ambiguous meaning or may not describe the mathematical concept, process, or skill in a highly accurate manner using appropriate and accurate mathematical language*. That is, porridge words act as a device to facilitate mathematical thinking. In communicating mathematical thinking, even through the use of porridge words, students are engaging in mathematical discourse.

Mathematical Discourse and Discourse Analysis

Andreas (2011) argues that mathematical discourse differs from other forms of discourse due to the nature in which mathematics is communicated. Communicating mathematical thinking and understanding is multi-modal, involving various forms of communication and semiotic systems. These forms include but are not limited to words, symbols, graphs, drawings, and gestures (Andreas, 2011; Sfard, 2008). In communicating mathematics, andreas (2011) suggests students engage in both mathematical and generic discourses and explains that generic discourse may be relevant to the mathematical discourse that is occurring. In doing so, mathematical discourse is a dynamic and interactive process that can provide opportunities for students to explore, explain, and deepen their mathematical understanding.

To investigate the occurrence of porridge words in students’ mathematical thinking, discourse analysis was employed. Discourse analysis examines spoken or written texts to understand the ways words or phrases function in a particular context (Paltridge, 2008). Adopting a pragmatic approach, whereby understanding what students are saying rather than what the words or phrases mean in the “most literal sense” (Paltridge, 2008, p. 3) is used to understand the meaning behind what a student has said. In this way, the spoken text is analysed to identify porridge words and how they are used by students. In terms of mathematics, students may also have a “linguistic repertoire” (Paltridge, 2008, p. 29) that they use when communicating their mathematical understanding. A student’s linguistic repertoire then depends on the domain in which the mathematical language is used, such as the primary classroom and the interactions with others. Paltridge (2008) used the term “speech communities” to describe a group of people that interact with each other, using the same language that includes common geographical, cultural, age, and social factors (p. 28). In terms of social factors, Paltridge (2008) describes that the linguistic repertoire of a member will depend on who we are interacting with, the social context, the topic function and goal of the discussion or conversation as well as the formality of the setting and the status of each of the members. To understand the relationship between what is spoken and the meaning, pragmatics is employed as a means to study the context in which the spoken act has occurred. In this way, the context of the situation is crucial in interpreting and understanding what has been said.

Context

The study was conducted at a small inner-regional South Australian State School. The school had a total student population of 36 students separated into two classes. A junior class comprising Reception (first year of primary school), to year 2 inclusive. An upper class comprising of students in Years 3–6 inclusive. Flexible grouping was used for several learning areas including mathematics. The school is located in a small regional town, with an Index of Community Socio-Educational Advantage (ICSEA) value of 942 (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2023). ICSEA is calculated by adding a) the social education advantage, b) remoteness, and c) percentage of indigenous students. The social education advantage is determined by parental occupation, school education, and non-education levels. ACARA (2023) reports that the ICSEA measure provides the opportunity to compare Australian Schools where the median value is 1000 and the standard deviation is 100. The school has an ongoing strong community relationship, maintaining close connections with community groups and promoting the sharing of the school's facilities. The school population has experienced almost a 30% increase between 2020 to 2023 (ACARA, 2023).

Method

Children were withdrawn from class to complete a drawing and semi-structured interview. A drawing prompt was read to each child, outlining the requirement for children to "draw themselves doing mathematics" (Quane et al., 2021) with further instructions stating that children needed to include their face and that the focus of the drawing could be any aspect of mathematics. The drawing prompt guided students to show the mathematics that they were doing, thereby providing unique insights into how they communicate the mathematics depicted in their drawing. The same prompt was read to all children to ensure consistency. Children were prompted to write about their drawing and participated in a semi-structured interview that asked clarifying questions about what was drawn. To prompt students to communicate the mathematics that they had depicted in their drawing, students were asked "Can you explain the maths that you have drawn in your picture?". The semi-structured interviews were transcribed verbatim by the researcher including pauses, laughter, and filler words such as um, ah, er. The transcripts and audio recordings were reviewed numerous times for initial understanding to identify the occurrence of porridge words or phrases and the context in which the porridge words were used. Each interview transcript was uploaded to the NVivo 12 software, which provides "powerful processes of indexing, searching, and theorizing" (Creswell, 2012, p. 243). NVivo 12 was used to create nodes, cases, and case classifications to explore the data further in preparation for the coding process. The coding process identified words and phrases used by students that had an ambiguous meaning and therefore, constituted a porridge word or phrase.

Each drawing, written description, and transcribed interview were viewed initially as three separate pieces of data and then as a collection of work. Two main coding systems were then employed. First, a systematic analysis using the principle of atomism was used to examine each drawing. Once the drawing was examined at the atomic level, drawings were viewed holistically (Quane et al., 2019). The process of analysing at the atomic and holistic levels was repeated with the child's written description and interview responses. Second, the data generated from the three individual data collection techniques were analysed, they were combined to form a more holistic and comprehensive picture of children's mathematical thinking and, in particular, their use of porridge words. An inductive approach was used to identify porridge words within the generated data (Pezzica et al., 2016). To ensure that the data was consistently coded for the occurrence of porridge words, the researcher and an educator coded the data independently. Cohen's Kappa and the percentage of agreement were calculated using SPSS. Complete agreement between the two raters, that is 100% agreement (Fleiss et al., 2004), was achieved for the identification of porridge words.

Participants

An information letter and consent form were sent home via their child. The response rate 64%, with 13 male (57%) and 10 female (43%) students with all year levels except reception represented in the sample. Five students identified as Aboriginal or Torres Strait Islander. Table 1 shows the distribution of participating students by year level.

Table 1

Participant Numbers

Year level	**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Male	3	3	0	1	2	4	13
Female	1	2	1	3	1	2	10

Findings

The first level of analysis was to identify the porridge words in the generated data. A total of 34 different porridge words or phrases were identified with 17 words being used once by different students (Table 2). D1 (Figure 1) used the most porridge words (n = 11), whereas there was a single child (Year 1, male) who did not use any porridge words. The most common porridge word was *math* or *maths* with 13 children ambiguously using the word, followed by the word plus or plusses (n = 6) and the phrase figured out (n = 4). At the surface level, some words classified as porridge words will appear to be used correctly. However, the examination of the intent of the use of these words, such as diameter, counting, and measure reveals that they are indeed used ambiguously.

Table 2

Occurrence and Frequency of Porridge Words

Porridge word	***n***	**Porridge word**	***n***	**Porridge word**	***n***
A blocks and B blocks	1	Figured out	4	Plus/plusses	6
Add	1	Getting answers	1	Practicing	1
Algebra	1	Groups	1	Problems	2
Big	1	Growing	1	Pop them/Put them/put some	2
Break	1	Make them	1	Shape	1
Break the numbers in Half	1	Making	1	Stuff	2
Building	1	Math/maths	13	Sums	3
Bunch/bunch of	2	Mental math	1	Take away	2
Counting	5	Measure	2	Times/timsing	2
Diameter	1	Numbers	4	Those/them	2
Equalling	2	Number maths	2	Thing	1
Find out	1			Working out	1

Note. n denotes the number of students.

The second level of analysis examined the intent and context in which the porridge words were used. Math or Maths were used as porridge words, particularly by younger students to provide a general description of what they had depicted in their drawing. For example, D5 (year 3) states “um, ah, I’m doing maths and I’d doing plusses” and similarly D7 (Figure 2) states she is “doing my maths” and later adds “I’m doing sums”. D7 uses the word “sums” as a porridge word to describe “the sums are two times eight and twelve times nine”.

“Figure out” was used to describe the process of solving a problem, deriving an answer, or identifying and classifying an object. For example, D21 (Figure 6) used the phrase “figure out” to describe her process of identifying and classifying the type of angle she depicted in her drawing, “Ah, just sitting, (pause), like just looking trying to figure out the best way, trying to concentrate.” D10 used the phrase “figure out” to describe the process of determining the next number in a number pattern increasing by fives “it’s um, where you, um (pause) try to figure out what’s the answer and not guessing”. D6 used the phrase multiple times to “figure out equations” and to describe an authentic situation of determining the number of carpet tiles required for a games room:

> Well we are finished now, but when we were doing the games room I was helping my dad built it and we had to figure out um cause we were putting carpet tiles down on the floor because I was getting my cast off and I was sad that I had to miss a day of school so he said I could do the maths and figure out cause they were a metre square each and I had to figure out the dia (sic), the diameter of the room and figure out how many boxes we needed of carpet tiles.

In all instances, students hesitated before using the phrase “figure out” by either pausing or using filler words such as ‘um’ or ‘ah’. As such the hesitancy may be an indication of students attempting to process their thoughts and communicate their understanding clearly but stumble, unable to find the most appropriate mathematical language. This is also the case in D6’s use of the word diameter to describe the dimensions of a rectangular room.

There were varying degrees of invisible thinking from students from not being able to articulate their thinking “I can’t really describe it” (D8, Figure 3) or “I don’t really know how to explain it” (D21), to students who gave an indication that they were thinking but providing little or no description or explanation. For example, “I did it in my head and I also know my two times tables pretty well” (D7). Porridge words were further categorised as either having a conceptual intention or a process intention. Invisible thinking also manifested in students’ partial descriptions of the mathematics they had shown in their drawing. For example, D17 (Figure 4) provided the following description of adding decimals:

> There was these, thousandths, hundredths, and tenths, and then there were these ones and tens. We had to pop them into (pause) add the numbers up and then after over 10 we had to put the one over to the next.

In D17’s description, we see how important language plays in describing and explaining mathematical thinking. In attempting to describe his thinking, D17 has used several porridge words (“pop them into” and “put the one over to the next”) to describe what he has done. The use of porridge words broadly describes the processes used, but D17 has yet to provide a clear conceptual explanation of the mathematics he has shown, rendering part of his thinking invisible. Continuing with porridge words to describe processes, D20 (Figure 5) uses several porridge words and phrases to describe her recollection of grouping and regrouping numbers:

> So, if the teacher said if you have 200 and 20 tens and twenty fives then you had to make them take away some and put them in the tens and put some of the tens in the tens, I mean ones into the tens.

In examining, D20’s use of porridge words, we can see that she is attempting to describe the place value of numbers as well as attempting to explain the process of regrouping. Here the intention of the phrases “make them”, “take away some”, “put them” and “put some” may be initially unclear. However, examining D20’s drawing we can see that she had depicted a range of Multiple Attribute Blocks, depicting these blocks as units (ones), longs (tens), and flats (hundreds). It appears that the porridge words used by D20 are evidence of early emergent informal mathematical language.

Figures 1–6

Student Drawings: Draw Yourself Doing Mathematics, Write About Your Drawing

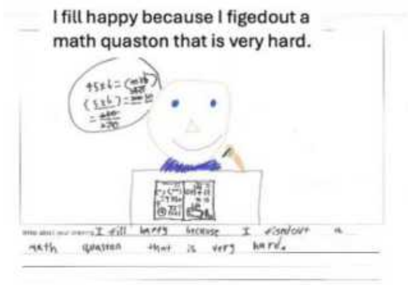

Figure 1: D1, Year 5, Male

Figure 2: D7, Year 4, Female

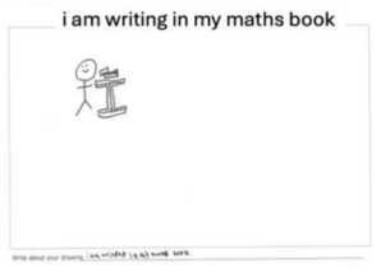

Figure 3: D8, Year 6, Male

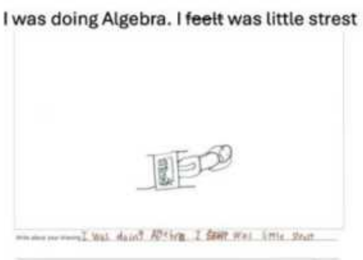

Figure 4: D17, Year 6, Male

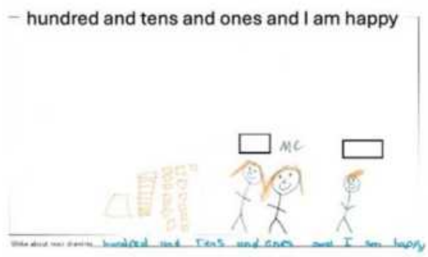

Figure 5: D20, Year 4, Female

Figure 6: D21, Year 6, Female

D1's drawing (Figure 1), written description and interview responses show many examples of porridge words. D1 provides a partial explanation of the mathematics he has depicted.

Interviewer: Can you tell me about your picture

D1: My drawing is me working on a maths question that I didn't understand that was in the mental math book before but now that I have understood it. When I first got the question the first week that we had maths I got a question like 37 times 5 and I had no idea how to do that but then I figured out that I needed to break the numbers in half, I had to break the 30 away from the 7.

Interviewer: Yes

D1: So, I have 30 and I have 7. The number which would either be 5 times that number

Interviewer: Yes

D1: So then 5 times 30 would be, I would take off the 0 then it would be 5 times 3, 15 then I would add the 0 on which would be 150 I would write that down in the first box. Then I'd do the unit which would be 7 times 5 then I would add whatever that number is to the other number to get the answer

Interviewer: And how do you feel about that strategy?

D1: I feel it is a very good strategy to figure out big numbers times a little number

First, D1 was able to describe the mathematical procedure that he used to multiply a two-digit number by a single-digit number. In doing so, D1 used several porridge words including "break", "take off", "figured out", "box", "little numbers, and "big numbers". D1 used "break" to describe the procedure he has used in two different ways. Initially, D1 described that he needed to "break the numbers in half" and started to explain how he did this by adding "I had to break the 30 away from the 7". In attempting to explain how he partitioned the number 37, D1 used the word "half" to indicate partitioning the numerical value of the tens and units, which contradicts the true meaning of half. Further probing would have provided the opportunity to clarify D1's understanding of partitioning. D1 then launches into describing further how he multiplied the partitioned numbers by five, again using a series of porridge words including "I would take off the 0" and then returning to his description "I would add the 0 on". In this description, we gain insights into the procedure that D1 has applied, and we are starting to see a conceptual explanation of the mathematics in terms of D1 reference to "(t)hen I'd do the unit". It appears that the use of porridge words is helping D1 describe the procedure and providing an opportunity to develop the conceptual understanding to explain his thinking.

Discussion

Communicating mathematical thinking is multifaceted and the results from this study have shown that students are developing their mathematical linguistic repertoire via the use of porridge words. Most students (n = 22, 96%) engaged in invisible thinking either in their drawing, written description, or discussing their drawing as a means to attempt to communicate their mathematical thinking. Further analysis of the occurrence of invisible thinking revealed that students use invisible thinking in several ways. Returning to the work of de Bono (1971) helps us make sense of possible reasons why students communicate mathematical thinking that is invisible. First, thinking may be "intermediate impossible" which de Bono (1971, p. 139) describes as an idea that is not right but acts as a transitional point to another idea that is right. D17 could be considered an example of "intermediate impossible" where he describes his thinking using emergent language. It appears that many students knew that they were not using the most appropriate mathematical term but found familiar words to continue their descriptions or explanations of the mathematics that they depicted.

Second, ideation and conceptual understanding forms slowly over time as new knowledge is acquired. The data analysis suggests that the use of porridge words which may result in invisible thinking is a sign that students are hunting for mathematical clues to help them explain their mathematical thinking to others. de Bono (1971) classified clues into three types: (1) Clues that are obvious to everyone—but may still be misinterpreted; (2) Features that are obvious to everyone but do not become clues unless some significance is attached to them; and (3) Clues that are not at all obvious and have to be worked on (pp. 170–171).

de Bono (1971) describes clues have three distinct purposes. First, clues can be used "to suggest ideas" to help understand unfamiliar concepts by generating ideas, and engaging in noticing to focus on elements or features to understand the significance. Second, clues can "confirm ideas" to determine whether an idea fits with the schema that is developing and as such can be situational, or memory-based. Third, clues can be used to "exclude ideas" by eliminating possibilities that prove that thinking does not fit the situation, concept, or that thinking may need modifying (pp. 170–171). In this way porridge words "can serve as a starting point for a new line of thought" and in doing so, can provide insights into both conceptual understanding and possible alternate conceptions.

Conclusion

Communicating mathematical thinking is a core part of 'doing mathematics' providing insights into students' conceptual understanding of mathematics. However, the processes involved in communicating mathematical thinking can often be invisible. This paper reported a small-scale qualitative study examining primary students' mathematical thinking. The use of children's drawings, written descriptions, and interview responses provided illustrative case studies that moved beyond the identification of the aspects of mathematical thinking to understanding how mathematical thinking is enacted in the primary years. The study was situated in a small regional South Australian school which delimits the study to a very specific context. Studying students at a single school enriched the identification and classification of students' mathematical thinking across the primary years of schooling. The use of de Bono's (1971) porridge words may resonate with a wider context and prove to be invaluable in capturing students' mathematical thinking. An aim of applying porridge words to students' mathematical thinking was to make an ambiguous aspect of mathematical practice transparent and relatable. Further research into how why students use porridge words is warranted in a larger variety of contexts. Having participants review their explanations and their use of porridge words would yield further information about the use of porridge words. Additionally, research that explores how teachers use porridge words and how they address the occurrence of porridge words in a class context is recommended.

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