CANADIAN MATHEMATICS EDUCATION STUDY GROUP

GROUPE CANADIEN D'ÉTUDE EN DIDACTIQUE DES MATHÉMATIQUES

PROCEEDINGS / ACTES
2022 ANNUAL MEETING / RENCONTRE ANNUELLE 2022

Virtual Meeting
May 27 – 29, 2022

EDITED BY:
Jennifer Holm, *Wilfrid Laurier University*
Charlotte Megroureche, *Manchester Metropolitan University*

Proceedings of the 2022 Annual Meeting of the
Canadian Mathematics Education Study Group /
Groupe Canadien d'Étude en Didactique des Mathématiques
are published by CMESG/GCEDM.
Published in June 2024.

ISBN: 978-1-7771235-3-6

PROCEEDINGS OF THE 2022 ANNUAL MEETING OF THE CANADIAN MATHEMATICS EDUCATION STUDY GROUP / ACTES DE LA RENCONTRE ANNUELLE 2022 DU GROUPE CANADIEN D'ÉTUDE EN DIDACTIQUE DES MATHÉMATIQUES

45th Annual Meeting
Virtual Meeting
May 27 – 29, 2022

CONTENTS / TABLES DES MATIÈRES

LISA LUNNEY BORDEN v *Introduction*

vii *Schedule / Horaire*

PLENARY LECTURES / CONFÉRENCES PLÉNIÈRES

EDITH PETITFOUR 5 *Quel enseignement de la géométrie pour les élèves dyspraxiques ?*

WORKING GROUPS / GROUPES DE TRAVAIL

JEANNE KOUDOGBO & MARC HUSBAND 23 A • *Contenu et pratiques pour la formation initiale et continue des enseignants : Un regard plus approfondi sur les potentiels, les défis, les pièges et les perspectives / Content and practices for pre-service and in-service teacher education: A deeper look into the potentials, challenges, pitfalls, and prospects*

GURPREET SAHMBI, MAHATI KOPPARLA & MAHTAB NAZEMI 35 B • *Critical mathematics working group: Changing mathematics to fit our whole selves / Les mathématiques critiques : On change les mathématiques pour s'adapter à nous-mêmes*

OLGA FELLUS, STEVEN KHAN & STÉPHANIE LA FRANCE 41 C • *Weaving identity in mathematics education: Fads, fictions, fibers, and freedoms / Le tissage d'une identité dans l'enseignement des mathématiques : les modes, les histoires, les ficelles et les libertés*

LAUREN DEDIEU & ANALIA BERGÉ 59 D • *Assessment in undergraduate mathematics / Évaluation en mathématiques au postsecondaire*

DORIS JEANNOTTE & HELENA P. OSANA 65 E • *Matériel de manipulation dans l'apprentissage et l'enseignement des mathématiques au primaire / Manipulatives in elementary mathematics teaching and learning*

JUDY LARSEN, JIMMY PAI & MÉLANIE TREMBLAY 75 F • *Facilitating learning mathematics online / Favoriser l'apprentissage des mathématiques en ligne*

NEW PHD REPORTS / PRÉSENTATIONS DE THÈSES DE DOCTORAT

FATIMA ASSAF 91 *Understanding multilingual learners' mathematical experiences and meaning making in a Canadian educational setting*

NATHALIE BISAILLON 99 *Développement du sens du nombre et de la numération : élaboration d'un outil d'évaluation et d'une séquence didactique*

PAMELA BRITTAIN 107 *Addressing math content knowledge and math anxiety in a teacher education program*

TYE CAMPBELL 115 *Examining discourse practices that support middle grade students to learn mathematics in small groups*

JUDY LARSEN 119 *Mathematics teaching and social media: An emergent space for resilient professional activity*

MINNIE LIU 127 *Mathematical modelling – reducing reality or reducing complexity?*

LIXIN LUO 135 *Towards recursive mathematics curricula: A complexified hermeneutic journey*

MAVIS OKYERE 143 *Affordances of culturally responsive teaching through Adinkra symbols on the emergence of students' mathematical proficiency*

MATHIEU THIBAULT 151 *Aperçu d'une recherche-formation à l'enseignement des probabilités du secondaire avec des outils technologiques*

KWESI YARO 159 *Understanding African immigrant families' support for their children's mathematics learning in Canada*

APPENDICES / ANNEXES

167 A • *Working Groups at each Annual Meeting*

175 B • *Plenary Lectures at each Annual Meeting*

179 C • *Proceedings of Annual Meetings*

INTRODUCTION

Dr. Lisa Lunney Borden – President, CMESG/GCEDM
St. Francis Xavier University

Another year of Covid-19 meant another virtual meeting but in true CMESG fashion, we made the best of the situation and found ways to learn together and share connections. Having learned a lot from our 2021 virtual meeting, the executive was able to once again offer a virtual program that, although reduced from our normal offerings, ensured an opportunity for spirited discussions, meaningful learning, and celebrating our newest members.

In our virtual plenary, we welcomed Edith Petitfour who joined us from France to share ideas about how to create more inclusive practices relating to the learning of geometry. Later in our small group discussion sessions many interesting questions were generated that created a fruitful follow-up discussion. The Working Groups were well-attended and well-facilitated. There were many rich ideas generated that are reflected in Working Group reports in these proceedings. It is no simple task to manage a Working Group, and even more complex when trying to navigate it online with multiple strategies and tools for collecting ideas and encouraging small group conversations. Working Groups explored questions of teacher learning, critical mathematics education, identity, assessment in post-secondary mathematics, and the use of manipulatives in mathematics. I always appreciate how these Working Group conversations generate important ideas that influence the work we do in our research and teaching. And speaking of academic work, one of the highlights of every CMESG is to welcome our new PhD graduates. This year we had ten new PhD presentations, and we continued the process of having them pre-record sessions so that we could focus our time together on conversations and questions. We will continue to do this when we return to in-person conferences to allow more time for discussing and celebrating their research accomplishments. And of course, no CMESG would be complete without informal time to socialize and catch up with old friends and new. The virtual hangout sessions were also well-attended and offered members a great experience to engage socially with one another.

The challenge of a virtual meeting is that there is no local organizing committee to share the load with respect to planning so the executive took the lead on making this conference happen. Special thanks to outgoing president, Peter Liljedahl, and members of the executive team Limin Jao, Alayne Armstrong, Bernardo Galvão-Sousa, Claudia Corriveau and Laura Broley. It is no small task to organize such a wonderful event. Having joined in the process nearing the end, after becoming president-elect, I saw first-hand how hard folks were working to make the conference a success.

Une autre année de Covid-19 signifiait une autre réunion virtuelle, mais dans la plus pure tradition du GCEDM, nous avons tiré le meilleur parti de la situation et trouvé des moyens d’apprendre ensemble et de partager des connexions. Ayant beaucoup appris de notre réunion virtuelle de 2021, l’exécutif a été en mesure de proposer un programme virtuel qui, bien que réduit par rapport à nos offres habituelles, a à nouveau garanti une opportunité pour des discussions animées, des apprentissages significatifs et pour la célébration de nos nouveaux membres.

Lors de notre plénière virtuelle, nous avons accueilli Édith Petitfour qui nous a joints depuis la France pour partager des idées sur la manière de créer des pratiques plus inclusives liées à l’apprentissage de la géométrie. Ensuite, lors des sessions de discussion en petits groupes, de nombreuses questions intéressantes ont été soulevées et ont donné lieu à une discussion de suivi fructueuse. Les groupes de travail ont été bien fréquentés et bien animés. De nombreuses et riches idées ont été générées et sont reflétées dans les rapports des groupes de travail dans ce compte-rendu. La gestion d’un groupe de travail n’est pas une tâche simple, et elle est encore plus complexe lorsqu’il s’agit de naviguer en ligne avec de multiples stratégies et outils pour collecter des idées et encourager les conversations en petits groupes. Les groupes de travail ont exploré les questions de la formation des enseignants, de l’enseignement critique des mathématiques, de l'identité, de l'évaluation des mathématiques dans l'enseignement postsecondaire et de l'utilisation du matériel de manipulation en mathématiques. J'apprécie toujours la façon dont les conversations de ces groupes de travail génèrent des idées importantes qui influencent le travail que nous faisons dans notre recherche et notre enseignement. En ce qui concerne le travail universitaire, l'un des temps forts de chaque GCEDM est l'accueil de nos nouveaux doctorants. Cette année, nous avons eu dix présentations de nouveaux docteurs et nous avons poursuivi le processus de pré-enregistrement des sessions afin de pouvoir concentrer notre temps ensemble sur les conversations et les questions. Nous continuerons à le faire lorsque nous reviendrons aux conférences en personne, afin de consacrer plus de temps à la discussion et à la célébration de leurs réalisations en matière de recherche. Et bien sûr, aucun GCEDM ne serait complet sans un temps informel pour socialiser et retrouver les anciens amis et les nouveaux. Ces sessions de socialisation virtuelle ont également été bien suivies et ont offert aux membres une expérience formidable pour s'engager socialement les uns avec les autres.

Le défi d'une réunion virtuelle est qu'il n'y a pas de comité d'organisation local pour partager la charge en ce qui concerne la planification, de sorte que l'exécutif a assumé la réalisation de cette conférence. Nous remercions tout particulièrement le président sortant, Peter Liljedahl, et les membres de l'équipe exécutive, Limin Jao, Alayne Armstrong, Bernardo Galvão-Sousa, Claudia Corriveau et Laura Broley. L'organisation d'un événement aussi merveilleux n'est pas une mince affaire. Ayant rejoint le processus vers la fin, après être devenue présidente élue, j'ai vu de mes propres yeux à quel point les gens travaillaient dur pour faire de la conférence un succès.

Horaire

Jeudi 26 mai	Vendredi 27 mai	Samedi 28 mai	Dimanche 29 mai
	11h00 – 12h00 **Session pour nouveaux membres**		
13h00 – 14h30 **Rencontre de conseil d'administration de FLM**	13h00 – 15h00 **Plénière** **Présentation des groupes de travail** **Introduction**	13h00 – 15h00 **Groupes de travail**	13h00 – 15h00 **Groupes de travail**
15h00 – 16h00 **FLM Q&R (anciennement connu sous le nom Amis de FLM)** Bienvenue à tous	15h00 – 16h00 **Pause**		15h00 – 16h00 **Séances *ad hoc***
	16h00 – 18h00 **Groupes de travail**	16h00 – 18h00 **Nouvelles docteures**	16h00 – 18h00 **Petites groupes – discussion de la plénière** **Q&R avec la conférencière** **Rapports des groupes de travail**
	Activité sociale : Zoom	**Activité sociale : Gather Town**	**AGA**

Schedule

Thursday May 26	Friday May 27	Saturday May 28	Sunday May 29
	11:00 – 12:00 **Newcomers Session**		
13:00 – 14:30 **FLM Board of Directors Meeting**	13:00 – 15:00 **Plenary** **Working Group Presentations** **Introduction**	13:00 – 15:00 **Working Groups**	13:00 – 15:00 **Working Groups**
15:00 – 16:00 **FLM Q&A (formally Friends of FLM)** Everyone welcome	15:00 – 16:00 **Break**		15:00 – 16:00 ***ad hoc***
	16:00 – 18:00 **Working Groups**	16:00 – 18:00 **New PhDs**	16:00 – 18:00 **Plenary Small Group Discussions** **Plenary Q&A** **Working Group Reports**
	Social activity: Zoom	**Social activity: Gather Town**	**AGM**

Plenary Lectures

Conférences plénières

QUEL ENSEIGNEMENT DE LA GÉOMÉTRIE POUR LES ÉLÈVES DYSPRAXIQUES ?

WHAT GEOMETRY TEACHING FOR DYSPRAXIC STUDENTS?

Édith Petitfour[1]
Université de Rouen Normandie
Laboratoire de Didactique André Revuz

Dans l'enseignement de la géométrie plane en France, les constructions instrumentées ont une place importante, notamment au cycle 3 (élèves de 9-11 ans). Les programmes précisent ainsi que :

> *Les situations faisant appel à différents types de tâches (reconnaître, nommer, comparer, vérifier, décrire, reproduire, représenter, construire) portant sur des objets géométriques, sont privilégiées afin de faire émerger des concepts géométriques (caractérisations et propriétés des objets, relations entre les objets) et de les enrichir. Un jeu sur les contraintes de la situation, sur les supports et les instruments mis à disposition des élèves, permet une évolution des procédures de traitement des problèmes et un enrichissement des connaissances. (MEN, 2020, p. 97)*

Selon les programmes scolaires, utiliser des instruments pour analyser une figure ou réaliser des tracés doit donc contribuer à l'acquisition de connaissances géométriques. Cependant, cette méthode d'enseignement n'est pas adaptée aux élèves dyspraxiques, mis en échec lorsqu'ils doivent manipuler du matériel.

Nos travaux de recherche visent à proposer un moyen d'accès à des apprentissages géométriques pour ces élèves. Nous précisons tout d'abord ce que nous entendons par dyspraxie et donnons quelques illustrations des conséquences en classe de ce handicap pour les élèves dans le domaine de la géométrie. Nous présentons ensuite un cadre d'analyse du processus d'accès aux concepts géométriques par la construction instrumentée à partir duquel nous avons élaboré un dispositif alternatif d'enseignement de la géométrie. Nous présentons enfin ce dispositif avec ses fondements et son fonctionnement dans différents environnements de travail.

CONSÉQUENCES DE LA DYSPRAXIE EN GÉOMÉTRIE

Nous qualifions de « dyspraxiques » les élèves porteurs d'un *trouble développemental de la coordination*. Le DSM-5[2] (American Psychiatric Association, 2013) caractérise ce trouble avec les critères suivants :

a) *The acquisition and execution of coordinated motor skills is substantially below that expected given the individual's chronological age and opportunity for skill learning and use. Difficulties are manifested as clumsiness (e.g., dropping or bumping into objects) as well as slowness and inaccuracy of performance of motor skills (e.g., catching an object, using scissors or cutlery, handwriting, riding a bike, or participating in sports).*

---

[1] Normandie Univ, UNIROUEN, Université de Paris, Univ. Paris Est Creteil, CY Cergy Paris Université, Univ Lille, LDAR, 76000 Rouen, France edith.petitfour@univ-rouen.fr

[2] Diagnostic and Statistical Manual of Mental Disorders fifth edition

b) *The motor skills deficit in Criterion A significantly and persistently interferes with activities of daily living appropriate to chronological age (e.g., self-care and self-maintenance) and impacts academic/school productivity, prevocational and vocational activities, leisure, and play.*
c) *Onset of symptoms is in the early developmental period.*
d) *The motor skills deficits are not better explained by intellectual disability (intellectual developmental disorder) or visual impairment and are not attributable to a neurological condition affecting movement (e.g., cerebral palsy, muscular dystrophy, degenerative disorder).* (American Psychiatric Association, 2013, p. 74)

La dyspraxie se manifeste ainsi dans les habiletés gestuelles et affecte les performances scolaires des élèves. Nous donnons quelques exemples de ses conséquences en géométrie issus de nos observations.

Commençons par une tâche simple, celle de relier deux points à l'aide d'une règle, réalisée par Cyril, élève dyspraxique de 10 ans. Cet élève effectue trois essais successifs sans parvenir à un résultat satisfaisant (Figure 1a). Il commence par positionner sa règle un peu trop en dessous des deux points (problème de contrôle visuel), trace, puis se rend compte que le tracé ne convient pas (essai 1). Il réajuste le positionnement de sa règle, mais elle bouge quelque peu au moment du démarrage du tracé (problème de dosage dans la pression de stabilisation de la règle par la main) ; il achève le tracé en s'écartant de la règle pour atteindre le second point (essai 2). Il repositionne la règle, la maintient fermement, mais démarre le tracé un peu après le premier point et l'achève plus loin que le second en allant au-delà de la règle (problème de contrôle de la vitesse de tracé) (essai 3). Ainsi, Cyril ne parvient pas à gérer simultanément les gestes qui mèneraient à la production qu'il souhaite.

Nous pouvons observer, pour ce même type de tâche réalisé par Line, élève dyspraxique, des choix de manipulation peu propices à la réussite du tracé, par exemple, la règle maintenue à son extrémité par trois doigts et un tracé le long de son bord inférieur, la main crispée sur le crayon tenu verticalement (Figure 1b). En conséquence, la règle bouge, le crayon s'écarte de la règle pendant le tracé, le crayon crisse, fatigue et douleurs musculaires s'ensuivent pour l'élève. Notons aussi que le bord inférieur de la règle n'a été ajusté que sur un des deux points. Tout comme Cyril, Line ne parvient pas à organiser ses gestes pour manipuler la règle de façon efficace afin d'obtenir le tracé souhaité.

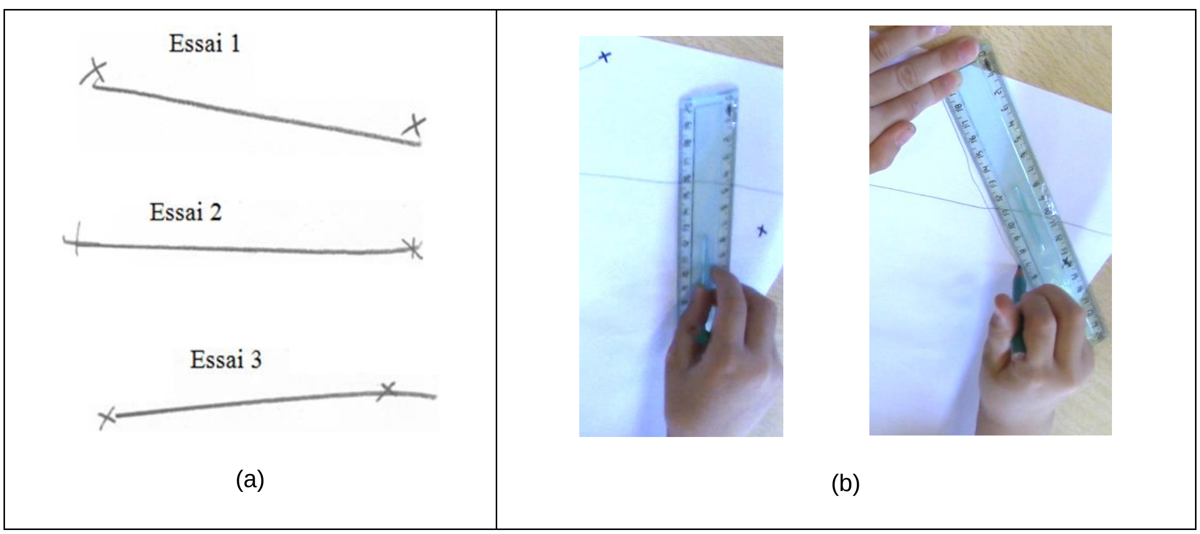


Figure 1. Tracés de segments à la règle.

Continuons par un exemple de manifestation des difficultés d'une élève dyspraxique de 11 ans, Marion, avec un extrait d'une de ses évaluations faites en classe (Figure 2) : à gauche la construction du symétrique du point L par rapport à la droite (d), à droite celle du symétrique de la droite (△) par rapport à la droite (d). Le codage des angles droits et les traits de compas révèlent une bonne prise en compte des propriétés de la symétrie pour réaliser les constructions. Dans ses commentaires, le professeur relève la *justesse*[3] de la technique utilisée (« oui, tu as compris »), mais sanctionne le

[3] Nous qualifions de *juste* une figure construite avec l'utilisation de propriétés géométriques portées par des instruments appropriés. Ici, l'élève a construit un angle droit en utilisant l'angle droit de son équerre, elle ne l'a pas réalisé en plaçant sa règle visuellement.

manque de précision dans le positionnement des instruments (« mal prolongé », « l'équerre est mal placée »). Comme pour Cyril et Line, nous observons un écart entre le projet de tracé et la production obtenue.

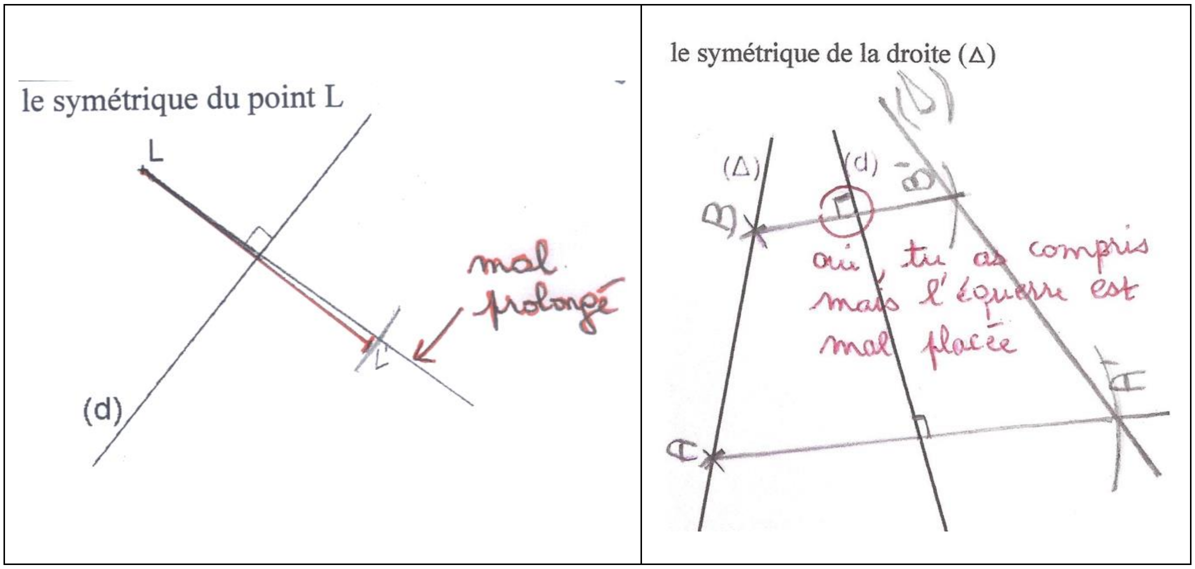


Figure 2. Constructions géométriques.

Terminons enfin par la résolution d'un problème géométrique réalisé par Thierry, élève dyspraxique de 11 ans (Figure 3). Même si cet élève a bien utilisé les instruments (équerre et compas) permettant de produire les propriétés géométriques voulues, son dessin final obtenu est peu lisible pour être support du raisonnement. De plus, une forte dysgraphie compromet toute communication par écrit.

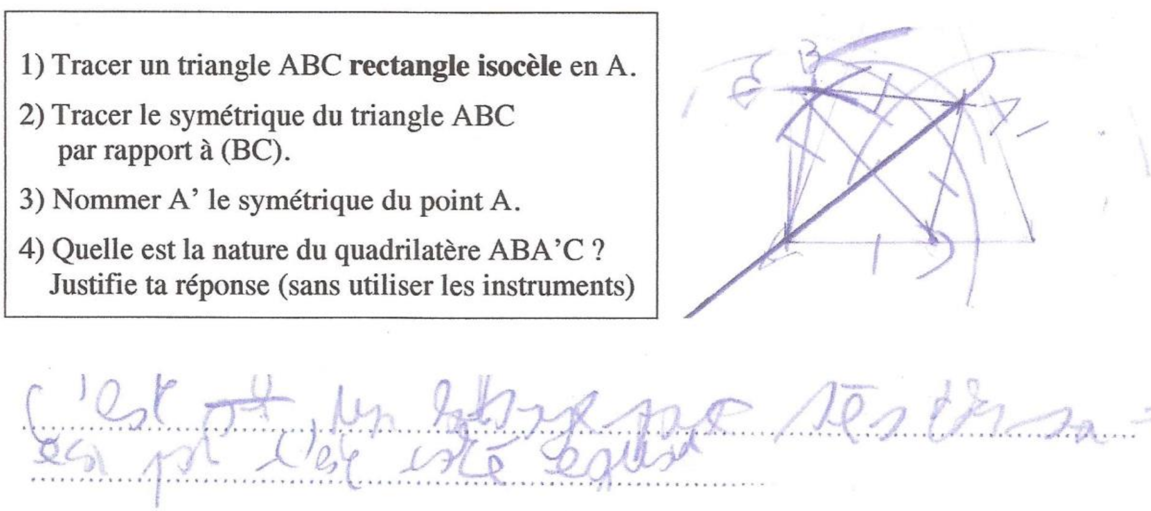


Figure 3. Résolution d'un problème géométrique.

Ces quelques exemples illustrent les difficultés que les élèves dyspraxiques rencontrent dans des travaux géométriques sollicitant des manipulations d'instruments. Il ne s'agit pas de difficultés de compréhension qui pourraient faire obstacle au raisonnement. Ainsi que le soulignent Mazeau et Le Lostec (2010), l'élève dyspraxique est mis en difficulté par la méthode d'enseignement, le matériel pédagogique utilisé, et non par les connaissances ou le concept à acquérir. Cela nous a amenées à chercher d'autres moyens d'enseigner la géométrie en nous interrogeant d'abord sur les apports que pouvait avoir la construction instrumentée dans les apprentissages géométriques. Nous explorons cette question dans la partie suivante.

ANALYSE DU PROCESSUS D'ACCÈS AUX CONCEPTS GÉOMÉTRIQUES

Nous présentons notre *cadre d'analyse de l'action instrumentée* (Petitfour, 2017a) en donnant déjà un aperçu des concepts des sciences cognitives sur lequel il s'appuie. Nous mettons ensuite en évidence les connaissances en jeu dans le processus d'accès aux concepts géométriques à l'aide du cadre d'analyse.

CADRE D'ANALYSE DE L'ACTION INSTRUMENTÉE

Apports des sciences cognitives

Dans le champ de la neuropsychologie (Mazeau et Pouhet, 2014), une action, ou enchaînement de gestes, est un ensemble intentionnel de mouvements coordonnés dans le temps et dans l'espace, défini par l'intention de son auteur qui en décide l'exécution en fonction de sa finalité. L'action comporte un versant cognitif et un versant moteur. La phase cognitive de l'action comprend ses aspects préparatoires : l'intention d'agir (le sujet se représente le but poursuivi), l'intention motrice qui comprend deux phases imbriquées, à savoir la planification (organisation temporelle de l'action) et la programmation (organisation spatiale et motrice), des régulations (simulations anticipatrices de l'action et ajustements en cours de réalisation de l'action) et la prise de décision du passage à l'acte. La phase motrice de l'action découle de la prise de décision du sujet : le processus moteur se déroule, avec la mise en route des organes effecteurs, sensoriels et moteurs et le résultat prévu se produit.

Dans le champ de l'ergonomie cognitive, l'approche instrumentale (Rabardel, 1995) permet d'analyser le rapport instrumental du sujet. Un artefact, objet matériel ou symbolique avec des caractéristiques permettant d'assurer l'accomplissement de buts spécifiques, devient un instrument au cours d'un processus de genèse instrumentale qui consiste en l'élaboration de schèmes d'utilisation de cet artefact. Les schèmes (Vergnaud, 1990) constituent l'organisation invariante de la conduite du sujet pour une classe de situations, sur le plan de l'action tout comme sur celui de l'activité symbolique. Ils comportent des règles de production des actions, ainsi que des anticipations des effets à obtenir. Au sein de ces schèmes d'utilisation, les schèmes d'action instrumentée sont relatifs aux tâches principales pour lesquelles l'artefact est un moyen de réalisation, et les schèmes d'usage sont orientés vers des tâches secondes, relatives à la gestion des caractéristiques de l'artefact et qui peuvent dans certains cas comprendre des buts propres.

Action instrumentée

Dans le domaine de la géométrie, nous entendons par action instrumentée l'action d'un sujet qui, dans un environnement de travail, utilise un objet technique pour produire un objet graphique représentant un objet géométrique ou pour analyser une propriété d'un objet graphique traduisant une relation géométrique. Lorsque l'environnement de travail est numérique, les objets techniques sont matériels (clavier d'un ordinateur, par exemple) et numériques (outils d'un logiciel de géométrie dynamique, par exemple), les objets graphiques sont sur un écran. Dans le cas de l'environnement papier-crayon, les objets techniques sont uniquement matériels (équerre, compas, etc.) et les objets graphiques sont tracés sur une feuille de papier.

Au niveau cognitif, nous considérons que le projet de réaliser une action instrumentée est constitué par une double intention de la part du sujet : l'intention d'agir et l'intention motrice. L'intention d'agir concerne une anticipation de l'action dans laquelle le sujet se représente le but poursuivi (produire ou analyser l'objet graphique représentant l'objet géométrique) et le projet de l'action à réaliser avec l'objet technique pour atteindre ce but. Le projet de l'action consiste en la planification d'un enchaînement d'actions élémentaires (choisir l'objet technique, le positionner, tracer ou analyser une propriété) s'appuyant sur des schèmes d'utilisation de l'objet technique. L'intention motrice, générée par l'intention d'agir, est constituée de la programmation et de la planification de l'action contextualisée dans un environnement avec un objet technique particulier. La programmation de l'action s'appuie sur des schèmes d'usage de l'objet technique. Au niveau moteur, se réalise l'exécution de l'action instrumentée principale et des actions périphériques à cette action si nécessaires.

La Figure 4 modélise l'action instrumentée telle que nous venons de la présenter. L'environnement est représenté par un rectangle au sein duquel se trouvent le corps du sujet, les objets techniques et les objets graphiques. Les doubles flèches représentent les différentes relations existantes.

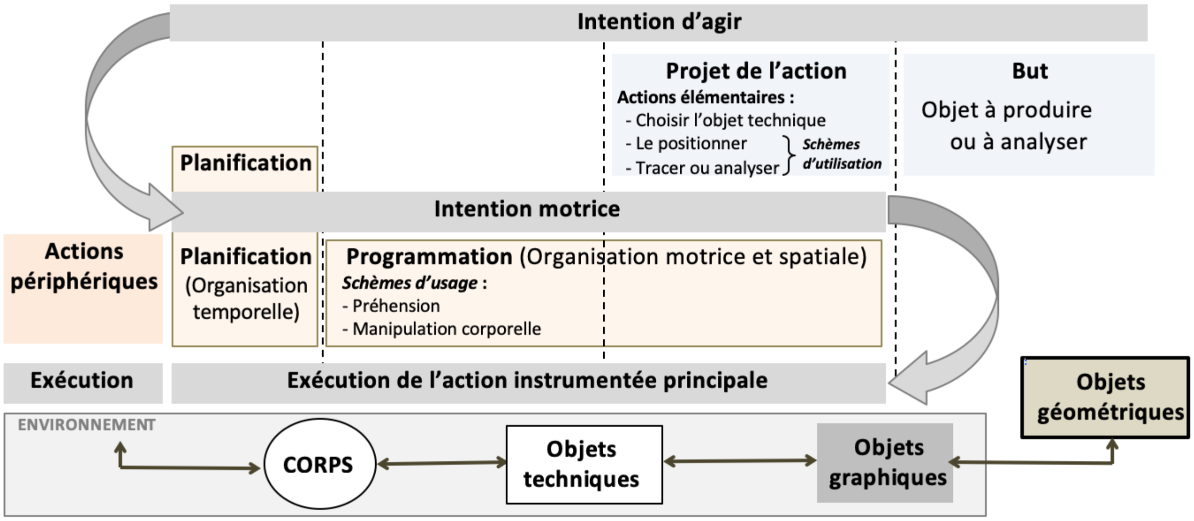


Figure 4. Modélisation de l'action instrumentée.

CONNAISSANCES EN JEU

Nous exposons les connaissances en jeu au sein d'une action instrumentée en les illustrant à partir d'une action ayant pour but le tracé du segment [AB], les deux points A et B étant donnés.

Au niveau de l'intention d'agir

Les *connaissances techniques* portent sur la fonction de l'objet technique et ses schèmes d'utilisation construits dans les processus de genèse instrumentale (Rabardel, 1995). Par exemple pour la règle, une fonction est celle de permettre le tracé de traits droits, une autre celle de vérifier des alignements. Pour le tracé du segment [AB], on fixe la règle sur les points A et B et on trace le long de la règle avec le crayon d'un point à l'autre.

Les *connaissances graphiques* portent sur les informations graphiques pertinentes à prélever sur les objets graphiques et à interpréter géométriquement. Elles sont reliées aux tracés, aux codages et aux notations. Par exemple, pour le tracé du segment [AB], il faut savoir qu'un point isolé est représenté par une croix à côté de laquelle est écrit son nom et que le point se situe graphiquement à l'intersection des branches de la croix. Il faut savoir qu'un segment est représenté par un trait droit limité par les deux points qui sont ses extrémités.

Les *connaissances géométriques* sont relatives à la définition des objets géométriques et aux relations qui peuvent exister entre eux (appartenance, perpendicularité, alignement, etc.). Par exemple, pour le tracé du segment [AB], il faut savoir ce qu'est un point, un segment.

Des *connaissances spatiales* et *organisationnelles* sont sollicitées au niveau de l'intention d'agir. Au niveau visuo-spatial, un repérage des objets graphiques est nécessaire et au besoin le recours à un processus de déconstruction dimensionnelle (Duval, 2005), par exemple le repérage d'un point comme extrémité d'un segment, sommet d'un polygone ou intersection de lignes. Il s'agit également de repérer des positions relatives d'objets graphiques à mettre en lien avec des propriétés portant sur des objets géométriques, par exemple, de repérer un alignement possible de trois points. Au niveau organisationnel, une planification des actions élémentaires (d'abord choisir l'objet technique, ensuite le positionner et enfin tracer ou analyser) est également nécessaire.

Au niveau de l'intention motrice

Les *connaissances manipulatoires* sont relatives à la coordination des mouvements et ajustements posturaux réalisés avec l'objet technique dans l'espace, ainsi qu'à la manière de manipuler l'objet technique avec précision et efficacité tant sur le plan matériel que sur le plan corporel dans l'espace de travail. Pour le tracé du segment [AB] à la règle, il faut, par exemple, tenir compte de l'épaisseur de la mine du crayon en décalant un peu la règle des points, il faut aussi une posture du corps adaptée pour faciliter le tracé, un certain dosage dans les appuis et la vitesse de tracé, une position adéquate des mains sur la règle pour la maintenir tout en traçant, etc.

Les *connaissances organisationnelles* concernent la planification des actions en concevant l'organisation selon un plan déterminé. Certains gestes sont simultanés, par exemple, maintenir la règle tout en déplaçant le crayon le long, d'autres sont successifs. Des actions périphériques à l'action instrumentée doivent aussi être réalisées en amont comme se procurer crayon et règle, tailler son crayon, organiser son espace de travail.

Les *connaissances spatiales* concernent la réalisation d'une analyse visuelle pour prélever des informations spatiales (repérage d'entités graphiques, repérage de parties de l'objet technique, repérage de parties corporelles, repérage d'éléments de l'espace extracorporel). Elles concernent aussi la représentation des relations spatiales et leur interprétation (anticiper la position relative de l'objet technique, des objets graphiques et du corps, anticiper la production graphique finale sur le support). Pour le tracé du segment [AB], par exemple, il faut repérer les deux points A et B, anticiper le lieu du tracé pour placer la règle, effectuer des choix pour pouvoir réaliser le tracé dans une position confortable (par exemple éviter un croisement de mains pour tracer).

DISPOSITIF ALTERNATIF POUR ENSEIGNER LA GÉOMÉTRIE

RÉFLEXION SUR UNE ALTERNATIVE

À la lumière de la caractérisation des troubles des élèves dyspraxiques (partie 1) et de l'analyse du processus d'accès aux concepts géométriques (partie 2), nous pouvons voir que les difficultés des élèves dyspraxiques se situent au niveau de l'intention motrice de l'action instrumentée et qu'elles se répercutent dans la réalisation effective de cette action. Mazeau (2005, 2020) précise que, malgré un entraînement, même intensif, ces élèves ne sont pas en capacité d'automatiser certains gestes utiles à la réussite de leurs actions, ce qui va les mettre dans une situation de double tâche à laquelle ils ne pourront faire face.

> *L'enfant dyspraxique est donc un enfant qui, en dépit d'un enseignement et d'un entraînement habituels, ne peut pas, en raison d'une atypie développementale, inscrire cérébralement le schéma de certains gestes*. (Mazeau, 2005, p. 11)
>
> *Si la tâche de bas niveau [tels la lecture ou le graphisme] n'est pas automatisée—ce qui est le cas du jeune dys—absorbant toute la charge mentale disponible, il ne lui restera plus assez de ressources pour la tâche seconde qui est généralement la tâche cible lors d'un apprentissage*. (Mazeau, 2020, pp. 144-145)

Nous avons donc écarté les pistes de remédiation qui consisteraient à surentraîner les élèves à utiliser leurs instruments, même en leur proposant du matériel ergonomique (règle antidérapante munie d'une poignée, compas avec système de blocage des branches, etc.). Nous pointons d'ailleurs un risque de malentendu pour les élèves sur les enjeux d'apprentissage dans des séances qui pourraient leur être proposées pour améliorer leurs constructions instrumentées, avec une centration sur l'acquisition de connaissances manipulatoires plutôt que de connaissances géométriques. En témoigne cet extrait de l'intervention de la professeure de mathématiques de Marion (dont nous avons présenté des constructions en figure 2) lors d'une séance d'aide personnalisée : « Attends avant de tracer, regarde toujours si ton équerre, là tu ne vois pas bien, voilà, parce que, doucement, doucement, ne va pas trop vite, là on ne voit pas bien hein, tu vas un peu trop loin, on ne voit pas du coup si t'es, attends, attends avant de tracer, voilà, là t'es bien le long du segment, maintenant il faut que tu prévoies, attends, calme-toi, tu veux aller trop vite, il faut que tu prévoies que ton crayon va avoir une certaine épaisseur, d'accord ? Donc là quand tu as posé… ».

Nous n'avons pas non plus investi la piste de l'exploitation d'outils numériques allégeant les aspects manipulatoires, dans des conditions de classe où un élève dyspraxique utilise ces outils tandis que le reste de la classe travaille dans l'environnement papier-crayon. Nos observations en classe et nos échanges avec des enseignants nous ont,en effet, conduites à penser que ce type d'adaptation aurait peu de chances de répondre aux objectifs visés (Petitfour, 2015a).

Nous avons élaboré un dispositif de travail en exploitant le fait que les élèves dyspraxiques n'ont pas de difficulté spécifique au niveau de leur *intention d'agir*, précisément là où se développent les connaissances géométriques. En outre, nous avons supprimé de l'action instrumentée des élèves dyspraxiques tous les aspects relatifs à *l'intention motrice* et à l'exécution de l'action, pour leur permettre d'exprimer leur *intention d'agir*, en appui sur leurs compétences préservées que sont le raisonnement, la mémoire et le langage oral. Le dispositif de travail peut être mis en œuvre dans un environnement numérique, avec un logiciel de géométrie dynamique, par exemple, ou dans l'environnement papier-crayon. Nous présentons d'abord ce dispositif avec ses fondements et son fonctionnement. Nous l'illustrons ensuite avec un exemple d'expérimentation menée avec deux élèves, hors classe (Petitfour, 2015b).

TRAVAIL EN DYADE

Le dispositif de travail en dyade (Petitfour, 2017b, 2022) vise à offrir des opportunités d'apprentissages géométriques aux élèves dyspraxiques. Il vise aussi à réduire, pour tout élève, le saut cognitif existant dans l'enseignement entre une géométrie instrumentée (pratique de dessins aux instruments) et une géométrie plus discursive (élaboration de programmes de construction, démonstration). Il est basé sur un travail géométrique dans lequel deux élèves collaborent dans le cadre de constructions instrumentées, un élève étant instructeur et l'autre constructeur. Ce dispositif est ancré dans une conception de l'apprentissage comme phénomène social (Vygotski, 1931/1978) : la formation des concepts mathématiques se réalise à travers des interactions sociales, dans un travail conjoint autour de la résolution d'un problème.

Le principe consiste à remplacer les situations d'actions instrumentées pour l'élève dyspraxique par des situations au plus proches de l'action, mais sans action effective, tout en produisant des effets analogues au niveau des apprentissages géométriques. Au sein de la dyade, l'élève dyspraxique, instructeur, exprime son *intention d'agir* par des instructions langagières, accompagnées de gestes. L'autre élève, constructeur, exécute les actions instrumentées demandées. L'élève dyspraxique éprouve alors ces actions par leur observation, sans se préoccuper de leurs caractéristiques manipulatoires et organisationnelles. Il bénéficie aussi d'une rétroaction concordante avec ses instructions, sur laquelle peut s'appuyer sa réflexion. Le dispositif fait ainsi le sacrifice d'un développement de l'autonomie matérielle en classe de l'élève dyspraxique, au profit d'une autonomie de son activité intellectuelle centrée sur les apprentissages géométriques.

Nous apportons maintenant des précisions sur la communication envisagée entre instructeur et constructeur. Notons que nous concevons le langage et les gestes non seulement comme moyen de communication, mais aussi comme ayant un rôle constitutif dans la pensée, ainsi que l'expriment ces auteurs :

> *Thought is not merely expressed in words; it comes into existence with them.* (Vygotski, 1934-1986, p. 218)
>
> *Gestures do not just reflect thought but have an impact on thought. Gestures, together with language, help constitute thought.* (McNeill, 1992, p. 242)

<u>Quel langage pour les instructions ?</u>

Le langage géométrique permet d'exprimer le but final de l'action instrumentée, à savoir l'objet ou la relation géométrique que l'on souhaite produire ou analyser avec l'instrument. Cependant ce langage ne peut pas être maîtrisé d'emblée au début de l'apprentissage d'un concept géométrique. L'usage du langage géométrique est encouragé progressivement, mais au départ nous proposons l'emploi d'un *langage technique* (Petitfour, 2017b) permettant de formuler le projet de l'action (cf. Figure 4). Dans le domaine de la géométrie, ce langage est relatif à l'usage des instruments en lien avec les propriétés géométriques dont ils sont porteurs. Il permet d'exprimer la mise en relation de tracés géométriques avec un instrument donné pour produire un nouveau tracé ou vérifier une propriété géométrique. Pour un tracé donné, il s'agit de préciser l'instrument à utiliser, son positionnement et le tracé souhaité. Notons qu'un guidage manipulatoire du type « Avance un peu l'équerre » ou « Stop » est exclu du langage technique, de même que les termes spatiaux comme « à gauche » ou « en dessous ».

Par exemple, une droite d et un point M étant donné, le tracé de la perpendiculaire à la droite d passant par le point M (formulation en langage géométrique) correspond aux instructions suivantes en langage technique : (1) Prends l'équerre. (2) Place un côté de l'angle droit de l'équerre sur la droite d et l'autre côté de l'angle droit sur le point M. (3) Trace une droite passant par le point M le long de ce côté de l'équerre. Ce langage ne doit pas être confondu avec un langage à visée manipulatoire (« Maintiens fermement l'équerre », « Décale un peu l'équerre pour tenir compte de l'épaisseur de la mine du crayon ») ou organisationnelle (« Taille ton crayon »).

<u>Quels gestes pour communiquer ?</u>

Suivant McNeill (1992) en ce que les gestes, en collaboration avec le langage, aident à constituer la pensée, nous proposons d'associer au langage oral l'emploi de *gestes mathématiques* (Petitfour, 2016), à savoir tout mouvement corporel porteur d'une signification en lien avec les mathématiques. En géométrie, ces gestes peuvent évoquer un objet géométrique, une relation ou propriété géométrique, comme la symétrie de figures (Figure 5a : retournement d'une main pour la superposer sur l'autre) ou des droites sécantes (figure 5b : les deux index suivent chacun une direction jusqu'à se croiser). Ils peuvent être liés à un objet graphique (figure 5c : l'élève utilise ses mains pour relier un point aux extrémités d'un segment). Ils peuvent permettre de mettre en lien les objets graphiques (des éléments du dessin) avec l'objet technique (l'instrument) : sur la Figure 5d, des gestes de parcours montrent un côté de l'angle droit de l'équerre à mettre sur la droite *d* et des gestes de pointage le sommet de l'angle droit de l'équerre à mettre sur le point *M*. Ils peuvent évoquer enfin une manipulation d'un objet technique : l'élève mime une utilisation du compas avec l'instrument (Figure 5e) ou avec ses doigts en guise de compas (Figure 5f) ; la Figure 5a peut aussi évoquer l'action de plier une feuille pour analyser la symétrie de deux figures par transparence.

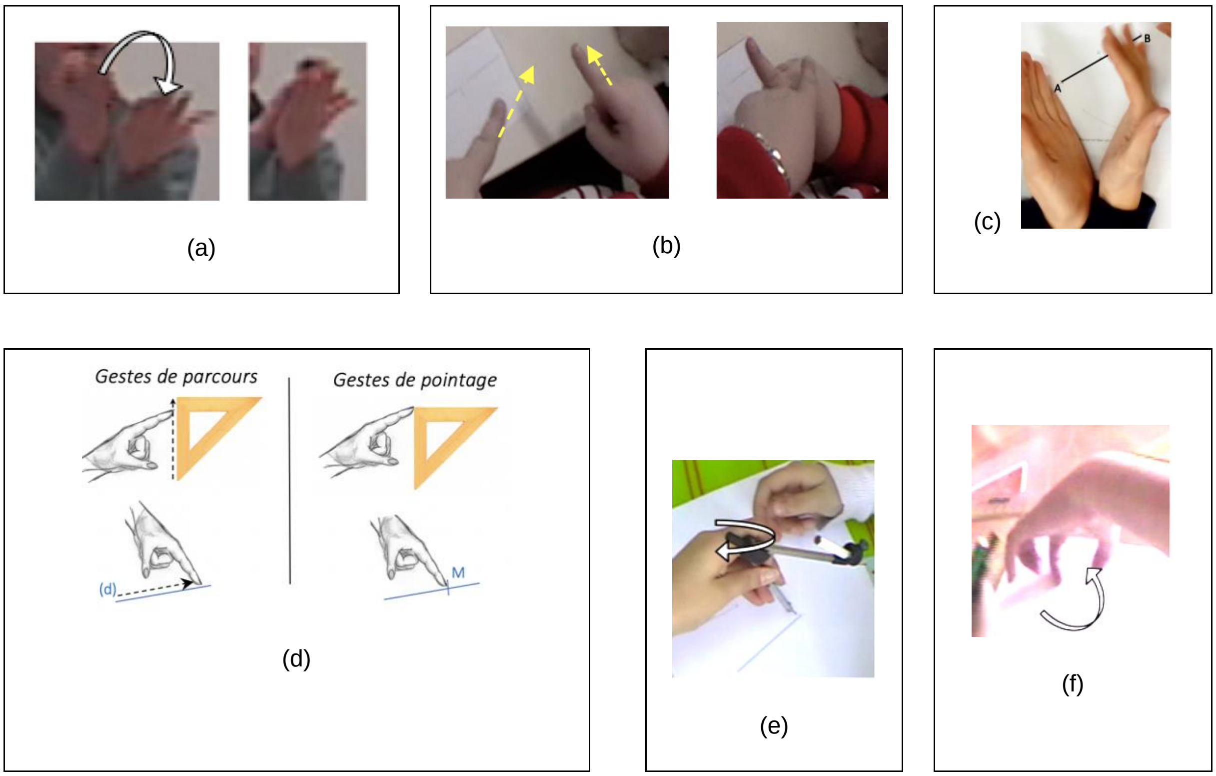


Figure 5. Exemples de gestes mathématiques.

<u>Quelles rétroactions ?</u>

Le constructeur manipule les instruments en suivant les instructions données au fur et à mesure par l'instructeur : il prend l'instrument, le positionne et réalise le tracé comme demandé. Il s'assure que ce qu'il fait corresponde bien à ce que souhaite l'instructeur. Il prend à sa charge la précision des tracés. Lors de l'étape de positionnement de l'instrument, il suit scrupuleusement les instructions, tout en essayant de ne pas répondre aux attentes de l'instructeur si elles n'ont pas été formulées explicitement. Il s'agit ainsi pour lui d'agir en « faisant le moins probable » sans toutefois « faire preuve de mauvaise foi ». Le but est d'impliquer pleinement le constructeur dans la mise en jeu des connaissances géométriques lors de la construction : il doit réfléchir à la validité des instructions permettant d'obtenir une *figure juste* (cf. note 3 de bas de page). Le but est aussi de renvoyer des rétroactions immédiates sur la qualité des formulations proposées par l'instructeur, ce dernier devant tenter de les améliorer en levant les implicites, si besoin.

Reprenons l'exemple, du tracé de la droite perpendiculaire à la droite *d* passant par le point *M*, une droite *d* et un point *M* étant donné. Si l'instructeur demande de placer l'équerre sur la droite d, le constructeur peut la positionner, par exemple, comme sur la Figure 6a : il place bien un côté de l'équerre (donc l'équerre) sur la droite *d* mais pas un côté de l'angle droit de l'équerre comme souhaité implicitement par l'instructeur. Cela amène ce dernier à ajuster sa demande. S'il demande ensuite le placement d'un côté de l'angle droit de l'équerre sur la droite d, le constructeur fera en sorte de ne pas faire passer l'autre côté de l'angle droit sur le point *M* comme attendu, puisque cela n'a pas été demandé (Figure 6b).

Figure 6. Le constructeur agit en « faisant le moins probable ».

EXPÉRIMENTATION DU DISPOSITIF

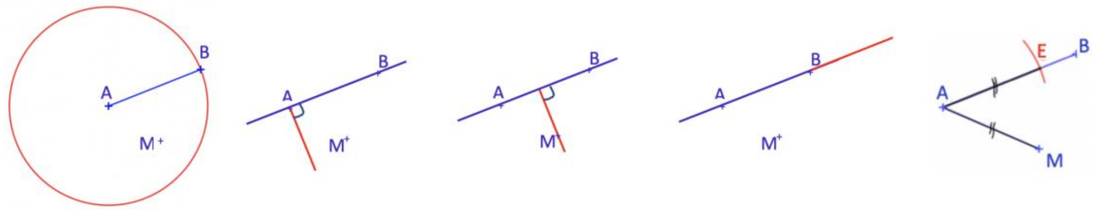
Nous présentons une mise en œuvre du dispositif de travail en dyade réalisée dans l'environnement papier-crayon expérimenté avec deux élèves de 11 ans : Marion, diagnostiquée avec une dyspraxie visuo-spatiale, et Bonnie, élève de sa classe. Les troubles des praxies entraînent pour Marion des difficultés dans la planification, la programmation et l'anticipation de ses actions tel que nous l'avons présenté dans la partie 1. La figure 2 donne un exemple de ses constructions instrumentées réalisées lors d'une évaluation en classe. Les troubles visuo-spatiaux ont pour conséquences en géométrie des difficultés dans la lecture de textes et dans l'appréhension globale des figures. Ainsi que l'explique Mazeau :

> *Les yeux ne peuvent pas explorer la figure, ou le modèle, de façon cohérente et régulière. Ils « sautent » sans logique d'un endroit à un autre, se posant à plusieurs reprises sur certains secteurs et en ignorant d'autres, sans percevoir l'ensemble ni l'organisation spatiale des diverses parties.* (dans Crouail, 2009, p.60).

L'expérimentation a été réalisée hors classe, tout en étant en lien étroit avec les séances de géométrie de la classe auxquelles nous assistions pour faire des observations. Nous présentons le déroulement des deux premiers temps de l'expérimentation permettant d'initier les élèves au travail en dyade et un extrait du troisième temps dans lequel ce dispositif est mis en œuvre avec la réalisation d'une construction géométrique.

Dans un premier temps, nous revenons avec les deux élèves sur le rôle des instruments (règle non graduée, compas et équerre) et leurs usages spécifiques. Nous proposons une appropriation du vocabulaire technique en associant aux termes techniques une exploration gestuelle des parties d'instruments à mettre en relation avec les tracés géométriques. Ainsi, on parcourt le « bord droit de la règle » avec le doigt, bord qui serait à mettre sur un segment que l'on souhaiterait prolonger ou sur deux points d'une droite que l'on voudrait tracer. On désigne avec le doigt la « pointe du compas », pointe que l'on associerait au centre d'un cercle que l'on voudrait tracer ; on parcourt « l'écart pointe-mine du compas » en ligne droite, longueur qui correspondrait au rayon du cercle. On parcourt avec la main l'« angle droit de l'équerre », que l'on distingue du « sommet de l'angle droit de l'équerre » pointé avec le doigt, et enfin on parcourt avec le doigt un « côté de l'angle droit de l'équerre » (Figure 5d).

Dans un deuxième temps, nous introduisons les règles de communication entre instructeur et constructeur à partir de cinq types d'actions instrumentées élémentaires (Figure 7). Une combinaison de plusieurs actions de différents types permettra par la suite la construction d'une figure complexe avec règle, compas et équerre comme instruments disponibles. Le constructeur dispose des instruments et de la figure de départ : le segment ou la droite (AB) et le point M, représentés en bleu. L'instructeur dispose d'un énoncé constitué de la figure de départ et d'un tracé en rouge à faire réaliser au constructeur. Un codage (en noir) marque la relation de perpendicularité (Figures 7b et 7c) et d'égalité de longueurs (Figure 7e). Chaque action instrumentée est réalisée par Marion et Bonnie en tant que constructrice et en tant qu'instructrice. Lorsque Marion est constructrice, elle positionne les instruments, mais à la place de tracer, elle utilise des gestes : parcours avec un doigt le long de l'équerre ou de la règle, mime de tracé avec ou sans compas (Figures 5e et 5f). Les gestes ont l'avantage de ne pas laisser de traces qui seraient remises en cause pour un manque de précision.



(a) (b) (c) (d) (e)

Figure 7. Actions instrumentées élémentaires.

Le troisième temps consiste à construire, dans un travail en dyade, le symétrique d'une droite (Δ) par rapport à une droite (*d*), sur une figure où les droites (*d*) et (Δ) sont représentées, ainsi qu'un point A appartenant à la droite (Δ) et le point A', symétrique du point A par rapport à la droite (*d*). Ce type de tâches de construction du symétrique d'une droite a déjà été rencontré en classe par Bonnie et Marion (cf. sur la Figure 2 la construction réalisée par Marion un mois et demi plus tôt). Compte tenu des troubles visuo-spatiaux de Marion, nous faisons le choix de transmettre la consigne à l'oral pour ne pas ajouter une tâche de lecture à la tâche géométrique proposée. Nous avons différencié les deux droites par de la couleur (l'axe de symétrie est en rouge) et présentons la figure de départ à l'aide de gestes (parcours des droites, pointage des points) accompagnant le discours. La figure 8 représente ces modalités de passation de la consigne.

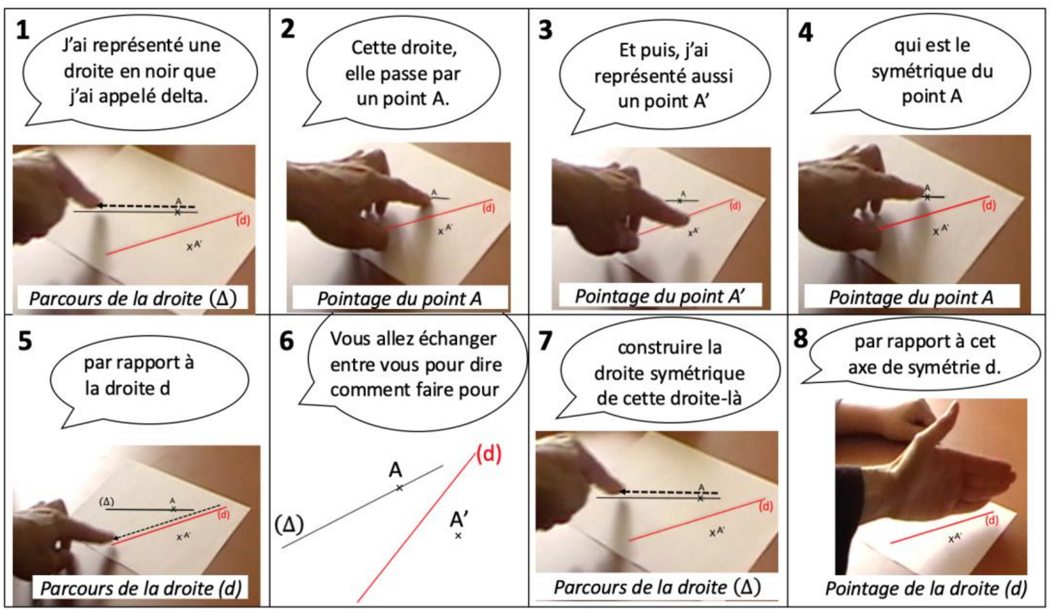
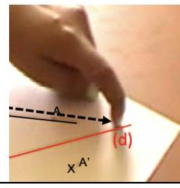
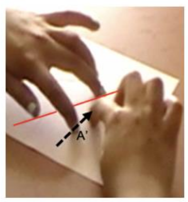


Figure 8. Passation de la consigne.

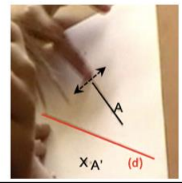
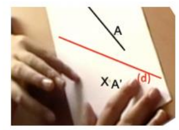
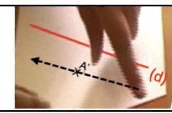
Le travail se déroule ensuite en articulant des échanges libres (sans contrainte sur le langage) sur les techniques de construction envisagées, des mises en œuvre de technique en dyade instructeur-constructeur et des discussions entre élèves sur la validité de ce qui est produit. Notons que, dans cette expérimentation hors classe, nous sommes intervenues parfois dans les échanges, non pas pour valider les propositions des élèves, mais pour encourager leurs expressions et justifications. Nous formulions par exemple des demandes du type « Attends, laisse-la terminer son idée », « Continue ton idée », « Que penses-tu de sa méthode ? », « Qu'est-ce qui ne va pas d'après toi dans sa méthode ? », « Justifie pourquoi c'est bien la 'bonne' droite ».

Nous illustrons à présent le fonctionnement du dispositif de travail en dyade en commentant les interactions entre Bonnie et Marion au début de leur résolution du problème.

Bonnie et Marion annoncent immédiatement l'une et l'autre qu'elles savent comment faire, dès la fin de la passation de la consigne. Bonnie donne une technique de construction qui revient à prolonger la droite (Δ) jusqu'à son point d'intersection avec l'axe de symétrie pour tracer ensuite la droite qui passe par ce point et le point A' (Figure 9a). Marion réagit en contestant cette technique : « Mais en fait Bonnie, tu peux la tracer où tu veux ta droite, tu peux la mettre comme ça, comme ça, comme ça, pour qu'elle touche le point. ». Elle accompagne son discours par des gestes (analogues à ceux de la Figure 8, vignette 8), montrant avec sa main différentes directions de droite qui passent par le point d'intersection que Bonnie propose de construire. Marion donne ensuite sa technique de construction, revenant à tracer le symétrique d'un deuxième point de la droite (Δ) pour tracer la droite qui passe par ce point symétrique et le point A' (Figure 9b).

Technique proposée par Bonnie	
Ben, on prend une règle, on prolonge jusqu'à arriver jusqu'à l'axe de symétrie	
après on prend sa règle, on trace une droite et on essaye que la droite elle fasse pareil que là, qu'elle atterrisse sur le point d'intersection entre l'axe de symétrie et la droite delta.	

(a)

Technique proposée par Marion	
Moi, j'dis qu'il faut mettre un deuxième point du côté, plutôt du bas de la droite, même si c'est infini, euh de la droite qui nous est représentée	
et euh on fait le symétrique	
et après on, ben on trace euh les deux points	

(b)

Figure 9. Techniques de construction.

Nous soulignons dans cet épisode le réinvestissement dans les interactions des élèves de la dimension gestuelle introduite lors des deux premiers temps de travail et aussi utilisée dans la passation de la consigne. Leur expression langagière seule (Figure 9) ne suffit pas pour savoir quelle droite chacune propose de construire, mais complétée par leur production de gestes, elle le permet. En particulier ni Bonnie ni Marion ne mentionnent le point A', mais toutes deux le pointent et font passer leur doigt sur ce point dans leur geste de parcours de la droite symétrique envisagée. Les deux élèves peuvent ainsi engager une réflexion commune sur le problème géométrique qui leur est posé, sans obstacle langagier qui limiterait le raisonnement : difficile de se centrer sur un bon usage du langage géométrique en même temps que d'explorer et d'exposer des pistes de solutions d'un problème. Sont écartés aussi les obstacles liés au tracé que rencontrerait Marion si les premiers échanges pour résoudre le problème s'appuyaient sur des tracés instrumentés, étant donné son handicap. Notons qu'antérieurement nous avions expérimenté avec elle le dessin à main levée comme support potentiel de la réflexion (Petitfour, 2015b, pp. 365-367), mais son usage ne s'est pas avéré concluant, bien au contraire.

À la fin des premiers échanges, le désaccord persiste entre les deux élèves sur la validité de la technique proposée par Bonnie. Nous leur demandons alors de tester les deux techniques avec les instruments. La phase de travail en dyade démarre avec Marion, instructrice de la méthode qu'elle envisage, et Bonnie constructrice. La Figure 10 retrace le début de leurs interactions.

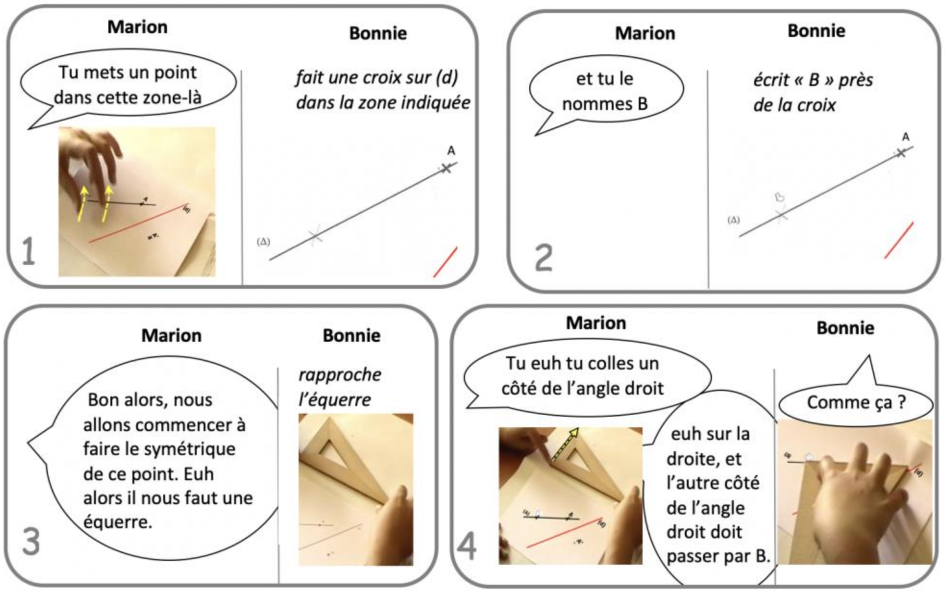


Figure 10. Travail en dyade Marion (instructrice)—Bonnie (constructrice).

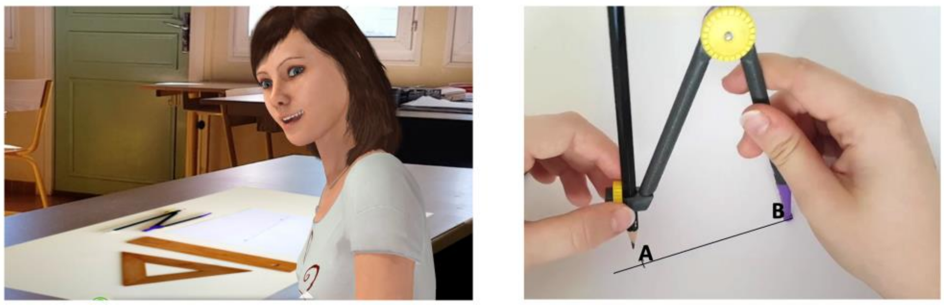
Marion indique par le geste la partie de la droite (Δ) sur laquelle elle souhaite que Bonnie mette un point (Figure 10, vignette 1). En plus de l'indication nécessaire de la droite à laquelle le point demandé doit appartenir, elle fait probablement en sorte que le point choisi soit éloigné du point A. Dans la formulation de sa technique (Figure 9b), elle avait en effet précisé de mettre le second point « plutôt du bas de la droite ». Elle fait ainsi appel à une connaissance manipulatoire (il faut que les deux points soient suffisamment éloignés l'un de l'autre pour obtenir un tracé précis) à laquelle elle avait associé des connaissances graphiques et géométriques (le tracé de (Δ) est la représentation d'une droite, il est limité alors que la droite est infinie). Précisons que cette prise en charge de l'aspect manipulatoire n'est pas attendue de la part de l'instructeur, mais de celle du constructeur. Marion nomme ensuite « B » le point introduit (Figure 10, vignette 2). Nous avions encouragé les élèves à procéder ainsi, à l'occasion de l'action instrumentée élémentaire de la Figure 7c, pour alléger les formulations langagières du langage technique, de même que l'on procède dans le langage géométrique. Marion annonce alors son intention : faire le symétrique du point B (Figure 10, vignette 3). Cette information peut permettre à Bonnie d'anticiper les tracés, en rectifiant si besoin la position du point B sur la droite (Δ) pour que son point symétrique ne sorte pas de la feuille support. Elle peut aussi l'aider à apporter des rétroactions à Marion sur ses instructions (agir en « faisant le moins probable »). Après avoir demandé l'équerre, Marion en précise le positionnement : « un côté de l'angle droit sur la droite et l'autre côté de l'angle droit doit passer par B » (Figure 10, vignette 4). L'instruction est incomplète, Marion n'a pas précisé de quelle droite elle parlait (l'axe de symétrie (d)), ni ne l'a montrée en la parcourant du doigt. Bonnie propose alors un positionnement qui répond à l'instruction donnée, en choisissant la droite (Δ) et interroge Marion pour savoir si cela convient. Marion pourra ainsi apporter la précision manquante si ce n'était pas le positionnement qu'elle envisageait.

La présentation de ce court extrait des interactions entre Bonnie et Marion laisse entrevoir une certaine potentialité du dispositif de travail en dyade : les deux élèves peuvent s'engager dans la résolution d'un problème de construction instrumentée, mettre en œuvre leurs connaissances, les faire évoluer. L'obstacle de la manipulation des instruments est levé pour l'élève dyspraxique, laissant place à un travail géométrique.

Suite à cette première expérimentation en dehors de la classe, nous avons poursuivi le travail en collaboration avec des enseignants de classes ordinaires de cycle 3 (élèves de 9-11 ans) en « faisant vivre » le dispositif dans le contexte de la classe, en l'adaptant à ses contraintes, afin qu'il produise les effets escomptés au niveau des apprentissages géométriques pour tous les élèves. Nous avons pu observer une appropriation du langage technique non aisée pour certains élèves instructeurs et des rétroactions pas toujours adaptées de la part de certains constructeurs. Des enseignants étaient également demandeurs de pouvoir parfois faire travailler l'élève dyspraxique seul. Nous avons alors imaginé un travail en dyade élève-avatar, préalable à la dyade élève-élève, visant à une appropriation du langage technique, des règles de fonctionnement de la dyade et du bon usage des instruments. Nous en donnons un aperçu dans la partie suivante.

DYADE ÉLÈVE—AVATAR

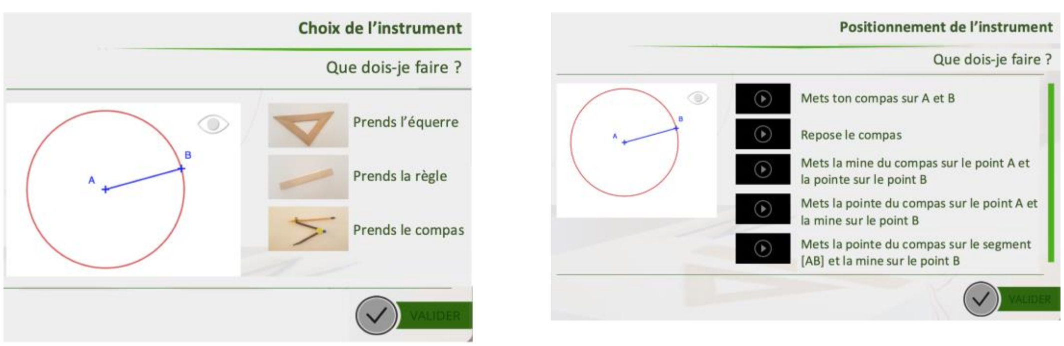
Un simulateur d'interactions humaines créé avec le logiciel Virtual Training Suite[4] (VTS) a déjà été utilisé comme ressource dans le contexte de la formation d'enseignants en mathématiques (Emprin et Sabra, 2019). Avec ce même logiciel et en collaboration avec Fabien Emprin, nous avons alors conçu un simulateur permettant des interactions entre un élève utilisateur et un agent conversationnel virtuel (l'avatar) pour produire un dessin géométrique sous les instructions de l'élève. Nous avons utilisé un avatar réaliste (Figure 11a) pour mettre l'élève au plus proche des conditions d'un travail en dyade qu'il vivrait en classe dans l'environnement papier-crayon. La réalité est aussi rendue par le décor d'une salle de classe dans laquelle l'avatar est installé comme à côté de l'élève instructeur (Figure 11a). Elle l'est également par des vidéos de mains d'une personne réalisant les actions demandées (Figure 11b). L'angle de vue est celui de l'avatar, orienté comme l'élève instructeur se trouvant face à l'écran : l'élève peut ainsi avoir l'impression d'être lui-même en train de manipuler les instruments.



(a) (b)

Figure 11. L'avatar dans le simulateur.

L'élève peut, dans un premier temps, regarder les instruments à disposition et avoir des informations sur les termes spécifiques à utiliser pour l'équerre (« sommet de l'angle droit de l'équerre », « côté de l'angle droit de l'équerre ») et le compas (« mine du compas », « pointe du compas »). Dans un second temps, il choisit un énoncé parmi cinq actions instrumentées élémentaires à réaliser (celles de la Figure 7). Il reçoit la consigne, orale et écrite, de « réaliser le tracé rouge sur la figure de départ » (en bleu). Il doit d'abord choisir l'instrument parmi l'équerre, la règle et le compas (Figure 12a), puis préciser son positionnement en choisissant une instruction dans une liste de formulations d'élèves issues de nos premières expérimentations en classe (exemple Figure 12b). Les propositions sont écrites, mais peuvent être oralisées si l'utilisateur le souhaite, ce qui a été prévu notamment pour les élèves qui auraient des difficultés de lecture (élèves dyslexiques par exemple).



(a) (b)

Figure 12. Choix des instructions.

Une fois l'instruction de positionnement de l'instrument choisie, l'élève en voit la réalisation par l'avatar, qui observe la règle du « moins probable » : l'avatar fait exactement ce qu'on lui dit de faire, il ne décode pas les implicites.

[4] https://seriousfactory.com/virtual-training-suite/

L'élève peut alors revenir sur son choix ou demander à l'avatar de tracer s'il pense que ça convient. Dans ce dernier cas, l'élève voit en vidéo la réalisation d'un tracé à partir du positionnement choisi pour l'instrument.

Nous avons encouragé l'inhibition, à savoir la capacité à ne pas donner la réponse la plus évidente sans se demander si une solution plus appropriée pourrait exister, en imposant systématiquement à l'élève de valider ses choix ou de les confirmer (« Est-ce que je trace ? », « Est-ce que tu penses que le tracé est correct ? »). Lorsque l'élève précise que le tracé réalisé est correct, il peut voir le tracé attendu se superposer au tracé produit s'il était correct (ou ne pas se superposer sinon). L'avatar formule alors en langage géométrique le tracé produit (dans l'exemple de la Figure 12, l'avatar dit : « Bravo, tu as bien tracé le cercle de centre A et de rayon AB »). L'élève peut ensuite enchaîner avec un autre tracé ou s'arrêter. L'enseignant peut récupérer par la suite une synthèse des activités réalisées par l'élève.

Notons que la simulation informatique enlève certains aspects des interactions humaines, ce qui peut avoir des effets positifs. En effet, le fait que l'ordinateur ne manifeste ni impatience ni jugement peut contribuer à mettre en confiance l'élève utilisateur, notamment lorsqu'il est en situation de handicap. Ce dernier peut mener sa réflexion à son rythme, bénéficier d'interactions avec l'avatar moins chargées d'émotions qu'elles pourraient l'être avec un pair. De plus, comme l'ordinateur fait ce qu'on lui demande sans surinterprétation, l'élève peut mieux comprendre la nécessité d'être explicite dans ses instructions. L'utilisation du simulateur présente quelques inconvénients cependant. Nous avons dû abandonner le développement de la dimension gestuelle de la part de l'instructeur en accompagnement du langage : les gestes ne peuvent être pris en compte par le logiciel utilisé, seules des instructions langagières le sont. En outre, la liberté de formulation est restreinte pour l'élève puisqu'il doit faire une sélection dans une liste finie : il n'y trouvera pas nécessairement celle qu'il aimerait tester.

Un travail exploratoire étudiant la contribution du simulateur d'interactions humaines ainsi conçu pour enseigner la géométrie aux élèves dyspraxiques nous a conduites à établir une preuve de concept (Emprin et Petitfour, 2021). Nos premiers résultats expérimentaux indiquent qu'une alternance des dyades élève-avatar et élève-élève présente un intérêt pour enseigner la géométrie aux élèves dyspraxiques, mais aussi pour tout élève. Ces résultats restent à confirmer par de nouvelles expérimentations.

RÉFÉRENCES

American Psychiatric Association. (2013). *Diagnostic and statistical manual of mental disorders* (5th ed.). Author.

Crouail, A. (2009). *Rééduquer dyscalculie et dyspraxie. Méthode pratique pour l'enseignement des mathématiques*. Elsevier Masson.

Duval, R. (2005). Les conditions cognitives de l'apprentissage de la géométrie : développement de la visualisation, différenciation des raisonnements et coordination de leurs fonctionnements. *Annales de Didactique et de Sciences Cognitives*, *10*, 5–53. IREM de Strasbourg.

Emprin, F., & Sabra, H. (2019). Les simulateurs informatiques : ressources pour la formation des enseignants de mathématiques. *Canadian Journal of Science, Mathematics and Technology Education*, *19*(2), 204–216.

Emprin, F., & Petitfour, E. (2021). Using a simulator to help students with dyspraxia learn geometry. *Digital Experiences in Mathematics Education*, *7*, 99–121.

McNeill, D. (1992). *Hand and mind: What gestures reveal about thought*. University of Chicago Press.

Mazeau, M., & Le Lostec, C. (2010). *L'enfant dyspraxique et les apprentissages. Coordonner les actions thérapeutiques et scolaires*. Elsevier Masson.

Mazeau, M., & Pouhet, A. (2014). *Neuropsychologie et troubles des apprentissages chez l'enfant. Du développement typique aux « dys »* (2e éd.). Elsevier Masson.

Mazeau, M. (2005). Reconnaître une dyspraxie. *Réadaptation*, *522*, 8–13.

Mazeau, M. (2020). Les troubles des apprentissages en 2020. *Contraste*, *51*, 139–159.

MEN. (2020). *Programmes du cycle 3, en vigueur à la rentrée 2020. BOEN n° 31 du 30 juillet 2020*. Ministère de l'éducation nationale, France.

Petitfour, E (2015a). Enseignement de la géométrie à des élèves dyspraxiques visuospatiaux inclus en classe ordinaire. *Recherches en éducation*, *23*, 82–94.

Petitfour, E. (2015b). *Enseignement de la géométrie à des élèves en difficulté d'apprentissage : étude du processus d'accès à la géométrie d'élèves dyspraxiques visuo-spatiaux lors de la transition CM2-6ème*. [Thèse de doctorat, Université Paris Diderot—Paris]. https://hal.archives-ouvertes.fr/tel-01228248

Petitfour, E. (2016). Enseignement de la géométrie à des élèves dyspraxiques : étude du processus d'accès à la géométrie par la construction instrumentée. In T. Barrier & C. Chambris (Eds.), *Actes du séminaire national de didactique des mathématiques* (pp. 137–153). ARDM, Paris.

Petitfour, E. (2017a). Outils théoriques d'analyse de l'action instrumentée, au service de l'étude de difficultés d'élèves dyspraxiques en géométrie. *Recherches en Didactique des Mathématiques*, *37*(2-3), 247–288.

Petitfour, E. (2017b). Enseignement de la géométrie en fin de cycle 3. Proposition d'un dispositif de travail en dyade. *Petit x*, *103*, 5–31.

Petitfour, E. (2022). Quel accès aux apprentissages géométriques pour les élèves dyspraxiques ? *Au fil des maths*, *545*, 5–13. APMEP.

Rabardel, P. (1995). *Les hommes et les technologies. Approche cognitive des instruments contemporains*. Armand Colin.

Vergnaud, G. (1990). La théorie des champs conceptuels. *Recherches en Didactique des Mathématiques*, *10*(2.3), 133–170.

Vygotki, L. S. (1931/1978). *Mind in society. The development of higher psychological processes*. (M. Cole, V. Jolm-Steiner, S. Scribner, & E. Souberman, Eds.). Harvard University Press.

Vygotski, L. S. (1934-86). *Thought and language*. MIT Press.

Working Groups

Groupes de travail

CONTENU ET PRATIQUES POUR LA FORMATION INITIALE ET CONTINUE DES ENSEIGNANTS : UN REGARD PLUS APPROFONDI SUR LES POTENTIELS, LES DÉFIS, LES PIÈGES ET LES PERSPECTIVES

CONTENT AND PRACTICES FOR PRE-SERVICE AND IN-SERVICE TEACHER EDUCATION: A DEEPER LOOK INTO THE POTENTIALS, CHALLENGES, PITFALLS, AND PROSPECTS

Jeanne Koudogbo, *Université de Sherbrooke*
Marc Husband, *St. Francis Xavier University*

PARTICIPANTS

JJ Bosica	Canan Gunes	Alexandria Middlemiss
Pamela Brittain	Sara Hamad	Kaitlyn Pollock
Tye Campbell	Erica Huang	Evan Throop Robinson
Katryne Dubeau		

****English version to follow.****

INTRODUCTION

Partant du fait que le monde de l'éducation est de nos jours en pleines transformations, tout comme les recherches en didactique des mathématiques et les pratiques en formation initiale ou continue, le groupe de travail A s'est penché, dans ces conditions, sur ces problématiques du point de vue des divers acteurs concernés dont les chercheuses ou chercheurs en didactique des mathématiques, les formatrices ou formateurs d'enseignant.es, les enseignant.es en formation initiale ou continue mais également les élèves. Rappelons que les problématiques entourant la formation initiale ou continue intéressent les recherches en didactique des mathématiques (Bednarz, 2012; Davis & Renert, 2013; Lajoie & Tempier, 2019; Morin, 2008; Rapke et al., 2020).

La visée principale du groupe de travail consistait à porter un regard plus approfondi sur les potentiels, les enjeux, les défis, les pièges et les perspectives et ce, à partir du contenu et des pratiques en formation initiale et continue des personnes enseignantes. Pour cela, le GT s'est focalisé sur ces éléments qui prévalent actuellement en formation initiale et continue à l'enseignement des mathématiques dans différents contextes et niveaux scolaires. Précisément, nous avons circonscrit, mon collègue Marc Husband et moi-même, trois questions fondamentales pour servir de points de discussions dans le cadre des trois journées de travail du groupe thématique, tenues en ligne les 27, 28 et 29 mai 2022.

En résumé, ces trois questions englobent d'une part, les incontournables pour le développement des compétences nécessaires à la profession enseignante, d'autre part, la manière de développer chez les personnes enseignantes un esprit critique face à l'usage des TIC et du numérique et, finalement, les avenues à explorer en termes de perspectives pour l'arrimage de la recherche à la pratique enseignante. Ainsi, chaque question a servi à impulser les activités du GT ainsi qu'à nourrir les discussions selon chaque journée de travail. Nous présentons dans les lignes qui suivent des synthèses des activités réalisées lors de ces trois journées remplies de questionnement, de discussions et de réflexions. Mais avant cela, brossons un portrait des personnes participantes.

PORTRAIT DES PERSONNES PARTICIPANTES

Notre groupe de travail a accueilli plusieurs personnes participantes. Nous leur avons d'emblée demandé de se présenter en indiquant leur institution d'attache, leur rôle et leurs intérêts. De cette présentation, il en ressort que ces personnes sont issues, majoritairement, des provinces anglophones et sont rattachées à différentes institutions universitaires, voire scolaires. Elles semblaient aussi bien passionnées, par la didactique des mathématiques, l'enseignement et l'apprentissage.

DÉVELOPPEMENT DES COMPÉTENCES POUR LA PROFESSION ENSEIGNANTE

Dans cette première partie, les discussions, dès la première journée, ont lieu autour de la question 1. En effet, après avoir « brisé la glace », nous avons lancé les travaux en les invitant à répondre à la question 1 :

> *Quels sont les éléments incontournables en formation initiale pour développer chez les futur.es enseignant.es les compétences professionnelles permettant de faire face aux défis et problématiques actuels en enseignement des mathématiques ?*

Nous avons retenu comme modalités de travail d'utiliser l'exploration, individuellement, suivie de la discussion en sous-groupes et, finalement, le retour en grand groupe.

EXPLORATION ET TRAVAIL INDIVIDUEL

D'abord, lors de la phase exploratoire, chaque participant-e devait réfléchir à la question posée et y répondre de manière individuelle. Pour ce faire, nous avons utilisé l'outil numérique, le Jamboard, afin que les réponses y soient inscrites sur des autocollants de couleur rose. De plus, une contrainte était que chaque personne s'assure de n'utiliser qu'une note autocollante par idée. Nous présentons les réponses dans le Jamboard ci-dessous (Figure 1). Notons que les réponses sont presque toutes en anglais. Nous synthétisons plus loin les idées développées en fonction des sous-groupes que nous avons constitués dans la deuxième phase du travail.

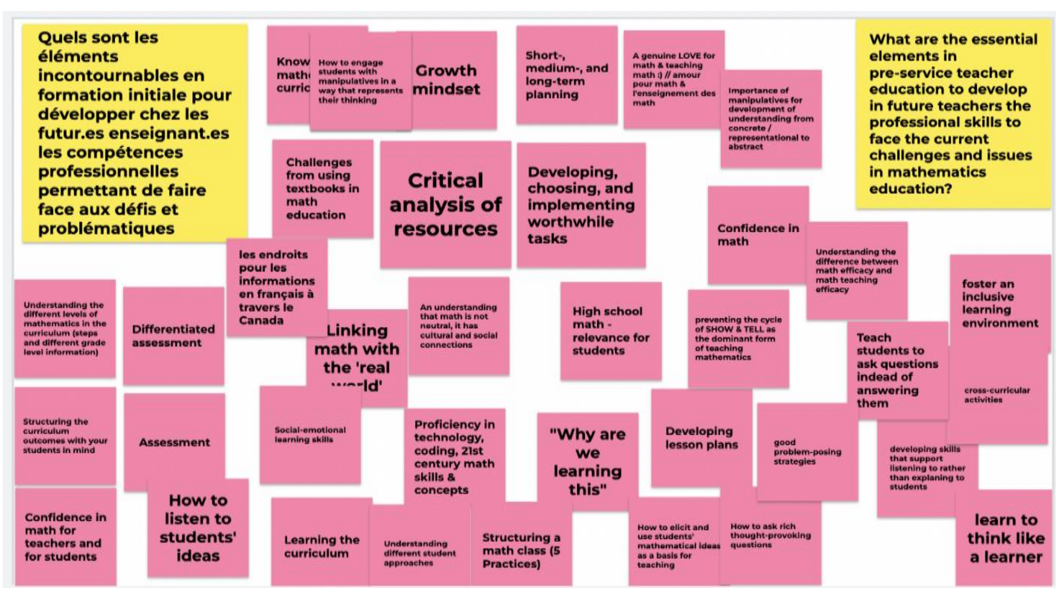


Figure 1

MODALITÉS DU TRAVAIL EN GROUPES ET PRÉSENTATION DES RÉSULTATS À L'ENSEMBLE DES PARTICIPANT-ES

Une fois que toutes les personnes participantes ont rempli le Jamboard avec leurs idées respectives inscrites sur les autocollants, constatant une saturation des données (aucune autre nouvelle idée n'apparaissant sur le Jamboard), nous avons alors scindé de façon aléatoire l'ensemble des participant-es en trois groupes, puis nous avons copié le Jamboard (et son contenu) pour le coller dans l'interface de chaque sous-groupe.

Ainsi, nous avons demandé à chacun des groupes constitués d'utiliser les données de leur nouvelle Jamboard afin de les traiter, les analyser et les classifier en créant des thèmes. Enfin, dans le développement des thèmes, des autocollants de différentes couleurs ont été utilisés au sein des groupes en vue de discriminer et de décrire chaque thème. Les groupes ont finalement présenté à l'ensemble des personnes participantes leurs résultats.

ÉLÉMENTS INCONTOURNABLES : PERSONNES ENSEIGNANTES ET APPRENANTES ET LES MATHÉMATIQUES

En gros, pour les trois groupes, les éléments considérés en formation initiale comme étant incontournables pour développer les compétences professionnelles permettant aux personnes enseignantes de faire face aux défis et aux problématiques actuels en enseignement des mathématiques touchent les trois pôles du système didactique (Brousseau, 1998), à savoir : la personne enseignante, la personne apprenante ou l'élève, ainsi que les mathématiques, discipline à enseigner et à apprendre; en plus des dimensions psychoaffectives.

Plus particulièrement, l'un des thèmes principaux concerne ainsi le traitement de **l'anxiété des acteurs concernés**, c'est-à-dire, l'enseignant-e et l'élève (Groupe 1 et 3). D'ailleurs, plusieurs études en didactique des mathématiques ont révélé la portée de la dimension affective en enseignement et en apprentissage des mathématiques (Adihou, 2011; Lafortune et al., 2002).

Un autre thème touche **la compréhension des besoins des élèves** par la personne enseignante. Pouvoir développer des compétences qui aident à avoir l'écoute active plutôt qu'à toujours donner l'explication aux élèves. Ainsi il est primordial de saisir là où l'élève en est dans ses apprentissages semble important pour que l'enseignant puisse faire des choix conséquents pour aider l'élève. Dans ce même ordre d'idées, un autre thème suggère que **les mathématiques puissent être applicables**. En ce sens, la personne enseignante devrait être capable de formuler judicieusement des questions en mathématiques (Groupe 1) et d'amener les élèves à en faire autant (Groupe 2). Mais encore, la personne enseignante devrait être capable de proposer **des situations permettant d'appliquer les mathématiques à la vie courante**, en établissant des liens avec la culture et la société (Groupe 2), ce qui pourrait contribuer à réduire l'anxiété évoquée précédemment.

Par ailleurs, l'action pédagogique étant au cœur des pratiques d'enseignement, une **maîtrise des connaissances mathématiques** semble indispensable. Les travaux de Adihou (2011) et de Morin (2008) abondent dans ce sens, car, les mathématiques posent un problème aux personnes enseignantes, notamment à propos de leur compréhension et de leur maîtrise des concepts à enseigner. Ainsi, **le sentiment d'auto-efficacité** et **l'autosatisfaction liée aux compétences en mathématiques** de l'enseignant ou de l'élève s'avèrent non négligeables (Groupe 2 et 3). De plus, **la maîtrise du programme d'études** ou **de formation mathématique** est nécessaire à la planification (Groupes 1 et 2). En ce sens, **la planification de situations d'enseignement** au cœur de l'action pédagogique est retenue par l'ensemble des trois groupes comme étant un thème important. D'ailleurs, **le développement de l'esprit critique** face aux ressources (Groupes 1 et 2) ainsi que **le recours à la manipulation** semblent incontournables à la conceptualisation (concret- représentation-abstrait) (groupes 1 et 3). Également, **l'évaluation des apprentissages** des élèves compte (groupes 1 et 2). Finalement, seul un groupe a retenu le thème concernant la **maîtrise des compétences technologiques et mathématiques du 21e siècle**.

Somme toute, nous avons ainsi pu comparer lors de la mise en commun, les résultats produits. Pour plus de pertinence, nous montrons, dans le tableau suivant (Taubleau 1-3), une copie du Jamboard de chaque groupe, avec les thèmes circonscrits, les données pour créer les thèmes et les idées introduites en en langue anglaise.

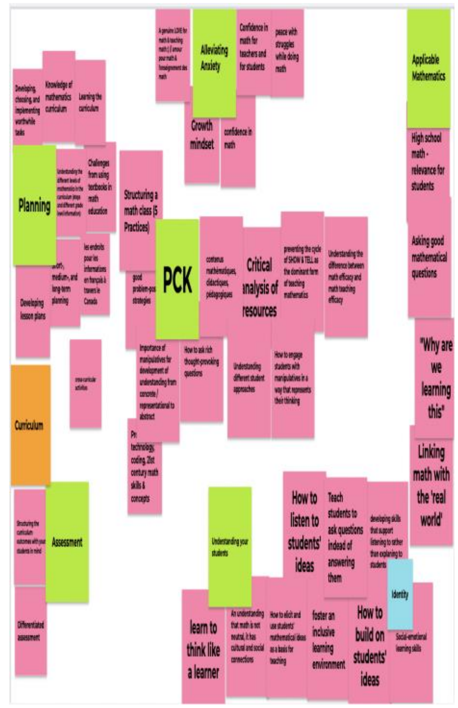
Thèmes	Exemples d'idées	Organisation des thèmes: Jamboard
Alleviating Anxiety	Confidence in math for teachers and for students peace with struggles while doing math	
Understanding your students	developing skills that support listening to rather than explaining to students	
Applicable Mathematics	Asking good mathematical questions	
PCK (Pedagogical Content knowledge)	Critical analysis of resources How to ask rich thought-provoking questions How to engage students with manipulatives in a way that represents their thinking	
Planning	Understanding the different levels of mathematics in the curriculum (steps and different grade level information) Developing, choosing, and implementing worthwhile tasks	
Assessment	Structuring the curriculum outcomes with your students in mind	

Tableau 1. Contenu du Jamboard du Groupe 1.

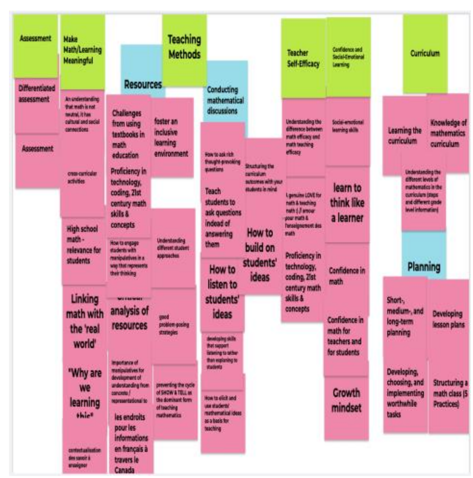
Thèmes	Exemples d'idées	Organisation des thèmes: Jamboard
Assessment	Assessment	
Make Math/ Learning Meaningful	An understanding that math is not neutral, it has cultural and social connections	
Teaching methods	Resources Proficiency in technology, coding, 21st century math skills & concepts Conducting mathematical discussions Teach students to ask questions instead of answering them	
Teacher self-efficacy	Understanding the difference between math efficacy and math teaching efficacy	
Confidence and Social-Emotional Learning	Social-emotional learning skills	
Curriculum	Understanding the different levels of mathematics in the curriculum (steps and different grade level information) Planning Short-, medium-, & long-term planning	

Tableau 2. Contenu du Jamboard du Groupe 2.

<table>
<tr><th>Thèmes</th><th>Exemples d'idées</th><th>Organisation des thèmes: Jamboard</th></tr>
<tr><td>Pedagogical skills</td><td>Lived
preventing the cycle of SHOW & TELL as the dominant form of teaching mathematics

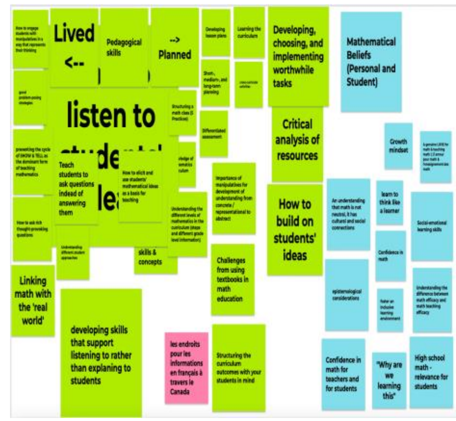
Planned
Importance of manipulatives for development of understanding from concrete / representational to abstract</td><td rowspan="2">
</td></tr>
<tr><td>Mathematical Beliefs (Personal and Student)</td><td>Understanding the difference between math efficacy and math teaching efficacy
Confidence in math for teachers and for students</td></tr>
</table>

Tableau 3. Contenu du Jamboard du Groupe 3.

DÉVELOPPEMENT DE L'ESPRIT CRITIQUE FACE AUX OUTILS NUMÉRIQUES

Pour démarrer les activités de la deuxième journée, nous avons demandé aux participants de réfléchir à la question 2 :

> *Comment les formateurs, formatrices d'enseignant.es peuvent-ils aider les enseignant.es en formation initiale et continue à réfléchir de manière critique à la multitude d'influences extérieures à l'ère de la démocratisation du numérique (par exemple, les médias sociaux) afin qu'ils puissent faire la distinction entre les matériels à visée non éducative—et ceux ayant un potentiel éducatif ?*

DOCUMENT VIDÉO

Concrètement, nous avons utilisé un document vidéo pour servir de point d'amorce à la discussion. Cette vidéo est extraite de TikTok et téléchargeable à ici.

Dans cette vidéo, une personne enseigne un contenu mathématique en chantant avec les élèves. La situation porte sur la résolution d'une équation de premier degré : $2x – 1 = 11$.

MODALITÉS DU TRAVAIL EN GROUPES ET PRÉSENTATION DES RÉSULTATS

Pour réaliser le travail sur la vidéo, nous l'avons organisé selon plusieurs étapes et modalités (individuelles, sous-groupes, grand groupe) pour enfin répondre à la deuxième question. Ainsi, en premier lieu, les personnes participantes ont été invitées à : 1) visionner individuellement la vidéo sans aucun a priori, sans aucun point de vue didactique ou pédagogique, en adoptant par exemple, la posture d'une personne étudiante en formation initiale; 2) inscrire leurs idées dans le Jamboard; et 3) comparer les idées entre elles. En deuxième lieu, les personnes participantes devaient visionner de nouveau la vidéo, en ayant une loupe didactique. Dit autrement, en empruntant une perspective propre à la didactique des mathématiques, en recourant ainsi à des éléments théoriques, didactiques, pédagogiques ou mathématiques, pour analyser le contenu de la vidéo et inscrire les idées ou les questions dans le Jamboard (voir plus loin, dans le tableau, ces idées introduites, dont certaines sont en langue française).

De ce fait, il nous a été possible, lors du premier visionnement de la vidéo, d'amener les personnes participantes à documenter les perspectives auxquelles elles pourraient s'attendre à observer chez un enseignant en formation initiale. Plusieurs éléments ressortent du visionnement et sont en lien direct avec la forme de l'enseignement, le caractère ludique, inspirant, astucieux ou motivant, voire novateur ainsi que la qualité de l'enseignante.

Lors du deuxième visionnement de la vidéo, il nous été possible de voir émerger chez les personnes participantes diverses questions et constats sur la vidéo, plus ancrés dans la didactique des mathématiques. Notons que les idées développées s'inscrivent, en majorité, dans des perspectives de l'enseignement des mathématiques, dans le contexte anglophone (par exemple, la théorie de Pirie Kieren, semblait plus familière). Cependant, la théorie des situations didactiques de Brousseau (1998) a été aussi introduite, dans une certaine mesure. Comparativement aux idées développées lors du premier visionnement, dans ce cas-ci, des commentaires fondés sur une analyse critique ont été formulés à propos des trois composantes du système didactique (élève- enseignant-e-savoir).

Ainsi, du point de vue des élèves, ont été questionnées, la place de l'élève, son rôle et son engagement dans la situation, l'auto-efficacité en mathématiques, l'activité mathématique de l'élève, sa compréhension et sa représentation de la situation. Du point de vue de l'enseignante, la pertinence de la forme de l'enseignement utilisée a été questionnée en fonction du choix conséquent à effectuer pour enseigner, de l'usage de manière critique des ressources, du type de routine à adopter, entre autres. Du point de vue des mathématiques à enseigner, des éléments conceptuels propres à la relation d'égalité ont été évoqués, la portée des trucs mathématiques, en plus de la manière d'amener les élèves à apprendre. De plus, *la situation adidactique* dans la théorie des situations de Brousseau semble indispensable pour amener les élèves à s'engager dans des problèmes et à les résoudre de façon autonome.

Finalement, les discussions ont surtout porté sur la manière dont il est possible de soutenir les enseignants pour établir le lien entre la théorie et la pratique en analysant de manière critique le contenu, ainsi que les ressources, disponibles, pour que ceux-ci soient utiles à l'enseignement et l'apprentissage des mathématiques.

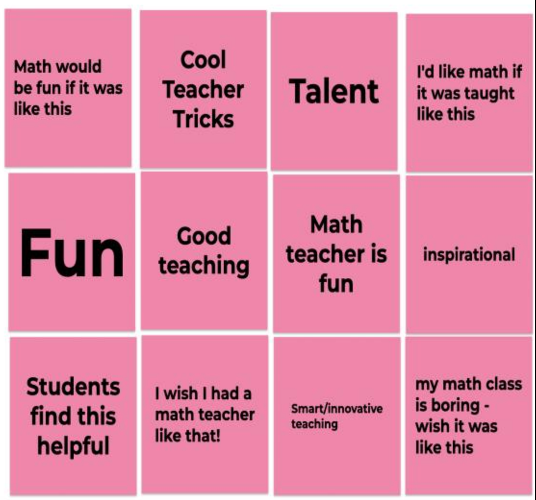
Réponses attendues de futurs enseignant-es	Analyse dans une perspective didactique
	What are the students doing? What are the routines to mathematize in the classroom? Are the students increasing their own level of mathematics self-efficacy by rapping watching their do math? No agency in the students. Is this what teaching should be, or just a part of it? Should teachers strive to write raps and entertain? Écoutez vos élèves—ils sont des EXCELLENTS ressources Upside down teaching? Did the students come up with the rule "what you do to one side you do to the other" or was this rule given to them? What are some ways that an image of balance could be recorded? Avoir la confiance dans votre créativité How does the activity engage students with their understanding and provide opportunity to represent their own thinking? Usage des ressources Penser aux ressources d'une façon critique Situations adidactiques What earlier images are connected to the rule (e.g., "What you do to one side you do to the other")

Tableau 4. Idées de personnes étudiantes en formation initiale et dans une perspective didactique.

PERSPECTIVES POUR ARTICULER LA RECHERCHE ET LA PRATIQUE

Durant la troisième et dernière journée, nous avons débuté les activités en traitant principalement la question 3 :

> *Quelles sont les perspectives qui pourraient être partagées en matière de formation initiale et continue pour promouvoir une meilleure articulation entre la recherche et la pratique ?*

Pour se pencher sur cette question, nous avons utilisé les mêmes modalités en invitant les personnes participantes à y réfléchir individuellement puis en groupe. Notons que les discussions ont été très fructueuses. Nous synthétisons quelques idées importantes dans les lignes qui suivent en les reprenant dans leur forme initiale, car elles reflètent les perspectives actuelles lorsqu'il s'agit de dresser un pont entre la recherche et la pratique. Nous organisons, toutefois, les idées selon des unités de sens pour plus de pertinence.

DU POINT DE VUE DE LA FORMATION MATHÉMATIQUE, DE LA PENSÉE ET DU RAISONNEMENT (ENSEIGNANT-ES/ÉLÈVES)

- En tant que formateurs d'enseignants et chercheurs, nous devons également réfléchir à la manière de rendre nos recherches plus accessibles et plus faciles à mettre en œuvre dans la salle de classe.

- Prendre conscience de ses limites
- Importance du bagage expérientiel
- Réflexion sur nos idées et pensées
- Mettre les enseignants à la place des élèves. Ils font des mathématiques; ils *mathématisent* le monde dans lequel nous vivons (trouver les mathématiques dans les expériences quotidiennes).
- Travailler avec les idées de leurs collègues : écouter et inciter uniquement. Par opposition aux critères qui leur sont donnés.

CONSIDÉRATIONS DES DIMENSIONS SOCIO AFFECTIVES ET CULTURELLES

- Empathie avec les élèves lorsque vous apprenez quelque chose de nouveau—les adultes ont tendance à oublier ce que vous ressentez
- Importance des mathématiques dans différentes cultures
- Savoir que la critique constructive n'est que cela—elle est là pour aider et non pour diminuer (pour l'élève et les enseignants)

CONSIDÉRATIONS DIDACTIQUES ET MATHÉMATIQUES EN ENSEIGNEMENT-APPRENTISSAGE

- Le statut de l'erreur : l'erreur est une forme de connaissance
- Faire le lien entre les maths et le monde éducatif (moins isolé)
- Mettre en œuvre une ou deux pratiques étudiées à la fois; processus itératif.
- Développer un esprit critique envers la salle de classe, le curriculum, les pensées des élèves…
- Développer un esprit critique envers les ressources disponibles

DU POINT DE VUE DES TYPES DE COLLABORATION : ACTEURS CONCERNÉS, DISCIPLINES

- Collaborer avec d'autres personnes dans différentes spécialisations (comme les langues, les arts, etc.)
- Collaboration entre les mathématiciens et les éducateurs
- Collaboration interdisciplinaire
- Communauté signifie les uns francophone, anglophone et Première Nation, et d'ailleurs
- Comprendre l'importance des relations entre les universités et les communautés
- Prendre plus de temps à faire des relations avec les communautés alentour de nous-autres
- Inclure les étudiants dans la recherche en éducation que le professeur fait
- Introduction aux étudiants à la recherche

CONCLUSION

Pour conclure, précisons que ces trois journées de travail ont permis aux personnes participantes d'échanger autour des trois questions, en menant des analyses réflexives pertinentes en fonction de leur formation, de leurs expériences universitaires, d'enseignement ou de recherche. Elles ont permis également d'ouvrir sur des perspectives en matière de formation, d'enseignement, d'apprentissage et de collaboration.

****English version****

INTRODUCTION

The aim of the Working Group was to take a more in-depth look at the potentials, issues, challenges, pitfalls and prospects, based on the content and practices in the initial and continuing education of teachers. The Working Group is interested in these aspects from the point of view of researchers, teacher educators and prospective and practicing teachers. The following three questions were used to plan activities and provoke small and whole group discussions for each working day:

Day	Guiding question
One	What are the essential elements in pre-service teacher education to develop in future teachers the professional skills to face the current challenges and issues in mathematics education?
Two	How can teacher educators help preservice and in-service teachers think critically about the myriad of outside influences in the era of digital democratization (e.g., social media) so that they can distinguish between materials considered limited and those with educational potential?
Three	What perspectives should be shared in pre-service and in-service teacher education to support the bridging of research and practice? In other words, how do we do this?

DAY 1

We instructed the group to individually think about the guiding question:

> *What are the essential elements in pre-service teacher education to develop in future teachers the professional skills to face the current challenges and issues in mathematics education?*

Each participant recorded responses on a pink sticky note using Jamboard. We asked everyone to ensure they used one sticky note for each idea. See Figure 2.

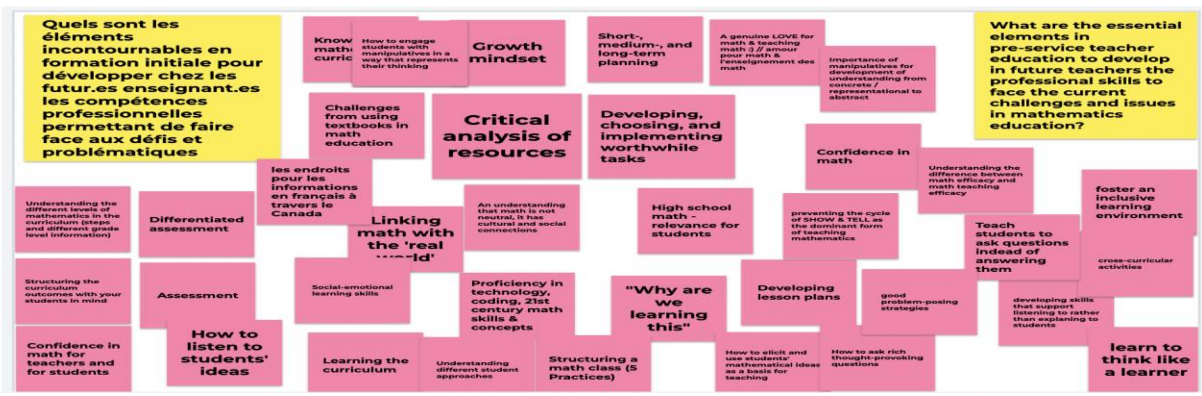


Figure 2.

Once all the participants filled the sticky notes with their ideas and no more stickies were appearing on the Jamboard, we randomized the group into three smaller groups and copied the Jamboard for each group to use as data. Each group was tasked with analyzing the data, sorting it, and creating themes. When themes developed, the groups used different colour sticky notes to describe the theme. Groups reported back to the whole group, and we compared each groups' findings. Below is a copy of each group's Jamboard (Tables 5 to 7) and the themes and examples of data used to create the theme.

Themes	Example of comments	Jamboard
Alleviating Anxiety	Confidence in math for teachers and for students peace with struggles while doing math	
Understanding your students	developing skills that support listening to rather than explaining to students	
Applicable Mathematics	Asking good mathematical questions	
PCK (Pedagogical Content knowledge)	Critical analysis of resources How to ask rich thought-provoking questions How to engage students with manipulatives in a way that represents their thinking	
Planning	Understanding the different levels of mathematics in the curriculum (steps and different grade level information) Developing, choosing, and implementing worthwhile tasks	
Assessment	Structuring the curriculum outcomes with your students in mind	

Table 5. Group 1 responses.

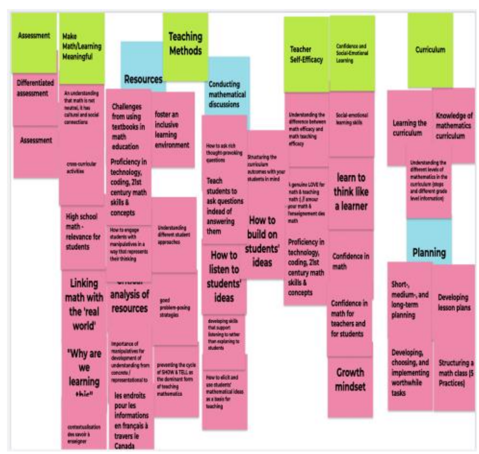
Themes	Example of comments	Jamboard
Assessment	Assessment	
Make Math/ Learning Meaningful	An understanding that math is not neutral, it has cultural and social connections	
Teaching methods	Resources Proficiency in technology, coding, 21st century math skills & concepts Conducting mathematical discussions Teach students to ask questions instead of answering them	
Teacher self-efficacy	Understanding the difference between math efficacy and math teaching efficacy	
Confidence and Social-Emotional Learning	Social-emotional learning skills	
Curriculum	Understanding the different levels of mathematics in the curriculum (steps and different grade level information) Planning Short-, medium-, and long-term planning	

Table 6. Group 2 responses.

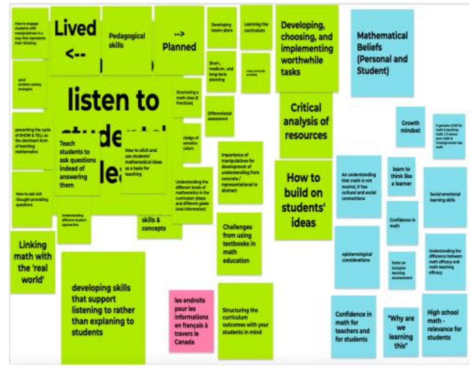
Themes	Example of comment	Jamboard
Pedagogical skills	Lived preventing the cycle of SHOW & TELL as the dominant form of teaching mathematics Planned Importance of manipulatives for development of understanding from concrete / representational to abstract	
Mathematical Beliefs (Personal and Student)	Understanding the difference between math efficacy and math teaching efficacy Confidence in math for teachers and for students	

Table 7. Group 3 responses.

DAY 2

To consider the guiding question *How can teacher educators help preservice and in-service teachers think critically about the myriad of outside influences in the era of digital democratization (e.g., social media) so that they can distinguish between materials considered limited and those with educational potential?*, we watched the video twice from two different perspectives. In this video, a person teaches math content by singing with students. The students are singing about the equation to solve: $2x – 1 = 11$.

During the first viewing, we asked participants to document perspectives they might anticipate hearing from a pre-service teacher. These perspectives were captured in the comments feature in Zoom, and some of them were documented as sticky notes in the table below. During the second viewing, we asked participants to frame questions and observations about the video using a conceptual/theoretical perspective in mathematics education that was familiar to them (e.g., Pirie Kieren Theory). In small groups, participants discussed how they might support teachers to critically analyze the contents in the video and other online resources that are widely available and easily accessed by teachers. In other words, we used the activity to talk about ways to bridge research and practice.

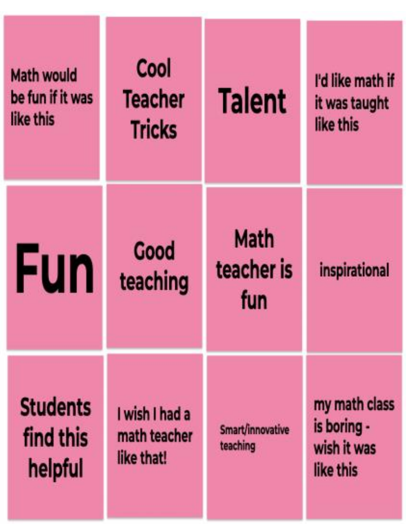
Anticipated responses from pre-service teachers	Questions and observations from a mathematics education perspective
	What are the students doing? What are the routines to mathematize in the classroom? Are the students increasing their own level of mathematics self-efficacy by rapping watching their do math? No agency in the students. Is this what teaching should be, or just a part of it? Should teachers strive to write raps and entertain? Écoutez vos élèves - ils sont des EXCELLENTS ressources Upside down teaching? Did the students come up with the rule "what you do to one side you do to the other" or was this rule given to them? What are some ways that an image of balance could be recorded? Avoir la confiance dans votre créativité How does the activity engage students with their understanding and provide opportunity to represent their own thinking? Usage des ressources Penser aux ressources d'une façon critique situations adidactiques What earlier images are connected to the rule (e.g., “What you do to one side you do to the other”)

Table 8.

DAY 3

To address the question *What perspectives should be shared in pre-service and in-service teacher education to support the bridging of research and practice? In other words, how do we do this?*, we used the same strategy of inviting the participants to think about it individually and then in groups.

The following ideas were shared via Jamboard:

- As teacher educators and researchers, we also need to think about how to make our research more accessible and easy to implement in the classroom?
- Implement one or two researched practices at a time; iterative process
- Putting teacher's in the student position. They are doing mathematics
- Mathematizing the world that we live (finding the math in everyday experiences)
- Working with their colleagues ideas: listening, and prompting only. As opposed to criteria being given to them
- Empathie avec les élèves lorsque vous apprenez quelque chose de nouveau—les adultes ont tendance à oublier ce que vous ressentez
- Prends consciences de ses limites
- Réflexion de nos idées et pensées
- Développer un esprit critique envers la salle de classe, le curriculum, les pensées des élèves, etc.
- Le statut de l'erreur est une forme de connaissance
- Collaborer avec d'autres personnes dans différentes spécialisations (comme les langues, les arts, etc.)
- Collaboration entre les mathématiciens et les éducateurs
- Faire le lien entre les maths et le monde éducatif (moins isolé)
- Collaboration interdisciplinaire
- Importance des mathématiques dans différentes cultures
- Importance du bagage expérientiel
- Sachez que la critique constructive n'est que cela—elle est là pour aider et non pour diminuer (pour l'élève et les enseignants)
- Communauté signifie les uns francophone, anglophone et Première Nation, et d'ailleurs
- Comprendre l'importance des relations entre les universités et les communautés
- Prenez plus de temps à faire des relations avec les communautés alentour de nous-autres
- Inclure les étudiants dans la recherche en éducation que le professeur fait
- Introduction aux étudiants à la recherche
- Développer un esprit critique envers les ressources disponibles

In summary, comments from participants reflect four different categories for bridging research and practice: a) focusing on student thinking, b) social-cultural perspectives, c) pedagogical approaches for teaching and learning mathematics, and d) collaboration.

SOME REFERENCES / RÉFÉRENCES SUGGÉRÉES

Adihou, A. (2011). Enseignement/apprentissage des mathématiques et souffrance à l'école. Actes du Premier colloque international « La souffrance à l'école » du Cercle interdisciplinaire de recherches phénoménologiques. *Les Collectifs du Cirp*, *2*, 90–102. http://www.cirp.uqam.ca/documents%20pdf/collectifs/10_Adihou_A.pdf

Andrà, C., Rouleau, A., Liljedahl, P., & Di Martino, P. (2019). An affective lens for tensions emerging from teacher professional development. *For the Learning of Mathematics*, *39*(1), 2–6.

Bednarz, N. (2012). Formation mathématique des enseignants : état des lieux, questions et perspectives. In J. Proulx, C. Corriveau, & H. Squalli (Dir.), *Formation mathématique pour l'enseignement des mathématiques : pratiques, orientations et recherches* (pp. 13–54). Presses de l'Université du Québec.

Brousseau, G. (1998). *Théorie des situations didactiques*. La Pensée Sauvage.

Davis, B., & Renert, M. (2013). Profound understanding of emergent mathematics: Broadening the construct of teachers' disciplinary knowledge. *Educational Studies in Mathematics*, *82*(2), 245–265. https://doi.org/10.1007/s10649-012-9424-8

Durpaire, F. (2014). L'école et les cinq mutations du Temps-monde. Dans M. M. Cresta (Éd.), *École et mutation: Processus, expériences, enjeux* (pp. 33–42). De Boeck Supérieur.

Lafortune, L., & Massé, B., avec la collaboration de Lafortune, S. (2002). *Chères mathématiques : susciter l'expression des émotions en mathématiques*. Presses de l'Université du Québec.

Lajoie, C., Tempier, F. (2019). Introduction au numéro spécial sur les dispositifs de formation à l'enseignement des mathématiques. *Revue canadienne d'enseignement des mathématiques, sciences et technologies / Canadian Journal of Sciences, Mathematics and Technology Education*, *19*, 83–86. https://doi.org/10.1007/s42330-019-00052-y

Morin M.-P. (2008). Les connaissances mathématiques et didactiques chez les futurs maîtres du primaire : quatre études de cas. *Canadian Journal of Education*, *31*(3), 537566.

Rapke T., Husband M., & Bourrie H. (2020) Blurring the border between teacher education and school classrooms: A practical testing activity for both contexts. In N. Radakovic & L. Jao (Eds.), *Borders in Mathematics Pre-Service Teacher Education* (pp. 163–184). Springer, Cham. http://doi.org/10.1007/978-3-030-44292-7_8

Seeley, C. L. (2017). Turning teaching upside down. *Educational Leadership*, *75*(2), 32–36. https://www.ascd.org/el/articles/turning-teaching-upside-down

CRITICAL MATHEMATICS WORKING GROUP: CHANGING MATHEMATICS TO FIT OUR WHOLE SELVES

Gurpreet Sahmbi, *University of Toronto*
Mahati Kopparla, *University of Calgary*
Mahtab Nazemi, *Thompson Rivers University*

PARTICIPANTS

Fatima Assaf
Lisa Lunney Borden
Maria-Josée Bran Lopez
Sophie Burill
France Caron
Joelle Cateri Balaski
Olive Chapman
Bernardo Galvão-Sousa
Michelle Hawks
Jennifer Holm
Limin Jao
Olivia Lu
Asia Majeed
Mavis Okyere
Gale Russell
Min Seo
Laura Shepherd

INTRODUCTION

Critical mathematics helps us to "read and write the world with mathematics" (Gutstein, 2016, p. 455). In this Working Group, we invited participants to reflect on their experiences teaching and learning mathematics, both as mathematicians and mathematics educators. Drawing from sociocultural theories of learning and identity and critical race theory in education, we explore the importance of counter-narratives to centre our lives and lived experiences in mathematics teaching and learning. Who we are as mathematics teachers and learners has everything to do with how we teach, learn, and relate to one another, our students, and to mathematics. Together we explored and considered ways of nurturing students to see themselves as capable doers of mathematics so that we can each bring our whole selves to contexts of teaching and learning. Our overall aim in this Working Group was to push each other to think about how we might change mathematics (as it is commonly [mis]understood) to fit ourselves, instead of changing ourselves to fit mathematics.

Over the span of three days, we explored the meaning of critical mathematics for our*selves*, and how it can be used to change mathematics for us, rather than change our selves for mathematics. Our discussions for each session were as follows:

- **Day 1**: We explored our relationships with mathematics and the nuances of how our mathematical stories differ from each other, and our students
- **Day 2**: We explored the subtleties of cultural contexts and identity and how these shape our engagement with mathematics
- **Day 3**: We explored how mathematics could help us articulate and take action on problems within our communities

Our design and facilitation of the Working Group was aimed at creating opportunities for critical reflections and discussions about teaching and learning mathematics. Our intention was to open up a space for challenging conversations and continued reflections even following the Working Group. During each of the sessions, we split our time between individual activities, small group discussion and whole group sharing.

DAY 1

During the first day, we started with brief introductions of the participants and facilitators. The goal of the first session was to identify our personal relationships with mathematics and begin to think about the need for critical approaches to engaging with mathematics.

EQUATION OF LIFE

The first activity with the group was to write an equation relevant to our life. As there are so many things in our lives that we think about mathematically, this activity was designed to encourage the participants to really uncover how deeply mathematical thinking is embedded in every aspect of our lives. We, as facilitators, provided the following equations as examples to model the activity.

Gurpreet: $y = a(x-h)^2 + k; h \geq 0, k \geq 0$ AND strawberries + (old*hard)cheese = joy

> *I've been working with a lot of 10th graders, and we've been looking at the vertex and there was a lot of talk of happy face versus sad faces, to help them think about how any of these parameters matter. And so for me, vertex form of a quadratic relation felt relevant. But also, a realistic one that really is my everyday is strawberries plus, old hard cheese, the older and harder the better gives me a lot of joy if I'm having a bad day.*

Mahati: $\sin^2 x + \cos^2 x = 1$

> *I'm the one, the sine and cos functions are different parts of my life, which are usually going up and down, doing whatever they want, but then they all come together to kind of make my life.*

Mahtab: $6\mathrm{y} - 5\mathrm{i} > 3(2\mathrm{y} - 5\mathrm{u})$ solve for i

> *If you haven't solved it I encourage you to, the equation will have a nice message if you solve for i.*

The participants wrote equations relevant to their lives and noted that even simple equations involving basic concepts like addition and multiplication could express complex ideas and philosophical discussions about life. Though we may not do them intentionally, we all create models for ourselves intuitively, not only to describe what we do, but also to help us achieve what we want. These explicit equations could help us to see ourselves, our identities, our lifestyles, or mindsets in math. Through these equations, math could help us express to others a different way of how we are thinking. Putting our ideas into an equation could be a different way of expression when it might be difficult to describe our ideas in words.

INTRODUCTION TO CRITICAL MATHEMATICS

Building on the recommended readings for the session by Gutstein (2016), a brief overview of critical mathematics was provided for the participants. Our intent was to deeply engage with the term 'critical mathematics' and begin to think about what it could look like, feel like, and be like in our classrooms. Critical mathematics approaches provide opportunities to look at the world through mathematics, use mathematical tools to understand the world, and imagine possibilities of liberation and social justice with mathematics. Challenging the historical stance of mathematics as an objective or neutral discipline, critical mathematics could make space for diverse mathematical literacies associated with local communities and socio-political realities of the mathematics doers. During this section, we highlighted the need to think about our identity in mathematics, identities of our students, relationships with social justice, and our relationship with mathematics teaching and learning.

The participants were placed in small groups to discuss their relationship with mathematics, particular, the guiding prompts were as follows:

- What role has mathematics played in your life?
- Why and how have you continued to do mathematics?
- What are your mathematical turning points?

Most of the participants in our session associated positively with mathematics. In their small group discussions, they highlighted the positive emotions associated with mathematics and recalled cherished moments of doing mathematics with their families as reasons for continued engagement with mathematics. Through discussions of their personal stories and journeys in the field of mathematics, they saw that their love for mathematics did not come from regimented mathematical instruction at school, but from explorations like puzzles, finding patterns, or trying to make sense of mathematical concepts. They enjoyed thinking and reasoning through puzzles and problems and the opportunities to use math in fun and creative ways. Some referred to the satisfying feeling of solving the equation and getting to the solution as a motivation to continue doing mathematics.

Participants also discussed the gatekeeping power of mathematics and the dominant rhetoric of being able to do mathematics as a source of status or power. Participants who were "good at" mathematics had opportunities to play and learn with mathematics. However, they recognized that many of their peers might not have had opportunities to develop a similar feeling of love for mathematics. Building on the work of Richard Hoshino (2015), they started to think about what it means to be good at math and compared the experiences of those deemed good at math versus those who were not. These external qualifications of being able to do mathematics well translated into varying levels of opportunity to engage with mathematics.

HOMEWORK

Over any beverage of their choosing, whether it is water, tea, coffee, ethanol, or the water vapor in the air, we encouraged the participants to take some time before our next session to reflect more on your relationship with mathematics and specifically start to think about how that impacts their teaching. Specifically, why do they teach math the way they do. Maybe, the way they teach math had changed over the years, maybe it had stayed the same. While there is no right or wrong answer here, we just wanted them to start thinking about that connection.

DAY 2

During the second day, our session began with a brief check in with the participants and a recap of the previous day for any new participants in the Working Group. The goal of the session was to dive deeper into critical mathematics and connect it to ourselves. The first activity was to think about the communities that we might be a part of, either the ones we are born into or those that we chose to be a part of (as shown in Figure 1).

Figure 1.

PRIVILEGE AND MARGINALIZATION

Through small group discussions, the participants were invited to think about how their membership in some communities could give them power and privilege, while their membership in some other communities might cause them to feel marginalized or oppressed.

The participants had conversation around context and shifting context. Due to the normalized privileges in the field that often go unnoticed, mathematics spaces were implicated in perpetuating inequity. One of the groups explained

> *When we are in different spaces and places, sometimes we feel and notice our differences or similarities in different ways depending on who is around us and sometimes we cannot actually pinpoint what it is that makes us feel different. We also brought it into the classroom, how do we acknowledge that then how do we kind of recognize the similarities or differences amongst our students and I think we kind of came back to that relationality piece and creating a different sort of context for our students so that we can acknowledge and honor all of those differences and similarities.*

Some other themes of discussion were as follows:

Mathematics and gender: Sometimes in the math world, being a female and identifying as female is seen as a deficit or seen as less than. More broadly, the predominant use of hetero-cis-normative contexts in the mathematics spaces were problematized. While the relationship between mathematics and gender is still approached through a binary understanding of gender, there is little to nothing written about non-binary and trans people.

Mathematical ability and pedagogy: Very often, future teachers do not identify as being good at math. They have been told that they cannot do mathematics and they know what it means to struggle and have those doors closed. However, they also bring hope to change and shift the pedagogical approaches in the field because they are more willing to think about what they can do to change that dynamic for their own students in the future.

Mathematics and mathematics education: In some spheres, mathematics educators are seen as less than by the mathematicians in the field. The affiliation with mathematics or mathematics education could be a source of privilege. Thinking about the intersections and intricate connections between the two fields, the participants talked about wanting to challenge that privilege and to try to keep creating a more open space for the two to exist.

Rigid disciplinary boundaries: A lot of institutions still see mathematics as either belonging in a Bachelor of Arts or a Bachelor of Science, but there is so much missed opportunity for interdisciplinary work because fields like psychology, philosophy, music and poetry are very connected to mathematics. Some of the great mathematicians of the past came from artistic and creative spaces. However, the rigid classification and labeling of what qualifies as mathematics itself is an oppressive stance that excludes certain people and knowledge from the official mathematics field.

INTERSECTIONALITY AND STORY PROBLEMS

Following the discussion about the various identities of the participants, they were introduced to the construct of intersectionality through an illustrated guide, examples and reference to Kimberlé Crenshaw's (e.g., Crenshaw, 2016) work. The intent of the discussion was to highlight that oppressions are interlinked and cannot be solved alone. To deeply engage with intersectionality and identify the little pieces of our identity that unconsciously get embedded in mathematics, the participants were invited to write a mathematical story problem that involves dividing fractions and involves a particular context that might be unique to them. Some of the problems written by the participants were as follows:

> The cord that attaches the drones of a bagpipe together measures 1.8 meters. If that cord is divided into five equal sections, how long will each section be? And then if the cord needs to be divided into three smaller equal sections and two larger equal sections, how long might each of the sections be?
>
> In 2019, the whole planet emitted more than 36 gigatons of CO2. In that same year, Canada, with a population of 37 million, emitted 740 megatons of CO2. How much greater would the global CO2 emissions have been if they had followed the Canadian model?
>
> First assume that a percentage M% of the population believe that the Moon landing is fake and that E% of the population believes the Earth is flat. What fraction of the Moon landing disbelievers are also flat Earth believers? What are the smallest and largest possible fractions? Maybe also assume that M>E.
>
> If the cats eat 2/3 of a cup of food each day, and there are 8 1/2 cups of food left in the bag, how many more days will it be before I have to go buy cat food again?

By sharing these story problems, we encouraged the participants to keep thinking about how we identify and how this plays out in our engagement with learning and teaching mathematics. As we were getting familiar with each other in the Working Group, we tried to guess who wrote which problems and discussed how we guessed. The activity of guessing made apparent the different ways we insert parts of ourselves into our engagement.

HOMEWORK

Equity does not equal equality. Find evidence to support that this is a true statement and be prepared to prove this to our group.

DAY 3

During the last day, we focussed on thinking about tangible ways to engage with critical mathematics and designing ways to bring back to their classrooms. The session started with briefly revisiting the homework. The participants shared various examples of how they understood equity and equality as different concepts. For example, one of the participants explained, "I go to a grocery store and that is equal for everyone. But is it equitable? Can everyone have the same access to fresh food, probably not."

EQUITY ISSUES

The participants were then prompted to individually think about issues of equity in their communities that they were passionate about or wanted to explore further. Then, they engaged in small group discussions to share their ideas and think about

- How can mathematics support in understanding and describing the concern?
- What is your positionality as you approach this issue?
- Who are/could be unfairly impacted by this issue? How can mathematics be utilized to explore possible inequities?

In small groups, participants shared their personal stories and concerns that their communities were experiencing. Through sharing their ideas, they identified the overarching concern for mathematics educators. Whether it is systemic racism or climate change or injustice in the world, what is our responsibility to use math in a way to be able to understand and explore those issues? What is our responsibility as mathematics educators to prepare the next generation of teachers to be ready to address those issues in the classroom and not to be treating math like falsely neutral?

MATHEMATICS TOWARDS JUSTICE

For the remainder of the session, the participants engaged in designing mathematical exploration or a learning task that was based around the concern that they had identified. They were encouraged to think about how they might bring it into their classroom and engage with the problem over a timeline that suited their needs. They were provided the following guiding prompts:

- How can we mobilize mathematics towards designing potential solutions?
- How can we create space for sharing and learning from diverse opinions and lived experiences?
- What forms of mathematical literacies are given preference? What knowledges or literacies are being ignored?

By the end of the session, most of the participants were developing different ways of mobilizing critical approaches to mathematics in their teaching practice. While the activity was a trigger to engage in critical reflections for some participants, they were excited to continue working on their learning tasks following the Working Group. Some other participants were already extensively working on these concerns and shared their experiences and ongoing work. The major themes that emerged were as follows:

Deconstructing math anxiety: Some of the participants problematized the term 'math anxiety.' They explained that students' anxiety has nothing to do with what mathematics actually is. However, they are anxious about being wrong on the formulas, rules and structures imposed in the math classrooms. Engaging with mathematics critically and going beyond neutral computations may alleviate the anxiety experienced by students in mathematics spaces. We discussed the use of strategies such as storytelling and mathematical autobiography to parse out student experiences with mathematics and unpack all of the baggage that comes with the inequities in the system that they grew up with.

Mathematical modeling: Through discussions about their ongoing endeavors towards equity and social justice through mathematics, some participants recognized the invaluable role of mathematical modeling in supporting these conversations. We discussed the varying curricular requirements of mathematical modeling in different parts of Canada and the varying emphasis in elementary, middle and high school contexts.

Critical mathematics in higher education: Some participants from mathematics departments highlighted their ongoing efforts to challenge mathematics as a gatekeeper for student success in higher education and engaging in active conversations around equity, diversity and inclusion. They shared ideas such as book clubs and development of Open Education Resources for students to begin conversation around what decolonization of university mathematics spaces could look like.

SUMMARY

The Working Group provided a space to engage in critical dialogues about our identities and approaches to engaging in mathematics teaching and learning. Participants from diverse geographical locations and a variety of mathematics spaces including K-12, higher education, and teacher education were able to come together to envision critical approaches to mathematics. Collectively, we thought about '*what mathematics can do for us*' and contribute towards developing a better world or a different kind of society. As we concluded the Working Group, we recognized that we had only taken a few steps in our continuous journey to reimagine mathematics to fit our whole selves. Having just gotten to the 'good stuff' by the end of Day 3, the facilitator and participants were eager to find ways to continue these important conversations in their own capacities.

REFERENCES

Crenshaw, K. (2016). *The urgency of intersectionality* [video]. TED Talk. https://www.ted.com/speakers/kimberle_crenshaw

Gutstein, E. (2016). "Our issues, our people—math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education*, *47*(5), 454–504.

Hoshino, R. (2015). *The math olympian*. FriesenPress.

Ladson-Billings, G. (2021). Does That Count? How Mathematics Education Can Support Justice-Focused Anti-Racist Teaching and Learning. Journal of Urban Mathematics Education, 14(1B), 1-5.

ADDITIONAL RESOURCES

Battey, D., & Leyva, L. A. (2016). A framework for understanding whiteness in mathematics education. *Journal of Urban Mathematics Education*, *9*(2), 49–80.

Esmonde, I., Blair, K. P., Goldman, S., Martin, L., Jimenez, O., & Pea, R. (2013). Math I am: What we learn from stories that people tell about math in their lives. In B. Bevan, P. Bell, R. Stevens, & A. Razfar (Eds.) *LOST opportunities: Learning in out-of-school time* (Vol. 23, pp 7–27). Springer, Dordrecht. https://doi.org/10.1007/978-94-007-4304-5_2

Gutiérrez, R. (2009). Embracing the inherent tensions in teaching mathematics from an equity stance. *Democracy and Education*, *18*(3), 9–16.

Martin, D. B., Gholson, M. L., & Leonard, J. (2010). Mathematics as gatekeeper: Power and privilege in the production of knowledge. *Journal of Urban Mathematics Education*, *3*(2), 12–24.

McGee, E. O., & Hostetler, A. L. (2014). Historicizing mathematics and mathematizing social studies for social justice: A call for integration. *Equity & Excellence in Education*, *47*(2), 208–229.

Zavala, M. (Host). (2020, April). *What is Luis Leyva doing in Nashville?* (No. 9) [Audio podcast episode]. In TODOS. https://www.podomatic.com/podcasts/todosmath/episodes/2020-04-09T16_31_51-07_00

WEAVING IDENTITY IN MATHEMATICS EDUCATION: FADS, FICTIONS, FIBERS, AND FREEDOMS

LE TISSAGE D'UNE IDENTITÉ DANS L'ENSEIGNEMENT DES MATHÉMATIQUES: LES MODES, LES HISTOIRES, LES FICELLES ET LES LIBERTÉS

Olga Fellus, *University of Ottawa*
Steven Khan, *Brock University*
Stéphanie La France, *University of Alberta*

PARTICIPANTS

Glen Aikenhead
Alayne Armstrong
Judy Bicep
Edward Doolittle
Susan Gerofsky
Frédéric Gourdieu
Marta Kobiela
Minnie Liu
Cynthia Nicol
Kathy Nolan
Jamie Pyper
Elena Polotskaia
Tsubasa Saito

INTRODUCTION

> *"Identity is worthy of being more than a fad…There is certainly evidence in our writing that we are not always talking about the same thing when we talk about identity."* (Darragh, 2016, p. 29)
>
> *"The United Kingdom is still one of the few advanced nations where it is socially acceptable—fashionable even—to profess an inability to cope with [mathematics]."* (Williams, 2008, p. 3 as cited in Epstein, Mendick, & Moreau, 2010)
>
> *"Learning and knowing the stories of people provides insight into their identities…Stories provide the context for understanding, feeling, and interpreting identities, which gives* voice *for people."* (Berry, 2021, vii)
>
> *"[S]ucceeding in school mathematics is less a matter of learning mathematics content than it is a quest for a positive sense of identity in mathematics class."* (McFeetors & Mason, 2005, 17)

Identity research in mathematics education constitutes a large and diverse body of studies operating from multiple paradigms, positionalities, and commitments. There have been several integrative literature reviews and studies on the topic of identity that provide epistemological responses, theoretical frameworks, and methodological approaches to investigating identity in mathematics education. For example, Darragh (2016), pointed to the lack of definition and theoretical compatibility across research in the field; Fellus (2019), conducted a theory-driven literature review suggesting a quadripartite model to understanding identity; and Langer-Osuna and Esmonde (2017), described and organized research approaches in the field as positional, narrative, discursive, psychoanalytic and performative. A ZDM Special Issue (Volume 51 in 2019) along with a recent NCTM Research Monograph (Langer-Osuna & Shah, 2021) showcase a broad spectrum of conceptualizations and operationalizations of the construct of identity in mathematics education research. Pulling these threads together, we concur with Martin (2021), who argues that despite diverse and specific lines of research, "all mathematics education research is about identity, even when the researchers make no such claims" (p.129). While some may wish to qualify "all" in "all mathematics education research," and while others may agree, to a lesser degree, with the centralized role identity plays in learning/teaching mathematics, we conceptualize identity as being a pivotal and central hub in the context of mathematics education. We are interested in the following questions:

- What does the concept of identity in mathematics education mean?
- Why is the concept of identity relevant to the future of the field of mathematics in general and mathematics education in particular?
- How might we harness the concept of identity in our work as researchers, practitioners, learners, parents, curriculum writers, and policy makers?
- How are mathematical identities socioculturally and discursively co-constructed?
- What shared vision and values inform judgments of who can do and who cannot do mathematics?
- How are mathematical identities privileged and/or socioculturally marginalized and what might be the implications of such socially and culturally entrenched processes?
- How does understanding mathematics identity as simultaneously interdependent and multidimensional—auto/biographically through experiences and their interpretation; discursively through language and other semiotic artefacts; authoritatively through opportunities to develop ownership over mathematical ideas; socioculturally through available identities to align with or reject, intersectionalities, and imagination—provide avenues of possibilities and hope for mathematics learning and futures in mathematics?
- Where might we boldly go next?

These questions and others were our initial guides for the exploration of identity over the three sessions at the CMESG gathering in Spring 2022 and in our discussions while preparing these proceedings.

Conceptualizing mathematics identity as dynamic, complex, in flux, and at times tangled, we organized each day around different, but closely associated, identity-related strands. Here, we use the term mathematics identity as a term to encapsulate identity making in the context of school mathematics as well as personal and professional identities related to doing mathematics across life-world contexts. We chose to focus the conversation in our Working Group on three sub-threads: autobiographical identity, socioculturally available identities, and authorial identity. Thus, on Day 1, Working Group participants examined perceptions of mathematics identity and autobiographical identity work. On Day 2, we examined socioculturally available identities in mathematics. On Day 3, the focus was on a critical look at freedom and responsibility in developing a voice and authority in mathematics. In what follows, we describe the activities and responses of participants on each day.

DAY 1: PERCEPTIONS OF MATHEMATICS IDENTITY AND AUTOBIOGRAPHICAL IDENTITY WORK

We invited the group members to individually and collectively think about identity in metaphorical terms. We intended for participants to consider particular strands, and possible implications for practice within their respective contexts.

PULLING THE THREAD: SETTING THE CONTEXT

Day 1 began with introductions prompted by inviting participants to describe their interest in the topic, something that they already knew about and something that they hoped to unpack in the next two days. Following self-introductions, participants were invited to share a crucial moment or observation that contributed to their personal relation to mathematics (McAdams, Josselson, & Lieblich, 2001). This led to an activity of using metaphor to frame mathematics identity. The two tasks of narration and metaphor would guide the following two days in our Working Group's meeting. Our rationale for structuring the ensuing activities and content around participants' interest was that we wanted to surface and untangle the complex and diverse nature of identity making. Some of the participant responses are shown in the box in Figure 1.

"I work with EDI and decolonizing curriculum in teacher education. Identity is important, in a sense of understanding where we are in our self-identities. There is **a feeling that I am losing my identity as a maths educator.**"

"I've noticed over many years that when I tell people I am a maths educator, the responses are largely negative in regards to maths. I also find in my work that **educators believe they have a need to 'save' students from the math rather than thinking who can be mathematical beings**."

"I joined projects with a background in science education and continue to focus my work in Indigenous education for example with the Sami people in Norway. I got hooked on maths education because I was offered a chance to do a review and couldn't believe what had been accomplished in Saskatchewan and across other Canadian provinces in Indigenous education. With regards to identity and self-identity, **there are similar conversations** in science education."

"I am a professor in maths education at the elementary level. I am here because I want to learn more about maths identity **including what 'fad' means**."

"I am in maths education in the elementary department. I am here because the more I work in the field the more I see it is important in the field of maths ed and teacher education. I know some **about positioning**."

"My interest is in **culturally responsive pedagogy**. I am interested because of the language used in the write up and title. I was also drawn by authors of recommended reading."

"I am interested in the working group because I want to learn more about ideas in identity and learn **how to move from fixed conception to more fluid understanding and conceptualisation**. Hoping that the research around this topic will help think about my own work in culturally responsive pedagogy."

"I am a maths educator in post secondary, interested in maths education for immigrant students. I am interested in the session because I know of **situations where stereotyping has provided tensions with mathematics due to felt-stereotype conflict. I am interested in how to navigate these situations**."

"I am interested in learning about how **I can encourage more students in mathematics**."

"As someone who is interested and involved in teaching future maths educators, it brought back to mind **questions about maths anxiety and how that shapes and is shaped by identity** especially in pre-service teacher education."

"I'm interested in this working group because **I am still trying to understand what my self-identity looks like. As soon as I feel I have a grasp, it seems that my identity is constantly changing.** When interacting with the environment, there are many things that change about myself and I want to know more about how my self-identity can impact my students and colleagues."

"I am a professor of mathematics on Treaty 4 and a member of the Mohawk nation. I am interested in Indigenous maths and indigenizing maths and the relation between Indigenous peoples and mathematics. **I find that mathematics disturbs and changes the identity of Indigenous peoples. They often sacrifice some of their identity in order to be successful in maths.** I wonder how do we work around this problem? How do we teach maths in a way that does not provide this tension? How do we change that students often do bare minimum in maths?"

Figure 1. Responses to the question "What brings you here?" (we have removed participants' names).

From the responses in Figure 1, we note that participants came to the group for a variety of reasons with a range of perspectives and prior experiences. Of particular note for us was a common focus on operationalizing the work on identity to provide insight into better serving students in school, pre-service teachers, specific groups, and oneself. The responses also reflect tension around continuing salvific narratives that characterize parts of education and the way that mathematics (as a discipline and set of experiences) disturbs the identity of some students who have to choose or perceive a need to sacrifice some of their identity in order to succeed in mathematics.

An emergent question was what the identity of being treaty people means for us as mathematics educators. As we look around in the present moment, we are gratified to see some examples of this work (e.g., Meyer & Aikenhead, 2021; Stavros & Murphy, 2019). How can mathematics identity help improve pedagogical approaches and instructional strategies in order to create change and develop multiple genres of mattering? How can understanding mathematics identity help us become agents of change in mathematics education? These questions and others were linked to our thinking of the concept of identity and how best to engage in mathematical activity in ways that do not negate other identities. We wanted to attend particularly to the relational aspects of mathematics education. This includes the relationship between one's own mathematics identity and others' identities even as one's identity shifts and responds—all in relation to mathematics as a discipline, to teaching and learning of mathematics, and to culturally responsive contexts.

ACTIVITY 1: DISCUSSING IDENTITY THROUGH METAPHOR

Metaphor is an important way that complex ideas can be made accessible through language. Thinking about metaphors and identity making in mathematics generated diverse frameworks that we used to co-construct a network of themes.

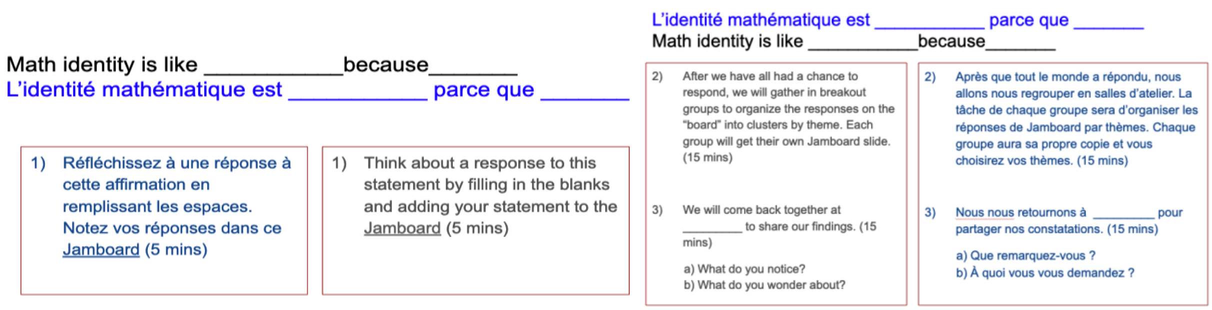


Figure 2.

We found that the metaphors that the participants offered often juxtaposed maths identity as unique and empowering with maths identity as a barrier to developing a can-do/growth mindset in mathematics. For example, « L'identité mathématique est comme une carapace qui peut nous protéger d'un ennemi ou nous permettre de nous avancer avec confiance » (WG participant). This metaphor compares mathematics identity with a shell, one which can protect us or enable us to act with confidence. In contrast, the metaphor "Maths identity is like knowing, and being clear about one's relationship to mathematics. 'I am not a maths person' because I see myself this way" showcases how mathematics identities can manifest self-attributions of temporary or long-term identities as doers and users of mathematics.

We feel that this tensioned aspect of identity is particularly relevant to mathematics. The discussion in the smaller breakout groups brought to light common threads. Other main themes included conceiving identity through nature/ecosystem metaphors, metaphors around permanence, complexity in identity, and metaphors around readily observable characteristics.

While many of the themes that were developed were common, participants had used different qualifiers to describe them. For the theme of permanence, one group used two categories—"fluid and changing" and "fixed and defined"—whereas another group combined the two into one and dubbed it "shifting versus rigid." Similarly, we found the terms "visibility" and "how you see yourself versus how others see you" as terms to discuss identity as performance. Some unique themes in metaphor emerged as well. For example, one group framed mathematical identity as both restraining or freeing, which speaks to the possibility of autonomy and agency in identity making (Boaler & Greeno, 2000; Lange, 2010). Another group arranged the metaphors referencing our natural world together. Understanding mathematics identity through place or ecology might suggest that interdisciplinarity is influential and important in conversations around mathematics identity; in other words, we might be cautious about isolating mathematics identity as unique from other disciplines. Interestingly, one group did not include labels in their theme organization, so we are hesitant to speculate on whether there is overlap between their themes and the labels used in other groups. This wide array of metaphors point to the conceptual complexity of the term, which is not surprising given how diverse and inconsistent the language around identity tends to be in the research (Daragh, 2016; Sfard, 2019).

The original Jamboard link is found here.

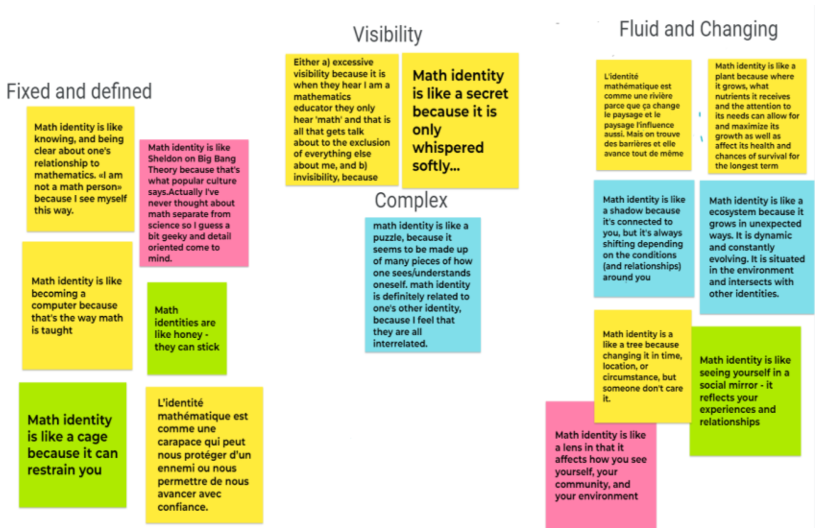


Figure 3.

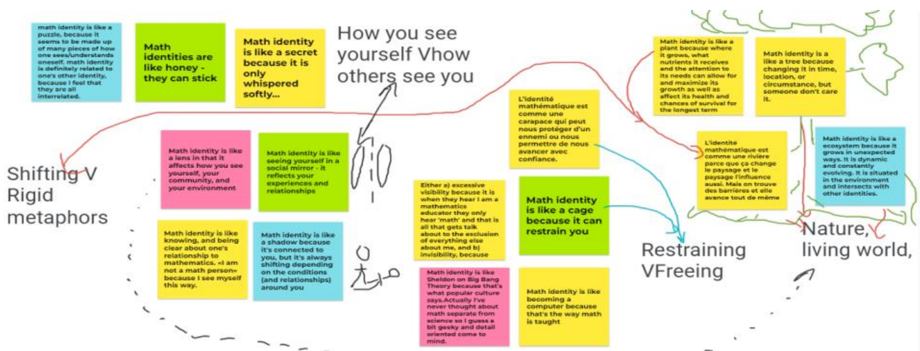


Figure 4.

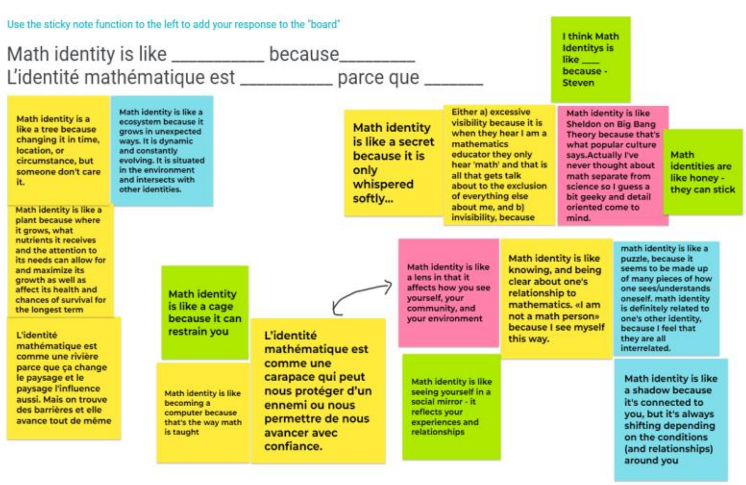


Figure 5.

ACTIVITY 2: SITUATING MATHEMATICS IDENTITY IN TERMS OF KEY TURNING POINTS

In order to situate the discussion of identity in the context of mathematics, we asked our participants to draw on and narrate a memorable experience they had in their mathematics schooling. This would set the stage for discussing mathematics identity as autobiographical, narrated and storied on Day 2. These autobiographical narratives elicited many identity-related experiences of teacher-student interaction—both positive and negative—indicating the importance of the impact of teachers on co-constructing students' mathematical identity. These key turning events may be experiences of interaction or moments of realization—what we call crystalizing moments. Given the personal nature of these turning points, we have chosen to leave participant quotations out of the proceedings document. These key turning points, however, are noteworthy in the way they are impactful and unpredictable, which showcases the complex nature of mathematics identity making throughout the schooling process.

DAY 2: FREEDOM TO CHOOSE FROM AVAILABLE NARRATIVES

PULLING THE THREAD

Considering the multiple strands of identity making that we wanted to pull together, and following the introduction of Day 1, the identity-related thread for Day 2 focused on socioculturally available narratives about mathematics and mathematicians, who can or cannot do mathematics, and what it means to do mathematics. Following the examination of socioculturally available narratives about mathematics, we planned to delve into the strand of developing an authorial voice in mathematics through freedoms to *choose* or construct new narratives in Day 3. In other words, Day 2 was about exploring the current landscape, and Day 3 would be about re-imagining the landscape.

In setting up the activities for Day 2, we followed the general query: *What are some of the socioculturally available identities and how can we effectively surface these in our work with students*?

> *According to our definition, identities are stories about persons, and not just any stories, but only those that are, first, reifying—they speak about what a person is and has rather than what she does; second, significant, that is, they are considered by the storyteller as crucial, indeed, as defining features of the person; and third, endorsed by their author, that is, they are seen by the storyteller as reflecting the real state of affairs in the world, and thus as reliable guides for her future actions.* (Sfard, 2019, p. 557)

This quote from Anna Sfard's work is helpful as it provides an ontological definition to identity as stories. Identity that is ontologically understood as stories of multiple kinds that are co-constitutive and interactive. These stories include personal experiences and their interpretation (auto/biographical), stories people tell about themselves and others as doers of mathematics (discursal), stories of developing an authorial voice in mathematics (authorial), and socioculturally available stories about mathematics (sociocultural) (Fellus, 2019; Fellus et al., 2022). Taken together, these stories provide a framework of reference for justifying engagement in and with mathematics and developing new storylines.

ACTIVITY 1: *ORANGE IS THE NEW BLACK*

In setting up the discussion for Day 2, participants watched a brief clip from the Netflix series *Orange is the New Black* Season 3 Episode 7 and collectively annotated it. In the scene, Gloria Mendoza (played by Selenis Levya) is being visited by her teenage son, Benito (played by Tyler Alvarez). Gloria is concerned about Benito's disengagement in school and failing maths. In response to his declaration of hating fractions and seeing no use for them, she tries to provide a real-world example of fraction addition in the context of cooking. We notice that even though Benito's age is not disclosed (we assume he is in his mid-teens), the mathematical concept he is working on is fractions—a topic that is introduced in early grades. We thought this piece of information is interesting given the (mis)representation of mathematics in the popular media landscape.

Benito complains: "I hate fractions! I ain't never gonna use it!" to which Gloria responds enthusiastically, "Oh yea? What about when the recipe calls for you to double it?" As we attempted to dig into the subtexts of the scene, we asked the members of the Working Group to consider what identities, mathematics or otherwise, they identified in the clip. Participants noted the presence of a Latina character, and a mother who takes on the role of a maths teacher. The intersections of these identities are showcased in the scene with the added complexity of being incarcerated, which suggests a divergence from and a convergence of the stereotypical relation between mathematics identity and being an inmate. In contrast, Benito performs an all-too-familiar mathematics identity: one of disdain for and doubt about the relevance of the mathematics he is learning (see Clarke, 2005). His mother becomes his teacher as she attempts to put the mathematical content in a real-life context, presumably in an effort to equip her son with a better understanding of the connection between mathematical ideas and everyday life and to motivate him to apply himself to his schoolwork.

Questions arose pertaining to who can do mathematics. What kind of mathematics? And when is mathematics used? These questions led to a discussion about the challenges of facing stereotypical deficit framings of Latinx-mathematics that has been recorded in studies (e.g., Gutiérrez et al., 2011).

Figure 6. Screen grab from Kogen (2015).

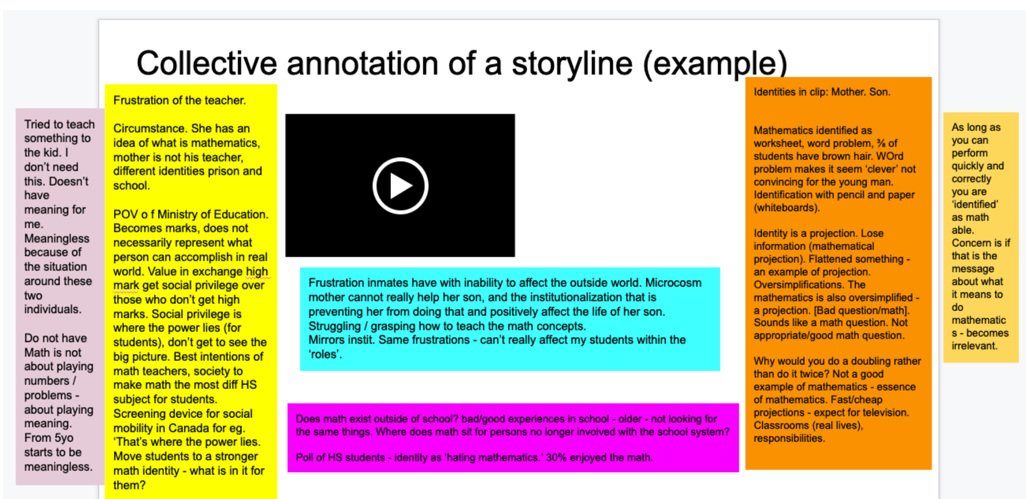


Figure 7. Collective annotation of a storyline (example).

Socioculturally available narratives about mathematics. In discussing the clip, a participant who was familiar with the show noted that one of the major themes is, "the frustration of the inmates not having the ability to change anything in the outside world. We have this microcosmos where the mother can't really help because she has been institutionalized and so can't help her son the way she wants to. She doesn't have maths teacher training." Unfortunately, too often, the sociocultural narratives available to us offer flattened representations of mathematics. One participant noted that despite the different contexts; there is a superficiality to the approach to mathematics, and that in this superficial framing, meaning and relevance are lost, as it is showcased in the clip by Benito's behaviour.

A powerful question came up in discussing why mathematics starts to be meaningless so early on: "What kind of identity can be built if for 20 minutes you just stand and count from 1 to 100?" thus surfacing the association between class rituals and the development of mathematics identity.

Socioculturally available narratives about mathematicians. A conversation took place on the question of Who can do mathematics? A common sentiment was that the identity of being a professor of mathematics or a mathematics educator is funnelled into a narrow view of numbers and functions. Available narratives propagate stereotypes. And identification with gender, ethnicity, racial group, age group, etc. is, more often than not, overlooked. One participant exclaimed "People look at me and their brain explodes. They think of all the maths that blew them away in highschool… [and] of the people that were teaching them without compassion or care for the human endeavour of learning mathematics. [They were] clearly dismissed by the maths teachers in my school." This showcases the idea that available narratives provide storylines for people to align with or reject. A lot has to match—all of the pieces of one's mathematical identity have to align with what available narratives say it means to be a doer of maths. Another participant noted that the identity of oneself and that of mathematics can get conflated, so that what people might see as mathematics informs how they perceive a doer of mathematics. For example, if maths is only numbers then identity as a doer of mathematics is heavily influenced by associations with calculations (and speed of calculation) using the basic operations and is "only numbers." The perception of who can do maths informs and is informed by the question what is mathematics? In unpacking this notion of what mathematics is, we found that the lived experience of participants was that perceptions of what maths means is "reified and fixed", as one participant put it. They reflected: "how do I think about it in relation to pre-service elementary school teachers?" "What do they perceive as mathematics?" To which many participants nodded. The common question became: "What counts as mathematical?" And how do we bring in more complexity into the conversation on identity, so that we can help our students identify as strong mathematicians or at least feel a sense of affiliation and a filiation with mathematics.

Socioculturally available narratives about school mathematics. In discussion, we asked: What might the series creators be communicating about the identity of mathematics as a field of study? We began by considering where maths exists. "Does maths exist outside of school?" As the participant who asked this question noted, if we define maths as school maths only, then we imply mathematics does not exist for those who are outside of the school system, which is problematic. At the same time, studies show that by and large, those that have long since left the school system, still identify mathematics as that which they learned in school. One participant pointed to the impact that the Ministry of Education has on our conceptions of mathematics. "How [does the Ministry] characterize a student who can do maths?" and commented that "for high marks, you get social privileges, which leads to health and other elements of privilege with good jobs… Maths as a screening device for social mobility." This participant added that there is a need to look at the variables that inevitably influence how mathematics is defined and how these definitions become pervasive in the educational system because they influence the intentions of the Ministry, teachers, and students. In our attempts to redefine not only school mathematics but mathematics in general, it is important to consider the political barriers and the buy-in from students as contributors or inhibitors to learners' developing mathematical identity.

Following this attention to socioculturally available narratives about school mathematics, we discussed stereotypical definitions of mathematics, and how those have potential to create tensions during the development of mathematics identities. A common sentiment was that mathematics is believed to be mostly about doing calculations fast, correctly, and easily and about problems that are out of context or unrelatable (Schoenfeld, 2016). If doing mathematics is about playing with problems and numbers that are out of context, then how can one build a story of themselves in connection to mathematics? "How many kids can do this?" One participant suggested the impact of this on pedagogical practice: "Mathematics is identified as worksheet and word problems like '⅜ of students have brown hair…etc.' These types of problems that attempt to provide opportunities for relevance ultimately fail because, among other things, they are unconvincing."

A connection that was made was to Francis Su's discussion of Christopher Havens—an incarcerated young man who found a purpose in mathematics and founded the Prison Mathematics Project. Notions of the cage and carapace from Day 1 also were surfaced during this activity. Steven noted one of the reasons he chose this clip and connected it to Christopher Havens and the quote offered by Francis Su from Simone Weil. Simone's brother, Andre Weil—a noted mathematician—was incarcerated during the second World War noted that this was a period of some of his best mathematics. The scene was also meant to serve as a reference point and pedagogical pivot (Ellsworth, 2005) for some of the discussion on Day 3.

ACTIVITY 2: SITUATING MATHEMATICS IDENTITY IN NARRATIVES TO BE READ DIFFERENTLY

> *« Chaque être crie en silence pour être lu autrement. »* (Weil, 1947/1988, p. 152)

Following the discussion around the Orange is the New Black clip, we set out in breakout rooms to annotate and reflect on the narratives of mathematics identity in pop culture, media, film or books. What are socioculturally available identities, and how can we effectively surface these in our work with students? We shared Anna Sfard's (2019) quote:

> *The moment they are told, identity stories begin spreading among people and, as a result, get a life of their own and stop being controllable by their authors—or by any other single person, for that matter. Moreover, whereas changing your ways of acting seems, at least on the face of it, to be a matter of your own decisions, you do not have the same agency upon your second- and third-person identities.* (p. 559)

We also presented Kasmana or Mathical's Mathematical Book prize winners as curated lists of texts and media around mathematics. Each group raised different texts that narrate positive mathematical identities. Amongst the groups, many narratives emerged. Richard Hoshino's (2015) *The Math Olympian*; Larry Niven's (1986) *Convergent Series*; Simon Singh's (2014) *The Simpsons and their mathematical secrets*; Yoko Ogawa's (2009) *The housekeeper and the professor*; Francis Su's (2020) *Mathematics for human flourishing*; and an internet influencer Kyne—a drag queen mathematician who popularizes mathematical stories via TikTok. Across these stories lie tales of sharing mathematical knowledge and experiencing mathematics in authentic non-contrived ways, overcoming hardship with and through mathematics, building loving relationships and interconnectedness with and through mathematics, and developing a sense of self as valuable and capable in meaningful ways with and through mathematics. These narratives, both fictional and non-fictional, act to re-write stereotypical engagements with mathematics. They offer alternative narratives for mathematics identity and on being read differently.

BEGINNING TO WRITE A DIFFERENT FICTION

En discutant des liens entre les activités et entre les concepts, nous avons tenté d'explorer ce que peut signifier être un mathématicien et être étudiant/élève en mathématiques. Nous avons remarqué qu'il n'y a pas une définition ou même de conception fixe. Il s'agit plutôt de plusieurs ficelles d'identités qui s'entremêlent de façon dynamique et complexe. Nous soulevons alors la question à savoir où trouvons-nous des actes mathématiques? Les mathématiques existent certainement en dehors de l'école et sont présentes et représentées dans les médias, dans les histoires, dans les comédies et dans les tâches quotidiennes. Ceci qui nous a conduits à une autre question : quelles sont les histoires populaires qui sont construites et qui construisent nos conceptions des identités mathématiques ?

Dans les discussions, nous avons rencontré un certain nombre de difficultés autour du discours sur l'identité. Notamment, relativement à la possibilité de précision lorsqu'une discussion a cours dans plusieurs langues : « it is tough in more than one language to be precise. » La précision se perd dans le processus de traduction et les métaphores utilisées ne se traduisent pas forcément. Tel que soulevé par une participante : « what does fad mean in this context ? » We use the term as a placeholder until we can refine the language.

On s'est demandé s'il y avait même un mot équivalent. Le problème d'équivalence ou de congruence nous amène à la question de la précision (Clarke, 2005). Il s'agit d'une vraie problématique ; we use metaphors to understand complex ideas, but metaphors are culturally and linguistically situated, and we run into the issue of complex ideas being 'lost in translation' because the metaphors do not translate and the precision is lost (e.g., Cole, 2009; Fellus & Glanfield, 2017).

The common threads that we find in the representation of mathematical identities in socioculturally available narratives are those of oversimplifications: oversimplifications of mathematical identity and oversimplifications of mathematics. These oversimplifications lead to flattened projections. One participant noted a significant challenge in talking about identity, that even the act of trying to characterize the term identity, is a form of flattening in and of itself. So, how do we go about presenting and enacting different narratives in ways that honour the complexities of mathematics identities? We began to talk about mathematics identity as something we care about for ourselves and for others. Maths identity is about relationships. It is about the struggle of building those relationships (House, 2021). Simone Weil's (1947) quotation helps clarify the connection: « Chaque être crie en silence pour être lu autrement, » (Simone Weil, 1947/1988, p.152) which is a central thread in Francis Su's (2020) *Mathematics for human flourishing*.

DAY 3: WEAVING FREEDOM AND RESPONSIBILITY INTO AUTHORING MATHEMATICAL IDENTITIES

Day 3 was our attempt to have participants begin to weave together some of the threads of the previous days' discussions. In particular, how stories provide insights into identities (Berry, 2021) and that much, if not all of our work in mathematics education could be thought of as being about identity in some way (Martin, 2021; McFeetors & Mason, 2005). This was intended to also bring out discussions around our concomitant ethical responsibilities for how we go about our work as educators, scholars, and researchers in authoring our own identities and in affecting the authorship of learners' identities. In preparation for the final day, participants were asked to view a clip of Deborah Ball's 2018 AERA Presidential address between 1h:19 and 1h:39. In the clip Ball quotes Gholson and Wilkes' (2017) who state that "*Identities serve as the organizing link between macro-structural forces and the face-to-face moments in which we all live*" and poses the question "*How do macro-structures play out in the micro-moments in Black and Brown children's experiences in classrooms?*" (1:19:42) in relation to historical legacies of slavery, racism, colonialism, sexism, genderism, and ableism. This was to be the focus for the final session. In addition, Maisie Gholson's and Charles Wilkes' (2017) paper "(Mis)taken identities" was the suggested reading at the end of Day 2 as it was the likely source of the quote in Ball's discussion and so provided additional context.

The section of the clip is titled 'Just Dreams' (Ball, 2018) and begins with a focus on the concept of discretionary spaces in mathematics classrooms. A discretionary space refers to the dozens of moment-by-moment decisions that teachers have to make, say, or not do, or not say something in response to the unfolding dynamics and utterances in the classroom in any given period of time. The context of the clip is a classroom moment in which Ball was the teacher, the classroom composition is 22 black children, four Latinx children, and four white children and the lesson goal is around understanding fractions as numbers on the number involving the prompt below. The focus is on the interactions between two students–Aniyah and Toni–both of whom are described as Black. Aniyah presents a response to the prompt below of "one-seventh" with an explanation that counts all the sections on the number line, Toni asks during a questioning period immediately following, "Why did you pick one-seventh?" and giggles while playing with her hair.

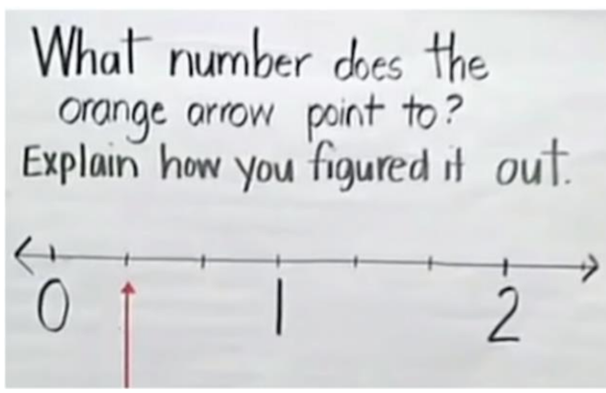


Figure 7. Screenshot of the mathematical prompt (Ball, 2018, 1:20:51).

Ball (2018) asks members of the audience what they might do next or what they think might happen. Ball offers three frequent responses and then engages with the question of what these different possible responses do to Toni. All three possible moves described excluding Toni from the discussion with one positioning her as "not listening" and another sidelining her mathematical point and contribution. In elaborating on what these moves do to Toni, Ball suggests that they all "signal to her that her contributions to the class are not appropriate or valuable," "her participation and mathematical attentiveness are made invisible or erased" and "she is seen as being a distraction" which together combine to "eclipse her humanity." From here Ball connects the micro-moments in this learning situation to the larger macro-environment in which there exists a significant differential rate of suspension between Black and White girls based on differential disciplining as a result of subjective judgements of the teacher. Ball's final question in our clip poses the question, "What would it take to disrupt the patterns through which a Black girl like Toni is marginalized?" She offers that it would require us to see "Toni's question as key to the class's work," which is related to a teacher's Mathematical Knowledge for Teaching, and secondly to "take as axiomatic the brilliance of Black children" which requires us to see Toni as mathematically capable and asking a mathematical question that she intends to ask. Ball's main point in this clip is that teachers have the power to use their discretion to intentionally and deliberately "disrupt the patterns through which Black girls are marginalized" by, for example, "acknowledging publicly the importance of Toni's question." Zooming out one more time to the profession, Ball identifies that there is a need to form a profession to work with discretion for justice that involves making the teaching profession more diverse and building professional education for the just and skillful exercise of discretion. The latter, Ball suggests, requires a teacher education curriculum that educates the whole self, develops professional orientations founded on ethics and care related to disrupting intersectional oppressions, and developing a repertoire of practices that effectively serve to counteract normalized practice.

Ball's (2018) clip was meant to play against the *Orange is the New Black* clip (Kogen, 2015) from the previous day showing a more typical mathematical interaction. Both focused on the types of identities authored by the students through interaction and the types of identities that could be discursively expressed by the teacher through the use of their discretion. The clip was also meant to draw attention to recursive implications of the repeated denials of a learner's humanity in relation to their developing mathematical identity as the micro-moments of lived experience sediment and consolidate into macro-structural inequities. It was intended to be read alongside the work of Gholson and Wilkes (2017) in which they describe cases of four children,

> *Ahmed Mohamed, Kiera Wilmot, Niya Kenny, and her unnamed peer [as] examples of children who have suffered from (mis)taken identities within the domains of science and mathematics. Each of these children was arrested—one charged with a felony—in their science and mathematics classrooms. The disciplinary identities of these children were mistaken with stereotypes of being a Black girl or a Muslim boy and, consequently, wrongly taken (i.e., mis-taken), in one case, quite violently. Mistaken identification is not merely a matter of sensational media headlines but has carceral implications. That is, children, like Ahmed, Kiera, Niya, and the unnamed young Black girl, are not only denied labels, such as mathematics doers, tinkerers, knowers, and legitimate observers, but inherit labels of academic disidentification (Osborne & Jones, 2011), such as troublemaker, criminal, and terrorist that serve to naturalize their trajectory out of mathematics and science and into the school-to-prison pipeline. Much has been said about the school-to-prison pipeline but <u>few have suggested the infrastructure that supports and maintains the school-to-prison pipeline is in fact the discipline of mathematics</u>.* (p. 229, underline added for emphasis).

Moreover, Gholson and Wilkes (2017) state that

> *We believe identity-based research in mathematics to be <u>a key site of disruption</u>, even in light of the overwhelming macro-structural forces that organize children's learning within the collective Black, such as residential segregation, school funding, and tracking policies. For example, consider that children's lives have been extinguished within micro-scales of time based on readings of an identity, like "Black boy." Think Tamir Rice and Trayvon Martin, two Black boys gunned down due to the perception of their personhood as thugs, predators, and criminals. These are just two recent and tragic examples. In this sense, identities serve as the organizing link between macro-structural forces and the face-to-face moments in which we all live. Identities—really our consumption of children's identities—can be extraordinarily powerful in a child's life trajectory. Our read and interpretation of who children are, what they are doing, and who they are becoming set into motion a constellation of constraints or affordances—at the interpersonal level—that structure interactions, activities, and, ultimately, life opportunities.* (pp. 232–233, underline added for emphasis)

FREEDOM AND RESPONSIBILITY

The theme of Day 3 was "Freedom and Responsibility for authoring mathematics identities." After viewing the video (and perhaps reading the paper) participants engaged with the prompts in Figure 9 for their discussion.

3.1 How do macrostructures of the historical legacies of multiple 'isms' (race, gender, sex, ability, age) play out in the micro-moments of diverse children's lives in the classrooms in Canada?

3.1 Comment les macro-structures des héritages historiques de multiples « ismes » (race, sexe, genre, capacité, âge) se manifestent-elles dans les micro-moments de la vie de divers enfants dans les salles de classe au Canada ?

*

3.2 What would it take to disrupt the patterns through which some identities are marginalized in your context?

3.2 Que faudrait-il pour perturber les schémas par lesquels certaines identités sont marginalisées dans votre contexte ?

*

3.3 What might a different set of patterns / discourses in instructor-student and student-student interaction look/feel/sound like?

3.3 À quoi un ensemble différent de schémas/discours dans l'interaction instructeur-élève et élève-élève pourrait-il se ressembler ?

*

3.4 What might be some of the concrete steps to realize and sustain equitable and inclusive patterns in mathematics classrooms?

3.4 Quelles pourraient être certaines des étapes concrètes pour réaliser et maintenir des schémas équitables et inclusifs dans les cours de mathématiques ?

Figure 9. Day 3 prompts.

The discussion and notes were recorded in a Jamboard (here) and screenshots of the three groups' discussion are in Figures 10–12.

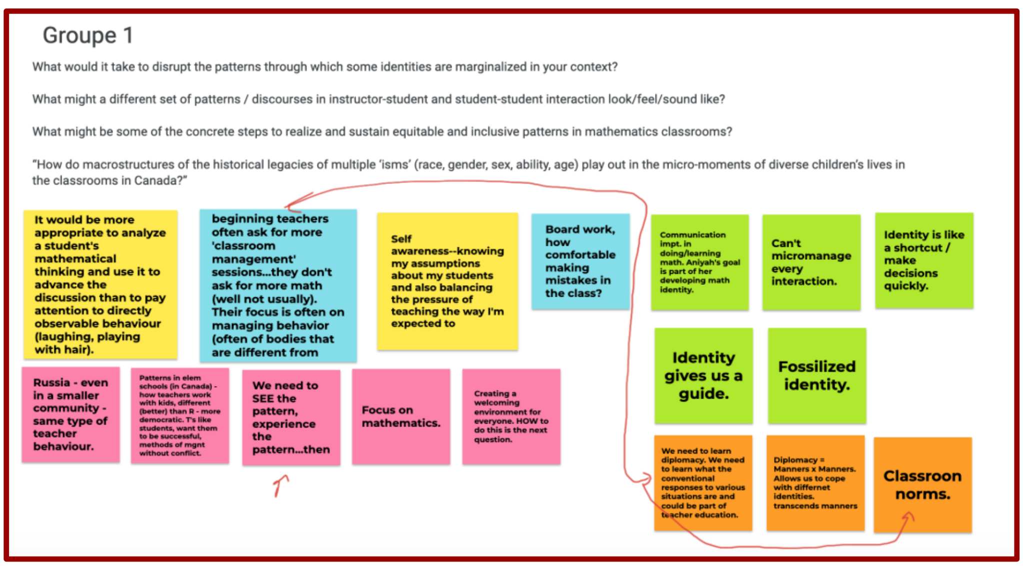


Figure 10.

From the first group's discussion (Figure 10), we wish to highlight some of the following threads. It was noted that beginning teachers often ask for more 'classroom management' sessions and not usually more content knowledge experiences in mathematics and the connection made that this managing of behaviour intersects with the identities of students as raced, gendered, and abled in ways that may differ from that of the teacher. There was a discussion about a need to better understand the patterns and experiences in Canadian classrooms and a connection to the literature on classroom norms and mathematical norms. Work on visualizing dialogue and interaction in mathematics classroom (e.g., McGarvey et al., 2022) may be relevant for further exploration and investigation.

There was an explicit suggestion that learning 'diplomacy' might be important in disrupting current patterns that marginalize some learner identities and allows us to cope with different identities in our spaces of learning. There was some discussion about what the teaching of diplomacy might entail with no resolution. We find this to be an interesting and novel suggestion that goes beyond current discussions about, for example, welcoming spaces and socio-emotional learning, and speaks to the heart of discretionary action. We believe this is something worth considering and expanding on in the future. The teaching and practising of diplomacy in the diverse discretionary spaces of mathematics education we believe does have potential to disrupt patterns by which some identities are marginalized and open space for the emergence and nurturing of other more humanizing patterns. At the same time, we note the tendency to instrumentalize research and the idea that identity categories can be a shortcut to rapid decision making is fraught outside of very specific situations as these often elide the complex intersectional positionalities that learners present to us in their being. Our discussion mirrored the previous in expressing caution.

Figure 11.

From the second group's discussion (Figure 11), we wish to highlight the following threads and connections. The notion of norms that was also raised in Group 1 is elaborated in terms of identifying behavioural expectations that are currently privileged in mathematics and reflecting on how those norms might function as exclusionary. We note the relevance of the discussion on politics and ethics in relation to identity as being central to the work of teaching and learning mathematics so that all learners have opportunities to develop positive mathematical identities without having to deny or feel the need to suppress or devalue other aspects with which their mathematical identities intersect across the locus of their lifeworlds.

In Group 2, cultural responsivity and decolonial lenses emerged as part of the discussion around identity. As a colonised country, with a legacy of denying the identities of Indigenous students, we discussed the ways in which we might focus our efforts to bring Indigenous perspectives to our framing of mathematics in schools and attune ourselves as educators to more holistic and community-oriented pedagogies, and simultaneously unlearn taken-for-granted notions of what it means to do or be good at mathematics. We find this to be an important response-ability (Hunter et al., 2020) in the development and nurturing of students' identities, particularly for Indigenous students and see a connection with the practice of diplomacy advocated for in Group 1.

Again, there were questions related to practical matters of what teachers might need in order to be able to respond to specific identities such as Indigenous learners of mathematics.

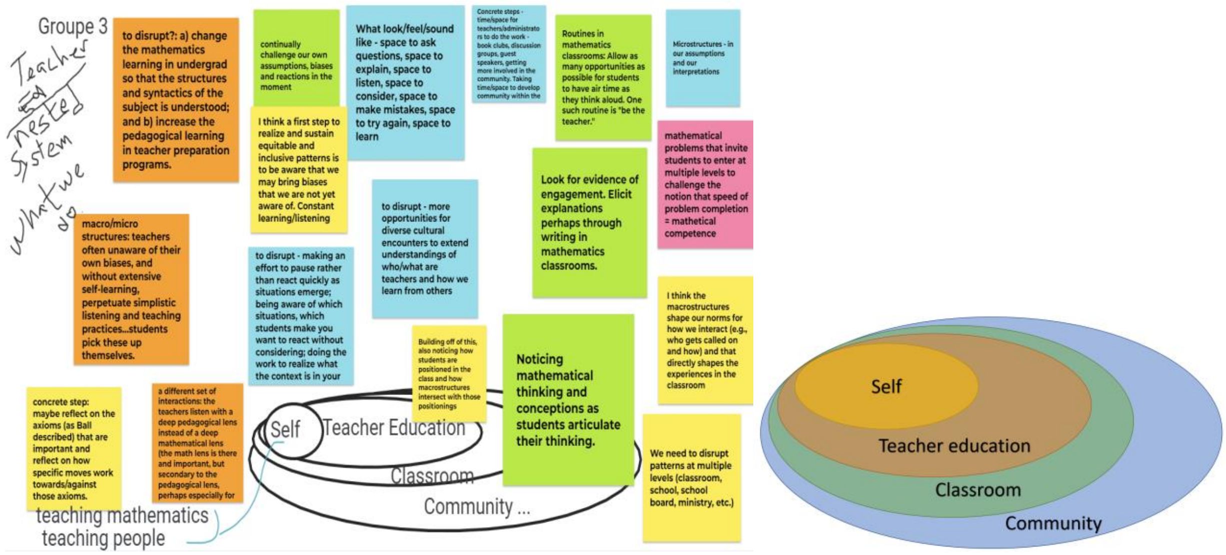


Figure 12.

What we notice about Group 3's discussion (Figure 12): From the third group's discussion, we wish to highlight the following as they managed to attend and record explicit suggestions to the critical prompts about imagining alternatives for more equitable outcomes in relation to identities. As shown in Figure 12, the group's conversation could be organized in a nested framework of inter-related, concentricdomains of action from the self, to teacher education, individual classrooms, and the wider community.

At the level of the self is the need to interrogate our own conscious and unconscious biases, to become aware of what these might be. This might include giving selves permission to pause more often for reflection individually and collectively as well as engaging in empathetic listening as a category of pedagogical and hermeneutic listening that is oriented by concerns for care and well-being and developing identities (Paley, 1986).

At the level of the classroom is the need to enact different sets of interactions (for example we see this extensively in many of the works of our colleagues in Canada who do in-classroom research with teachers). Encounters with different identities is one that spans the levels but situated here as it came up in connection with learners' encounters and developing understanding of who learners are. Open-ended and low-floor, high-ceiling tasks as well as routines provide learners with different affordances to engage and have air-time as they think aloud and discuss. Extending the idea of personal awareness of bias is the idea of noticing or becoming aware of the positionings of learners in individual classrooms and how the macrostructures intersect with those positionings to produce different identities.

At the level of the community, there is the need to reflect on the axioms in our field and how those taken for granted axioms work to marginalize and to articulate specific alternatives or strategies to counter the power of those axioms. Macro-structurally the importance and need for time and space for those in positions of power to do this work through various means such as book clubs, discussion groups, speakers and involvement in community was discussed as an important and under-utlized set of strategies for consciousness raising as well as disruption.

PULLING THE THREADS TOGETHER: WEAVING A TAPESTRY

Conceptually, mathematics identity carries a conceptual complexity into mathematics education. From the broad spectrum of work done, we selected strands of autobiographical mathematical identity, socioculturally available narratives about what mathematics is and what it is that mathematicians do, and freedoms to choose which narratives to align with or reject complex concept, but this work provides one example of how to pull together the threads. We weave together autobiographical identities, sociocultural identities to present one singular tapestry and suggest that there are many ways to craft identity. This tapestry presents questions that still remain to be explored.

Mathematics identity is inextricably linked with the doing of mathematics. In all conversations about maths identity that emerged in the groups, there were implications for what it means to do mathematics, who can do mathematics and who is considered a mathematician. We find we did not adequately address the implications of these conversations on classroom practice and pedagogical methods and suggest further explorations into the applications of this work. What practical approaches might emerge through an exploration of these discussions in terms of classroom practice? This question and the many others that participants expressed through reflection at the end of the three-day conversation are shown in the collaboratively created tapestry in Figures 13 and 14.

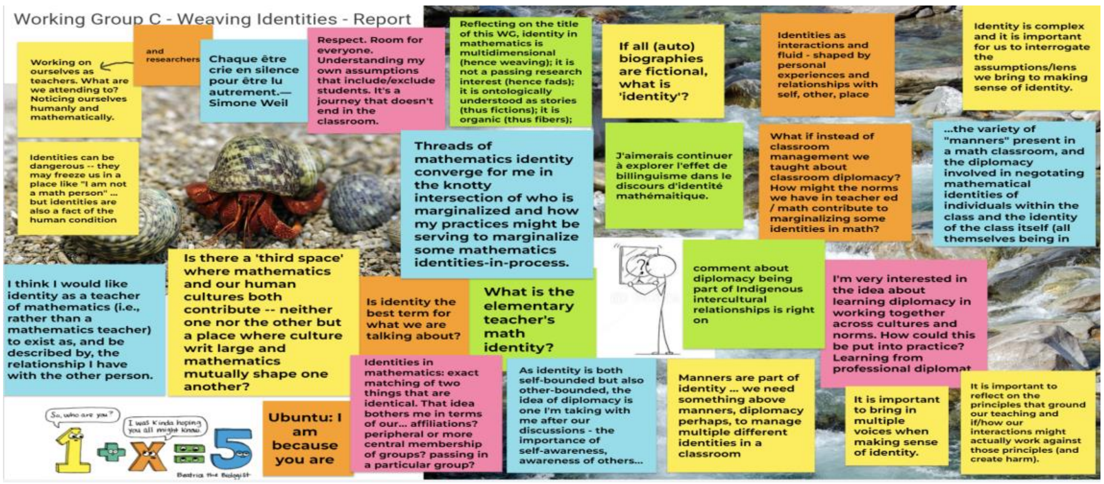


Figure 13.

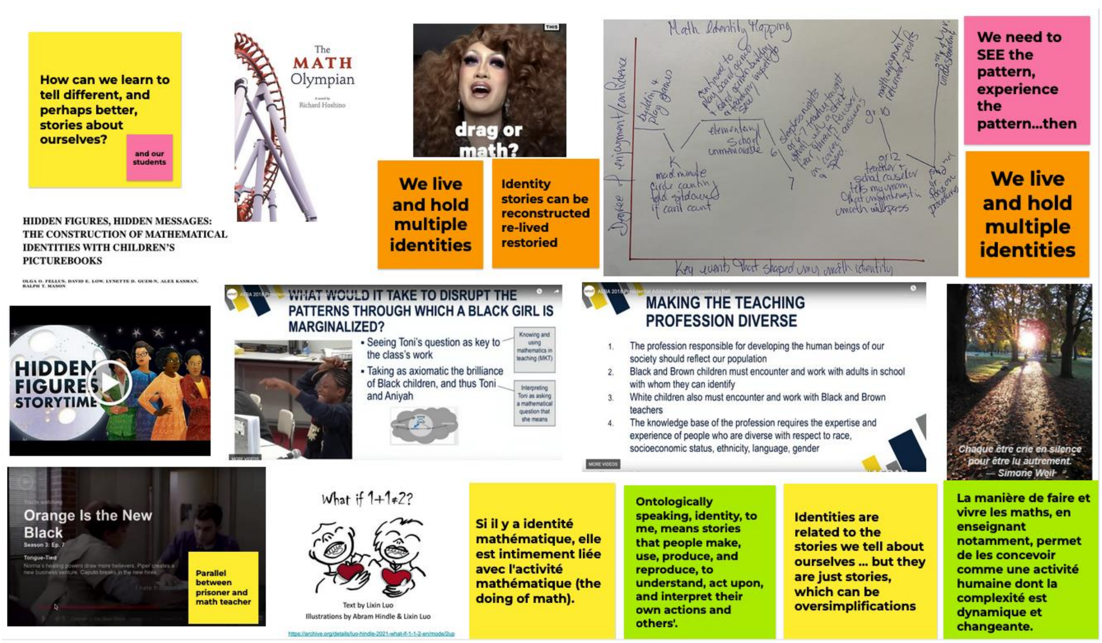


Figure 14.

LOOSE THREADS: A PLACE FOR STRINGS THAT LINGER

There were many places that we had planned to go but time did not permit. In this space we want to point to some of these as they may serve as toeholds for future exploration. The first is the urgent need for more work in mathematics education around the concept of intersectionality as it relates to complex identity positionings. Moore's (2021) work on queer identity and theory intersections in mathematics education and Bullock's (2018) on figure hiding and the narrow analytical frames of critical mathematical education are good examples and places to start. For Bullock, "Figure hiding [involves processes that] renders [some] identities and modes of thought in mathematics education and mathematics education research invisible" (p. 122) with intersectional analysis providing the methodological tools for critical mathematics education researchers (and others) to, "address the complexities of social and political realities and to identify ways that current approaches to research unintentionally participate in figure hiding" (p.142). Our work in this Working Group only just began to open up this complex terrain and is also an area worthy of further attention in our community.

A second important aspect of work in identity was surfaced by participants who acknowledged that there is similar work albeit sometimes emerging from different foundations in science education, teacher education, and cultural studies. However, and this is a critique articulated in multiple literatures inside and outside of mathematics education, there is as yet insufficient commerce among the theoretical, methodological, practical innovations made in these fields around identity. For example the concept of disidentification used in the quote by Gholson and Wilkes (2017) above (page 51 in this text) is borrowed from an important work of Jose Esteban Munoz (1999) which Morissey (2021) suggests is

> *a heuristic that provides critical scholars with a framework for theorizing the relationships between subject formation, ideology, politics, and power while also offering people from marginalized communities a way to navigate intersecting forms of oppression and enact agency. Scholars use disidentification to refer to performances that minoritarian subjects engage in to survive within inhospitable spaces, while nevertheless working to subvert them…[and that] [d]isidentification, by foregrounding identities and experiences of difference, offers…researchers a way to consider how one's life can be understood in relation to others, within the social structures that govern daily life, and within the ideological commitments that organize our experiences.* (lines 5–21)

These sorts of conceptual groundings were not part of our Working Group explicitly, though related ideas were surfaced during the three days for example in the discussion of the perceived need to sacrifice aspects of one's identity in the formation of mathematical ones. We encourage our colleagues to look outside of our near disciplinary homes, perhaps through interesting collaborations with colleagues in other fields who also study identity using different tools.

A discussion emerged amongst the Working Group leaders during the preparation of the proceedings around the US-based context of both Ball's clip (2018) and Gholson and Wilkes' (2017) examples. We wondered about the relevance of the term "the school-to-prison pipeline" in a Canadian context. A cursory internet search suggests that Prison Education programs in Canada are in need of increased funding and expansion (John Howard, 2020) as they reveal ongoing intersections with Indigenous racism and history of colonialism. We sense that there is a need for focused research in this area in a Canadian context including an initial literature review of mathematics education initiatives in the Canadian correctional system.

Inspired by Urie Brofenbrenner's (1979) work on the ecology of human development that offers a helpful model to considering the relationship between and among context-specific experiences and the development of the individual, it is difficult, and perhaps impossible, to think of mathematical identity as siloed, fragmented, and largely devoid of societal biases, ideas, and expectations. Brofenbrenner's concentric ecological system theory is helpful as it helps us see identity work in mathematics education as not only multidimensional but also intricately intertwined as each dimension renders mutually constitutive relationships between and among the other dimensions. In our CMESG three-day conversation on identity in mathematics, we chose to frame the discussions using the metaphor of the act of weaving.

REFERENCES

Ball, D. (2018). *AERA 2018 Presidential Address: Deborah Loewenberg Ball* [video]. YouTube. https://www.youtube.com/watch?v=JGzQ7O_SIYY&t=4740s

Berry III, R. Q. (2021). Identity: Understanding stories. In J. M. Langer-Osuna & N. Shah (Eds.), *JRME Monograph 17: Making visible the invisible: The promises and challenges of identity research in mathematics education* (pp. vii–viii). NCTM.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171200). Ablex.

Bronfenbrenner, U. (1979). *The ecology of human development: Experiments by nature and design*. Harvard University Press.

Bullock, E. C. (2018). Intersectional analysis in critical mathematics education research: A response to figure hiding. *Review of Research in Education*, *42*, 122–145.

Cole, M. (2009). The perils of translation: A first step in reconsidering Vygotsky's theory of development in relation to formal education. *Mind, Culture & Activity*, *16*(4), 291–295.

Clarke, D. (2005). Essential complementarities: Arguing for an integrative approach to research in mathematics classrooms. *Building Connections: Research, Theory and Practice*, 3–18.

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*(1), 19–33. DOI:10.1007/s10649-016-9696-5

Ellsworth, E. (2005). *Places of learning: Media, architecture, pedagogy*. Routledge.

Epstein, D., Mendick, H. & Moreau, M-P. (2010). Imagining the mathematician: young people talking about popular representations of maths. *Discourse: Studies in the Cultural Politics of Education*, *31*(1), 45–60, DOI: 10.1080/01596300903465419

Fellus, O. (2019). Connecting the dots: Toward a four-dimensional conceptualization and operationalization of identity in mathematics education. *ZDM Mathematics Education*, *51*(3), 445–455. https://doi.org/10.1007/s11858-019-01053-9

Fellus, O., & Glanfield, F. (2017). Reflections on the FLM preconference. *for the learning of mathematics*, *37*(1), 15–19.

Fellus, O., Low, D. E., Guzmán, L. D., Kasman, A., & Mason, R. T. (2022). Hidden figures, hidden messages: The construction of mathematical identities with children's picturebooks. *for the learning of mathematics*, *42*(2), 2–8.

Gholson, M., & Wilkes, C. (2017). (Mis)Taken identities: Reclaiming identities of the "Collective Black" in mathematics education research through an exercise in Black specificity. *Review of Research in Education*, *41*, 228–252. DOI:10.3102/0091732X16686950

Gutiérrez, M. V., Willey, C., & Khisty, L. L. (2011). (In) equitable schooling and mathematics of marginalized students: Through the voices of urban Latinas/os. *Journal of Urban Mathematics Education*, *4*(2), 26–43.

Hoshino, R. (2015). *The math olympian*. Friesen Press.

House, D. J. W. (2021). Dear Dr. D'Amour: Relational psychoanalysis at the heart of teaching and learning. *Journal of the Canadian Association for Curriculum Studies*, *19*(1), 137–145.

Howard, J. (2020). *Lack of education and training in Canadian prisons*. https://johnhoward.ca/blog/lack-of-education-and-training-in-canadian-prisons/

Hunter, R., Hunter, J., & Bills, T. (2020). Enacting culturally responsive or socially response-able mathematics education. In C. Nicol, J. Archibald, F. Glanfield, & A. J. Dawson (Eds.), *Living culturally responsive mathematics education with/in Indigenous communities* (pp. 137–154). Brill. https://doi.org/10.1163/9789004415768

Kogen, J. (Writer), Heder, S. (Writer), Kerman, P. (Writer), & Robinson, J.A. (Director). (2015, June 11). Tongue-tied (Season 3, Episode 7) [Netflix series episode]. In J. Kohan, S. Hess, M. Trim, L. I. Vinnecou (Executive Producers), *Orange is the New Black*. Tilted Productions in association with Lionsgate Television.

Kyne. (n.d.). *@onlinekyne*. TikTok.

Lange, T. (2010). "Tell them that we like to decide for ourselves"—Children's agency in mathematics education. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (CERME) (pp. 25872596). Institut National De Recherche Pédagogique.

Langer-Osuna, J. M., & Esmonde, I. (2017). Identity in research on mathematics education. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 637-648). NCTM.

Langer-Osuna, J. M., & Shah, N. (Eds.) (2021). *JRME Monograph 17: Making visible the invisible: The promises and challenges of identity research in mathematics education*. NCTM.

Martin, D. B. (2021). Refusing capital accumulation and commodification: A brief commentary on mathematics identity research. In J. M. Langer-Osuna & N. Shah (Eds.), *JRME Monograph 17: Making visible the invisible: The promises and challenges of identity research in mathematics education* (pp. 123–135). NCTM.

McAdams, D. P., Josselson, R. E., & Lieblich, A. E. (2001). *Turns in the road: Narrative studies of lives in transition*. American Psychological Association.

McFeetors, J., & Mason, R. (2005). Voice and success in non-academic mathematics courses: (Re) Forming identity. *for the learning of mathematics, 25*(3), 16–23.

McGarvey, L., Glanfield, F., Mgombelo, J., Thom, J., Towers, J., Simmt, E., Markle, J., Davis, B., Martin, L., & Proulx, J. (2022). Layering methodological tools to represent classroom collectivity. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 177–201). Alicante, Spain: PME.

Meyer, S., & Aikenhead, G. (2021). Indigenous culture-based school mathematics in action: Part 1: Professional development for creating teaching materials. *The Mathematics Enthusiast*, *18*(1), Article 9. https://doi.org/10.54870/1551-3440.1516

Moore, A. S. (2021). Queer identity and theory intersections in mathematics education: A theoretical literature review. *Mathematics Education Research Journal*, *33*, 651–687. https://doi.org/10.1007/s13394-020-00354-7

Morissey, M. E. (2021). Disidentification. *Oxford Research Encyclopedia of Communication*. https://oxfordre.com/communication/view/10.1093/acrefore/9780190228613.001.0001/acrefore-9780190228613-e-1180

Munoz, J. E. (1999). *Disidentifications: Queers of color and the performance of politics*. University of Minnesota Press.

Niven, L. (1986). *Convergent series*. Del Rey.

NowThis News. (2021). *Drag queen Kyne uses TikTok to teach math* [video]. https://www.youtube.com/watch?v=dy7fF0gDjIQ

Ogawa, Y. (2009). *The housekeeper and the professor*. Picador.

Paley, V. G. (1986). On listening to what the children say. *Harvard Educational Review*, *56*(2), 122–132.

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, *196*(2), 1–38.

Singh, S. (2014). *The Simpson's and their mathematical secrets*. Bloomsbury.

Sfard, A. (2019). Making sense of identities as sense making devices. *ZDM, 51*, 555–564. https://doi.org/10.1007/s11858-019-01058-4

Su, F. (2020). *Mathematics for human flourishing*. Yale University Press.

Stavros, S. G., & Murphy, M. S. (2019). Identity-making through Cree mathematizing. *Canadian Journal of Education*, *42*(3), 692–714. https://www.jstor.org/stable/10.2307/26891581

Weil, S. (1947/1988). *La pesanteur et la grace. Les classiques des Sciences Sociales.* http://classiques.uqac.ca/classiques/weil_simone/pesanteur_et_grace/pesanteur_et_grace.html

ASSESSMENT IN UNDEGRADUATE MATHEMATICS

ÉVALUATION EN MATHEMATIQUES AU NIVEAU POSTSECONDAIRE

Lauren DeDieu, *University of Calgary*
Analia Bergé, *Université du Québec à Rimouski*

PARTICIPANTS

Matthew Cheung	Derek Postnikoff	Frosina Stojanovska-Pocuca
Amenda Chow	Allyson Rozell	Natalia Vasilyeva
Kseniya Garaschuk	Pam Sargent	Brand Yarnton
Kim Koh	Asmita Sodhi	Gaitri Yapa

INTRODUCTION

In this Working Group, we reflected on the characteristics of a well-designed assessment in post-secondary mathematics and the variables that influence its design. At our three two-hour Working Group Zoom sessions, we explored the following questions:

- Do we aim for our exams to emphasize the execution of known operations and procedures, their application to new situations, or a combination of both?
  Vise-t-on dans la conception des évaluations l'exécution d'opérations et de procédures connues, une combinaison de celles-ci, la résolution d'une nouvelle situation ?
- Do we include theoretical questions on our exams (e.g., theorems and proofs)? Why?
  Vise-t-on à inclure des questions théoriques dans un examen ? (Par exemple, énoncer un théorème et le démontrer). Pourquoi ?
- Besides learning the necessary mathematical content, what do we hope students take away from our courses? What assessments support these learning objectives?
  Outre l'apprentissage du contenu mathématique nécessaire, qu'espérons-nous que les étudiants retirent de nos cours ? Quelles évaluations soutiendraient ces objectifs d'apprentissage ?
- How can we ensure consistency in grading assessments?
  Comment pouvons-nous assurer la cohérence de la notation des évaluations ?
- What would be the characteristics of an 'ideal' exam?
  Quelles seraient les caractéristiques d'un examen « idéal » ?
- How can we use assessments to foster the development of metacognitive skills in our students?
  Pouvons-nous favoriser le développement de compétences métacognitives chez nos étudiants par le recours aux évaluations ? Comment ?
- Can we use assessments as a tool to actively engage students in the course and boost motivation? How?
  Pouvons-nous utiliser les évaluations comme un outil pour engager activement les étudiants dans leur cours et stimuler leur motivation ? Comment ?

- To what extent should mathematical communication be emphasized in assessments? How can we help students cultivate this skill?
  Dans quelle mesure la communication mathématique doit-elle être mise en valeur dans les évaluations ? Comment pouvons-nous aider les élèves à cultiver cette compétence ?

DAY ONE: FRIDAY

PART 1—WHAT IS ASSESSMENT?

At our first session, we introduced ourselves to each other and explored the term 'assessment.' Participants split into two breakout rooms where they examined three definitions of assessment found in the literature (Coppé, 2018), and then discussed what assessment means to them. Both groups agreed that at the university level, assessment is very focussed on grades and found it interesting that the three definitions of assessment did not mention grades. They felt that at younger levels, assessment is more about learning, and would like university assessments to be more about learning and less about grades. Overall, our Working Group felt that the main role of assessment is to provide feedback to both students and instructors. We felt that the act of assessment should be less teacher-centred and more student-centred; in particular, assessments should provide students the opportunity to evaluate their own learning. Participants also discussed the value of peer feedback and felt that this should be incorporated into assessments.

PART 2—TRANSFERABLE SKILLS

Psychological determinants, such as motivation, can have a strong impact on students' learning and success in mathematics (Linnenbrink & Pintrich, 2002). While the primary role of assessments may be to provide feedback about students' learning, we wondered if we could also use assessments to help boost motivation. In this session, participants broke up into two breakout rooms and explored the following questions:

- How can we use assessments as a tool to actively engage students in the course?
- How can we use assessments as a tool to boost motivation?

Group members discussed their personal strategies for using assessments to boost motivation/engagement. Here are some highlights:

Scientist Interviews: Invoking purpose in a discipline (e.g., helping students to understand why the content is important, how the discipline helps humanity) can help boost motivation (Lang, 2016). In a first-year calculus course, the instructor could interview people who work in the life sciences and then align the assignments with interviews. This would provide motivation to learn the content because students could see that people actually use the math that they are learning.

Group Exams: In a group exam, students typically work on a portion individually and a portion in a group. These group exams boost engagement because the group has a very well-defined goal and students are motivated to work together to achieve this goal.

Feedback without Grades: After an assessment, the instructor posts feedback about the assignment, but no grade. Students are asked what they think their grade should be. This provides motivation for students to read the feedback and learn from their mistakes.

We concluded Day One by exploring mathematical communication. Participants shared what mathematical communication means to them, the extent to which mathematical communication is emphasized in their courses, the benefits of emphasizing mathematical communication, and where in the curriculum mathematical communication should be emphasized. Participants shared that mathematical writing helps students improve their understanding of logic and their ability to articulate complex ideas clearly—both of which are important skills in basically every field of work. They also shared that it takes the focus away from just getting the 'right answer' and instead emphasizes the learning process. This discussion about the transferable skills related to mathematical communication was meant to set the stage for an activity in the upcoming days where groups choose a transferable skill that is important to them and design an assessment that helps students develop that skill.

DAY TWO: SATURDAY

PART 1—GRADING

We began Day Two by discussing our philosophies behind grading assessments. To set the stage, groups were given a linear algebra prompt with a sample student solution. They were asked to grade it and discuss their rationale. The conversation then broadened to discuss grading more generally. Some themes that emerged from this discussion are outlined below.

Scaffolding: Group members said that they would be careful about putting a computational question with too many steps on a test, as there are so many places where students can go wrong, which could make grading and assigning partial credit challenging. Instead, they would identify the key areas where they would like students to demonstrate mastery and design multiple shorter questions that hit upon each of these learning goals.

Conceptual Understanding: Most group members agreed that they cared more about students being able to demonstrate a conceptual understanding of the material than they did about students being able to do computations. As such, their assessment questions, and the way that they assign partial credit would reflect this.

Strategies for Multiple Graders: There was discussion about the challenges of ensuring consistency when you have multiple Teaching Assistants (TAs) grading the same question. The grading software *Crowdmark/Gradescope/Plom* make it easier to view samples of students' work to get a feel for common mistakes in order to create a more robust rubric and to help communicate expectations to TAs. This software also allows the instructor to review grading to make sure that the TAs are assigning partial credit in a reasonable way.

Instilling Confidence Though Partial Marks: When students in your course have low mathematical confidence, being generous with partial marks when students demonstrate understanding and good reasoning can help boost confidence. One group member said that they tell their graders to always resolve indecision between grades in the students' favour.

PART 2—TRANSFERABLE SKILLS

The second half of Day Two began with an independent reflection:

> *"Besides learning the necessary mathematical content, what do you hope students take away from your courses?"*

The transferable skills identified were summarized into four major categories:

Confidence: We want our students to develop confidence in their mathematical abilities and leave the course with positive feelings towards math.

Metacognitive Skills and Positive Habits of Mind: We want our students to develop metacognitive skills (i.e., accurately assess what they do and do not know and modify their behaviour as a result) and other positive habits of mind related to learning (e.g., growth mindset, embracing failure, grit).

Mathematical Communication and Mathematical Thinking: We want our students to develop their mathematics communication and mathematical thinking skills (e.g., logic, numeracy).

Math in Everyday Life: We want our students to see math in everyday life (i.e., the application of math to the real world).

Group members then began to brainstorm about what assessments they could use to promote this development and support these learning objectives.

DAY THREE: SUNDAY

PART 1—MAKING AN ASSESSMENT

We created four groups—one for each of the transferable skills categories identified on Saturday. Group members chose to join the group that most resonated with them and worked to create an outline for an assessment that addressed their transferable skill. Each group's work is summarized below.

Confidence: The confidence group developed an assignment where students would work in groups to solve math problems each week. Additionally, individual students would submit a reflection about their successes and challenges (e.g., one thing you did well and one thing you would like to improve) every week. This reflection would enable the instructor to see individual students' levels of confidence before and after the math assessment task. Having students reflect on one thing they did well and one thing they need to improve on would hopefully have the benefit of helping students see their growth throughout the course.

The group recommended that the instructor provide examples of a reflection or of past students' work in order to clarify expectations. They also suggested that the instructor could assign badges to students (e.g., active learner, discussion board superstar) throughout the course in order to boost confidence. They recommended Open Badges (2022).

Metacognitive Skills and Positive Habits of Mind: This group developed an assessment for students who are just starting their university career. After every class, they would respond to a prompt that was designed to help them develop metacognitive skills and positive habits of mind. For example, "What were the key ideas of this lesson? How do you feel right now? What were the hardest ideas of this lesson? How do I know I have made a mistake? I am very frustrated right now, why is that?" The questions would change each week so that the assessment would not be repetitive. The group suggested using prompts similar to Francis Su's (2020) 7 exam questions for a Pandemic (or any other time). The focus of the responses should be on feelings and struggles and should not be a summary of the math they did that day. Examples of what it is like to write about your feelings and struggles would be provided.

This would be an ongoing assessment, where students would contribute to it every class and submit it at the end of the semester. It would be graded based on completion and not according to a complicated rubric. It would either be worth bonus marks or a small percentage of the course grade. A few weeks into the semester, students would submit a snippet of their choice, and this would be peer-reviewed. Peers would be provided guidelines of what they should be looking for (e.g., is it clear and easy to read?). For equity purposes, the assignment would not focus on English proficiency; in particular, it could be in point form or paragraphs. The focus would be on the articulation of the process. Students would be encouraged to seek support from campus resources, such as the writing centre, to discuss how they can articulate better. This would hopefully help to get rid of the misconception that you only use these resources when you are in trouble or failing.

The main learning outcomes of this assessment would be for students to

- articulate the key concepts,
- articulate their thought processes,
- explain the key concepts and their thought processes to peers, and
- have a real experience with academic support on campus.

Through this assessment, students would begin to build positive habits of mind where they think about how they think. The hope is that they would carry these habits forward to future courses and to other areas of their life.

Mathematical Communication and Mathematical Thinking: This group developed an expository writing assignment, where students would explain a concept to a someone with less content knowledge than they have. These prompts may be built into a larger assignment. For example, first students might show that a function is discontinuous at a point, and then they would explain what it means to be discontinuous at a point to a peer without using mathematical notation. This assessment would help students develop their ability to communicate mathematical ideas in words, and they would also gain practice in using formulas and precise mathematical language.

The group suggested doing exercises like these in assignments throughout the course so that students have the opportunity to learn and improve their communication skills throughout the semester. If the instructor wanted to incorporate peer feedback, a discussion board could be used to share responses. This group also suggested giving exemplars to demonstrate expectations and noted that non-expert examples could carry more weight than expert ones.

Math in Everyday Life: This group developed an assessment to help students see how math can be applied to the real world. In particular, the motivation was for students to see how math prevents us from falling into error/calamity by exploring a trendy relevant application. An example of such an application may be, “How has math saved the pandemic?”

At the beginning of the semester, students would brainstorm in class to get a list of about 20 subtopics related to the overarching theme (e.g., lockdowns and rates of change, vaccination and probabilities, impact of lockdown on the economy, etc.). Throughout the semester, the professor would highlight the mathematical concepts related to the 20-ish topics generated by the students (e.g., in a probability lesson, why one dose and not two doses of a vaccine). At the end of the semester, students would pick one of these topics and present the applications and mathematics behind it (e.g., provide a short list of real-world issues and a list of mathematical concepts from the course and ask students to choose one of each and explain how the mathematical concept helps with the real-world issue). This could take the form of a question on the final exam, a bonus assignment, a presentation, an essay, or a combination of these (e.g., both a group presentation and a question on the final exam).

If there is time, students could also use technology in a purposeful way. For example, students could find data related to the pandemic online, generate curves, and in turn have a better understanding of functions and how they change over time. This group had a class of approximately 100 students in mind when they designed this assessment. They suggested that it could potentially be tied to multiple courses a math major may take.

PART 2—TOLERABLE FAILURE

We concluded the day by discussing failure. Group members responded to the prompt,

> *Do you think that there exists a certain percentage of ‘tolerable failure?’ As a teacher, do you have in mind a percentage of failure that you could accept and a percentage that would be unacceptable to you?*

The responses are outlined in Figure 1.

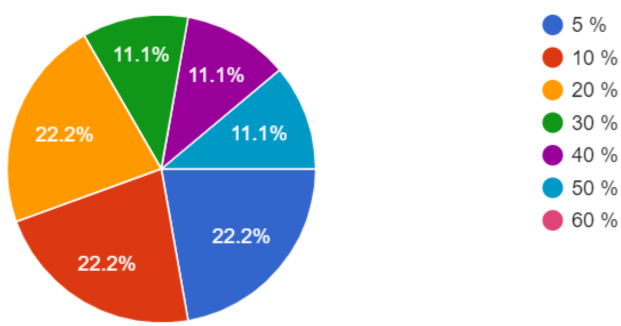


Figure 1.

The high variance in the responses can most likely be attributed to the fact that context matters significantly. One group member shared that the tolerable level of failure will depend on the percentage of students who actually have the prerequisites to be in the course (which is not always the case). Another group member shared that it depends on the type of course; for example, they would expect a higher failure rate in a test-based math course than in a course about education. Other group members pointed to institutional constraints. For example, some departments scale grades in multi-section courses, and others have a policy that if over 30% of students fail a course then the professor needs to justify it. Other group members said that they compare performance on tests to the previous year; if the average seems off, then they review their test and adjust grades if necessary.

CONCLUSION

Our Working Group had many rich and fruitful discussions concerning assessment. There was a very strong emphasis on how we can use assessments to support students (e.g., boost confidence and motivation, support their metacognitive development). We also left with lots of food for thought, such as “What would things be like if we did not have grades? Could grades be assigned without an assessment? Can we motivate students to learn without grades?” Thank you very much to all of our group members for your active engagement and wonderful contributions!

Notre groupe de travail a eu de nombreuses discussions riches et fructueuses concernant l'évaluation. L'accent a été mis sur la manière dont nous pouvons utiliser les évaluations pour soutenir les étudiants (par exemple, pour renforcer leur confiance et leur motivation, pour favoriser leur développement métacognitif). Nous sommes également repartis avec de nombreuses pistes de réflexion, telles que : « Comment seraient les choses si nous n'avions pas de notes ? Des notes pourraient-elles être attribuées sans évaluation ? Peut-on motiver les élèves à apprendre sans notes ? » Merci beaucoup à tous les membres de notre groupe pour votre engagement actif et vos merveilleuses contributions !

REFERENCES

Coppé, S. (2018). Évaluation et didactique des mathématiques : vers de nouvelles questions, de nouveaux travaux. *Mesure et évaluation en éducation*, *41*(1), 7–39. https://doi.org/10.7202/1055895ar

Documentation: Open Badges/Learn/Introduction. (2022, Mar. 4). In *UBC Wiki*. https://wiki.ubc.ca/Documentation:Open_Badges/Learn/Introduction

Lang, J. M. (2016). *Small teaching: everyday lessons from the science of learning* (1st ed.) Jossey-Bass.

Linnenbrink, E. A. & Pintrich, P.R. (2002). Motivation as an enabler for academic success. *School Psychology Review*, *31*(2), 313–327. https://doi.org/10.1080/02796015.2002.12086158

Su, F. (2020, Apr. 26). *7 Exam questions for a pandemic (or any other time)* [blog post]. Francis Su. https://www.francissu.com/post/7-exam-questions-for-a-pandemic-or-any-other-time

MATÉRIEL DE MANIPULATION DANS L'APPRENTISSAGE ET L'ENSEIGNEMENT DES MATHÉMATIQUES AU PRIMAIRE

MANIPULATIVES IN ELEMENTARY MATHEMATICS TEACHING AND LEARNING

Doris Jeannotte, *Université du Québec à Montréal*
Helena P. Osana, *Concordia University*

PARTICIPANTS

Nathalie Bisaillon	Manon LeBlanc	Vanessa Radzimski
Claudia Corriveau	Lixin Luo	Diana Royea
Viktor Freiman	Joshua Markle	Jo Towers
Anjali Khirwadkar	Marshall Morgan	

INTRODUCTION

The use of physical objects, also known as "manipulatives," in the teaching and learning of elementary mathematics is not new. The writings of Froebel and Montessori at the end of the 19th and start of the 20th centuries can attest to this observation. Nonetheless, recent research on the effects of manipulatives is far from conclusive (e.g., Carbonneau et al., 2013; Lafay & Osana, 2021; McNeil & Jarvin, 2007), implying that the complexity of their role in teaching and learning is not yet fully captured by current theoretical and conceptual frameworks.

In this Working Group, we discussed the use of manipulatives at the elementary level (Grades 1 through 6). We began on the first day by exploring a variety of theories that can inform the study of manipulative use in the classroom. On the second day, we investigated the interplay between teaching practice and student thinking through the notion of "affordance" (Gibson, 1979; Greeno, 1994) and "didactic variables" (Brousseau, 1981). Finally, on the third day we thought about the design of new manipulatives that could generate a deeper theoretical understanding of how physical objects can be used in the classroom to support teaching practice and student learning in mathematics.

DAY ONE / JOUR UN

On Day 1, we aimed to explore some ideas about manipulatives that frequently emerge in educational discourse and some theories that could help us make sense of the use of manipulatives in elementary classrooms. Some of those ideas are linked to mathematics learning specifically, others to classroom management in general. For example, one belief discussed is that learning mathematics with manipulatives is important only for some students, usually younger students or ones that encounter difficulties in school. The research does not entirely support this notion (Carbonneau et al., 2013), however. Linked to this first belief is the abstract/concrete duality. We discussed the fact that the use of manipulatives is sometimes justified by the idea that it can help students to render abstract mathematical concepts more concrete, a justification that may find its roots in an interpretation of Bruner's (1966) theory of instruction.

Another idea that was discussed was related to the attributes of manipulatives and how manipulatives are used in the classroom, which were seen as possible learning hindrances. For example, time or classroom management can discourage teachers from using them to the desired extent. In addition, the perceptual features of manipulatives (e.g., color), previous ways the manipulatives were used, and the opportunities students are given to play with them can distract students from learning (Adrien et al., 2022; McNeil et al., 2009; Osana et al., 2018). Participants also mentioned how manipulatives can be linked to creativity as students, with different background knowledge from adults, explore their uses, an aspect that can perhaps leverage subsequent learning.

We then explored three theories: cognition, embodied cognition, and commognition. The Working Group leaders briefly presented each perspective to the participants, which served to provide important nuance to subsequent discussions.

COGNITION

The cognitive perspective places focus on children's mental (i.e., internal) representations of mathematical ideas and concepts. Images and objects, sometimes called "external knowledge representations" (Belenky & Schalk, 2014) are intended to "stand for" abstract mathematical concepts that are targeted in instruction. In this sense, manipulatives can be considered symbols in the sense that they are representational (Uttal, 2003). Aligned with this symbolic view, theories of analogical reasoning (English, 2004; Gentner & Colhoun, 2010) can explain how manipulatives can support students' mathematical abstractions. When children are presented with analogous cases, such as the two external representations of 134 in Figure 1, they can be encouraged to make comparisons of their structural similarities through a process called "structure mapping" (Gentner, 1983). In this example, processes of structure mapping can support the development of key principles of the base-ten numeration. It is for this reason that English (2004) has referred to manipulatives as "pedagogical analogies."

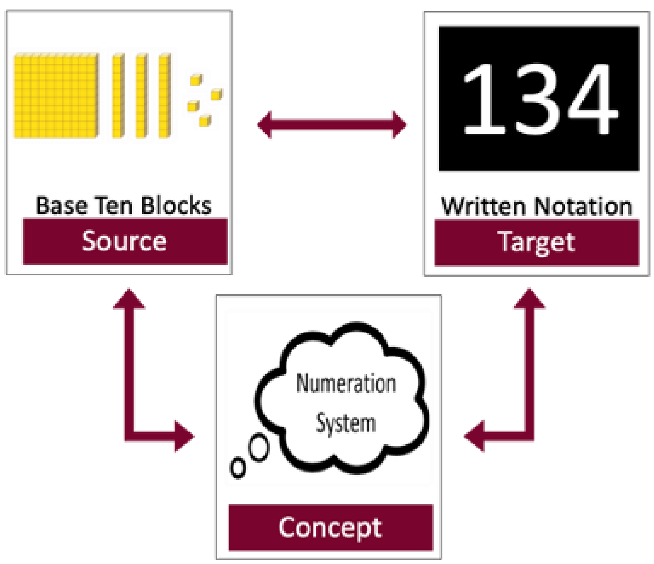


Figure 1. Abstracting numeration concepts through structure mapping between two analogous external representations.

EMBODIED COGNITION

Embodied cognition (Abrahamson et al., 2020) is a theory that rests on the notion that thinking and learning emerge from action and perception. Viewing cognition as tightly linked to the sensorimotor system depicts human thought as "embodied," grounded in physical perception and action with objects in the environment (Glenberg, 2008). Evidence from cognitive psychology and neuroscience shows that mathematical processes, including counting, number comparison, and estimation, are intertwined with perception (e.g., Goldstone et al., 2010) and motor activity (e.g., Martin & Schwartz, 2005). In their theory of physically distributed learning, Martin and Schwartz (2005) argued that actions with objects allow for re-interpretations of mathematical concepts, which in turn impact further actions with the objects in a hand-over-hand fashion.

COMMOGNITION

The non-mentalist theory of commognition (Sfard, 2008) focuses on patterns of participation within a given mathematical community. Learning is defined as the development of mathematical discourses (mathematical activity as communication), e.g., changes in patterns of communication (vocabulary, routines, visual mediator, and accepted narratives). Those communication patterns are called routines, and thus learning occurs through what Lavie et al. (2019) call routinization. Within this perspective, it is possible to study the use of manipulatives in mathematical classrooms as routines. Indeed, manipulatives are not only visual mediators, but their use can nurture the use of particular vocabulary and narratives and the development of particular patterns of communication.

QUESTIONS AND THEORIES

After briefly exploring the theories, the group formulated different questions that could be tackled from the different perspectives. Here is a partial list of the questions that were generated:

- How do children learn to use manipulatives?
- How do manipulatives enable learners to solve problems?
- What are the characteristics of the manipulatives mobilized by the students? What role do they play in the student's mathematical activity?
- How can manipulatives help students regulate their learning?
- How does the context of the problem influence the positioning of manipulatives and vice versa?
- How do children connect different representations of the same idea through the use of manipulatives?

DAY TWO / JOUR DEUX

On Day 2, we explored two tasks through the concepts of didactic variables and affordance. Didactic variables are parameters, linked to the design of a teaching situation, which can vary according to the teacher's choices, and that can lead to a change in the student's mathematical activity (Brousseau, 1981).

Brousseau defined three categories of didactic variables:

- related to conditions (e.g., the choice to have students work in groups of three),
- related to the use of tools (e.g., the choice to use a single sheet of paper in a group activity or the choice to use LEGO® bricks to explore concepts of division)
- related to the problem (e.g., the choice to use fractions in a problem, rather than whole numbers).

Gibson's (1979) theory of affordances has been used to investigate the role of physical features of the environment on human behavior. Nathan et al. (2013) defined the term "affordance" as the physical and perceptual properties of an object that direct and constrain a user's actions in a specific environment. Researchers who study the impact of manipulatives on students' learning in mathematics have relied on Gibson's theory to explain how children interpret, use, and learn from external knowledge representations (e.g., Adrien et al., 2022; Lafay et al., 2022).

LEGO® EXPLORATION

The first task we explored as a group was to examine the display of LEGO® bricks presented in Figure 2 and to consider the question, "Keeping in mind our conversation of didactic variables and affordances, where is the mathematics in this configuration?"

Figure 2. Brick configuration for the LEGO® exploration on affordances.

The discussion that ensued demonstrated that the mathematical concepts that surfaced depended in large part on the perceptual affordances, particularly that of color and the studs in this example, that were salient for each participant. Some of the mathematics that the participants extracted from the display were:

- a sequence of squares (i.e., 4, 9, 16, 25), which may or may not show 2 x 2, 3 x 3, 4 x 4, and 5 x 5, respectively
- unequal parts of a whole, which could generate a number of ideas related to fractions (e.g., in term 2, the red brick is what fraction of the whole?)
- the number of blocks in the sequence (i.e., 1, 3, 2, 7): Is there a mathematical pattern here?
- concepts related to scaling and similarity
- concepts related to the increases in area as a function of the pattern of bricks and/or studs
- concepts related to generalization: Does the 1 x 1 brick in the top left corner of the second and fourth terms figure into a mathematical generalization?

The focus of the first part of the discussion on the mathematics that emerged from the analysis lay squarely on the perceptual affordances in the display. For instance, the observation that the pattern illustrated a sequence of squares was based on the affordance of the studs but not color. On the other hand, the focus on the unequal parts of a whole (e.g., identifying fractional parts of any given square) was primarily based on the color of the bricks. In other words, the physical attributes of the LEGO® bricks impacted the mathematical structure that emerged, which differed by participant.

Furthermore, several questions emerged from the analysis of the LEGO® display that also appeared to have emerged from the perceptual affordances in the configuration. Interestingly, such questions focused on the social aspects behind the configuration, namely the intentions and the thinking processes of the person who constructed the display: What guided the choice of bricks? What was the intent of these choices? There appears to be a lack of regularity in the pattern: Was the person perhaps distracted by the size or color of the bricks? Why were the colors selected in this way? Was it purposeful or random? Was there a pedagogical intent behind the choices of bricks? If the purpose was to show a pattern of squares, might there be a better way to display them in a way that makes the mathematical pattern easier to detect?

THE GARDEN TASK

To continue our reflection on manipulatives using the concepts of variable and affordance, we explored a task (Figure 3) designed in collaboration with teachers in the MathéRéaliser project (see Corriveau & Jeannotte, 2015).

> *Your neighbor wants to make a garden and plant carrots, tomatoes, and lettuce. He wants to give half the area to carrots and give a larger area to tomatoes than lettuce. He wonders what fraction of the garden to reserve for each of the vegetables. Offer two solutions.*
> *Ton voisin veut faire un jardin et y planter des carottes, des tomates et de la laitue. Il veut accorder la moitié de la superficie aux carottes et accorder aux tomates une plus grande superficie qu'à la laitue. Il se demande quelle fraction du jardin réserver à chacun des légumes. Propose-lui deux solutions.*

Figure 3. The garden task.

The participants were encouraged to explore the task with different physical or virtual manipulatives with the following two prompts in mind:

- Analyze the task provided in terms of didactic variables and affordances;
- Think of ways students can use manipulatives to explore the task.

This exploration allowed us to discuss the different meanings of fractions in the context of the problem and/or with the use of manipulatives. In the context of the problem, the fraction could mean a part of an area, where the garden is then represented by a continuous whole. However, any type of manipulative could be used to solve this problem; it is not necessary to use manipulatives that represent a continuous whole. The question about linking different representations seems important here as it becomes important to reconcile the different representations and meanings in the context of the garden and in mathematics.

The choice of the whole combined with the choice of manipulatives is also linked to the possible partitioning strategies. Indeed, if one uses pattern blocks and chooses the hexagon to represent the garden, the possible partitions are not the same as with colored squares (see Figure 4). The freedom in the choice of manipulatives and the whole led to different partitioning; the freedom can be considered itself a didactic variable.

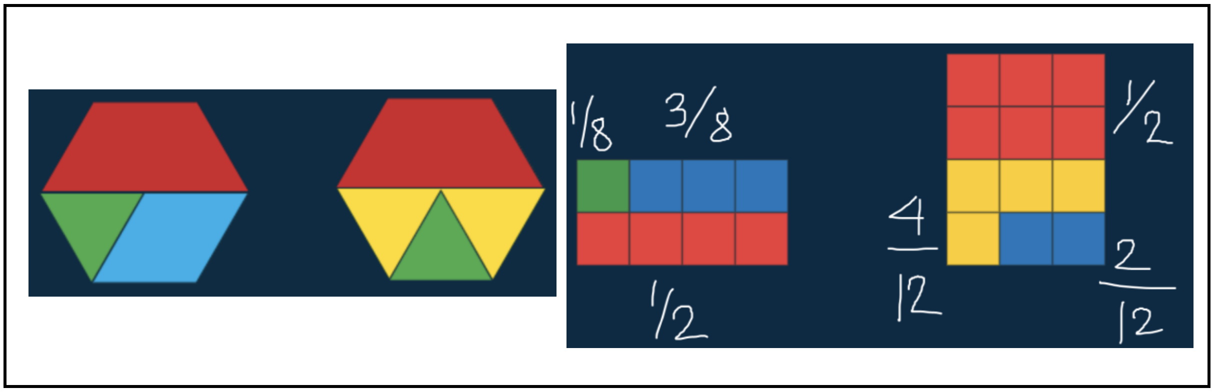


Figure 4. Two answers from participants for the garden task.

The different colors or 'blocks' sometimes allow for perceptions of the partitions used or the different parts constituting the gardens, constituting two other variables that could be taken into account when designing a task with manipulatives.

Manipulatives could be used to solve the task, communicate, or justify the answer. However, the manipulatives alone may not suffice to communicate the answer to this problem even if they represent the garden itself. We noticed that mathematical symbols or oral communication were needed to share the solutions. The different affordances linked to the manipulatives are necessarily present but do not suffice for the generation of the solution (Figure 4).

DAY THREE / JOUR TROIS

DATA EXPLORATION

We started Day 3 by watching three short videoclips of students solving the garden task in pairs. In each of these short clips, we were able to observe how students exploited different attributes of the manipulatives in creative ways, ways that may differ from the designer's intention. In the first clip, two third graders were solving the garden task using green multi-link cubes (Figure 5).

Figure 5. Two third graders using green multi-link cubes to solve the garden task.

The students could not rely on color to distinguish the different parts of their gardens. As seen in Figure 5, they chose to identify their partitions by placing rows of cubes on the top of their garden. As the participants discussed the video, we highlighted that the choice made by the teachers and researchers to give access to only one type of block did not stop the students from solving the task, but rather pushed them to be more creative while explaining important aspects of fraction concepts as equipartitioning.

In the second clip, two fifth graders were solving the garden task using printed cardboard pattern blocks (Figure 6). In this case, the students used smaller pieces as a token to identify the partition of the garden that was formed of ten hexagons. They also relied on the spatial organization as they separated both halves of the garden.

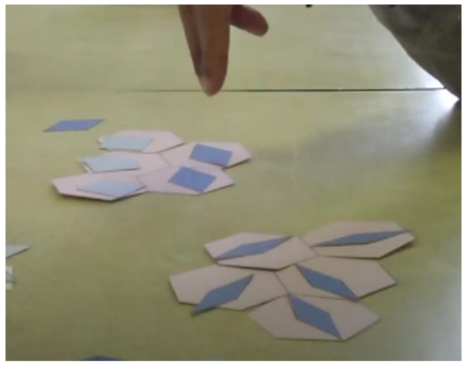


Figure 6. Two fifth graders using cardboard pattern blocks to solve the garden task.

In the third clip, two fifth graders were solving the Garden task using wooden Cuisenaire® Rods (see Figure 7). They used orange rods to 'draw' the contour of their rectangle-shaped garden and green and blue rods to identify the partitions of the garden in half and in $^1/_6$ and $^2/_6$. They had no trouble justifying their partitioning.

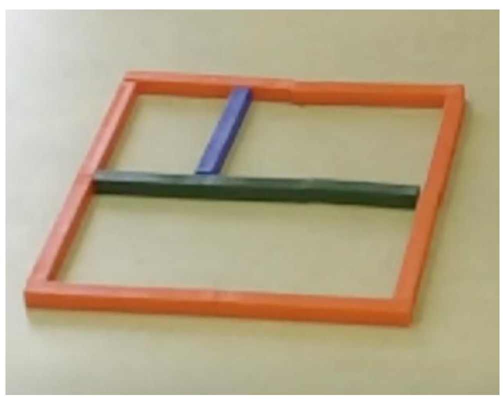


Figure 7. Two fifth graders using Cuisenaire® Rods to solve the garden task.

What we noticed in the three short videoclips is how students used the perceived attributes of the manipulatives differently than what was intended by the designers of the manipulatives. Each pair of students was able to arrive at a valid solution and communicate it using the manipulatives provided. In other words, they were able to generate acceptable narratives with the tools provided to them through exploration.

EXPLORING THE FUTURE

In the final segment of Day 3, we chose to think about the future of manipulative use. How could we imagine new ways to use manipulatives? We projected ourselves in two different directions. The first is virtual reality (VR). How could we think about the use of manipulatives exploiting VR? Thinking of proportionality, we imagined ourselves not only moving the manipulatives virtually but moving with the manipulatives. Several questions emerged: How could we exploit different affordances of virtual reality to design learning opportunities?

Second, new technologies could allow not only better access to educational tools but to the design of those manipulatives themselves. Teachers could design the manipulatives they need to create specific, targeted learning opportunities in their classrooms and create them on 3D printers. It could allow them to exploit colors and shapes differently than what is otherwise available to them. Greenstein et al. (2020) provided a glimpse of what this kind of experience could bring to a prospective teacher's experience. The design of manipulatives not only opens the possibility for the students to explore mathematics concepts differently, but also opens new avenues of exploration for teachers. In addition, if students use manipulatives in innovative ways, could we imagine them designing their own manipulatives? What could happen if students became the designers of their own learning tools?

CONCLUSION

To conclude this report, we attempt a synthesis of what the participants took away from this Working Group. Participants were asked to take the last 30 minutes of the final day to synthesize the main messages that emerged from the activities and discussions. Participants wrote take-away messages and posted them on the Jamboard. The final collection of Jamboard stickies is presented in Figure 8.

Figure 8. Take-away messages from participants posted on the Jamboard.

One point that emerged from the discussion is that the perceptual features of manipulatives act as affordances that can impact mathematical activity. Perceptual affordances can support and also constrain, the discovery, understanding, and justification of the concepts that are brought to bear in activity. Similarly, the notion of didactic variable emerged as a factor when considering the relation between the teacher's instructional goals and students' interactions with the materials that are provided. Didactic variables, such as the numbers in the problems or constraints on the types of manipulatives provided, can potentially change the way students use materials, which in turn is influenced by their perceptual affordances. The discussion highlighted the interplay between affordance and didactic variable in ways that could guide future research in important directions.

Some time was spent discussing a broader conceptualization of affordance to include the notion of 'affordance spaces.' For instance, the students using the green cubes to think through the Garden task (i.e., Figure 5) used the same cubes in different ways to solve the problem. The participants observed the students in the videoclip adapt their environment in ways that achieved their problem-solving goals, leading to different affordances in the 'space.' Students' communications and negotiations as they worked on the task together were important elements of the affordance space in this example.

Other discussion points on manipulatives included the observation that students do not interpret or use manipulatives in the same ways that adults do. The notion of creativity arose as well; being creative in mathematics is important, and manipulatives could provide mechanisms to leverage that creativity in the context of learning and problem-solving. The fine line between students' 'usual ways of doing' and creativity when using manipulatives was an important consideration. How can we allow students to be creative/use their creativity while fostering learning? Finally, the participants shared their thoughts about the apparent complexity that teachers must manage when using manipulatives in the mathematics classroom. Managing classroom activity to leverage affordances and didactic variables involves, among others, assessing students' interpretations of the materials, keeping track of how students adapt (and re-adapt) their environments to make sense of the mathematics at hand, and providing support for student learning is at the core of this complexity.

REFERENCES

Abrahamson, D., Nathan, M. J., Williams-Pierce, C., Walkington, C., Ottmar, E. R., Soto, H., & Alibali, M. W. (2020). The future of embodied design for mathematics teaching and learning. *Frontiers in Education*, *5*, Article 147.

Adrien, E., Osana, H. P., & Uttal, D. H. (2022). *Costs and benefits of telling children the quantitative meaning of manipulatives* [poster presentation]. International Mind, Brain and Education Society, Montreal, QC, Canada.

Belenky, D. M., & Schalk, L. (2014). The effects of idealized and grounded materials on learning, transfer, and interest. *Educational Psychology Review*, *26*, 27–50.

Brousseau, G. (1981). Problèmes de didactique des décimaux. *Recherches en didactique des mathématiques*, *2*(3), 37–127.

Bruner, J. S. (1966). *Toward a theory of instruction*. Harvard University Press.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, *105*(2), 380–400.

Corriveau, C., & Jeannotte, D. (2015). L'utilisation du matériel en classe de mathématiques: quelques réflexions sur les apports possibles. *Bulletin AMQ*, *55*(3), 32–49.

English, L. D. (2004a). Mathematical and analogical reasoning. In L. English (Ed.), *Mathematical and analogical reasoning of young learners* (pp. 1–22). Erlbaum.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive science*, *7*(2), 155–170.

Gentner, D., & Colhoun, J. (2010). Analogical processes in human thinking and learning. In B. M. Glatzeder, V. Goel, & A. A. C. von Müller (Eds.), *Towards a theory of thinking: Building blocks for a conceptual framework* (pp. 35–48). Springer.

Gibson, J. J. (1979). *The ecological approach to visual perception*. Psychology Press.

Glenberg, A. M. (2008). Embodiment for education. In P. Calvo & A. Gomila (Eds.), *Handbook of cognitive science: An embodied approach* (pp. 355–372). Elsevier.

Goldstone, R. L., Landy, D. H., & Son, J. Y. (2010). The education of perception. *Topics in Cognitive Science*, *2*(2), 265–284.

Greeno, J. G. (1994). Gibson's affordances. *Psychological Review*, *101*(2), 336–342.

Greenstein, S., Jeannotte, D., Fernandez, E., Davidson, J., Pomponio, E., & Akuom D. (2020). Exploring the interwoven discourses associated with learning to teach mathematics in a making context. In A. I. Sacristán, J. C. Cortés-Zavala, & P. M. Ruiz-Arias (Eds.), *Proceedings of the 42nd meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 840–844). Cinvestav. https://doi.org/10.51272/pmena.42.2020

Lafay, A., & Osana, H. P. (2021). Manipuler des objets permet-il toujours de développer la pensée mathématique de l'enfant? In G. Hilaire-Debove & N. Joyeux (Eds.), *Proceedings of the XXIème rencontres de l'Unadréo* (pp. 11–32). OrthoEdition.

Lafay, A., Osana, H. P., & Levin, J. R. (2022). Does conceptual transparency in manipulatives afford place value understanding in children with mathematics learning disabilities? *Learning Disability Quarterly*, *46*(2), 92–105. https://doi.org/10.1177/07319487221124088

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in Mathematics*, *101*(2), 153–176. https://doi.org/10.1007/s10649-018-9817-4

Martin, T., & Schwartz, D. L. (2005). Physically distributed learning: Adapting and reinterpreting physical environments in the development of fraction concepts. *Cognitive Science*, *29*, 587–625.

McNeil, N., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory Into Practice*, *46*(4), 309–316.

McNeil, N. M., Uttal, D. H., Jarvin, L., & Sternberg, R. J. (2009). Should you show me the money? Concrete objects both hurt and help performance on mathematics problems. *Learning and Instruction*, 19(2), 171–184.

Nathan, M. J., Srisurichan, R., Walkington, C., Wolfgram, M., Williams, C., & Alibali, M. W. (2013). Building cohesion across representations: A mechanism for STEM integration. *Journal of Engineering Education*, *102*(1), 77–116.

Osana, H. P., Przednowek, K., Cooperman, A., & Adrien, E. (2018). Encoding effects on first-graders' use of manipulatives. *Journal of Experimental Education*, *86*(2), 154–172.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Uttal, D. H. (2003). On the relation between play and symbolic thought: The case of mathematics manipulatives. In O. Saracho & B. Spodek (Eds.), *Contemporary perspectives in early childhood* (pp. 97–114). Information Age.

FACILITATING LEARNING MATHEMATICS ONLINE

FAVORISER L'APPRENTISSAGE DES MATHÉMATIQUES À DISTANCE

Judy Larsen, *University of the Fraser Valley*
Jimmy Pai, *Ottawa-Carleton District School Board*
Mélanie Tremblay, *L'Université du Québec à Rimouski*

PARTICIPANTS

Josh Bekker
Stephanie Boragina
Laura Broley
Gordon Hamilton
Simon Lavallée
Clinton Loo
Doug McDougall
Joyce Mgombelo
Susan Oesterle
Peter Taylor
Mathieu Thibault
Simon Tse

INTRODUCTION

Mathematics education research is robust with findings and framings around what is important when considering how to facilitate the learning of mathematics. We know learning mathematics is centered around student thinking and doing (e.g., exploring, collaborating, generalizing, justifying). Various research (e.g., Boaler & Greeno, 2000) has highlighted the importance of valuing student agency in mathematics (e.g., growth mindset, sense of belonging), which promotes positive mathematical identities and contributes to a deep understanding of mathematical concepts and problem-solving abilities. These findings have been enacted in face-to-face settings for many years and in many contexts.

La pandémie associée au SARS-CoV-2 a toutefois propulsé la tenue d'activités d'enseignement-apprentissage en mathématiques (E-A-M) dans des environnements d'apprentissage entièrement en ligne, et ce, peu importe l'intérêt ou même l'expertise que nous pouvions avoir. Bien que des recherches aient été menées sur l'E-A-M à l'aide de la technologie (e.g. Drijvers, 2013; Hoyles, 2018; Sinclair & Yerushalmy, 2016) et dans des environnements en ligne (p. ex. Borba & Llinares, 2012; Meyer, 2015; Stahl, 2009; Taranto & Arzarello, 2020), le passage forcé à l'enseignement en ligne a donné lieu à de nouveaux défis et de nouvelles opportunités qui ont motivé de plus récentes recherches. Parmi elles, certaines se sont intéressées à documenter les défis et les tensions rencontrées par les enseignant·e·s (e.g., Huang et al., 2020; Tremblay & Delobbe, 2021), d'autres ont documenté les approches adoptées pour faciliter l'apprentissage des mathématiques en ligne (Liljedahl & Larsen, 2021; Trenholm & Peschke, 2020) et les expériences vécues par les élèves ainsi que les étudiant·e·s dans ces contextes d'apprentissage (e.g. Radmer & Goodchild, 2021). Cela dit, il est encore nécessaire d'interroger et de réfléchir collectivement aux différents aspects à considérer pour favoriser et faciliter l'E-A-M en ligne. Le présent groupe de travail s'est donc d'abord intéressé aux principes et valeurs qui colorent les choix et actions didactiques au sujet de l'enseignement des mathématiques. Les approches et moyens retenus en enseignement en présentiel ont été discutés pour amorcer la réflexion sur les conditions permettant de faciliter l'E-A-M en ligne.

Our Working Group title was meant to involve both a focus on *facilitating learning* online, and *learning mathematics* online, with *learning* being what is favoured when engaging with mathematics at a distance. As such, our key guiding questions in this Working Group included:

1. What do we value when we think about creating conditions conducive for facilitating student learning? How are these values informed by our experiences in face-to-face learning settings, and how could they be adapted for an online setting? / Quelles valeurs et plus largement qu'est-ce qui supporte nos choix lorsque nous pensons aux conditions favorisant l'apprentissage des élèves et des étudiant·e·s ? Comment ce qui précède se traduit dans nos expériences en contextes d'apprentissage en salle de classe et comment cela peut ou pourrait être adapté aux contextes d'apprentissage en ligne ?
2. What do we value when we think about learning mathematics? How are these values informed by our experiences in face-to-face learning settings, and how could they be adapted for an online setting? / Quels fondements et valeurs supportent nos choix lorsqu'il s'agit de réfléchir et d'actualiser l'apprentissage des mathématiques ? Comment se traduisent ces valeurs et fondements dans nos expériences d'enseignement-apprentissage en salle de classe ? Comment pourraient-ils être adaptés aux contextes d'enseignement-apprentissage en ligne ?
3. Keeping in mind the values and goals that drive mathematics teaching and learning in our spaces, what challenges arise when moving from face-to-face to online? /En gardant à l'esprit les valeurs, fondements et visées motivant l'E-A-M, quels défis se posent lors du passage de l'apprentissage en salle de classe à l'apprentissage en ligne ?

Within these, we aimed to also pursue more specific points of interest such as

4. How do we build safe, brave, and identity-affirming learning environments online that encourages students to think and do mathematics together? / Comment assurer la mise en place d'environnements d'apprentissage en ligne favorisant le bien-être et la participation des élèves et étudiant·e·s, espaces qui favorisent le développement de manières d'être et de faire des mathématiques ensemble ?
5. How do we leverage tasks and feedback to support students doing, thinking, and communicating together? / Comment tirer profit des tâches et de la rétroaction pour aider les élèves et étudiants à faire, à penser et à communiquer ensemble ?

DAY 1/ JOUR 1 LEARNING MATHEMATICS ONLINE/APPRENDRE LES MATHEMATIQUES EN LIGNE

The theme of the first day was to unpack our views on learning mathematics and to discuss how those views inform learning mathematics in an online setting. To begin this discussion, members were invited to engage in a series of activities: the first welcomed members to the group, the second engaged members in doing mathematics online, and the third was to identify what members value about learning mathematics and how this can transfer to an online setting.

INITIAL THOUGHTS ABOUT ONLINE TEACHING

As people joined the virtual meeting, they were asked to share what brought them to this Working Group on a shared Google Document and to draw a picture of how they feel about online learning into a Desmos Activity (see Figure 1). A conversation ensued to elaborate on these introductions, with various comments pointing to the challenging nature of traversing between the demands of technological challenges, student needs, and personal circumstances.

Figure 1. Member drawings of how they feel about online teaching (posts anonymized).

DOING MATHEMATICS ONLINE WITH THE USE OF A JAMBOARD FOR SMALL GROUP COLLABORATION

The Working Group then proceeding to the first collaborative activity that was designed to elicit an online learning experience with a mathematical task. In this activity, participants were shown several numbered pictures of one group leader's child and were asked to indicate which picture showed a mood they most identified with. The numbered pictures were arranged in a specific manner to pose a problem named *mood connector* (see Figure 2). The task involved connecting each image at first arbitrarily. Once connected, the two values were to be multiplied together. The aim of the task was to find larger sums of products for these connections.

Pour travailler cette tâche, des groupes aléatoires ont été formés au moyen de l'application Zoom, dans laquelle la réunion avait lieu. Un espace de collaboration sur Google Jamboard a été partagé avec tous les membres. Prenant la forme de tableaux blancs interactifs, ses diapositives facilitent la collaboration en temps réel. Divers outils permettent de dessiner, d'insérer des images et de rédiger du texte. Le Jamboard utilisé pour cette tâche avait plusieurs pages sur lesquelles avaient préalablement été insérées des images numérotées et disposées dans un ordre prévu, afin que chaque groupe puisse trouver une page sur laquelle collaborer. Une tâche complémentaire avait pour visée de discuter de comment a été vécue la résolution dans cet espace numérique et de réfléchir aux décisions pédagogiques connexes prises par les responsables du groupe.

Members were brought back to the main room to discuss their experiences and reflections. Notably, groups approached the task in different ways and began to think of ways to extend the problem and change it by changing the order of the numbers, changing the numbers, and deciding to use straight or curved lines to connect the numbers. There was a fascination with how the task and the tools used to represent ideas led to creative mathematical thinking. Some groups also discussed the intentionality of using the faces of one co-leader's child, and how these kinds of decisions impact the task. Figure 2 shows one group's Jamboard where the photos with numbers are shown along with notes about ways to change the problem.

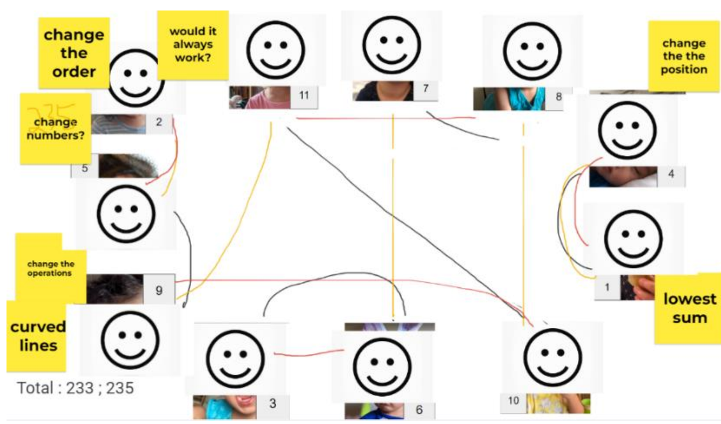


Figure 2. Jamboard from the mood connector task (original faces covered for privacy).

IDENTIFYING WHAT 'LEARNING MATHEMATICS' MEANS TO GROUP MEMBERS

Après cette première activité d'amorce jumelant des ingrédients pouvant susciter l'engagement des participant·e·s (problème mathématique ouvert, collaboration avec recours à la technologie sans avoir besoin de temps d'appropriation de celle-ci), les responsables ont tissé des liens entre les intentions du groupe et les différentes combinaisons d'expressions contenues dans le titre retenu : « Facilitating Learning Mathematics Online ». Une discussion a été lancée dans un Wooclap invitant à définir l'apprentissage des mathématiques. Le nuage de mots présenté à la Figure 3 a été l'occasion de réagir à l'importance collective accordée au travail commun d'élaboration d'idées mathématiques.

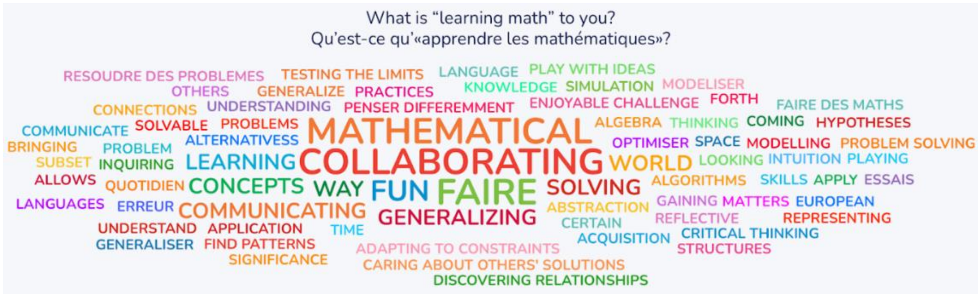


Figure 3. Wordcloud on "What is 'learning math' to you?"

Members then further discussed which aspects of the Wooclap results were important to them in breakout groups. The intent of this activity was to lay the foundation for values we hold that can then translate to online learning environments. Members were asked to make notes on the *Padlet* provided that was structured to allow members to create themes as headings on a column with comments below the themes coming from any group. Figure 4 shows part of the *Padlet* to reveal the structure in which members were working.

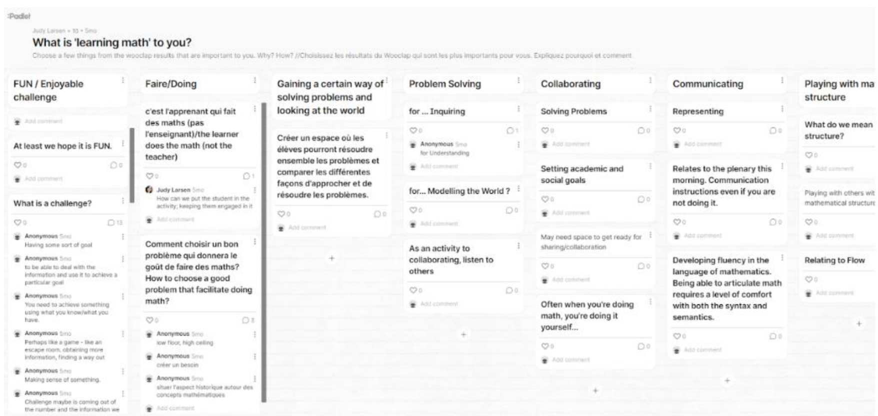


Figure 4. Padlet to further develop the question of "What is 'learning math' to you?"

We summarize the results of this discussion in Table 1 to reveal the key themes emerged.

As the day ended, participants were asked to share some of their own examples from teaching mathematics online to bring the next day towards an online teaching exchange of resources.

Key themes about what learning math means to us	Notes discussed about these themes
A fun and enjoyable challenge, at least we hope!	A challenge is about having some sort of goal, to deal with information and use it to achieve a goal or to make sense of something. It comes out of the kinds of information offered and can exist in either individual or collective settings. We want it to be a fun and enjoyable challenge because we hope it makes the mathematics engaging and that enjoyment would overshadow anxiety when working with challenges. We see collaboration and social interaction as necessary in this aim.
Doing/faire	Nous souhaitons trouver des façons d'engager mathématiquement les élèves en sélectionnant des problèmes ou tâches plus ouvertes qui ont le potentiel d'engager tous les élèves dont ceux en difficulté sur des activités mathématiques.
Problem-solving	We want learners to engaging in problem-solving so that they pursue inquiries towards understanding, model the world around them, and to develop collaboration skills. We want to help learners gain skills for problem-solving and to develop new ways of looking at the world.
Collaborer en/pour communiquer	Nous cherchons des façons de permettre la résolution de problèmes en collaboration puisque nous reconnaissons que cette modalité, notamment par la nécessité de communiquer mathématiquement, est une occasion supplémentaire de développer la compréhension mathématique en coordonnant les aspects syntaxique et sémantique des objets en jeu.
Ways of being	Learning mathematics is also about developing ways of being in the world around us. It helps us relate and consider various aspects of our world.
Humaniser les apprentissages	Une approche bienveillante pourrait porter nos actions afin que nos élèves aient le goût d'oser, qu'ils se donnent le droit à l'erreur et qu'ils prennent des risques. Faire de la classe un lieu où les idées mathématiques de toutes et de tous pourront être exprimées et où sera valorisée la diversité des procédures de résolution développées individuellement ou en équipe.
Caring about others' solutions	We want learners to care about each other, about each others' solutions, and to care about mathematics. Caring relates to appreciating that you can learn from others and that they can learn from you. It also means spending time with mathematics and understanding that solutions can come from various vantage points and understanding how another person arrived at a solution is insightful.

Table 1. Key themes about what learning mathematics means to group members.

DAY 2/JOUR 2 FACILITATING LEARNING MATHEMATICS ONLINE/FAVORISER L'APPRENTISSAGE DES MATHÉMATIQUES À DISTANCE

The theme of the second day was on the ways in which learning mathematics can be facilitated in an online setting. Similarly to the first day, we began with a collaborative learning activity that engaged members in experiencing doing mathematics online and followed it up by discussion about how learning mathematics can be facilitated online, with opportunities to share from members' own teaching experiences. However, the tools used in the collaborative activity and the focus of the discussions were different than on the first day.

DOING MATHEMATICS ONLINE WITH THE USE OF CONCEPTBOARD AND GOOGLE EARTH

The second day's opening activity introduced the use of ConceptBoard as a space for online collaboration. ConceptBoard is a virtual whiteboard with an infinite canvas unlike Jamboards, which are limited to slides. To introduce ConceptBoard, members were asked to practice using its tools in order to indicate through scribbling the locations of where they reside physically on a treaties map of Canada (see Figure 5). The purpose of this was twofold: to consider the locations of members on the treaty map and to try using the ConceptBoard tools themselves.

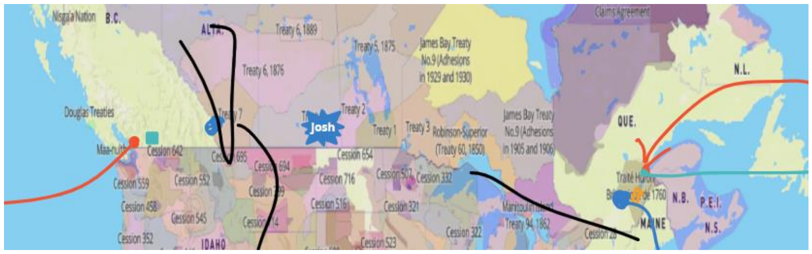


Figure 5. Treaties map with locations of members included via ConceptBoard annotation.

Une tâche collaborative impliquant l'utilisation de Google Earth a ensuite été lancée. L'objectif de la tâche était de trouver un endroit où les trois responsables du groupe de travail pourraient se réunir physiquement de façon équitable. Les outils de Google Earth ont été présentés, comme le montre la figure 6 ci-dessous, où les trois emplacements des résidences des coresponsables sont indiqués. Les membres ont ensuite été répartis au hasard dans deux salles de discussion pour travailler la tâche. Les membres ont été encouragés à utiliser divers outils en ligne comme Google Earth, les prix des vols en ligne, les données sur les émissions de CO2, etc., tout en prenant des notes sur le ConceptBoard.

Figure 6. Google Earth image of the three locations that the group task used.

Each of the groups began similarly by considering various parameters by which fairness could be achieved (e.g., accessibility, ease of travel, weather, distance, cost, when the travelling occurs, likeability of the location, and how many people must travel in each case). However, each group developed different criteria for determining fairness of the chosen location for meeting. We detail some of the thinking from each group on this task as it shows the diversity in mathematical thinking that may present itself in an online environment.

- Le groupe 1 a d'abord pensé à choisir un endroit où une seule personne devrait parcourir une longue distance, mais a ensuite tenu compte du fait que cette seule personne qui devrait voyager le plus loin a un jeune bébé. Ils ont ensuite travaillé à déterminer les distances de déplacement entre les emplacements pour discuter de la façon dont la distance pourrait être partagée. Ils ont également discuté de la notion générale d'ouverture d'un problème et de ce à quoi devrait ressembler le processus de définition d'un problème pour mobiliser les personnes apprenantes. Le présent problème proposé leur semblait un peu trop ouvert. Cependant, ils ont convenu que le processus de questionnement associé à cette phase de définition et même de tentative de résolution était de nature mathématique.
- Group 2, on the other hand, moved right into finding information about various flights and considering costs and CO_2 emissions for each option of travel. They also discussed whether perhaps the co-leaders would prefer to all travel to a more likeable location such as Hawaii rather than finding a middle ground of meeting in Saskatchewan during winter for example. This developed into questioning if everyone has passports for travel abroad and other real-world criteria that could play a role. When shifting into a purely mathematical approach, they discussed that finding the centre of the triangle that forms all three of the locations would be desirable. However, this point may not be a likeable location and therefore may not be

the best solution for the problem. The group then discussed how valuable the experience of working on a complex mathematical modelling problem such as this one was particularly with the tools available online. The question was brought up about whether we should let students also determine the level of complexity of the task since that was part of the process in this task but that it may not lead to specific curricular outcomes desired.

Tous les groupes ont défini des critères d'équité, utilisé divers outils en ligne pour trouver une solution et discuté de l'ouverture d'un problème pour mobiliser les personnes apprenantes. Le groupe 1 a estimé qu'il y avait trop d'ouverture dans le problème, tandis que le groupe 2 a jugé que cette ouverture était utile pour trouver des solutions concrètes. Les notes de chacun des groupes sur le ConceptBoard sont présentées ci-dessous dans la figure 7.

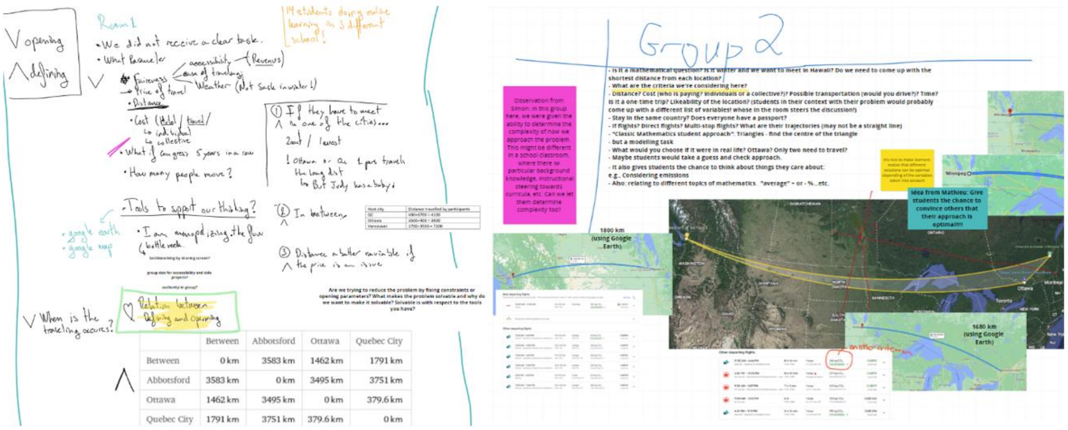


Figure 7. Written work on Google Earth problem on ConceptBoard from two groups.

A discussion about member's experiences of the Google Earth task and the way it was facilitated ensued. The key themes emerging from this discussion were related to the task design as well as breakout group size and composition.

- En ce qui concerne la conception des tâches, les membres discutent de l'influence et des risques potentiels de l'ouverture d'une tâche pour susciter l'expression de raisonnements mathématiques. Pour un groupe, la tâche semblait trop ouverte, tandis que pour l'autre, c'est l'ouverture qui la rendait intéressante. Comme les membres ont reconnu avoir d'abord dû mener un travail de définition de la tâche avant de s'engager dans sa résolution, une réflexion a été menée sur la façon dont les élèves s'engageraient dans cette phase de définition du problème et sur les potentiels freins que cela entraînerait pour certains. Il y a aussi eu une discussion sur la façon dont les expériences récentes des élèves, leur contexte propre et les relations entre les élèves dans la résolution pouvaient influencer les façons d'approcher et de résoudre un tel problème. Un membre a d'ailleurs formulé d'intéressantes questions: "Are we trying to reduce the problem by fixing constraints or opening parameters? What makes the problem solvable and why do we want to make it solvable? Solvable is with respect to the tools you have? Do students have access to the tools? Does the tool define the solution more than the people?"
- In terms of breakout group size and composition, members considered these as factors in why Group 2 seemed to make more progress on the task than Group 1. The team dynamics for groups of five were very different between the groups. In Group 1, five felt too big, and in Group 2, it felt just right. Some factors in the groups' relative success in moving through the task involved aspects such as fluency with using online tools, personal problem-solving approaches, leadership in the group, and personal ways of learning. The leaders shared that they decided on using two groups of five to ensure enough diversity among individuals in the groups; but this seemed to work better for one team than for the other. Perhaps there was too much diversity among the ideas in one group that took away from deciding on how to move forward. One member noted, "I have certainly experienced a group of three that feels 'big' and a group of three that feels 'small.'" This led to other comments about how leadership in breakout groups arises, how sometimes people will 'drop' from engaging in a group perhaps because they need more space or independent think time, or how an unfamiliarity with online tools can be a barrier. As another member noted, "who is in the room steers where it goes," and yet another brought up the idea from Towers et al. (2013) on students as 'pollinators.'

This discussion resulted in emerging key themes for facilitating learning mathematics online such as paying attention to how tasks are designed in terms of opening and defining, how groups are chosen, how many members are in a group, and acknowledging that students sometimes need some space to think before coming back to group learning.

ONLINE TEACHING EXCHANGE AND DISCUSSION ABOUT TEACHING MATHEMATICS ONLINE

Pour discuter davantage des moyens favorisant l'apprentissage des mathématiques en ligne, un échange a suivi au cours duquel les membres ont été invités à partager et à présenter des exemples de leur propre expérience d'enseignement des mathématiques en ligne. Des applications comme le Julia Robinson Mathematics Festival (2022; par exemple, avec une version animée de l'énigme de la traversée de la rivière) et le Exploding Dots de James Tanton (n.d.) ont été mentionnées et discutées, en abordant la façon dont les apprenants s'y sont engagés. Alors que plusieurs membres partageaient leurs tâches, leurs exemples et leurs expériences, des idées d'enseignement en ligne ont émergé. Par exemple, l'attention portée au moment où certaines voix dominent tandis que d'autres voix sont tues ou manquées ou le travail d'accompagnement dans la reconnaissance de la divergence des idées et des façons de les faire converger selon la dynamique de groupe.

The online teaching exchange served as a backdrop for the final summary discussion of the day that was guided by the following questions: *What are different ways of doing mathematics online? How do we support them? How do our choice of media allow or disallow, or impact the kind of mathematics that students do? What are the affordances and constraints?*

Various points emerged from this discussion as summarized in the points below.

- Many group members remarked that the different modes, environments, and tools of teaching and learning are undoubtedly consequential with respect to what teachers and what students do. For example, asynchronous and synchronous online spaces both lead to different sets of considerations and possible actions. The mechanics of a particular app or program along with the various intentions built into it will also change what students and teachers can control and engage with. These differences can have mathematical implications, for example, in terms of what mathematical doing is readily accessible within the realm of the apps or programs. These differences can also have social implications, for example, how students interact with the program, with the teacher, and with one another in a virtual environment.
- The role of technology was also addressed as a tool that can not only 'amplify' (to do more and faster), but also to reorganize (to do something differently than without it's affordances) (Pea, 1985). When facilitating learning mathematics online, the group agreed there are such affordances due to technology, but also new barriers to consider such as those pertaining to group dynamics and functioning. Every tool offers different affordances and constraints that need to be considered (e.g., Zoom allows one 'voice' at a time verbally but also allow for backchannel commentary via the chat function, Jamboards or ConceptBoards allow for multiple learners to annotate at once with annonymity but may miss certain learners' ideas if they do not have access to touch-screen technology).
- Members also discussed how these virtual spaces influence our ability to build a safe, brave, and identity-affirming learning environment and what pedagogical moves are needed to establish such an environment.

DAY 3/JOUR 3 IMPROVING THE STUDENT EXPERIENCE OF LEARNING MATHEMATICS ONLINE/AMELIORER L'EXPERIENCE ETUDIANTE D'APPRENTISSAGE DES MATHEMATIQUES EN LIGNE

The theme of the third day was to consider the overall student experience of learning mathematics online and to use these perspectives towards prioritizing and summarizing the key emergent themes in this Working Group.

DISCUSSION ABOUT STUDENT EXPERIENCES WITH ONLINE LEARNING

An NBC News video showing commentary from students feeling stressed by online learning during the pandemic was shown to the group to initiate a discussion about student experiences in online learning settings (see Figure 8).

Figure 8. Screenshot of NBC News video about students feeling stressed by online learning (NBC News, 2021)

While the overall messaging in the video prompt revealed negative perspectives towards online learning from students, members were quick to point out that these were responses made during the pandemic and that the reports of online learning as being more stressful could have been not only to do with the shift to online learning, but also a result of the pandemic conditions more broadly. The group discussed how the need to shift to online learning came at a difficult time when all other aspects of normal daily life and personal safety were being challenged and renegotiated. The question was posed to consider what online learning will look like in a post-pandemic world. The forced shift to online learning during the pandemic inevitably normalized online learning as a viable option. This leaves us with deciding on how to structure online learning spaces based on the demands and needs of the learning situation. As such, the group debated on the affordances and drawbacks offered by online learning contexts.

En termes d'affordances, le groupe a identifié plusieurs façons où les contextes d'E-A en ligne peuvent améliorer les expériences d'apprentissage. Un exemple a été donné où, au lieu d'organiser une conférence pour plusieurs centaines de personnes étudiantes, il existe maintenant des alternatives selon les besoins. Par exemple, une vidéo en ligne peut bénéficier à celles qui souhaitent prendre une pause, revenir en arrière et la rejouer. Une leçon offerte en comodalité peut aussi permettre aux personnes étudiantes de se déplacer si cela les aide à se concentrer, mais à la fois de participer si elles doivent être à la maison pour s'occuper de leurs enfants ou pour d'autres raisons. Les paramètres offerts en ligne permettent de garder l'anonymat, ce qui peut rendre les personnes étudiantes plus à l'aise de partager leurs idées dans certains cas. L'autonomie offerte dans un environnement en ligne peut également être souhaitable. Les pouvoirs et les responsabilités dans une salle de classe sont redistribués en ligne. La vision de l'école est elle-même en mouvance grâce aux nouvelles opportunités offertes. De nouvelles opportunités de s'engager s'ouvrent et de nouvelles contraintes émergent selon les contextes.

Inevitably, drawbacks of online learning also exist. One major drawback of online learning that the group discussed was around the limitations on possibilities for socializing, collaborating, and expressing care for one another in an online setting. While these can be in some ways compensated for, they are less likely to naturally occur due to constraints on the means for communication. Those who experienced teaching online also expressed the challenges with reading the room when students often turn cameras off. While assumptions about the lack of student engagement due to cameras being turned off should not be made since some students will be fully engaged even if their cameras are turned off, the inability to receive feedback makes it challenging for teachers to tailor their teaching to student needs as easily as when they can see facial expressions, gestures, and other cues. Creating a safe, brave, identity-affirming environment online can also become more difficult online since little moves such as smiling, putting your body closer to who is speaking, pointing across the room, etc. are unavailable. There are also issues around privilege and the considerations that need to be made for students with lower socioeconomic backgrounds who may have less access to both technologies and private workspaces to connect to classes without significant distractions. And, the use of automated testing and an over-reliance on online homework systems can also not only make it challenging for teachers to see the depth and authenticity of student understanding but can also inadvertently communicate to students that mathematics is about getting 'right' answers rather than showing processes and thinking.

Un autre facteur pris en considération est l'âge de la personne apprenante pour déterminer la préférence pour l'apprentissage en ligne ou l'enseignement en présence. À cet effet, Mathieu Thibault a présenté des résultats issus de la présentation de Nicolas Gagnon à la Conférence numérique (Gagnon, 2022), résultats obtenus par le biais d'un questionnaire en ligne complété par 19 898 étudiants de l'Université Laval (19 ans et plus) et où la moyenne de la proportion idéale de cours en présentiel décroît avec l'âge. Les résultats montrent que pour les personnes étudiantes de moins de 21 ans, plus de 70% de l'enseignement devrait être donné en présentiel. Le groupe convient que l'âge de la personne apprenante est un facteur à considérer parmi d'autres.

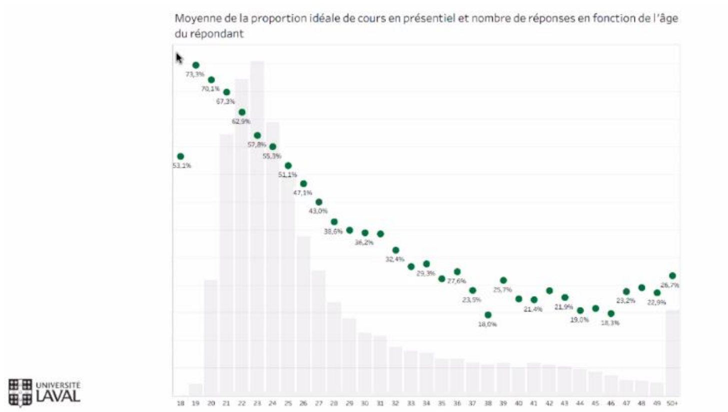


Figure 9. Questionnaire responses for interest in face-to-face learning by age.

Overall, the following points were noted on how to improve the student experience online:

- Allowing space for playing with mathematical structures as well as other structures
- Combiner des méthodes d'enseignement synchrone et asynchrone.
- Changing teaching practices to suit the context
- Réduire la charge cognitive en minimisant le nombre de liens à consulter
- Asking students what they expect from online learning and why they chose it
- Favoriser les opportunités d'apprendre en collaboration, notamment par le recours aux salles.
- Considering equity and access in the online setting
- S'appuyer sur les nouvelles opportunités pédagogiques créées dans le contexte de la pandémie afin de faire des choix pédagogiques qui prendront en compte les contextes et les façons différenciées d'apprendre.
- Humanizing the online learning space and making sure to show care for others, ourselves, and mathematics in the online learning space
- Se souvenir de la logique « ce qui n'est pas interdit est permis » lors de la conception d'activités d'apprentissage en ligne.

FINAL SUMMARY OF KEY POINTS FROM THE WORKING GROUP

As the Working Group ended, a final discussion was held to identify key themes from the Working Group's discussions. A Google Doc was used to allow members to add points during the discussion towards a collaborative set of key themes. The themes identified are as follows:

- La complexité d'usage de certains outils numériques peut être une barrière que l'on doit considérer.
- Task design needs to be intentional and the technological and mathematical structures involved need to be considered.
- Planifier les scénarios d'apprentissage en prenant en compte différents contextes, différentes valeurs et perspectives peut alimenter les moments ou trajectoires d'apprentissage où les apprentissages en ligne, en modalité synchrone ou asynchrone, seraient à préconiser.
- Online tools can help extend the world and offer affordances otherwise unavailable.
- Prendre en compte la diversité des façons d'approcher des problèmes et de les résoudre est à considérer dans l'enseignement en ligne.
- New pedagogical ecologies have arisen because of the prevalence of online learning, allowing for novel uses for tools and assessment strategies.

- L'apprentissage en ligne offre des moyens (p. ex. anonymat, accès, nouvelles façons d'interagir) mais aussi des limites (p. ex. approche bienveillante doit être réfléchie, façons de percevoir et d'accompagner l'engagement des élèves, etc.)
- Humanizing the mathematics learning experience (and mathematics, more generally) online is important (e.g., informal chats at the beginning of class, showing care for learners and mathematics, networking and collaborating).

CONCLUSIONS

Through experiencing doing mathematics collaboratively online during the Working Group meeting as well as sharing about various discussion points about experiences with facilitating mathematics online, the Working Group developed themes and new directions for further inquiry. As we reflect again on the initial guiding questions for the Working Group as well as the Working Group title, we see a few key emergent points to highlight.

The values we hold around what it means to facilitate student learning in mathematics remain as strong guideposts for both evaluating and designing online learning spaces for engaging with mathematics. The values discussed in this Working Group included focusing on process over product, humanizing mathematics, promoting student engagement, improving access to mathematics, and considering our pedagogical ecology in designing learning spaces for mathematics. Given these values, our attention was drawn to the affordances and drawbacks of using online tools for facilitating learning mathematics.

Nos conceptions sur la discipline, l'apprentissage de celle-ci et les valeurs qui portent son enseignement influencent la planification et la conception des espaces et des activités d'enseignement des mathématiques en ligne. Des composantes qui méritent d'être mises à l'avant furent alors discutées : l'importance de miser sur les processus (faire des mathématiques) et non uniquement les produits, l'humanisation des mathématiques, les moyens favorisant l'engagement des personnes étudiantes et des élèves, notamment, par la prise en compte de l'environnement pédagogique dans la conception d'espaces d'apprentissage pour les mathématiques. Le recours au numérique et aux environnements en ligne amène de nouvelles opportunités, mais aussi bien des défis.

On one hand, online tools improve access to learning in certain circumstances, offer novel structures for working with mathematics and with others, create anonymity for learners that can make them more comfortable, and can be a useful way to replace certain practices such as large lectures. On the other hand, teaching students how to play with mathematics, how to care for one another, and how to problem solve with each other can be more challenging in an online environment.

Building safe, brave, and identity-affirming learning environments online that encourage students to think and do mathematics together is the ultimate aim. Having this sort of mindset can serve as a launching point for designing new ways of facilitating mathematics online given the rich new pedagogical ecology that has emerged from the necessities of the pandemic. However, keeping a critical eye on our practices as we facilitate learning mathematics online is undoubtedly important.

In terms of questions for future pursuit, the Working Group developed the following:

- Que conserve-t-on de nos expériences d'enseignement en ligne maintenant que nous ne sommes plus forcés d'enseigner à distance ?
- What is specific to online learning that contributes to the enhanced pedagogical ecology it offers? And how can we extend this ecology metaphor to the ecology of tools, of students working together, etc.?
- Comment tirer parti de l'apprentissage en ligne pour améliorer les opportunités d'apprendre des personnes étudiantes et des élèves (p. ex. joindre d'autres classes, obtenir de la rétroaction) ?
- What are some different online ways that can help humanize the experience with mathematics?
- Quelles habiletés développent les personnes étudiantes et les élèves à travers des expériences d'apprentissage à distance ?
- How do we get students to 'play' online? What do they gain from 'playing' online (which may transfer to non-online)? What are the structures that make it effective online?

- Comment les environnements en ligne peuvent-ils offrir des espaces de résolution de problèmes ouverts où les personnes apprenantes auront plus d'opportunités de prise de décisions tant dans la problématisation du problème que dans sa résolution ?

REFERENCES

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing. *Multiple Perspectives on Mathematics Teaching and Learning*, *1*, 171–200.

Borba, M.d.C., & Llinares, S (2012). Online mathematics teacher education: Overview of an emergent field of research. *ZDM Mathematics Education*, *44*, 697–704.

Drijvers, P. (2013). Digital technology integration in mathematics education: Why it works (or doesn't). *PNA*, *8*(1), 1–20.

Gagnon, N. (2022, May 26). *Regard sur l'évolution des pratiques d'enseignement chez les enseignants et enseignantes du supérieur, 24 mois après le déclenchement de la pandémie mondiale de COVID-19* [conference présentation]. Numérique 2022, Montréal, Québec, Canada. https://numerique2022.hec.ca/programmation/evolution-des-pratiques/

Hoyles, C. (2018). Transforming the mathematical practices of learners and teachers through digital technology. *Research in Mathematics Education*, *20*(3), 209–228.

Huang, R., Tlili, A., Chang, T. W., Zhang, X., Nascimbeni, F., & Burgos, D. (2020). Disrupted classes, undisrupted learning during COVID-19 outbreak in China: application of open educational practices and resources. *Smart Learning Environments*, *7*, 1–15.

Julia Robinson Mathematics Festival. (2022). *Fun math for all*. https://jrmf.org/

Liljedahl, P., & Larsen, J. (2021). Building thinking classrooms online: A closer look at the types of tasks we use. *Virginia Mathematics Teacher*, *47*(1), 8–14.

Meyer, D. (2015). *Functionary: Learning to communicate mathematically in online environments* [Doctoral dissertation, Stanford University]. ProQuest Dissertations & Theses Global.

NBC News. (2021, February 16). *Students share struggles of online learning: 'I have never felt so much stress'* [online video]. https://www.nbcnews.com/now/video/students-speak-out-about-pressures-of-online-schooling-during-covid-101112901755

Pea, R. (1985). Beyond amplification: Using the computer to re-organize mental functioning. *Educational Researcher*, *20*(4), 167–182.

Radmer, F., & Goodchild, S. (2021). Online mathematics teaching and learning during the COVID-19 pandemic: The perspective of lecturers and students. *Nordic Journal of STEM Education*, *5*(1). DOI:10.5324/njsteme.v5i1.3914

Sinclair, N., & Yerushalmy, M. (2016). Digital technology in mathematics teaching and learning: A decade focused on theorising and teaching. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 235–274). SensePublishers Rotterdam.

Stahl, G. (2009). Studying virtual math teams. New York, NY: Springer Press.

Tanton, J. (n.d.). *Exploding dots*. G'day math. https://gdaymath.com/courses/exploding-dots/

Taranto, E., & Arzarello, F. (2020). Math MOOC UniTo: An Italian project on MOOCs for mathematics teacher education, and the development of a new theoretical framework. *ZDM Mathematics Education*, *52*(5), 843–858.

Towers, J., Martin, L. C., & Heater, B. (2013). Teaching and learning mathematics in the collective. The *Journal of Mathematical Behavior*, *32*(3), 424–433.

Tremblay, M., & Delobbe, A. M. (2021). Enseignement et évaluation des mathématiques à distance durant la Covid-19. *Canadian Journal of Learning and Technology / La revue canadienne de l'apprentissage et de la technologie* (CJLT/RCAT), *47*(4). https://cjlt.ca/index.php/cjlt/article/download/28098/20537/73987

Trenholm, S., & Peschke, J. (2020). Teaching undergraduate mathematics fully online: a review from the perspective of communities of practice. *International Journal of Educational Technology in Higher Education*, *17*(1), 1–18.

New PhD Reports

Présentations de thèses de doctorat

UNDERSTANDING MULTILINGUAL LEARNERS' MATHEMATICAL EXPERIENCES AND MEANING MAKING IN A CANADIAN EDUCATIONAL SETTING

Fatima Assaf
University of Ottawa

In the last six years, Canada has welcomed more than 1,200,000 immigrants and refugees; about half of them are school aged children and young adults (Citizenship and Immigration Canada, 2017). When newly arrived immigrant and refugee children enter our educational systems (i.e., Canadian in this case), they enter with a wide variety of previous experiences. Some have had interrupted or inconsistent schooling, some have been homeschooled, and still others have never attended school before. When these students come to our schools, they are confronted with a whole new experience, and with a new language as the means through which they interact with mathematics.

Language is recognized to play a crucial role in the teaching, learning, and doing of mathematics, an idea that is widely acknowledged by mathematics education researchers (Barwell, 2014a; Morgan et al., 2014; Moschkovich, 2017; Planas et al., 2018). The crucial role of language is even more important to consider in plurilingual[1] classrooms found in many Canadian schools. In such classrooms, the teaching and learning of mathematics is carried out in languages other than the student's home language. In the case of immigrants and refugees, where Arabic, Nepali, or Somali is often the newcomer's first language, English is often the language of instruction.

There are different views regarding multilingual[2] learners' success in such situations. Some believe that poor performance of multilingual learners in mathematics is attributed to the learner's limited proficiency in the language of instruction—suggesting that fluency in that language is necessary for the development of mathematical competency (Clarkson, 2006; Cummins, 2001; Halai, 2007, 2009). A review of the literature in this area challenges the idea that students need to have mastered the official language of instruction prior to learning mathematics (Barwell, 2005; Civil, 2008; Moschkovich, 2007). These researchers demonstrate that the knowledge of the language of instruction is only one aspect of becoming competent in mathematics and leads us to consider the multiple resources multilingual learners use to construct knowledge, negotiate meanings, and to communicate mathematically (Moschkovich, 2002, 2007). According to Gorgorió and Planas (2001), knowledge of the language of instruction is not simply or only a matter of reaching a survival competency in that language, but also attaining knowledge of the beliefs, values, and culture predominant in the host society. These learners may experience issues linked to conflicting social, cultural, linguistic, and political barriers, which are very often at odds with those previously experienced. Thus, learning and mathematical achievement emerge from a wide array of interrelated influences. I define influence as the capacity to affect someone's behaviour, actions, or views in a direct or indirect way, but which leads to the transformation of an experience. Some researchers claim "poor performance by multilingual learners thus cannot be attributed to the learners' limited language proficiencies in isolation from the wider social, cultural and political factors that infuse schooling" (Setati et al., 2008, p. 16).

The overall purpose of my study is to form an in-depth description and understanding of multilingual learners' mathematics learning within a Grade 2/3 classroom. The study addresses the following research questions:

What are the mathematical experiences of Grade 2/3 students in a plurilingual classroom in Ontario, Canada?

[1] I have taken plurilingual to imply any classroom with the presence of students who speak different languages, whether these languages are used or not, may be referred to as a plurilingual classroom.
[2] I use the definition of multilingual as articulated by Barwell (2003) to refer to any person in the presence of more than one language, even if they are not used, may be referred to as multilingual.

a) What are the influences on their mathematics experiences?
b) How do they negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

A BRIEF SUMMARY OF RELATED LITERATURE

I conducted an extensive review of the research literature in the area of language diversity and mathematics education which revealed a range of influences on multilingual students' learning of mathematics in various settings. This analysis prompted me to organize the influences that this research found into four categories: individual, pedagogical, institutional, and sociopolitical features of mathematics classrooms. These influences are important to understand, as they are highly interconnected and form a nested system of influences on students' experiences learning mathematics in an additional language.

Research on individual influences focuses on the characteristics of the individual learner that influence his or her mathematics learning. Much of this research has examined the role of the students' proficiency with the language of instruction as influencing their development of mathematical understanding. However, more current research has shown a shift away from thinking of the multilingual learner as being disadvantaged by being less fluent in the language of instruction towards valuing the multiple resources that the individual learner brings to the mathematics classroom from their home and previous learning experiences (Barwell, 2005; Moschkovich, 2014; Planas, 2014; Setati, 2005). In particular, such perspectives recognize the student as a valuable contributor of knowledge in the mathematics classroom, and also, recognize what influences the student's mathematics learning (Moschkovich, 2002; Elbers & Haan, 2005; Gorgorió & Planas, 2001).

A second key influence on students' learning of mathematics is characterized by teachers' pedagogical practices, as well as their views and experiences that shape and influence student's learning of mathematics. A review of the research literature suggests that the constraints and affordances that students experience in the classroom depend on the pedagogical practices of their teachers (Adler & Ronda, 2017; Kaur, 2013). For instance, Adler and Ronda (2017) argue that "learners' access to doing and talking mathematics is through their teacher" (p. 66). Hence, teachers play a fundamental role in students' access and engagement in mathematical discourse (Hunter & Hunter, 2018). Another influence on students' learning of mathematics is characterized by teachers' views and experiences that shape their pedagogical practices. Some researchers suggest that when teachers are confronted with a more diverse classroom, they tend to minimize the cultural differences among their students. The idea of minimizing cultural differences is further supported by teachers' conceptualization that mathematics knowledge is culture-free and universal (Abreu, 2005; Abreu & Gorgorió, 2007; César & Favilli, 2005). In such situations, teachers' understanding of mathematics as culture-free and universal prevents them from inviting and accepting alternate ways of solving problems, which can cause problems for children learning mathematics.

One additional strand of research on multilingual students' learning of mathematics focuses on institutional influences. Research in this area identify the school and the classroom as the institutions, and where, institutional influences impact mathematics teaching and learning. These institutional influences may legitimize the use of certain languages and disregard others (Barwell, 2012, 2018; Chronaki & Planas, 2018; Moschkovich, 2005; Norén & Andersson, 2016; Planas & Setati, 2009; Planas & Valero, 2016; Setati, 2005). According to Barwell (2012), "the pressure comes from a preference for a single school language, a standardized mathematical register, national and institutional language policies and the high status accorded to some natural languages and social languages" (p. 327). In an important way, diverse educational settings bring about differences in how languages are viewed and used in mathematics classrooms (Barwell et al., 2016; Moschkovich, 2014).

An additional and more recent strand of research on multilingual students' learning of mathematics focuses on sociopolitical influences. Sociopolitical influences relate to government or public affairs of a country or province or the policies that govern it. Setati (2005) suggests two categories of sociopolitical influences, each at a different level. One is at the macro level of provincial policy, and the second is at the micro level of the school or even classroom interactions. In either case, relations exist within and outside an educational setting that influence the actions, behaviors or beliefs of those within that setting, including school boards, schools, teachers, and students. Thus, when we talk about sociopolitical influences, we are dealing with issues related to educational policies which have an influence on teacher practice and, in turn, influence students' participation and attainment in mathematics education

(Barwell et al. 2016; Chronaki & Planas, 2018; Civil & Planas, 2011; Halai & Clarkson, 2016; Planas, 2018; Setati, 2005; Setati & Planas, 2012).

Although a review of the research literature on multilingual learners and mathematics suggests that researchers have observed many children in countries spanning the globe, there is still a small number of research in Canada (e.g., Barwell, 2014b, 2017, 2018, 2020; Takeuchi, 2015, 2016) that specifically focus on multilingual learners' mathematical experiences in a plurilingual educational setting. Hence, an objective of my research study is to build on this research to provide an in-depth description and understanding of multilingual learners mathematics experiences in a plurilingual educational setting, in Canada.

THEORETICAL PERSPECTIVES

This research adopts a sociocultural understanding of learning and development that considers "learning, thinking, and knowing…[as] relations among people engaged in activity in, with, and arising from the socially and culturally structured world" (Lave, 1991, p. 67). An account of Vygotsky's theory of human development identifies three key features that form the core of his theory including, a genetic method, cultural tools, and social interactions, as well as two fundamental constructs that have been widely considered by sociocultural researchers: internalization and the zone of proximal development. As part of the genetic method, Vygotsky acknowledges the influence of the history and the wider community or institution on the individual's participation in an activity. In particular, Vygotsky considered four basic lines in the development of behaviour: development in the species as a whole (phylogenesis); development over time in the wider culture of which the individual was becoming a member (cultural history); development through interactions in specific sociocultural settings (microgenesis); and finally, development over the life of an individual (ontogenesis). The second feature pertains to cultural tools, which Vygotsky notes "are culturally produced and represent cultural innovations and changes, [and] as we internalize their use, our thought is undeniably cultural in nature" (Nasir & Hand, 2006, p. 461). Commonly cited examples of cultural tools include "language, various systems of counting, mnemonic techniques, algebraic symbol system, work of art, writing, schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs; and so on" (Vygotsky, 1981a, p. 137; as cited in Wertsch, 1985, p. 79). The third feature relates to the idea that the individual and the society in which s/he participates are mutually constituted.

In terms of the two fundamental constructs; internalization and the zone of proximal development, Vygotsky viewed internalization as a process whereby certain aspects of patterns of activity that had been performed on the external (social) plane become executed on the internal (individual) plane. Vygotsky further argued that internal mental processes are created as a result of one's exposure to social processes. Hence, the emphasis is on the inherent relationship between the two planes of activity, and where internalization is addressed in consideration of the social origins of individual activity, which involves transformation rather than an identical replication (Wertsch, 1985). As for the zone of proximal development, Vygotsky (1978) argued that in order to understand the relationship between development and learning, we must determine at least two developmental levels. The first is the actual developmental level determined by what children can do on their own. The second is the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers (p. 86). Although Vygotsky specified "more capable peers", many researchers have argued that it is not necessary for there to be a person who is more capable than others because any person interacting with others could make a contribution that helps towards a solution (Forman & McPhail, 1993; Wells, 1999). Other researchers also add that such interactions are not necessarily exclusive to another human being but could include various artifacts or other things that support learning (Brown et al. 1996).

These understandings shifted the focus on learning from something that takes place exclusively inside a person's head to something that originates in social interactions and culturally organized activities (Ford & Forman, 2006; Lave, 1991; Wells, 1999). Hence, turning to sociocultural theories, we shift from what multilingual learners cannot do, and instead consider the multiple resources multilingual children use to communicate mathematically (Moschkovich, 2007; 2017). In Vygotskian terms, the focus is on "the potential for progress in what learners already know and do" (as cited in Moschkovich, 2012, p. 91).

RESEARCH SETTING AND PARTICIPANTS

Given my sociocultural perspective of learning and development, I wanted to conduct an ethnographic study to understand multilingual learners' mathematics experience and meaning making in a Canadian educational setting. As such, the study took place over seven months, with visits four to five times a week in a Grade 2/3 classroom in Ontario, Canada. All students in the classroom were invited to participate in the research project. Although some students moved and others arrived, ultimately, there were 18 students who participated in the project. All students' parents were either refugees or have immigrated to Canada. Also, 70% of the students were born outside of Canada, in Eritrea, Nepal, Somalia, Sudan, and Syria. All students spoke a first language other than English at home, including Arabic, Chin, Hindi, Nepali, and Somali. For students born outside of Canada, four came to Canada when they were between the age of one and four years old, and eight students arrived within the past 18 months at the time I was there and were between the ages of five and six. There was also one student from Somalia who arrived to Canada and joined the classroom in February 2019.

DATA COLLECTION

In order to address the research questions, this study consisted of three types of data: in class observations; video and audio recordings of students engaged in mathematical problem-solving activities; and follow-up interviews with the students.

The observations included notes written in the form of journal entries for each day I visited and interacted with the students in the mathematics classroom. I noted the ways the students and their teacher engaged with one another; the ways they interacted around mathematical problem-solving activities; the ways they produced oral or written explanations of their mathematical thinking; and of the ways they used the language(s) they know and other resources during mathematical discussions. The observation notes were analyzed to answer the first sub research question: What are the influences on multilingual learners' mathematics experiences?

The second component of the study focused on video and audio recorded data of students engaged in mathematics problem-solving activities. There are close to 50 video and audio recordings that capture students' interactions and thinking through their mathematical discussions and explanations as they worked together on mathematical problem-solving activities, as well as the ways they hesitated, made eye contact, used multiple resources, and gestures. These video and audio recorded data were analyzed to answer the second research sub-question: How do multilingual learners negotiate mathematical meaning during mathematical problem-solving activities in a plurilingual educational setting?

The third component of research included one-on-one interviews with students to better understand their experiences with and views of mathematics. The interviews were used for contextual information when answering the main research question (i.e., what are the mathematical experiences of multilingual learners), as well as the two research sub-questions.

All data sets (i.e., observation notes, video and audio recordings, and interviews) were used to address the main research question: What are the mathematical experiences of Grade 2/3 students in a plurilingual classroom in Ontario, Canada?

DATA ANALYSIS

Recognizing that learning and development are socially mediated, I used a microanalytic approach which allows for a detailed description of the social interaction within an immediate context (i.e., the Grade 2/3 mathematics classroom), and also the wider social processes of the community within which the school was situated (Bloome & Golden, 1982). Hence to build such a detailed description, one aspect of the analysis I focused on was my observation notes. I analyzed and presented the data using the four influences on students' learning of mathematics including, individual, pedagogical, institutional and sociopolitical features of the mathematics classroom. These four influences came from an extensive review of the research literature that I conducted in the area of language diversity and mathematics education. This analysis prompted me to organize the influences that this research found into these four categories.

In order to address the second research sub-question. I specifically focused on the video recordings of students' participation in mathematical problem-solving activities. Here, I drew on Gee's (2005, 2011) construct of building tasks to analyze "language-in-use". Language-in-use, according to Gee (2011) "is a tool, not just for saying or doing things, but also, used alongside other non-verbal tools, to build things in the world" (p. 88). Hence, whenever we speak or write, we are building or designing one of seven *things* in the world or in reality. These things are what Gee (2005, 2011) refers to as the "seven building tasks" of language, which include significance, activity, identity, relationships, politics, sign systems or knowledge, and connections. I have translated the building tasks to the context of mathematics education. These building tasks were all part of students' mathematical meaning making. In an important way, various themes started to emerge within each of the building tasks, which helped me to understand how the students negotiated meaning-making during mathematics activities.

The follow-up interviews were used for explanatory and contextual information for addressing all research questions. I paid particular attention to students' conceptualizations of "what is mathematics" or "what it means to do mathematics", as well as, their experiences learning mathematics at school, at the afterschool homework club, and at home. The analysis develops an in-depth understanding of students' previous and new mathematical learning experiences, and also, their reflections towards their participation in the mathematics classroom.

RESULTS

In response to the first research sub-question, I argued that teacher's pedagogical moves were significant to students' mathematics learning and the development of their identities. Through the teacher's pedagogical moves, the students engaged in mathematics learning that fostered the use of various strategies, concrete learning tools, and communication with others to reason their way to a solution. Hence, students' identities were constantly changing and evolving as they engaged with the mathematical practices of the Grade 2/3 classroom. In my response to the second research sub-question, I argued that as the students interacted with one another they utilized language to make meaning and to negotiate their mathematical understanding. Through the use of language, the students made certain terminologies, strategies, and tools significant; they portrayed their ways of using and developing mathematical knowledge and skills; they took on certain identities as they engaged with mathematics activities; they built relationships with other students, their teacher, or with mathematics; they distributed mathematical authority; and they privileged certain sign systems or knowledge over others. All of these revolved around students' interactions and their use of language to make meaning and to negotiate their mathematical understanding. Then based on the results of the two research sub-questions I addressed my main research question. In my response to the main research question, I argued that students' histories or previous experiences in specific sociocultural settings became part of their repertoire which informed their mathematics learning in the Grade 2/3 classroom. I also argued that students' interactions with one another strongly impacted their mathematics learning experiences as they were constantly negotiating their understanding by sharing their varied understandings and ways of thinking. Also, students' mathematics experiences were shaped by their use of language which emphasized the use of certain cultural tools to engage in mathematics in the Grade 2/3 classroom and elsewhere. Finally, I reiterated the important role of the teacher's pedagogical moves which closely aligned with a sociocultural perspective of learning and development and provided the context for the students' mathematics experiences. Ultimately, the detailed descriptions of multilingual learners' mathematics experiences and meaning making build on the small number of research in Canada (e.g., Barwell, 2014b, 2017, 2018, 2020; Takeuchi, 2015, 2016) that specifically focuses on multilingual learners' mathematical experiences in a plurilingual educational setting.

REFERENCES

Abreu, G. de (2005). Cultural identities in the multiethnic mathematical classroom. In M. Bosch, M. (Ed.), *Proceedings of the fourth Congress of the European Society for Research in Mathematics Education* (pp. 1131–1140). FUNDEMI IQS, Universitat Ramon Llull.

Abreu, G., de, & Gorgorió, N. (2007). Social representations and multicultural mathematics teaching and learning. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the fifth congress of the European society for research in mathematics education* (pp. 1559–1566). Department of Education, University of Cyprus.

Adler, J., & Ronda, E. (2017). Mathematical instruction in discourse matters. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into improvement in mathematics teaching and learning* (pp. 64–81). CRC Press.

Barwell, R. (2003). Patterns of attention in the interaction of primary school mathematics student with English as an additional language. *Educational Studies in Mathematics, 53*, 35–59.

Barwell, R. (2005). Working on arithmetic word problems when English is an additional language. *British Educational Research Journal*, *31*(3), 329–348.

Barwell, R. (2012). Heteroglossia in multilingual mathematics classrooms. In H. Forgasz & F. Rivera (Eds.), *Toward equity: Gender, culture, and diversity: Advances in mathematics education*. (pp. 315–332). Springer.

Barwell, R. (2014a). Mathematics and language. In P. Andrews, T. Rowland, & S. Brindley (Eds.), *Masterclass in mathematics education: International perspectives on teaching and learning* (pp. 75–86). Bloomsbury Publishing.

Barwell, R. (2014b). Centripetal and centrifugal language forces in one elementary school second language mathematics classroom. *ZDM—Mathematics Education*, *46*, 911–922.

Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *Journal of Mathematical Behavior*, *50*, 155–168.

Barwell, R. (2020). Learning mathematics in a second language: Language positive and language neutral classrooms. *Journal for Research in Mathematics Education*, *51*(2), 150–178.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J. N., Planas, N., Phakeng, M., Valero, P., & Villavicencio Ubillús, M. (Eds.) (2016). *Mathematics education and language diversity: The 21st ICMI study*. Springer International Publishing.

Bloome, D., & Golden, C. (1982). Literacy learning, classroom processes, and race: A microanalytic study of two desegregated classrooms. *Journal of Black Studies*, *13*(2), 207–226.

Brown, L. A., Ash, D., Rutherford, M., Nakagawa, K., Gordon, A., & Campione C. J. (1996). Distributed expertise in the classroom. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 188–228). Cambridge University Press.

César, M., & Favilli, F. (2005). Diversity seen through teachers' eyes: Discourse about Multicultural classes. In M. Bosch (Ed.), *Proceedings of the fourth Congress of the European Society for Research in Mathematics Education* (pp. 1153–1164). FUNDEMI IQS, Universitat Ramon Llull.

Chronaki, A., & Planas, N. (2018). Language diversity in mathematics education research: A move from language as representation to politics of representation. *ZDM—Mathematics Education*, *50* (2), 1101–1111.

Civil, M. (2008). Mathematics teaching and learning of immigrant students: An overview of the research field across multiple settings. In O. Skovsmose & B. Greer (Eds), *Opening the cage: Critique and politics of mathematics education* (pp. 127–142). Sense Publishers.

Civil, M., & Planas, N. (2011). Whose language is it? Reflections on mathematics education and language diversity from two contexts. In S. Mukhopadhyay & W. Roth (Eds.), *Alternative forms of knowing (in) mathematics: Celebrations of diversity of mathematical practices* (pp. 71–89). Sense Publishers.

Clarkson, P. C. (2006). Australian Vietnamese students learning mathematics: High ability bilinguals and their use of their languages. *Educational Studies in Mathematics*, *64*, 191–215.

Elbers, E. & de Haan, M. (2005). The construction of word meaning in a multicultural classroom. Mediational tools in peer collaboration during mathematics lessons. *European Journal of Psychology of Education*, *20*(1), 45–59.

Ford, M. J., & Forman, E. A. (2006). Redefining disciplinary learning in classroom contexts. In J. Green & A. Luke (Eds.), *Review of educational research* (Vol. 30, pp. 1–32). American Education Research Association.

Forman, A. E., & McPhail, J. (1993). Vygotskian perspective on children's collaborative problem-solving activities. In E. A. Forman, N. Minick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 213–229). Oxford University Press.

Gee, J. (2005). *An introduction to discourse Analysis: Theory and method* (2nd edition). Routledge.

Gee, J. P. (2011). *How to do discourse analysis: A toolkit*. Routledge.

Gorgorió, N., & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics, 47*, 7–33.

Halai, A. (2007). Learning mathematics in English medium classrooms in Pakistan: Implications for policy and practice. *Bulletin of Education & Research*, *29*(1), 1–15.

Halai, A. (2009). Politics and practice of learning mathematics in multilingual classrooms: Lessons from Pakistan. In R. Barwell (Ed.), *Multilingualism in mathematics classrooms: Global perspectives* (pp. 47–62). Cromwell press group.

Halai, A., & Clarkson, P. (Eds.) (2016). *Teaching and learning mathematics in multilingual classrooms: Issues for policy, practice and teacher education*. Sense Publishers.

Hunter, R., & Hunter, J. (2018). Opening the space for all students to engage in mathematical practices within collaborative inquiry and argumentation. In R. Hunter, M. Civil, B. Herbel-Eisenmann, N. Planas, & D. Wagner (Eds.), *Mathematical discourse that breaks barriers and creates space for marginalized learners* (pp. 1–22). Sense Publishers.

Kaur, B. (2013. Participation of students in content-learning classroom discourse: A study of two Grade 8 mathematics classes in Singapore. In B. Kaur, G. Anthony, M. Ohtani, & D. Clarke (Eds.), *Student voice in mathematics classrooms around the world* (pp. 65–88). Brill—Sense.

Lave, J. (1991). Situating learning in communities of practice. In L. B. Resnick, J. M. Levine & S. B. Teasley (Eds.), *Perspectives on socially shared cognition* (pp. 63–82). American Psychological Association.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: An overview of the research in the field. *ZDM—Mathematics Education*, *46*, 843–853.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning*, *4*(2&3), 189–212.

Moschkovich, J. (2007). Bilingual mathematics learners: How views of language, bilingual learners, and mathematical communication impact instruction. In N. Nasir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 89–104). Teachers College Press.

Moschkovich, J. (2012). How equity concerns lead to attention to mathematical discourse. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education* (pp. 147–163). Springer.

Moschkovich, J. (2014). Bilingual/ Multilingual issues in learning mathematics. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 57–61). Springer.

Moschkovich, J. (2017). Revisiting early research on early language and number names. *EURASIA Journal of Mathematics Science and Technology Education*, *13*(7b), 4143–4156.

Nasir, S. N., & Hand, M. V. (2006) Exploring sociocultural perspectives on race, culture, and learning. *Review of Educational Research*, *76*(4), 449–475.

Planas, N. (2014). One speaker, two languages: Learning opportunities in the mathematics classroom. *Educational Studies in Mathematics*, *86*, 306–320.

Planas, N. (2018). Language diversity builds mathematics learning as much as mathematics learning builds language diversity. In U. Gellert, C. Knipping, & H. Straehler-Pohl (Eds), *Inside the mathematics class: Sociological perspectives on participation, inclusion, and enhancement* (pp. 103–118). In G. Kaiser & B. Sriraman (Series Eds.), *Advances in mathematics education*. Springer International Publishing.

Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language: Lessons and directions from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing research in mathematics education: Twenty years of communication, cooperation and collaboration in Europe* (pp. 196–210). Routledge.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for research in Mathematics Education*, *36*(5), 447–466.

Setati, M., Molefe, T., & Langa, M. (2008). Using language as a transparent resource in the teaching and learning of mathematics in a Grade 11 multilingual classroom. *Pythagoras*, *67*, 14–25.

Setati, M., & Planas, N. (2012). Mathematics education across two language contexts: A political perspective. In O. Skovsmose & B. Greer. (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 167–186). Sense Publishers.

Takeuchi, A. M. (2015). The situated multiliteracies approach to classroom participation: English language learners' participation in classroom mathematics practices. *Journal of Language Identity & Education*, *14*(3), 159–178.

Takeuchi, A. M. (2016). Friendships and group work in linguistically diverse mathematics classrooms: Opportunities to learn for English language learners. *Journal of the Learning Sciences*, *25*, 411–437.

Vygotsky, L. (1978). *Mind in society*. Harvard University Press.

Wells, G. (1999). *Dialogic inquiry towards a sociocultural practice and theory of education*. Cambridge University Press.

Wertsch, J. V. (1985). *Vygotsky and the social formation of mind*. Harvard University Press.

DÉVELOPPEMENT DU SENS DU NOMBRE ET DE LA NUMÉRATION : ÉLABORATION D'UN OUTIL D'ÉVALUATION ET D'UNE SÉQUENCE DIDACTIQUE

Nathalie Bisaillon
Université de Montréal

PROBLÉMATIQUE

Depuis 2001, le Québec s'est doté d'un programme de formation en mathématiques au primaire basé sur le développement de compétences (Gouvernement du Québec, 2001). Ce programme met de l'avant la compréhension des élèves plutôt que l'acquisition de connaissances factuelles. Cette orientation du programme vise ainsi, en contexte d'apprentissage des mathématiques, le développement du sens du nombre. Le sens du nombre est la compréhension générale qu'un individu a du nombre et des opérations arithmétiques. C'est une forme de raisonnement, une capacité de réflexion sur les nombres, qui se manifeste par de la flexibilité dans le traitement et dans la manipulation des nombres ainsi que dans le choix des stratégies opératoires (Bobis, 2007; Northcore & McIntosh, 1999; Reys & Yang, 1998). Le sens du nombre se développerait tout au long de la vie (Greeno, 1991; Howden, 1989; McIntosh & Dole, 2000; Reys, 1994). L'une des premières manifestations de ce développement est la subitisation. La subitisation est la capacité de percevoir instantanément une petite quantité (Clements, 1999; Dehaene, 2003; Mandler & Shebo, 1982; Starkey & Cooper, 1980; Wynn, 1992a, 1992b). Elle est fortement liée aux habiletés visuospatiales qui sont associées à la capacité que les individus ont à générer, retirer, garder ou manipuler de l'information visuelle spatiale (Hawes et al., 2009). Ces habiletés permettraient éventuellement aux élèves de manipuler mentalement les nombres. Le sens du nombre et les habiletés visuospatiales sont deux importants prédicteurs de la réussite au primaire.

Une composante essentielle du développement du sens du nombre est la compréhension de notre système de numération, système qui permet de représenter et de nommer les nombres (Patenaude & Mathieu, 2019). Cet apprentissage pose des défis pour certains élèves du primaire. Dans les années 1980, Bednarz et Janvier-Dufour (1984a, 1984b, 1986, 1988) avaient observé que les élèves ont de la difficulté à voir et à comprendre le principe de groupement sous-entendu par l'écriture positionnelle. Plus récemment, Koudogbo et ses collègues (Koudogbo, 2013; Koudogbo et al., 2017) ont constaté qu'il ne semble y avoir aucun changement dans les performances des élèves par rapport à ceux des années 1980. Cependant, Bednarz et Janvier-Dufour (1984a, 1984b, 1986, 1988) ont constaté qu'il est possible d'accompagner les élèves dans leur compréhension de la numération en travaillant, entre autres, sur la perception de la pertinence du groupement. Plus récemment, les travaux de Jordan et ses collègues (Jordan, 2010; Jordan & Dyson, 2016) permettent d'ajouter que le développement des habiletés arithmétiques normalement développées dans la petite enfance, dont la subitisation, faciliterait aussi la compréhension de la numération. Peu de recherches s'intéressent aux conditions favorisant l'apprentissage de la numération en intégrant les habiletés arithmétiques précoces dans les séquences d'enseignement et d'apprentissage. De plus, certaines études s'intéressent au développement du sens du nombre chez les jeunes enfants et d'autres aux apprentissages qui sont liés à la numération, mais sans nécessairement associer ces deux aspects. L'objectif général de la présente recherche est donc de mieux comprendre comment se développe le sens du nombre et de la numération, de la petite enfance jusqu'à l'âge de 8 ans et de faire ressortir les conditions qui favorisent ce développement.

CADRE THÉORIQUE

Cette thèse s'intéresse au développement du sens du nombre menant à la compréhension de notre système de numération. La numération indo-arabe est un « système comprenant des symboles et des règles d'utilisation de ces symboles permettant d'écrire, de représenter et de nommer les divers nombres » (Patenaude & Mathieu, 2019, section Lexique Mathématique). Ce système de numération est multiplicatif, par la valeur attribuée à chaque position, et additif, en sommant toutes les valeurs de position. Sa structure s'appuie également sur le principe de groupement (groupes et groupes de groupes de 10) qui donne lieu au principe d'équivalence (une dizaine vaut 10 unités). Enfin, notre système est positionnel (le chiffre prend la valeur de la position).

La compréhension du système de numération et de ses différentes composantes est le fruit de la construction de connexions entre ses différentes représentations. Les élèves deviennent capables de faire des liens entre des représentations externes de ce concept qui peuvent être concrètes, imagées, symboliques ou orales et des représentations internes, qui sont celles qu'une personne évoque dans sa tête. Ces dernières peuvent être imagées, symboliques, verbales ou affectives (Duval, 1996; Hiebert & Wearne, 1992). Les études de Thomas et de ses collaborateurs (2002) permettent d'ajouter que ce sont les représentations internes imagées et dynamiques des quantités qui sont le plus associées à la compréhension des concepts de la numération.

DÉVELOPPEMENT DU SENS DU NOMBRE

Pour mieux comprendre comment se développe le sens du nombre jusqu'à cette compréhension de la numération, plusieurs études s'intéressant à certains moments du développement du sens du nombre ont été consultées. La synthèse de ces recherches a mené à l'hypothèse d'un continuum de développement du sens du nombre. Ce continuum s'apparente plutôt à des vagues qu'à un développement linéaire (Siegler, 2010) ; il fait ressortir les liens entre les aptitudes primitives des enfants en arithmétique et la compréhension de la numération ainsi que la construction de représentations mentales dynamiques et imagées. Cinq niveaux de développement du sens du nombre et de la numération ont été établis. Un âge possible a été associé à chacun des niveaux.

Au premier niveau, il est question des aptitudes présentes chez les nourrissons, soit les aptitudes de subitisation. Les jeunes enfants sont capables de reconnaître de façon instantanée trois ou quatre éléments ou moins (Dehaene, 2003; Houdé, 2004; Wynn, 1992a, 1992b) et de constater la différence entre deux collections lorsque les quantités de ces collections sont au moins du simple au double (Starkey & Cooper, 1980).

Au deuxième niveau, il s'agit du développement de la pensée additive, tel que proposé par Clark et Kamii (1996). Cette pensée se développerait autour de 5-6 ans (Brissiaud, 2005). Elle pourrait être caractérisée par la capacité de tirer profit des habiletés de groupitisation, cette capacité de compter plus rapidement lorsque les éléments sont placés en petits lots (Clements, 1999; Clements et al., 2019; Starkey & McCandliss, 2014). Les habiletés de groupitisation seraient fortement liées à la compréhension de la numération (Schleifer & Landerl, 2011; Starkey & McCandliss, 2014). À ce niveau, les élèves seraient capables de reconnaître des constellations ou d'en utiliser de nouvelles pour compter des quantités allant jusqu'à 20 et de se les représenter mentalement (Brissiaud, 2005; Clements, 1999; Clements et al., 2019). C'est aussi à cette étape qu'ils développeraient leurs habiletés de comptage séquentiel les menant au dénombrement et à une perception abstraite du nombre entier (Fuson, 1991; Gelman & Gallistel, 1978) ; ils développeraient alors une certaine flexibilité additive.

Le niveau trois du continuum correspond à la pensée multiplicative Clark et Kamii (1996) et à la pré-valeur de position (Jones et al., 1994) qui semble se développer autour de 6–7 ans. Pour ces auteurs, la pensée multiplicative permettrait désormais de considérer de façon simultanée deux informations sur le même objet, comme dans la tâche de l'inclusion des classes de Piaget (1947) ou dans les tâches associées à la disposition rectangulaire proposées par Battista et al. (1998). Cette pensée, plus abstraite, tirerait profit de la pensée additive, plus précisément des habiletés de groupitisation et de la prise de conscience du fait qu'organiser les quantités en petits groupes permet de compter plus vite. Il s'agit maintenant de faire des groupes organisés, soit des paquets de même quantité et, éventuellement, des groupes de groupes. La pensée multiplicative permettrait aux élèves de comprendre le rôle du groupement dans le système de numération (Bednarz et Dufour-Janvier, 1984a, 1984b, 1986; Koudogbo et al., 2017), les rendant capables de se représenter mentalement des quantités groupées (organisées en groupes de groupes) et de compter par bonds. Les enfants développeraient alors une certaine flexibilité multiplicative.

Le quatrième niveau est le passage à la dizaine et à la centaine (Jones et al., 1994; Jones et al., 1996) qui semble se développer autour de 7-8 ans. Les élèves doivent comprendre les règles d'équivalence du système de numération. En tirant profit des acquis liés à la pensée multiplicative, les élèves deviendraient capables de se représenter mentalement les quantités groupées en paquets de dix et de cent en utilisant du matériel représentant ces groupements (Jones et al., 1994; Jones et al., 1996; Thomas et al., 2002). Ils pourraient alors utiliser un comptage mixte, soit compter par bonds de un, de dix et de cent. À ce niveau, les enfants acquerraient une certaine flexibilité de représentation des nombres.

Le niveau cinq est la compréhension de la valeur de position du système de numération (Jones et al., 1994; Jones et al., 1996) qui se développerait autour de 8 ans. Selon ces auteurs, c'est à ce niveau que les élèves manipuleraient facilement les différentes quantités, aussi bien avec des représentations externes que des représentations internes, concrètement autant que symboliquement (Bednarz & Janvier-Dufour, 1984a, 1984b, 1986; Hiebert & Wearne, 1992; Lyons, 1982; Thomas et al. 2002; Reys, 1994). Ils pourraient composer et décomposer les nombres facilement selon la tâche qu'ils ont à accomplir. Ils seraient capables d'utiliser leur flexibilité de représentation et de faire des choix selon les opérations à effectuer.

CONDITIONS FAVORISANT LE DÉVELOPPEMENT DU SENS DU NOMBRE

Le choix des tâches et des conditions à favoriser pour faciliter le développement du sens du nombre a aussi été étudié dans la présente recherche. D'abord, cette thèse adopte une perspective socio-constructiviste, cognitiviste et didactique. Ainsi, il est important que l'élève soit actif cognitivement, qu'il développe des représentations à partir de ses connaissances antérieures et qu'il soit placé dans un état de déséquilibre cognitif, ce qui sera possible grâce aux tâches qui lui seront proposées (Legendre, 2004; Poirier, 2001; Rocheleau, 2009; Van de Walle & Lovin, 2007). Ces tâches doivent d'abord proposer un problème à résoudre aux élèves, ce qui constitue une occasion pour eux de réfléchir, de discuter de leurs stratégies et de leurs conceptions, afin de s'approprier les concepts (Braconne-Michoux & Marchand, 2021; Legendre, 2004; Poirier, 2001; Rocheleau, 2009; Van de Walle & Lovin, 2007). Les tâches donnent aussi des occasions aux élèves de réutiliser ces apprentissages (Noirefalise & Matheron, 2005). Dans ce contexte, il est donc important de miser sur le potentiel mathématique de l'élève, en tenant compte de son niveau de développement du sens du nombre (Mary & Squalli, 2021).

De plus, les études consultées ont permis de faire ressortir que les tâches doivent permettre de manipuler la quantité dans un mode concret ou imagé et dynamique, pour ensuite l'associer au symbolisme (Bednarz & Janvier-Dufour, 1984a, 1984b, 1986; Hiebert & Wearne, 1992). Il est important de permettre aux élèves de travailler d'abord avec du matériel au groupement apparent et accessible, c'est-à-dire qu'ils peuvent voir toutes les unités groupées, les défaire ou les refaire. Il est aussi important de travailler de façon dynamique et simultanée le développement des habiletés de calcul et le développement de la compréhension du système de numération (Bednarz & Janvier-Dufour, 1984a, 1984b, 1986). Pour faciliter la création de représentations mentales dynamiques et imagées, il est recommandé de développer des habiletés spatiales liées à la subitisation et à la groupitisation (Jordan, 2010; Jordan & Dyson, 2016) et de limiter le temps d'exposition aux différentes représentations externes, pour forcer la représentation interne (Yackel & Wheatley, 1990). Aucune des études consultées n'a proposé la construction d'outils visant le développement du sens du nombre dans ces conditions.

OBJECTIFS SPÉCIFIQUES

Le premier objectif spécifique de la recherche était d'élaborer un outil d'évaluation du développement du sens du nombre et de la numération ainsi qu'une séquence didactique, le tout appuyé sur un continuum du sens du nombre. Ces instruments s'adressent à des élèves de 8 ans et ont pour but d'identifier les élèves qui rencontrent des difficultés à apprendre certains concepts mathématiques et de les amener à développer leur sens du nombre et de la numération en leur donnant l'occasion de se donner des représentations mentales dynamiques et imagées à partir de leurs habiletés de groupitisation. Un deuxième objectif était de vérifier la viabilité en contexte des instruments grâce aux commentaires, critiques et suggestions de professionnels de l'éducation[1]. Finalement, le troisième objectif était d'améliorer ces deux instruments à la suite de la vérification de la viabilité en contexte.

MÉTHODOLOGIE

Une recherche-développement a été mise en place. Les travaux de Harvey et Loiselle (2009) et de Paillé (2007) ont servi de base pour élaborer les instruments didactiques. Chacune des étapes de ce type de recherche correspond à un chapitre de la thèse. Pour répondre au premier objectif de recherche, un outil d'évaluation a été construit. Il permet de situer l'élève sur ce continuum. Une séquence didactique a également été élaborée. Elle donne l'occasion à l'élève de

[1] La thèse a été élaboré au moment de la pandémie de la COVID 19. Une mise à l'essai en classe n'a pas été possible.

développer sa compréhension en suivant ce continuum et en se construisant des représentations mentales dynamiques et imagées. Cette séquence intègre tous les éléments identifiés préalablement comme favorisant le développement du sens du nombre. Ces instruments s'adressent aux élèves de la fin de la 2e année et du début de la 3e année, soit des enfants de 8 ans. Un deuxième objectif était de vérifier la viabilité en contexte de ces instruments auprès de professionnels de l'éducation. Treize professionnels de l'éducation ont accepté de participer au projet, dont des enseignants de classes ordinaires, des orthopédagogues et des conseillères pédagogiques en mathématiques ou en adaptation scolaire. Un questionnaire a été créé, comportant des questions sur l'outil d'évaluation, sur la séquence didactique et sur les instruments en général. Une entrevue d'une heure a eu lieu avec chacun des participants afin de colliger leurs réponses et de leur permettre de préciser certains éléments. Une analyse qualitative des données a ensuite été effectuée. Des grilles de codes ont émergé des données et ont été utilisées jusqu'à la saturation des données. L'analyse des commentaires a aussi permis d'améliorer les instruments.

PRINCIPAUX RÉSULTATS DE RECHERCHE

Les objectifs de l'étude ont été atteints : des instruments ont été créés, puis leur viabilité en contexte a été évaluée par des professionnels du milieu de l'éducation. Les commentaires des participants ont aussi permis d'améliorer les instruments, particulièrement par rapport à la structure de la séquence, aux descriptions des conduites des élèves et par rapport à certains éléments des consignes. De plus, selon les commentaires, l'outil d'évaluation permettrait d'évaluer le niveau de développement du sens du nombre des élèves et de dépister ceux qui rencontrent des difficultés au moment d'apprendre ces concepts. D'après ces commentaires, la séquence donnerait l'occasion aux élèves de développer leur sens du nombre, selon leur niveau de compréhension, et leur permettrait de construire des représentations mentales dynamiques et imagées. Les objectifs des deux instruments semblent atteints, selon les commentaires des participants.

Une analyse fine des commentaires fait ressortir que le sens du nombre n'occupe pas une assez grande place dans l'enseignement actuel de l'arithmétique au primaire ni dans le Programme de formation de l'école québécoise (Gouvernement du Québec, 2001). Pourtant, il demeure un prédicteur important de réussite scolaire. De plus, les participants soulignent que l'enseignement formel, actuellement présent dans les classes, met l'accent sur l'apprentissage des techniques de calcul écrit et sur la mémorisation. Les commentaires soulignent que ce n'est pas ce type d'enseignement qui est privilégié dans la séquence, mais plutôt l'enseignement par résolution de problèmes.

L'élaboration de l'hypothèse d'un continuum du développement du sens du nombre qui tire profit d'un croisement entre la didactique et les sciences cognitives est une contribution à l'avancement des connaissances de la didactique des mathématiques. Il s'agit aussi d'un apport original de cette thèse. Ce croisement a aussi permis de mieux comprendre le rôle des habiletés de groupitisation, fortement liées aux habiletés visuospatiales, et de souligner l'importance de la construction de représentations mentales imagées et dynamiques dans le développement du sens du nombre. L'une des forces de cette thèse est donc d'allier deux des principaux prédicteurs en mathématiques, soit le sens du nombre et les habiletés visuospatiales.

Pour ce qui est des retombées pour l'apprentissage ou sur l'enseignement des mathématiques, le continuum en constitue lui-même une source. En effet, si les enseignants s'y appuyaient pour planifier leurs activités d'enseignement, dès le préscolaire, leurs activités suivraient un itinéraire cognitif (Mary & Squalli, 2021) et permettraient d'aborder chacun des concepts clés, dans le bon ordre, ce qui aurait le potentiel d'accroître la réussite des élèves. L'intégration des conditions ressorties comme favorisant le développement du sens du nombre aurait aussi ce potentiel, tout comme celui de transformer éventuellement les pratiques.

RÉFÉRENCES

Battista, M. T., Clements, D. H., Arnoff, J., Battista, K., & Van Auken Borrow, C. (1998). Students' Spatial Structuring of 2D Arrays of Squares. *Journal for Research in Mathematics Education*, *29*(5), 503–532. JSTOR. https://doi.org/10.2307/749731

Bednarz, N., & Janvier-Dufour, B. (1984a). La numération : Les difficultés suscitées par son apprentissage. *Grand N*, *33*, 1–11.

Bednarz, N., & Janvier-Dufour, B. (1984b). La numération : Une stratégie didactique cherchant à favoriser une meilleure compréhension. *Grand N*, *34*, 1–17.

Bednarz, N., & Janvier-Dufour, B. (1986). Une étude des conceptions inappropriées développées par les enfants dans l'apprentissage de la numération au primaire. *European Journal of Psychology of Education*, *1*, 17–33. https://doi.org/10.1007/BF03172567

Bednarz, N., & Janvier-Dufour, B. (1988). A constructivist approach to numeration in primary school : Results of a three year intervention with the same group of children. *Educational Studies in Mathematics*, *19*(3), 299–331.

Bobis, J. (2007). From here to there: The path to computational fluency with multi-digit multiplication. *Mathematics: Essential for learning, Essential for life*, 53–59.

Braconne-Michoux, A., & Marchand, P. (2021). La géométrie dans l'espace : Une piste d'intervention auprès des élèves en difficulté ? In P. Marchand, A. Adihou, J. Koudogbo, D. Gauthier, & C. Bisson (Éds.), *La recherche en didactique des mathématiques et les élèves en difficulté* (p. 155–174). Les Éditions JDF inc.

Brissiaud, R. (2005). *Comment les enfants apprennent à calculer ?* RETZ.

Clark, F. B., & Kamii, C. (1996). Identification of multiplicative thinking in children in grades 1–5. *Journal for Research in Mathematics Education*, *27*(1), 41–51.

Clements, D. H. (1999). Subitizing: What is it ? Why teach it? *Teaching Children Mathematics*, *5*(7), 400–405. https://doi.org/10.5951/TCM.5.7.0400

Clements, D. H., Sarama, J., & MacDonald, B. L. (2019). Subitizing: The neglected quantifier. In A. Norton & M. W. Alibali (Eds.), *Constructing number: Merging perspectives from psychology and mathematics education* (p. 13–45). Springer International Publishing. https://doi.org/10.1007/978-3-030-00491-0_2

Dehaene, S. (2003). *La Bosse des maths*. Odile Jacob poche.

Duval, R. (1996). Quel cognitif retenir en didactique des mathématiques. *Recherches En Didactique Des Mathématiques*, *16*(3), 349–382.

Fuson, K. C. (1991). Relations entre comptage et cardinalité. In J. L. Bideaud, C. Meljac, & J.-P. Fisher (Éds.), *Les chemins du nombre* (p. 159–179). Presses universitaires de Lille.

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Harvard University Press.

Gouvernement du Québec. (2001). *Programme de formation de l'école québécoise, éducation préscolaire, enseignement primaire*. Ministère de l'Éducation.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education*, *22*(3), 170–218.

Harvey, S., & Loiselle, J. (2009). Proposition d'un modèle de recherche-développement. *Recherches qualitatives*, *28*(2), 95–117.

Hawes, Z., Moss, J., Caswell, B., Seo, J., & Ansari, D. (2009). Relations between numerical, spatial, and executive function skills and mathematics achievement: A latent-variable approach. *Cognitive Psychology*, *109*, 68–90.

Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education*, *23*(2), 98–122.

Houdé, O. (2004). *La psychologie de l'enfant*. Presses universitaires de France.

Howden, H. (1989). Teaching number sense. *Arithmetic Teacher*, *36*(6), 6–11.

Jones, G. A., Thornton, C. A., & Putt, I. J. (1994). A model for nurturing and assessing multidigit number sense among first grade children. *Educational Studies in Mathematics*, *27*(2), 117–143.

Jones, G. A., Thornton, C. A., Putt, I. J., Hill, K. M., Mogill, T., Rich, B. S., & Van Zoest, L. R. (1996). Multidigit number sense: A framework for instruction and assessment. *Journal for Research in Mathematics Education*, *27*(3), 310–336.

Jordan, N. C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*(2), 82–88. https://doi.org/10.1016/j.lindif.2009.07.004

Jordan, N. C., & Dyson, N. (2016). Catching math problems early: Findings from the number sense intervention project. In A. Henik (Ed.), *Continuous issues in numerical cognition: How many or how much* (p. 59–79). Elsevier inc.

Koudogbo, J. (2013). *Portrait actuel des connaissances d'élèves de troisième année de l'ordre primaire et de situations d'enseignement sur la numération de position décimale* [thèses de doctorat]. Université du Québec à Montréal. https://archipel.uqam.ca/5607/

Koudogbo, J., Giroux, J., & René de Cotret, S. (2017). La numération de position : Où en sont les connaissances d'élèves québécois ? *Canadian Journal of Science, Mathematics and Technology Education*, *17*(3), 199–218.

Legendre, M.-F. (2004). Cognitivisme et socioconstructivisme, Des fondements théoriques à leur utilisation dans l'élaboration et la mise en œuvre du nouveau programme de formation. In P. Jonnaert, & A. M'Batika (Éds.), *Les réformes curriculaires, Regards croisés*. Presses de l'Université du Québec.

Lyons, M. (1982). *Un modèle pour la compréhension des équations* [thèses de maîtrise non publié]. Université de Montréal.

Mandler, G., & Shebo, B. J. (1982). Subitizing: An analysis of its component process. *Journal of Experimental Psychology: General*, *111*, 1–21.

Mary, C., & Squalli, H. (2021). Miser sur le potentiel mathématique des élèves en difficulté : Fondements épistémologiques et didactiques. In P. Marchand, A. Adihou, J. Koudogbo, D. Gauthier, & C. Bisson (Éds.), *La recherche en didactique des mathématiques et les élèves en difficulté* (p. 14–30). Les Éditions JDF inc.

McIntosh, A., & Dole, S. (2000). Number sense and mental computation: Implications for numeracy. In Australian Council for Educational Research, *Improving numeracy learning: What does the research tell us* (pp. 34–37). Proceedings of the ACER research conference. ACER

Noirefalise, A., & Matheron, Y. (2005). *Enseigner les mathématiques à l'école primaire*. Vuibert.

Northcore, M., & McIntosh, A. (1999). What mathematics do adults really do in everyday life? *Australian Primary Mathematics Classroom*, *4*(1), 19–21.

Paillé, P. (2007). La méthodologie de recherche dans un contexte de recherche professionnalisante : Douze devis méthodologiques exemplaires. *Recherches qualitatives*, *27*(2), 133–151.

Patenaude, P., & Mathieu, P. (2019). *Lexique de mathématique*. Netmath. https://lexique.netmath.ca/systeme-de-numeration/

Piaget, J. (1947). *La représentation du monde chez l'enfant*. PUF.

Poirier, L. (2001). *Enseigner les maths au primaire*. Notes didactiques (ERPI).

Reys, B. J. (1994). Promoting number sense in the middle grades. *Mathematics Teaching in the Middle School*, *1*(22), 114–120.

Reys, R. E., & Yang, D.-C. (1998). Relationship between computational performance and number sense among sixth and eighth grade students in Taiwan. *Journal for Research in Mathematics Education*, *29*(2), 225–237.

Rocheleau, J. (2009). *Les théories cognitivistes de l'apprentissage*. https://oraprdnt.uqtr.uquebec.ca/pls/public/docs/GSC332/F766183874_Approche_cognitiviste_apprentissage2009_10_05.pdf

Schleifer, P., & Landerl, K. (2011). Subitizing and counting in typical and atypical development. *Developmental Science*, *14*(2), 280–291. https://doi.org/10.1111/j.1467-7687.2010.00976.x

Siegler, R. S. (2010). *Enfant et raisonnement, Le développement cognitif de l'enfant*. De Boeck.

Starkey, G. S., & McCandliss, B. D. (2014). The emergence of ‘‘groupitizing’’ in children's numerical cognition. *Journal of Experimental Child Psychology*, *126*, 120–137. https://doi.org/10.1016/j.jecp.2014.03.006

Starkey, P., & Cooper, R. G. (1980). Perception of numbers by human infants. *Science*, *210*, 1033–1035.

Thomas, N. D., Mulligan, J. T., & Goldin, G. A. (2002). Children's representation and structural development of counting sequence 1–100. *Journal of Mathematical Behavior*, *21*, 117–133. https://doi.org/10.1016/S0732-3123(02)00106-2

Van de Walle, J. A., & Lovin, L. H. (2007). *L'enseignement des mathématiques, L'élève au centre de son apprentissage*. ERPI.

Wynn, K. (1992a). Addition and subtraction by human infants. *Nature*, *358*, 749–750.

Wynn, K. (1992b). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*, 220–251. https://doi.org/10.1016/0010-0285(92)90008-P

Yackel, E., & Wheatley, G. H. (1990). Promoting visual imagery in young pupils. *The Arithmetic Teacher*, *37*(6), 52–58. JSTOR.

ADDRESSING MATH CONTENT KNOWLEDGE AND MATH ANXIETY IN A TEACHER EDUCATION PROGRAM

Pamela Brittain
OISE, University of Toronto

ABSTRACT

This study was an in-depth mixed methods study of a math content knowledge (MCK) course from a large scale, urban university's Faculty of Education teacher program. The focus of the research was to describe the effect of this course on the math content knowledge and math anxiety levels of pre-service elementary school teacher candidates. The participants of the study included the course creators, the instructors of the course and the teacher candidates enrolled in the course. Quantitative data, in the form of pre- and post-course math content knowledge tests and anxiety scales (collected by the course creators), and qualitative interviews were assessed to answer the following research questions:

- *What is the purpose of a mathematics content course in a teacher education program?*
- *What was the focus of the course?*
- *What are the perceived benefits and challenge of the MCK course?*
- *What effect did the course have on math content knowledge and math anxiety?*
- *How was success in the course evaluated and did the course meet expectations?*

Overall, this research found that the course had a significant positive effect on both improving the math content knowledge and decreasing the math anxiety levels of the teacher candidates enrolled in the course. It also helped the teacher candidates to improve their self-efficacy and confidence with mathematics. From an administrative perspective, the course seems to have met the expectations that the teacher candidates improve their scores on the diagnostic tests, and interviews with the teacher candidates indicated they found a direct benefit from the course as well. The study concluded with suggestions for improving the course in the areas of instructor selection, breadth and depth of topics covered in the course, linking the content to more formal pedagogy, and increasing the length of the course.

INTRODUCTION

Elementary teachers in Ontario are required to teach mathematics as a part of the curriculum on a daily basis. Yet, many elementary pre-service teacher candidates (PSTCs) self-report as having high levels of math anxiety and struggle with math concepts at the elementary level (Finlayson, 2014; Gresham, 2007; Novak & Tassell, 2017; Reid et al., 2018). Additionally, mathematics education in Ontario has become increasingly focused upon in the curriculum (Education Quality and Accountability Office [EQAO], 2019). This includes student testing in mathematics at the grades 3, 6 and 9 level to determine proficiency of students in the subject, and even discussions surrounding teacher qualification testing in the subject as well. Based on the amount of media coverage and resources dedicated to the subject of mathematics in our schools, this is clearly an area of intense and on-going interest to parents, teachers, and politicians alike. Yet, EQAO test results, which test student proficiency in Ontario, and PISA test results, which compare Canadian students to other countries, have shown a marked decline in student math grades and attitudes; leading this researcher, and many others, to question the reasons and how they can be addressed. Adding to these issues are the discussions and research surrounding topics related to beliefs about mathematics, mathematical abilities, and mathematics anxiety of both students, and their teachers.

Mathematics anxiety has been well studied: from its definitions to its proposed causes, and methods of addressing it. However, with all the research available on this topic, there is still no consensus on the issue. Researchers have spent decades studying math anxiety and the impacts it has on students, teachers, and the general population (Ashcraft & Krause, 2007; Chernoff & Stone, 2014; Harper & Daane, 1998) and countless research papers have been written on the proposed foundations of math anxiety and strategies or methods to address it (Humayun, 2016; Meier, 2015; Ramirez et al., 2016). Significant research has also taken place as to the role of math teachers in either lessening, or increasing, math anxiety's influence and impact on their students (Beilock et al., 2010; Gunderson et al., 2012; Soni & Kumari, 2017) including the impacts of female teachers' beliefs about math and how those are passed onto their female students.

Even the field of mathematics itself has been studied and researched extensively. It has had its place in education questioned (Muller, 2009), and its role, purpose, and instructional methods continuously researched, scrutinized, and debated (Government of Ontario, 1994; Ontario, 1965; Rankin, 2003). Governments have used math instruction and assessment as a political platform (Alphonso, 2017; Ontario Ministry of Education, 2019), and universities have questioned its place as a science, a philosophy, a language or simply a tool for other subjects to use (Muller, 2009). Mathematics has also had a long and complicated history in both our schools and in our society in general, even dividing teachers themselves on the best ways to teach the subject (Bueckert, 2017). Regardless of the debates that have taken place as to math's purpose, and its method(s) of instruction, one point remains consistent: that the field of mathematics holds a significant place in our school system and is a large (if somewhat contentious) part of the Ontario elementary and secondary school curriculum.

Finally, numerous studies have shown that math anxiety is a real and measurable issue that is experienced by a significant number of people in North America (Richardson & Suinn, 1972; Young et al., 2012). Furthermore, studies have indicated that both pre- and in-service elementary teachers tend to experience high levels of math anxiety and significant gaps in mathematics content knowledge (Reid & Reid, 2017; Stoehr, 2017). Other studies have also shown that a teacher's attitudes towards mathematics can be passed onto their students (Beilock et al., 2010; Gunderson et al., 2012), and that negative experiences by students in mathematics can cause them to abandon further studies in mathematics and careers that require math and can have a significant long-term effect on future wage earnings as well (Ahmed, 2018; Rose & Betts, 2004).

As research shows, strong math teachers at the elementary level will go a long way to ensuring we have strong future math students; opening up career and educational opportunities for them in the future (Finlayson, 2014; Lake & Kelly, 2014; Stoeher, 2017; Vinson, 2001). Furthermore, since mathematics holds such a vital role in both our education system and the future paths available for students' post-graduation, it is important to understand the role teachers play in teaching mathematics and how their math anxiety, math content knowledge, and approaches or attitudes towards math affect their students. This study attempted to answer some of these questions by looking at how one Faculty of Education has chosen to directly address the issues of math anxiety and math content knowledge (MCK) of teacher candidates by creating a course specifically focused on building MCK in first-year Masters of Education elementary teacher candidates.

BACKGROUND

The Math Content Knowledge (MCK) course in the Faculty of Education (FoE) was created by Lila and Henry, two faculty members within the Faculty. The idea for the creation of the course came from Lila's research during her Ph.D. studies along with her work as a faculty member. Lila's research, and her own experiences working as an instructor in the program, had shown that teacher candidates participating in her math pedagogy course, where she includes an emphasis on math content knowledge in addition to pedagogy, showed improvement in their math skills (through diagnostic and summative math content test results) and showed a significant decrease in their math anxiety levels. Henry, working in administration in the faculty at the time, saw these results and Lila recalls him remarking to her that "we need to do something like this, but scale it up. It cannot just be within your own classes that you teach" (Lila, interview, April 17, 2020).

The MCK course is a compulsory, non-credit mathematics course in a graduate level teacher qualification program at a Faculty of Education at a Canadian university. It is required for all teacher candidates enrolled in the Primary/Junior (PJ) and Junior/Intermediate (JI) divisions of the program. The course was designed to be a required part of the preservice program; a two-year graduate level program leading to a provincial certification. It is a pass/fail course taken for credit only, and teacher candidates may choose to write an exemption test to determine if their math skills are strong enough to be given credit without having to take the course. The course was scheduled for two hours per week and took place over 12 weeks.

The course began with a diagnostic test focused on numeracy and an Abbreviated Math Anxiety Rating Scale (AMARS). Teacher candidates then participated in 12-weeks of studying various topics in math numeracy that included hands-on assignments, weekly quizzes, and practice using Khan Academy. At the end of the course teacher candidates were required to repeat the same diagnostic test (referred to as the summative test in this paper) and the AMARS.

Numeracy skills were chosen as the basis for this course as, according to Reid et al. (2018), "[a]ll five math strands (i.e., number sense and numeration, measurement, geometry and spatial sense, patterning and algebra, and data management and probability) are interwoven and interconnected, however, number sense and numeration is the strand that is deeply embedded throughout" (p. 14).

METHODS

This study focused on understanding and evaluating the methods, effectiveness, and underlying principles of a math content knowledge course developed and implemented by a Canadian Faculty of Education. It was a mixed-methods study that included qualitative interviews with the various parties involved in the course such as course creators, course instructors, and teacher candidates. The interviews sought to understand the reasons behind the creation of such a course, how it was implemented by the instructors teaching it, and how it was received by the teacher candidates taking it. The quantitative portion included data collected through diagnostic and summative anxiety scales and math content knowledge tests completed by the teacher candidates at the start and end of the course. This data was then used to evaluate the effectiveness of the course in terms of potentially increasing the MCK of teacher candidates in the course, along with any potential changes in their mathematics anxiety levels.

While a mixed-methods methodology is often seen as additional work "given the added resources, time, and expertise required to conduct a mixed methods study" (McKim, 2017, p. 202), it also, according to Hurmerinta-Peltomäki and Nummela (2006), allows that "researchers were able not only to validate their findings through triangulation, but also often to give a deeper, broader and more illustrative description of the phenomenon" (p. 452). Additionally, they found that this type of research sometimes created contradictory information that created "puzzles" that needed to be solved and this "offered rewards in terms of the creation of new knowledge, which in turn increased the theoretical contribution of their studies" (Hurmerinta-Peltomäki & Nummela, 2006, p. 453). McKim (2017) also found that

> *[g]raduate students scored mixed methods higher with regard to perceived value and further explained that when done correctly, mixed methods has something for all readers, regardless of their philosophical worldview. They also stated that mixed methods is more rigorous than quantitative and qualitative methods.* (p. 213)

Since the course being studied in this paper was complex in both its design and implementation, along with the expectations of the course and the potential for differing viewpoints from those being interviewed (teacher candidates versus course instructors versus course creators), a mixed methods study seemed like a valid option. It was also assumed that there would be "puzzles" that would arise when comparing the qualitative data to quantitative data collected from the course as a whole and a mixed-methods methodology provided a framework for addressing and solving these puzzles.

In order to evaluate the quantitative data more effectively teacher candidate scores were broken into four tiers. Figure 1 shows the distribution of teacher candidates across each of the four tiers based on their diagnostic math content test scores. Of note here is that 51.1% of the teacher candidates did not score above 50% on the diagnostic test, and 80.1% did not score above 75%.

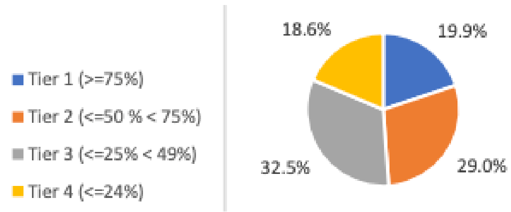


Figure 1.

FINDINGS

Overall, this research found that the MCK course was well received by participants and provided a positive benefit to nearly all of those interviewed. While there were some issues raised and suggestions made by the teacher candidates and instructors to improve the course, the general consensus from the participants in this study was that the benefits of the MCK course were significant. This study has provided some insights into the effectiveness, as well as the necessity of a course specifically focused on math content knowledge in a pre-service elementary teacher program.

The MCK course was established with the goal of increasing math content knowledge and decreasing math anxiety in PSTCs at the elementary level. The teacher candidates interviewed provided insights into how the course assisted them and also suggestions as to how the course could be improved. They provided positive feedback about the course and its effect, and this was supported by the quantitative data collected. While there was some outlying data, overall, the majority of teacher candidates in the course showed positive (and statistically relevant) improvements in both their math content knowledge and math anxiety.

A tiered analysis of the teacher candidates however, showed that these improvements were not universal across the teacher candidates and some benefited far greater than others. There were also some teacher candidates who showed increases in their math anxiety and nearly 45% showed a decrease of less than 10%. This information was not in line with previous studies and further investigation of the reasons is warranted. Additionally, only 12.8% of teacher candidates showed a less than 10% increase in their math content scores, with the majority (~70%) coming from the Tier 1 category (those who scored greater than 75% on their diagnostic test at the start of the course). However, an overall increase in math content test scores and a decrease in math anxiety were seen across the teacher candidates. Figures 3 and 4 show these overall changes (note: Year 1 and Year 2 here indicate the year in which the data was collected, not the year of study of the teacher candidates, all of whom took the course in Year 1 of their program).

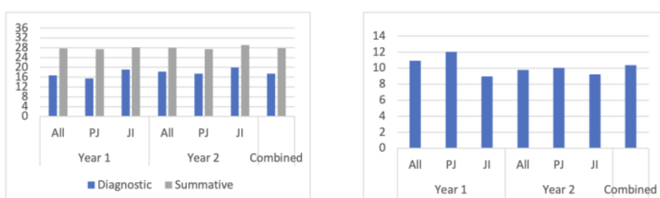


Figure 2. Diagnostic vs Summative Test Score Averages (out of 36) and Overall Change in Scores.

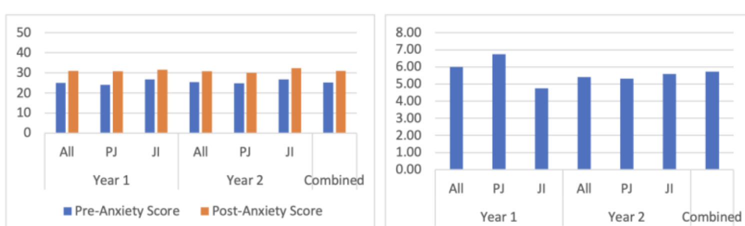


Figure 3. Pre-Anxiety vs Post-Anxiety Scales (out of 50) and Overall Changes.

The course instructors and course creators also had generally positive remarks about the course and its impacts. However, there was a difference in how each group evaluated success in the course. Many of the instructors were more concerned about creating a positive, welcoming environment where the teacher candidates felt safe to both learn and fail. They often evaluated the success of their teacher candidates based on how the teacher candidates reported feeling to the instructors. For the creators though, much of the success of the course was based on diagnostic versus summative tests on mathematics content and pre- and post-course anxiety scales.

The materials were created by the course creators and the lead instructor, and each class had access to the same materials. This was reflected in the quantitative data with very little differences being noticed between cohorts or instructors. This indicates that things like instructor background and teaching style may have had little influence on the achievement of the teacher candidates in the course and was instead more closely related to the materials provided. It could also be an indication that the instructors themselves had a commonality among them (such as creating a safe and welcoming environment for their teacher candidates) that was reflected in the scores as well.

Based on the findings, the following recommendations for the MCK course were suggested:

- Materials should be targeted to the different tiers as this would allow for those at different levels to feel more engaged in the content and may be more effective in addressing gaps in teacher candidate knowledge at the different levels. Those in lower tiers finished the course with significant gaps in their knowledge and may need a different experience than those in the higher tiers. This would also allow for a broader base of knowledge (outside of numeracy) for those who improved their numeracy skills early in the course.
- Course expectations should be made clearer at the start of the course in order to more accurately reflect the goals and focus of the course and prepare teacher candidates' expectations more accurately.
- An 'in-house' or tailored online platform for homework and practice of concepts should be used for all teacher candidates.
- The instructional methods of the course should be adjusted to be more in line with concepts being learned in the math pedagogy course. This would also involve having instructors skilled in different pedagogical methods of math instruction and not just skilled at mathematics.
- Alternative methods of assessment should be used in addition to quizzes, assignments, homework marks, and the summative content test.

FUTURE IMPLICATIONS

While this research was limited to one course at a single university, it has potentially wide- reaching implications. By showing the effectiveness of such a course on both increasing teacher candidate math content knowledge and decreasing math anxiety through a standardized course, it shows the potential to be extended to other institutions. By showing little variation in results between instructors, it could be surmised that the course could be taught elsewhere with similar results. A further study could be implemented at a different university utilizing the same materials and methods to determine if the MCK course could achieve similar levels of success in other environments. A further area of study related to this research could be an in-depth comparison of this MCK course to other courses and programs at other institutions and other methods of evaluating math content knowledge, such as large-scale teacher testing.

One additional area for study would be to determine if a MCK course affects gender and minorities differently. As there are already studies linking mathematics to issues of gender and equity, it would be interesting to see how this course relates to those issues as well.

Finally, a study on how the changes proposed to this course, when implemented, affect the results could be undertaken. Changes such as allowing teacher candidates to view their diagnostic test results and creating individually targeted, specific content based on these results; allowing instructors to teach using different, more student-centered or problem-based manner with the addition of manipulatives; or changes in the qualifications of instructors teaching the course. These changes could then be compared using this study as a base model for comparison. There is also potential for looking at how different teaching styles utilizing technology (such as adaptive learning, artificial intelligence, and micro- teaching) could have an effect on the effectiveness of this course; especially in addressing the impact of the course on the different tiers.

RESEARCHER REFLECTIONS

This research provided feedback to the creators of this course in order to improve its outcomes and to provide other faculties of education with a template to follow in their own institutions should they wish to implement a similar course or evaluate their own offerings in this area. This study also provided insights into how Math Content Knowledge (MCK) and Math Anxiety (MA) are closely correlated and the effects that this course has on both teacher candidate knowledge and confidence in the subject.

The qualitative aspects of this study also allowed teacher candidate participants to gain a deeper knowledge of their own history with math anxiety and gave them opportunities to form a greater understanding of these issues and reflect on their own learnings from the MCK course; allowing them to recognize these patterns in their own history and to potentially have a (positive) impact on their future interactions with their students. Course creators and instructors also had the opportunity to evaluate not only the course but their own involvement as well, allowing them to reflect and improve on their own teaching practices and make overall improvements to the course itself.

The diagnostic math content knowledge evaluations also showed a concerning insight into the math content knowledge of incoming elementary teacher candidates, with over 80% of them unable to score 75% or above on such a test. Without interventions like the MCK course, it is a concern that teacher candidates will enter the teaching profession without strong math content knowledge themselves, along with math anxieties that they could potentially pass onto their students.

While the MCK course still has some room for improvements, it is overall a good, solid, template for education programs looking to improve the math skills of elementary teacher candidates and to provide them with the tools to be effective teachers in the future. It is my belief that, rather than stressful qualifying tests, a focus on teaching fundamental math skills and building confidence (something the MCK course has proven itself capable of) is a far more effective course of action to create the teachers we are seeking for our schools.

Furthermore, I believe, like Lila, that mathematics education is one of the keys to equity in our schools and in our world. Many of the teacher candidates I interviewed for this study came from different backgrounds, and most were women. If these are the people our students are seeing as their early role models, then enabling them to be confident in mathematics, can help us start to change the way students see those in STEM and create strong, confident math students, encourage a more equitable education system and, by extension, a more equitable world. If we can create strong, elementary math teachers now, then hopefully we can create strong math elementary students in the future and eventually, hopefully, we will not need a MCK course because our students will already know the solutions.

REFERENCES

Ahmed, W. (2018). Developmental trajectories of math anxiety during adolescence: Associations with STEM career choice. *Journal of Adolescence*, *67*, 158–166. https://doi.org/10.1016/j.adolescence.2018.06.010

Alphonso, C. (2017, January). Ontario school boards want EQAO testing halted amid review. *The Globe and Mail*. https://www.theglobeandmail.com/news/national/education/ontario-school-boards-want-eqao-testing-halted-amid-review/article36802450/

Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, *14*(2), 243–248. https://doi.org/10.3758/BF03194059

Beilock, S. L., Gunderson, E. A., Ramirez, G., & Levine, S. C. (2010). Reply to Plante et al.: Girls' math achievement is related to their female teachers' math anxiety. *Proceedings of the National Academy of Sciences*, *107*(20), E80–E80. https://doi.org/10.1073/pnas.1003899107

Bueckert, K. (2017, February 9). How math education in Ontario is dividing educators—Kitchener-Waterloo—CBC News. *CBC News*. http://www.cbc.ca/news/canada/kitchener-waterloo/math-education-ontario-1.3974602

Chernoff, E., & Stone, M. (2014). An examination of math anxiety research. *Ontario Mathematics Gazette*, *52*(4), 29–31.

Education Quality and Accountability Office (EQAO). (2019). *Literature review of the empirical evidence on the connection between compulsory teacher competency testing and student outcomes*. Author.

Finlayson, M. (2014). Addressing math anxiety in the classroom. *Improving Schools*, *17*(1), 99–115. https://doi.org/10.1177/1365480214521457

Government of Ontario. (1994). For the love of learning: Report of the Royal Commission on Learning. http://www.edu.gov.on.ca/eng/general/abcs/rcom/main.html

Gresham, G. (2007). A study of mathematics anxiety in pre-Service teachers. *Early Childhood Education Journal*, *35*(2), 181–188. https://doi.org/10.1007/s10643-007-0174-7

Gunderson, E. A., Ramirez, G., Levine, S. C., & Beilock, S. L. (2012). The role of parents and teachers in the development of gender-related math attitudes. *Sex Roles*, *66*(3–4), 153–166. https://doi.org/10.1007/s11199-011-9996-2

Harper, N. W., & Daane, C. J. (1998). Causes and reduction of math anxiety in preservice elementary teachers. *Action in Teacher Education*, *19*(4), 29–38. https://doi.org/10.1080/01626620.1998.10462889

Humayun, A. (2016). The anxious child: Supporting students with anxiety and anxiety-related symptoms in the elementary classroom. [Master's thesis, Ontario Institute for Studies in Education of the University of Toronto]. TSpace. https://tspace.library.utoronto.ca/handle/1807/72203

Hurmerinta-Peltomäki, L., & Nummela, N. (2006). Mixed methods in international business research: A value-added perspective. *MIR: Management International Review*, *46*(4), 439–459.

Lake, V. E., & Kelly, L. (2014). Female preservice teachers and mathematics: Anxiety, beliefs, and stereotypes. *Journal of Early Childhood Teacher Education*, *35*(3), 262–275. https://doi.org/10.1080/10901027.2014.936071

McKim, C. A. (2017). The value of mixed methods research: A mixed methods study. *Journal of Mixed Methods Research*, *11*(2), 202–222. https://doi.org/10.1177/1558689815607096

Meier, K. (2015). Overcoming math anxiety: How does teaching math conceptually impact students learning math? [Master's thesis, Ontario Institute for Studies in Education of the University of Toronto]. TSpace. https://tspace.library.utoronto.ca/handle/1807/68713

Muller, J. (2009). Forms of knowledge and curriculum coherence. *Journal of Education and Work*, *22*(3), 205–226. https://doi.org/10.1080/13639080902957905

Novak, E., & Tassell, J. L. (2017). Studying preservice teacher math anxiety and mathematics performance in geometry, word, and non-word problem solving. *Learning and Individual Differences*, *54*, 20–29. https://doi.org/10.1016/j.lindif.2017.01.005

Ontario. (1965). Provincial committee on aims and objectives of education in the schools of Ontario. *Living and Learning: The Report of the Provincial Committee on Aims and Objectives of Education in the Schools of Ontario*. http://www.connexions.org/CxLibrary/Docs/CX5636- HallDennis.htm

Ontario Ministry of Education. (2019). *First year investment of Ontario's four-year math strategy announced*. https://news.ontario.ca/edu/en/2019/08/first-year-investment-of-ontarios-four-year-math-strategy-announced.html

Ramirez, G., Chang, H., Maloney, E. A., Levine, S. C., & Beilock, S. L. (2016). On the relationship between math anxiety and math achievement in early elementary school: The role of problem solving strategies. *Journal of Experimental Child Psychology*, *141*, 83–100. https://doi.org/10.1016/j.jecp.2015.07.014

Rankin, K. (2003). Ontario: Changing times, changing curriculum. *Orbit*, *33*(4). http://myaccess.library.utoronto.ca/login?url=https://search-proquest-com.myaccess.library.utoronto.ca/docview/213734396?accountid=14771

Reid, M., & Reid, S. (2017). Learning to be a math teacher: What knowledge is essential? *International Electronic Journal of Elementary Education*, *9*(4), 851–872.

Reid, M., Reid, S., & Hewitt, J. (2018). Nervous about numbers, math content knowledge and math anxiety of teacher candidates. *The MT Review*. https://mtrj.library.utoronto.ca/index.php/mtrj/article/view/29216

Rose, H., & Betts, J. R. (2004). The effect of high school courses on earnings. *The Review of Economics and Statistics; Cambridge*, *86*(2), 497–513.

Richardson, F. C., & Suinn, R. M. (1972). The mathematics anxiety rating scale: Psychometric data. *Journal of Counseling Psychology*, *19*(6), 551–554. https://doi.org/10.1037/h0033456

Soni, A., & Kumari, S. (2017). The role of parental math anxiety and math attitude in their children's math achievement. *International Journal of Science and Mathematics Education*, *15*(2), 331–347. https://doi.org/10.1007/s10763-015-9687-5

Stoehr, K. J. (2017). Mathematics anxiety: One size does not fit all. *Journal of Teacher Education*, *68*(1), 69–84. https://doi.org/10.1177/0022487116676316

Young, C. B., Wu, S. S., & Menon, V. (2012). The neurodevelopmental basis of math anxiety. *Psychological Science*, *23*(5), 492–501. https://doi.org/10.1177/0956797611429134

Vinson, B. M. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal*, *29*(2), 89–94. https://doi.org/10.1023/A:1012568711257

EXAMINING THE DISCOURSE PRACTICES THAT SUPPORT MIDDLE GRADE STUDENTS TO LEARN MATHEMATICS IN SMALL GROUPS

Tye Campbell
University of Alabama

The research presented at CMESG came from my dissertation research, which was recently published in *Mathematical Thinking and Learning* (Campbell, 2021). In the paper, I examined discourse practices that supported middle grade students to learn mathematics while working in small groups. There has been much research devoted to understanding how students develop learning opportunities in small groups. Research suggests that developing group roles and rules, for instance, are productive practices for small group learning (e.g., Boaler, 2006; Slavin, 1996). A growing body of studies has suggested that students need to be taught *how* to communicate in mathematics in order to develop learning opportunities (e.g., Laal & Ghodsi, 2012; Sfard & Kieran, 2001). Yet, research has not uncovered the talk moves, or discourse practices, that students can use to support their learning while working in small groups in mathematics. This study addressed this gap in research by identifying discourse moves that helped students to learn through small group interaction in mathematics.

THEORETICAL FRAMEWORK

Prior research reveals several theoretical lenses by which students develop learning opportunities in mathematics. In this study, I privilege learning through conflict as a theoretical lens. Sfard's (2008) commognitive conflict grounds this study. Commognitive conflict is "the encounter between interlocutors who use the same mathematical signifiers (words or written symbols) in different ways or perform the same mathematical tasks according to different rules" (Sfard, 2008, p. 161). In this study, I use a broader definition of conflict to include any situation wherein one or more learners communicate differing strategies to complete a task. Students can experience learning opportunities through conflict. For instance, when students communicate differing strategies, they can change their conceptions through discourse and argumentation. This study examines the discourse moves that students use during conflict that led to learning opportunities.

METHODS

Data from this study was culled from a larger research project wherein 77 middle grade students engaged in a three-day problem-solving unit. I acted as the teacher for the three-day problem-solving unit. Students worked on four problem-solving tasks that was first developed by Stylianides (2009). The tasks engaged students in meaningful problem-solving and simultaneously supported them to learn about the necessity of proof. Students were placed into groups of three to work on the problem-solving tasks, and they were audio-/video-recorded. I transcribed approximately 33 hours of audio-recordings.

After transcribing, I coded student data. First, I searched for all instances of conflict within the data (i.e., two or more students sharing different strategies for completing the task). I coded all such instances with the marker "conflict." Then, I examined whether the conflict led to a learning opportunity or did not lead to a learning opportunity. The instances which led to a learning opportunity were coded as "conflict leading to seized learning opportunities" while the instances that did not lead to a learning opportunity were coded as "conflict not leading to seized learning opportunities." Finally, I examined the conflicts within each category and constructed themes of discourse moves that supported students to develop learning opportunities during conflict and did not support student to develop learning opportunities during conflict.

FINDINGS

In total, I found 17 discourse moves that supported students to develop learning opportunities and six discourse moves that did not support students to develop learning opportunities. The discourse moves leading to learning opportunities were focusing on another learner's papers, slowing down cues, restating peers' ideas in own words, verbalizing specific points of confusion, capturing others' attention, asking if explanation is understandable, changing explanation to meet another student's needs, requesting explanation, challenging with evidence, seeking partners' opinion about possible solution, requesting and offering counterstrategies, defending ideas, soft disagreements, testing/checking idea, acknowledging validity of another student's strategy, gathering strategies, and building on another student's strategy. The discourse moves that did not lead to learning opportunities were avoiding criticism/overconfidence, ignoring others/not taking turns, making a best guess, rejecting or disagreeing without explanation, rude or inferior remarks, and verbalizing non-specific confusion. The reader is encouraged to read the larger manuscript for data and examples. However, I provide two examples of the discourse moves here (focusing on another learners' paper and avoiding criticism/overconfidence).

One discourse move that was productive was focusing on another learners' paper. Students exhibited this discourse practice when they moved their eye gaze to look at another student's paper when that student was talking. They stopped their own thought process and gazed at another learner's work. This may have helped them resolve conflict by minimizing distractions and giving attention to another idea.

One discourse move that was not productive was avoiding criticism/overconfidence. Some students avoided criticism in their work by using face-saving moves. Other students were overconfident in their own approaches which did not allow them to engage in meaningful conversation with others. This discourse move did not allow students to resolve conflict in ways that led to learning opportunities.

DISCUSSION

This study examined discourse moves that support students to develop learning opportunities during conflict. I uncovered 17 discourse practices that support conflict resolution and 6 discourse moves that hinder conflict resolution. This research can support teachers to understand how students should communicate in their classrooms. Teachers can explicitly teach students the discourse practices, or they can determine student-centered ways for students to learn productive discourse practices. This research can also motivate future research on how students should communicate while learning in small groups in mathematics.

REFERENCES

Boaler, J. (2006). How a detracked mathematics approach promoted respect, responsibility, and high achievement. *Theory into practice*, *45*(1), 40–46.

Campbell, T. (2021). Examining how middle grade mathematics students seize learning opportunities through conflict in small groups. *Mathematical Thinking and Learning, Advanced Online Publication*, 1–24.

Laal, M., & Ghodsi, S. M. (2012). Benefits of collaborative learning. *Procedia-social and behavioral sciences, 31*, 486–490.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity*, *8*(1), 42–76.

Slavin, R. E. (1996). Research on cooperative learning and achievement: What we know, what we need to know. *Contemporary educational psychology*, *21*(1), 43–69.

Stylianides, A. J. (2009). Breaking the equation "empirical argument=proof." *Mathematics Teaching*, *213*, 9–14.

MATHEMATICS TEACHING AND SOCIAL MEDIA: AN EMERGENT SPACE FOR RESILIENT PROFESSIONAL ACTIVITY

Judy Larsen
University of the Fraser Valley

ABSTRACT

Professional activity around mathematics teaching is vital in the improvement of mathematics education at all levels. The rise of social media allows education professionals to congregate through asynchronous communication without prompting, funding or mandate. In this study, I investigate the inner workings and nature of a particular social media collective, the Math Twitter Blogosphere (MTBoS), in which daily activity around mathematics teaching has occurred for almost ten years. To this end, I draw on tenets of complexity thinking *(Davis & Simmt, 2003; Davis & Sumara, 2006) and use my awareness as a MTBoS insider to enhance methodological design and analytical depth. My findings illuminate the co-acting influence of social capital and content capital on the resilience of ideational artefacts in the collective. As such, the results of this research indicate not only the popular topics within MTBoS, but also, features that drive ongoing and generative professional activity around mathematics teaching.*

INTRODUCTION

Professional activity around mathematics teaching is considered vital in the improvement of mathematics education at all levels (Borko, 2004). Research in mathematics education has identified various aspects of teacher professional development settings that make it effective at stimulating rich professional activity. For instance, activities should reflect and be driven by teacher needs and interests, and community building and networking should be at the core (Lerman & Zehetmeier, 2008). While many initiatives prove successful in engaging teachers in meaningful activity around mathematics teaching, there is growing attention on the sustainability of such activity. As such, informal settings where such activity occurs naturally have become of growing interest (e.g., Horn & Kane, 2015).

With the rise of social media technology, professional educators are turning to resources that are becoming increasingly available beyond the confines of institutional boundaries. In turn, many of the constraints of traditional forms of professional activity are being bypassed, allowing for informal professional activity to flourish. In some cases, collectives of professionals have formed. One such collective, referred to as the Math Twitter Blogosphere (MTBoS), has remained resilient for over ten years, with ongoing activity around mathematics teaching occurring daily via Twitter. MTBoS participants also have very promising statements about the possibilities for professional growth they experience and are often found suggesting that MTBoS is an effective space to share and develop ideas for teaching.

Although communication via Twitter is generally random and unprompted, MTBoS is treated as an established space, determined by participation from members who contribute and continue to use the MTBoS hashtag. For instance, contributors often refer to it as a place rather than a hashtag, and grow to expect like-minded peers to be available there (Larsen & Parrish, 2019). Studies have explored various facets of MTBoS, such as around content quality (Parrish, 2016), instances of negotiation (Larsen & Liljedahl, 2017), participant perspectives (Larsen & Parrish, 2019), and its capacity to support in-person conference events (Waddell, 2019). Taken together, these studies reveal strong potentialities of MTBoS as a tool for professional growth. While investigations of MTBoS to date have focused on various aspects of the user experience, none have investigated the co-activity between the various social and ideational structures at play within this setting. The unique nature of the Twitter medium and the social structures that allow supporting resilient professional activity around mathematics teaching require investigation. As such, examining what the MTBoS is as a structure that supports resilient professional activity around mathematics teaching developed into the central inquiry in my dissertation.

THEORETICAL FRAMEWORK

Although a variety of theoretical frameworks were taken into consideration when approaching MTBoS as a phenomenon of interest, the primary theoretical perspective was that of *complexity thinking* (Davis & Sumara, 2006). Complexity thinking provides the tools to describe a system of individual agents who seem to generate emergent macro-behaviours from individual autonomous actions. It is best suited for studying decentralized and bottom-up emergent learning contexts, where learning is treated as expanding the space of the possible and the “emergence of the as-yet unimagined” (Davis & Sumara, 2006, p. 135). Since the MTBoS has no central organization and is driven by individual professionals engaging in activity autonomously while simultaneously contributing to a collective that is often treated as a single entity, complexity thinking was the most well-aligned theoretical perspective for pursuing investigation into this context.

As a foundation of understanding complex systems, Davis and Sumara suggest six interdependent conditions necessary for complex emergence. They organize these into three complementary pairs (or tensions): specialization (diversity and redundancy), trans-level learning (neighbour interactions and decentralized control) and enabling constraints (randomness and coherence). Specialization involves diversities and redundancies among agents, trans-level learning involves the possibilities for individual ideas to ‘bump’ or interact with each other towards a greater whole, and enabling constraints consider the points of cohesion in the collective that maintain its structure while allowing for enough randomness for adaptation and learning. Although the focus in complexity thinking is primarily on ‘collective-knowing’, Davis and Sumara (2006) state, “the ideational network rides atop the social network” (p. 143). Since MTBoS was sometimes referred to as the ‘people of the MTBoS’ (a collection of parts), while other times as simply ‘the MTBoS’ (as whole in itself), my interest became focused on the connective layer between the individuals in the MTBoS and the collective of MTBoS that emerged.

As such, my specific research questions became as follows: *What is the nature of ‘MTBoS’ and how does it emerge from ‘the people of MTBoS’? And how and why does ‘MTBoS’ invoke a sustainable form of professional activity around mathematics teaching?*

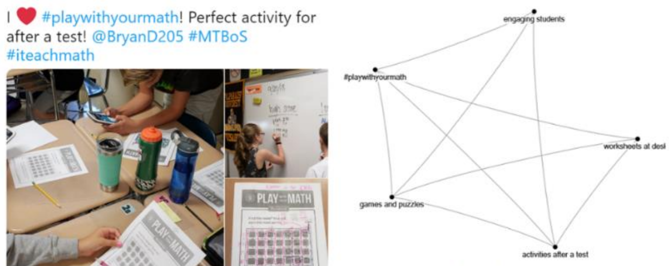
DATA

At ground level, a single tweet includes hyperlinked information about the user who publishes the tweet followed by the content they write. Content can include media, hyperlinks to webpages or to hashtags that organize other tweets around the given topic and is limited by a maximum on characters used and media uploads included. Tweets also include options for interacting via replies, retweets, and likes. Aside from the explicit statements made in each tweet, there are also implicit meanings that insiders in the collective can observe. To access these implicit meanings, I first had to become a participant myself. After several years of engaging as a participant in the MTBoS, I was attuned to the nature of data and was able to make an informed choice on data sampling. I chose to collect tweets with the term ‘MTBoS’ from one week in late September of 2018—after the back-to-school season but before exams. This choice was based on my view as an insider of what typically happens in this collective, and what happens when major events in the calendar year occur. I wanted to capture the most neutral timeframe possible. One week was also just long enough to see some consistency in the ideas posted about in that week, while not being too short and susceptible to the daily events in the work week. Even though one week seems like a short time, it resulted in 6146 tweets from almost 2948 users. Direct retweets in this set were removed to examine the space of possibility afforded in the MTBoS. This left 1493 tweets from 694 users. To make the dataset manageable, I then took a 30% random sample of this set, resulting in 444 unique tweets from 322 unique users. This formed my core data set.

METHODS

Guided by the nature of the Twitter communication medium along with tenets of complexity thinking, my research was organized into three parts: the ideational network, the social network, and feedback mechanisms. Findings from each part were then cross analyzed to identify the ways in which co-activity occurs both within these axes of complex emergence and across them. While it is impossible to share all the details in this paper, some key highlights are shared.

My first inquiry was into the ideational network for MTBoS in the week of data collection. This involved both constructing the ideational network, and then analyzing the tweets within it. To construct the ideational network, every tweet in the data set was analyzed through a process of recursive coding for both explicit and implicit *ideational artefacts*, a term used for the ideas being communicated by the tweet, along with their explicit or implicit relations. For example, tweet in Figure 1 below was coded as shown since '#playwithyourmath' was positioned as a task that 'engages students' when they work on it as a 'worksheet at their desks,' and as a 'game or puzzle' that is done as an 'activity after a test.'



Vertex 1	Vertex 2	Data Source
#playwithyourmath	activities after a test	5
#playwithyourmath	games and puzzles	5
#playwithyourmath	worksheets at desks	5
#playwithyourmath	engaging students	5
activities after a test	worksheets at desks	5
games and puzzles	activities after a test	5
engaging students	activities after a test	5
engaging students	games and puzzles	5
worksheets at desks	games and puzzles	5

Figure 1. Example ideational artefact analysis.

All 444 tweets were coded in this way, forming a dense ideational network that was visualized and further analyzed through network centrality measures by the Microsoft Excel NodeXL Pro (Smith et al., 2010) add-in. With an aim of identifying and examining the most prominent ideas in the space, a pruning process was applied to only consider the most frequent ideational artefacts and relations. Once a pruned version of the ideational network was constructed, the relations existing in this network were further examined through a qualitative analysis to identify trends and patterns in the ways in which ideational artefacts were related.

The second inquiry involved building the social network for MTBoS in the week of data collection. This also involved constructing the network and then analyzing tweets in the network. Since Twitter allows for unreciprocated follower relationships, each of the 322 users had a list of 'followers' and those they were 'following' that was gathered with the Twitonomy application (Diginomy, 2018). When a user 'follows' someone, it means they can see tweets from that user in their main feed. Rather than simply counting the numbers of followers and followings for each user from the global list of Twitter users, I limited these relations to only those that existed in the closed group of 322 users in the dataset by using a matching algorithm. This means each of the 322 users had in-degrees showing the number of incoming relations they had within the set, and out-degrees showing the number of outgoing relations they had within the set. Scatterplot visualization was used to identify user locations on the in-degree to out-degree coordinate plane. For both dimensions of in- and out-degree, cut-off values were identified to indicate relatively low and high values, resulting in four regions of social location: low in-degree and low out-degree (LL), low in-degree and high out-degree (LH), high in-degree and low out-degree (HL), and high in-degree and high out-degree (HH). The tweets made in each of these four regions were then analyzed qualitatively to determine patterns of activity in each.

The third inquiry involved examining the embedded feedback systems in the MTBoS. To this end, each tweet's number of 'likes', 'retweets', and 'replies' were tabulated, and low and high cut-off values were determined for each by considering their distributions. A three-way Venn diagram was developed to map out tweets representing the higher values of each of these three feedback responses, as well as those that received high values of two or more of these feedback responses. Tweets in each category were further analyzed qualitatively through recursive coding to identify themes associated with each form of feedback received.

A final cross-analysis was then performed to consolidate the findings of each of the above three inquiries and to build an overarching theory around how MTBoS forms a sustainable form of professional activity. To this end, the results of each of the investigations were considered in relation to each other at first with the three possible relations of ideational network and social network, ideational network and feedback systems, and social network and feedback systems, and then as all together towards an overall representation of how these findings work together.

RESULTS

The process of constructing an ideational network resulted in 1353 unique ideational artefacts linked with a total of 4183 relations. This resulted in an incredibly dense ideational network, so a pruning method was developed towards a final network that included the top 15 recurring ideational artefacts with all their connected relations (see Figure 2). The qualitative analysis of tweets that composed each relation in this network resulted in a categorical framework for relations to offer either ideational convergence or divergence, either explicitly or implicitly. Ideationally convergence was determined by the prevalence of redundancy among ideas while ideationally divergence was determined by the prevalence of diversity among ideas. Towards describing the opportunities these categories had on the system, Wenger's (1998) terms of *reification* (the process of attaining 'thingness') and *negotiation* (the social production of meaning) were used to describe the activity that ideational convergence or divergence created in the system, and explicit relations were termed as *active* while implicit relations as *inert*. Therefore, each relation could be classified as either offering active reification, inert reification, inert negotiation, or active negotiation. This coding is represented visually in Figure 2.

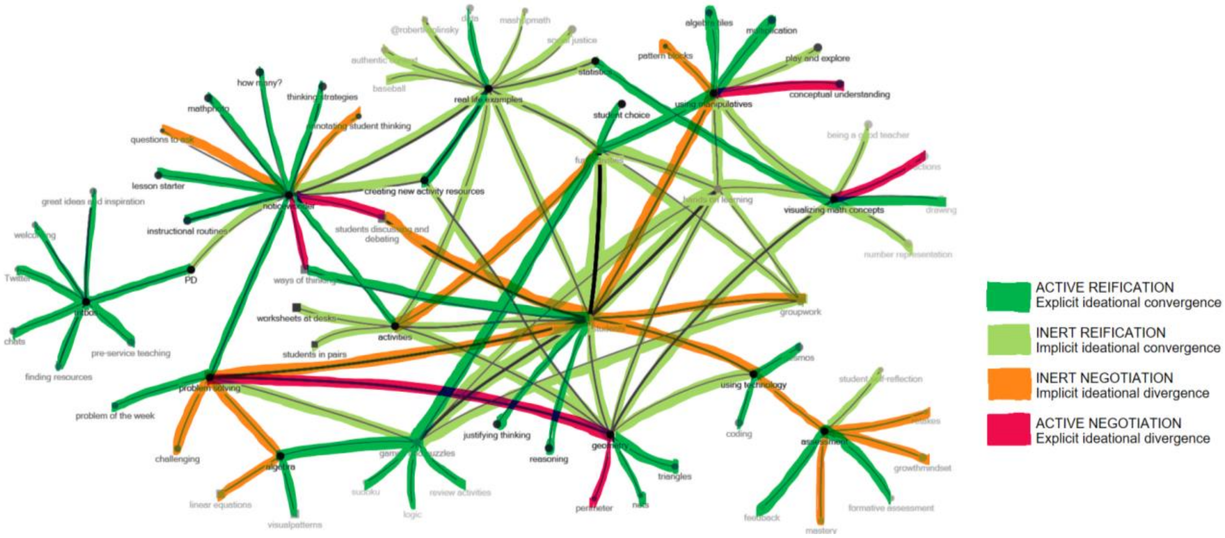


Figure 2. Top artefact network identified with types of relations.

Each artefact in this network was then considered based on the types of relations of which it was composed. While some artefacts, such as 'mtbos', were *reified* actively in all its relations, other artefacts, such as 'geometry', were *negotiated* actively in at least one relation. If an artefact had a significant amount of *inert* reification or negotiation, in that redundancies or diversities were not made explicit, then those artefacts were considered either *reifiable* or *negotiable*. This formed a spectrum for categorizing artefacts as *negotiated*, *negotiable*, *reifiable*, or *reified*.

An example of a negotiable artefact was that of 'assessment'. While it remained anchored in stability in its relations with 'feedback' and 'formative assessment', there were sources of negotiation available through the diversity among tweets involving 'assessment' linked with 'retakes' and 'growthmindset'. 'Growthmindset', a very active and popular hashtag in MTBoS and a term coined by (Dweck, 2008), permeates throughout the fabric of much of the content in MTBoS and was evidently agreed as important. However, the ways in which users tweeted about using 'growthmindset' approaches within 'assessment' practices varied, particularly around the locus of responsibility. For instance, the following two tweets come from different users who evidently both value 'growthmindset' implicitly, but who implement it within 'assessment' differently. While space for negotiation exists, it remains inert, as it did in much of the data.

> *Grading my first real SBG summative assessment in AP Stats. Loving the work and detail Ss put into their work! Also amazingly satisfying just writing feedback and noting areas of strengths and needed improvement instead of points. #MTBoS* (@DamionBeth, Sept. 26, 2018)

> *When a student comes to me a day before a test to say they aren't ready and request to take it another day, I say they can't earn full credit.. torn bc I know learning happens at wildly different paces, but they need to be prepared to handle deadlines #MTBoS* (@yungparabola, Sept. 26, 2018))

The overall lack of active negotiation in the network can be explained in part by the barriers to visibility of users' tweets created by the constraints of the system. As such, examining the social network of MTBoS was necessary. An analysis of tweets made by users in each of the four categories of social location that were constructed (*newcomers* (LL), *observers* (LH), *influencers* (HL), and *leaders* (HH)), indicated that not only did users in different social location have different levels of access to the ideational network, but they also tweeted in different ways. This points to the potentially symbiotic relationship between one's social location and how one tweets. For instance, leaders seemed to tweet in community-oriented, boundary-pushing, and relational ways that revealed a sense of social responsibility and ideational alignment to MTBoS. On the other hand, newcomers tweeted in practice-oriented, pragmatic, and generic ways. Overall, as users had higher awareness of others in MTBoS, their tweets were more ideationally aligned; and as users gained visibility in MTBoS, they showed more social responsibility.

Given this skewed access to the ideational network, examining the feedback mechanisms in MTBoS was pursued. The analysis of tweets that solicited high values of each of the three forms of positive feedback loops (i.e., via 'likes', 'retweets', and 'replies') revealed that highly 'liked' tweets showed users showcasing a sense of pride and accomplishment, highly 'retweeted' tweets showed utility and novelty, and highly 'replied' to tweets evidenced vulnerability and curiosity. Therefore, it may be seen that viewers predominantly used 'likes' to express celebration, 'retweets' to associate with the content, and 'replies' to interact with users. However, while these qualities were found in tweets that were highly responded to in each of these ways, they were also present amidst tweets that received little to no response. So, having these qualities seems to be necessary but insufficient to solicit these forms of feedback. Further inquiry into the tweets that received little feedback showed that they either did not invoke the qualities associated with receiving feedback, the content in them was overly redundant in the system, or the content was simply unseen. In fact, 80% of the tweets that received little to no response were made by newcomers, who had relatively low visibility or awareness in the ideational network of MTBoS.

As Davis and Sumara (2006) write, "the system itself 'decides' what is and is not acceptable" (p. 145). While control in a complex system is decentralized, and in turn, distributed, there are still ways in which possibilities for shared knowing in the collective are directed. This sort of control can be conceived of as a form of *authority*, in the sense of *authorship*, or a capacity to act in the consensual domain, rather than dominance. To examine how authority in MTBoS emerges, a series of cross-analyses between the findings from all three inquiries into the ideational network, the social network, and the feedback mechanisms was conducted and resulted in an overarching *theory of ideational authority*. This theory combines the findings from all three inquiries to show how the system in MTBoS emerges from a co-action between mechanisms of social capital and content capital. Not only does one's membership and social location amplify one's content, the ways in which one's published content aligns with the values of the ideational network and the features required by the system to receive feedback contribute to gaining authority. As such, ideational content, social locations of contributors, and forms of amplification through feedback are necessarily interwoven and inter-related. A summary of these findings is shown in Figure 3.

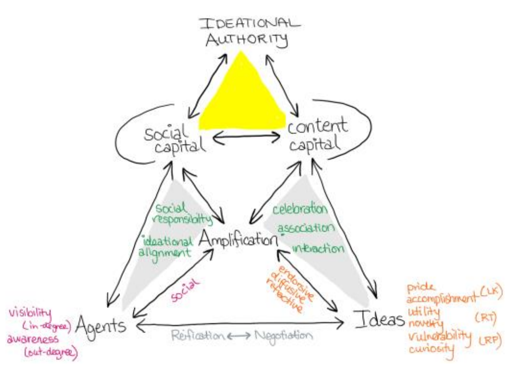


Figure 3. Theory of ideational authority for complex ideational networks as found in MTBoS.

This theoretical construct was used to then identify which ideational artefacts held authority in MTBoS. By applying weightings on ideational artefacts in the ideational network based on both social and content capital, artefacts such as 'assessment,' 'noticewonder,' and 'students discussing and debating,' were found to have more authorship than artefacts like 'games and puzzles,' 'hands on learning,' and 'fun activities,' which were prominent in the overall network.

CONCLUSION

In considering the findings of this series of investigations, which are outlined in more detail in (Larsen, 2019), it is possible to conclude that MTBoS is a complex collective of autonomous individuals who engage in public communication around mathematics teaching via social media. Although it is composed of a constantly in-flux population of users, it can be viewed as a unified collective that emerges as greater than the sum of its parts. This occurs through users creating content that contributes to reification and negotiation of ideas, which in turn become amplified in various ways through social and content capital. These processes contribute to an overall process of identifying aspects of the system that gain ideational authority, and therefore become strong sources of coherence in the system while allowing for further randomness to still occur. While the data in this study was taken from one week, these processes would continuously shift authority based on emerging and dynamic circumstances that the collective adapts to over time. In this way, it forms a sustainable form of professional activity around mathematics teaching. The implications of this research are vast, but at the core, they point to the aspects that should be considered when designing initiatives aimed at offering professional development opportunities.

REFERENCES

Beth, D. [@DamionBeth]. (2018, September 26). *Grading my first real SBG summative assessment in AP Stats. Loving the work and detail Ss put into their work!* [Post]. Twitter. Accessed January 7, 2019 from https://twitter.com/DamionBeth/status/1045127391796957186

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, *33*(8), 3–15.

Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education*, *34*(2), 137–167. http://www.jstor.org/stable/30034903

Davis, B., & Sumara, D. (2006). *Complexity and education: Inquiries into learning, teaching, and research*. Routledge.

Diginomy. (2018). *Twitonomy: Twitter #analytics and much more…* [web application]. Twitonomy. https://www.twitonomy.com/

Dweck, C. S. (2008). *Mindset: The new psychology of success*. Ballantine Books.

Horn, I. S., & Kane, B. D. (2015). Opportunities for professional learning in mathematics teacher workgroup coversations: Relationships to instructional expertise. *Journal of the Learning Sciences*, *24*(3), 373–418.

Larsen, J. (2019). *Mathematics teaching and social media: An emergent space for resilient professional activity* [Doctoral dissertation, Simon Fraser University]. SFU Summit Research Repository. https://summit.sfu.ca/item/19745

Larsen, J., & Liljedahl, P. (2017). Exploring generative moments of interaction between mathematics teachers on social media. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Conference of the International Group for the Psychology of Mathematics Education* (pp. 129–136). Singapore: PME.

Larsen, J., & Parrish, C. W. (2019). Community building in the MTBoS: Mathematics educators establishing value in resources exchanged in an online practitioner community. *Educational Media International*, *56*(4), 313–327.

Lerman, S., & Zehetmeier, S. (2008). Face-to-face communities and networks of practising mathematics teachers: Studies on their professional growth. In K. Krainer & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 3, pp. 133–153). Sense Publishers.

Parrish, C. W. (2016). *Supporting the development of teachers' attributes needed for the selection and implementation of cognitively demanding tasks through engagement with the MathTwitterBlogosphere* [Doctoral dissertation, Auburn University]. ProQuest Dissertations Publishing. https://www.proquest.com/openview/8d5cd5a3a3673435b504dbf3b690bb67/1.pdf?pq-origsite=gscholar&cbl=18750&diss=y

Smith, M., Milic-Frayling, N., Shneiderman, B., Rodrigues, E. M., Leskovec, J., & Dunne, C. (2010). *NodeXL: a free and open network overview, discovery and exploration add-in for Excel 2007/2010*. Social Media Research Foundation. https://www.smrfoundation.org

Waddell, G. (2019). *Mathematics teacher professional development via conference tweeting and blogging: A mixed methods analysis* [Doctoral dissertation, University of Nevada, Reno]. ProQuest Dissertations Publishing. https://www.proquest.com/openview/c443bbe453c72e722903dec9081414d6/1?pq-origsite=gscholar&cbl=2026366&diss=y

Ward, J. C. [@yungparabola]. (2018, September 26). *When a student comes to me a day before a test to say they aren't ready and request to take* [Post]. Twitter. Accessed January 7, 2019 from https://twitter.com/CoachHowardP2P/status/1045081938535567360

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press.

MATHEMATICAL MODELLING—REDUCING REALITY OR REDUCING COMPLEXITY?

Minnie Liu
Simon Fraser University

ABSTRACT

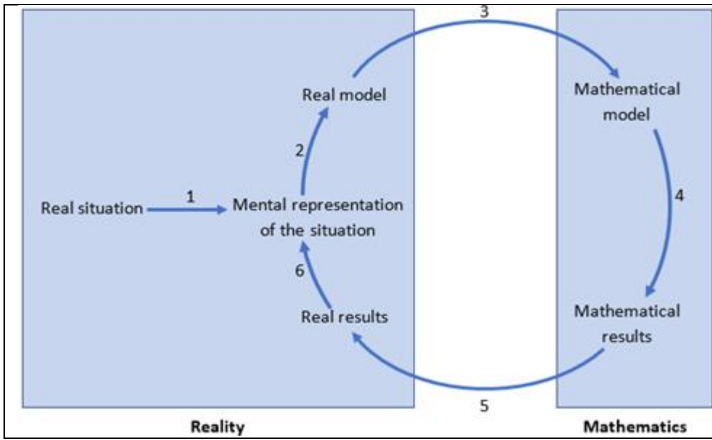
Mathematical modelling uses mathematics to solve real-world problems and can be used to prepare students for the challenges they face in the world. When describing mathematical modelling processes, researchers often emphasize the importance of extra-mathematical knowledge to highlight the relationship between reality and mathematics. In this study, I administered two rudimentary mathematics complex tasks, a type of tasks that present a complex situation but allow students to apply their well-worn mathematical tools to establish a solution, to two groups of junior secondary school students. These tasks allowed me to tip the balance between reality and mathematics in mathematical modelling to focus on modelling, and therefore to closely examine students' use of extra-mathematical knowledge when they solve such tasks. In this paper, I discuss the strategies students applied along with the reasons leading to their application of these strategies.

INTRODUCTION

In a 1992 article, Hewitt discusses the concept of "train spotting"—to solve problems by looking for patterns and paying attention to the numbers involved in the problem, but shy away from the richness of the problem. This results in students learning something mathematical about the patterns in the original situation but lose sight of the original situation. Hewitt suggests that what students seem to lack is not a fluency in their mathematical knowledge but a connection between their mathematical solution and the original situation, and the ability to recognize and use their knowledge related to the original situation to solve everyday problems. This disconnection between their solutions and the original problem also points to students' tendency not to apply their real-world knowledge and not to interpret their mathematical solution from a real-world perspective. Furthermore, this disconnection suggests that the production of a realistic and reasonable solution rests on both students' intra-mathematical skills (Blum & Borromeo Ferri, 2009), students' application of their mathematical skills to solve mathematical problems, and their extra-mathematical knowledge (Borromeo Ferri, 2006), students' interpretation and consideration of the situation and validation of their solution from a real-world perspective.

THEORETICAL CONSIDERATIONS

Extra-mathematical knowledge (EMK) is a term Borromeo Ferri (2006) uses in her modelling cycle to describe any knowledge or experiences that originate from outside of modellers' mathematical experiences. Borromeo Ferri's (2006) modelling cycle (Figure 1) contains two worlds, reality and mathematics, and highlights modellers' individual thinking processes through their actions (Borromeo Ferri, 2007, 2010). Starting with a real situation in reality, modellers build a mental representation of the situation (MRS), make assumptions, and draw on their EMK to build a real model based on their interpretation of the real situation. Afterwards, modellers move away from reality into the world of mathematics by applying their EMK to mathematize the real model into a mathematical one, utilize their intra-mathematical skills to produce mathematical results, use their EMK once again to interpret these results in terms of the context of the problem, and validate these results by comparing them to the original situation. The modelling process concludes if modellers decide the real results are acceptable. If not, modellers re-enter the modelling cycle and make adjustments and modifications to their work.



1 – understanding the task; 2 – simplifying/structuring the task, extra-mathematical knowledge (EMK) is used/needed; 3 – mathematizing, EMK is strongly needed; 4 – working mathematically, individual mathematical competencies is used; 5 – interpreting; 6 – validating

Figure 1. Borromeo Ferri's (2006) modelling cycle.

EMK is utilized throughout Borromeo Ferri's (2006) modelling cycle and plays a crucial role in connecting problems situated in reality and the mathematics used to solve problems. EMK may have little or no clear connections to mathematics, but it enables modellers to consider problem situations from a real-world perspective (Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2006). Without the application of EMK, modellers would likely produce solutions that are purely mathematical and embody what Maaβ (2006) refers to reality-distant modellers.

Recognizing the importance of EMK throughout the modelling cycle, this study investigates situations in which students have the required intra-mathematical skills but may not process sufficient EMK to solve tasks situated in reality. These tasks tip the balance of the modelling cycle away from mathematics by allowing students to use their rudimentary mathematical knowledge and skills to solve the tasks, and to focus on reality by allowing students to concentrate on the required EMK and their use of EMK throughout the modelling process to produce a realistic and reasonable solution. In particular, this study looks at students' behaviors during their process of solving these tasks situated in reality, including the actions and strategies they use to deal with situations where they have insufficient EMK.

PARTICIPANTS AND METHODS

The students in this study worked on a genre of tasks called *rudimentary mathematics complex tasks*, which can be considered a type of numeracy task (Liljedahl, 2010). These tasks have been designed specifically to meet the numeracy goals of the local curriculum and are crafted around "an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication abilities, and problem-solving skills that people need in order to engage effectively in quantitative situations arising in life and work" (Steen, 2001, p.7). The two tasks that the students worked on include: the *Design a New School* task (Figure 2) and the *Door Project* (Figure 3).

Design a new school

Your city is getting a new 11000m^2 middle school. It is going to be built on a lot (200m×130m) just outside of town. Besides the school, there will also be an all-weather soccer field (100m×75m), two tennis courts (each 15m×27.5m), and a 30-car parking lot on the grounds. The following requirements must be met:

all fields, courts, buildings, and parking lots must be no closer than 12.5m to any of the property lines.

any leftover property will be used as green space – grass, trees, shrubs.

good use of green space is an important part of making the school grounds attractive.

To help you with your design and layout you have been provided with a scaled map of the property (every square is 10m×10m). Present your final design on a copy of this map. Label all structures and shade the green space.

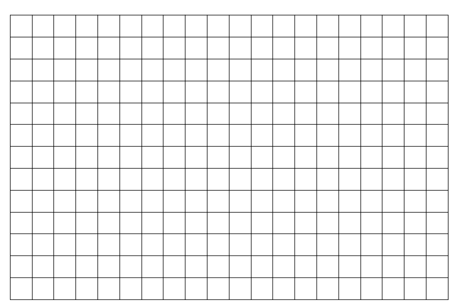


Figure 2. The Design a New School Task.

The door project
I have recently renovated my home office. As a final touch, I'm going to repaint and decorate my office door. The plan is to paint and then decorate the door with Starbucks gift cards. How much will this project cost me?

Figure 3. The Door Project.

Data for the research presented here was collected in a Grade 8 (age 12-13, n=13) and a Grade 9 (age 13-14, n=26) class in a secondary school in western Canada. Although it is not possible to know if the Grade 8's had seen numeracy tasks in their previous years, these were their first numeracy tasks in the Grade 8 school year. Conversely, the Grade 9's had experiences with numeracy tasks in Grade 8.

Students worked on the two tasks in randomly assigned groups of two to four during a 75-minute class. There were no instructions provided other than what can be seen in Figures 2 and 3. While the students worked, the teacher (the author) circulated naturally through the room and engaged in conversations with the students—sometimes prompted by her and sometimes prompted by the students. These conversations were audio recorded and transcribed. At the same time, photographs of students' work were taken, and students' finished work was collected. These, coupled with field notes summarizing the interactions as well as observed student activities, are used to build cases for each group of students. Each case is a narrative of students' task experience punctuated by significant moments of activity and emotive expression. These cases constitute the data.

Given the natural and unscripted nature of the teacher movement through the room, not all of the cases are equally well documented. Regardless, each of these cases were analyzed separately through the lens of modelling using Borromeo Ferri's (2006) modelling cycle, with a focus on the EMK involved during students' modelling process.

Focusing on students' strategies and their use of EMK to access the problems situations and reach a possible solution, this paper investigates "the transition process from MRS to real [model, where] an idealization and simplification of the problem takes place" (Borromeo Ferri, 2006, p. 92) as students consciously apply their EMK and make decisions about the situation, to simplify and idealize the situation in order to arrive at a real model. In particular, I intend to investigate the following:

> *What actions do students take to produce a real model based on the real situation? What are the challenges students experience during these processes that require them to engage with the problem from a realistic perspective?*

DATA AND ANALYSIS

Students employed various strategies to simplify the situation and to make the situation accessible to them. Their actions led them to the production of a real model and later on a solution. These actions are in line with modelling literature (for example, see Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2006, 2007). However, modelling literature only describes these strategies in general and under ideal circumstances, and do not provide details, including the reasons that led students to these strategies. During my analysis of data, I differentiated between two categories of simplification strategies: reducing reality and reducing complexity. In what follows I first present these two categories of simplification strategies. Afterwards, I discuss the reasons behind students' use of these strategies.

REDUCING REALITY

Reducing reality happens when students idealize and make assumptions about the situation and remove some negotiable aspects of the situation. These simplification and idealization processes make the problem accessible but retain the messiness and the complexity of the problem. The process allows students to proceed: to produce a real model and to arrive at a solution which fits or can be used to describe the original situation. The process of reducing reality is in line with modelling literature (for example, see Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2006, 2007). For example, while working on the *Design a new school* task (Figure 2), some students recognized that a parking lot involves parking spaces, a driveway that is linked to all parking spaces, and an entrance/exit, but did not fully understand the dimensions of these features. To make the problem accessible, they used the school staff parking lot as a template and made assumptions about these features to the best of their abilities and proceeded to generate a solution thereafter.

REDUCING COMPLEXITY

Reducing complexity happens when students change or remove the founding criteria or the non-negotiable aspects of the original real situation to avoid answering the problem and to look for a possible answer to the question as quickly as possible without fully considering the EMK involved in the question. Reducing complexity allows students to proceed: to find a way around the problem and to produce a solution. However, the solution does not fit the original problem well, because the nature of the problem is changed during the process of reducing complexity. For example, in the *Door project* (Figure 3), some students were unsure of the amount of paint and the cost of various tools required, and made the convenient assumption that they had free access to paint and tools in order to remove these considerations from the problem and to exclude them from their cost calculations to simplify the problem and to generate a solution as quickly as possible.

WHAT LEADS TO REDUCING REALITY AND REDUCING COMPLEXITY?

While both reducing reality and reducing complexity serve as ways for students to proceed to solve problem tasks, students applied these strategies based on the challenges they faced. In my analysis of students' strategies, I identified four possible reasons behind their application of these strategies. These are: mathematics class, work avoidance, fantasy or reality, and flow.

Mathematics class

Mathematics class is related to students' expectation to *do mathematics* in mathematics classes. While students may carry different ideas and understanding of what it means to *do mathematics*, it is safe to assume that some of their interpretations and assumptions of *doing mathematics* include various mathematical problems and exercises such as word problems, where students extract numerical values and apply mathematical computations to solve problems. For example, some students' first attempt to the *Door Project* (Figure 3) was to tile the door completely with gift cards. In doing so, some calculated the areas of the door and a gift card and divided the two areas to determine the number of cards required (Figure 4). In these scenarios, their treatment of the problem was very similar to that of word problems. While their approach and solution were not necessarily wrong, they neglected the context of the original problem and reduced a problem situated in reality into a set of mathematical computations.

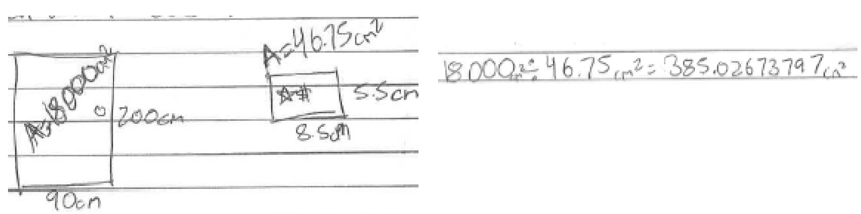


Figure 4. Mathematics class: students treated the *Door project* similar to a word problem.

Work avoidance

Work avoidance has strong ties with students' insufficient EMK. Both administered tasks allowed students to use their well-worn mathematical tools but required them to apply a large amount of EMK to produce a realistic solution. However, many students did not have sufficient EMK at the time of the study and therefore needed to overcome their insufficient EMK to proceed. While some students acquired additional EMK, others avoided EMK by simplifying the tasks. For example, the *Design a New School* task (Figure 2) required students to design a 30-car parking lot. While a parking lot can be considered an accessible item or an everyday object in this context, students in this study lacked the necessary EMK to create a realistic parking lot. To overcome their insufficient EMK, one of the groups attempted to designate the school's entire basement as their parking lot although they recognized and fully understood that that a much smaller area would be sufficient. In this scenario, they attempted to remove the 30-car constraint so their approach and solution would be considered acceptable in order to avoid the complexity of the problem and as a way to overcome their insufficient EMK.

<u>Fantasy or reality</u>

Some students believed that the problems presented represent a believable situation but also a fantasy, which led them to believe they have minimal responsibility to the decisions they made. For example, in the *Door Project* (Figure 3), some students used loaded gift cards ($5 per card) to tile the door and ended up spending thousands of dollars on the project. These students recognized that their decision was not cost efficient but decided to ignore this aspect of the problem. In this scenario, they treated the situation as a fantasy where their decisions had no real financial consequence (Figure 5). In other scenarios, students wanted to walk into Starbucks and remove gift cards for the project, and claimed that “it’s only stealing if you get caught” or “we will run out of there [so we won’t get caught]”. In reality, I doubt students would actually carry out these actions as they were conscious that their actions were inappropriate. Again, they treated the situation as some sort of fantasy where their decisions carried minimal responsibility. As a result, although students experienced tensions with their approach and solution, they chose to ignore these tensions.

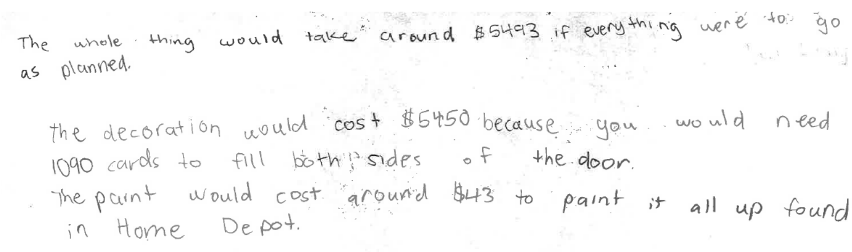


Figure 5. Fantasy or reality: students used loaded gift cards for the *Door project*.

<u>Flow</u>

The fourth reason to students’ decision to reduce reality or complexity has strong ties to flow theory (Csíkszentmihályi, 1990). The state of flow is created when there exists a balance between challenge and ability. When students are in flow, they are having the most optimal experience. Liljedahl (2018) extends Csíkszentmihályi’s (1990) flow theory to include a state of tolerance between flow and boredom, and a state of perseverance for the mundane between flow and frustration (Figure 6).

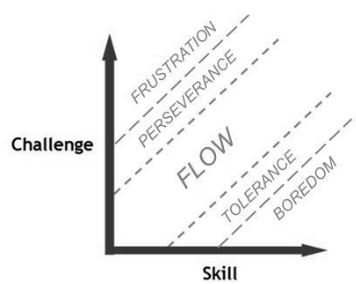


Figure 6. Liljedahl’s (2018) graphical representation of the balance between challenge and skill.

The states of tolerance and perseverance act “as buffers between flow and quitting by delaying the transition to boredom or frustration long enough for the imbalance between ability and challenge to be rebalanced. In the case of *tolerance*, this rebalancing was the result of an increase in complexity while in the case of *perseverance*, rebalancing could happen as a result of either a decrease in challenge or an increase in ability” (Liu & Liljedahl, 2019).

While working on the problems, students were not always in flow. In other words, there was an imbalance between challenge and ability at times. Because this imbalance often has roots in students’ abilities, or more specifically their insufficient EMK, sometimes it is possible to move students from the state of frustration and perseverance, where they were almost out of flow, back into flow by providing them with guidance and the EMK required to solve the problem so that the problem is now within their reach. This is a case of increasing students’ skills. At other times, when it was not possible to raise students’ abilities to bring them back in flow, students decrease the challenge to keep themselves within the band of flow. An example of students’ attempts to remain in flow by reducing the challenge is found in the *Design a New School* task (Figure 2). As one group worked on the parking lot, they lacked understanding of how to create a reasonable parking lot despite of all the help provided. Instead of giving up on the problem, the group reduced the challenge by using the grid provided as guidelines to create a layout of the parking lot (Figure 7) and ended up creating parking spaces much bigger than what was required. Alternatively, one group remained in flow by increasing their skills. In this case, they used the staff parking lot as their template to help them design a 30-car parking lot.

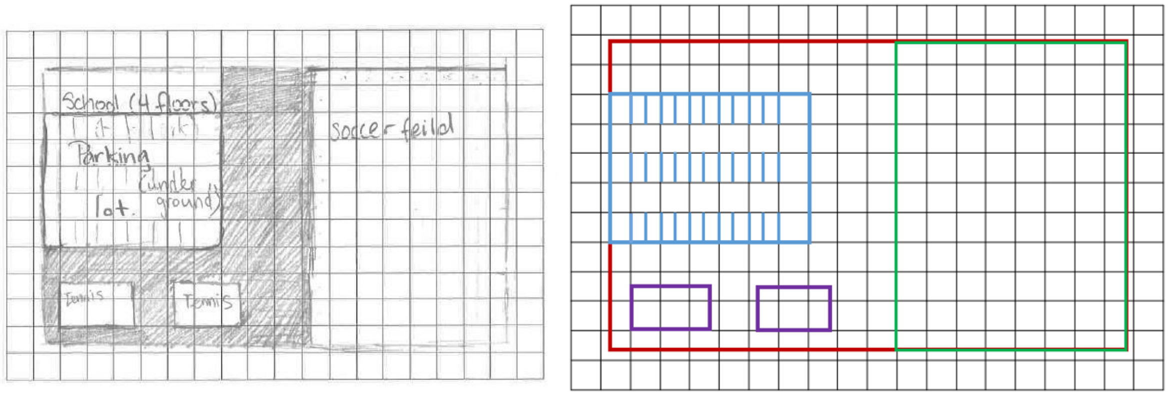


Figure 7. Flow: students attempted to stay in flow and used the grid as guidelines to create a parking lot. Each parking space measures 5m by 10m, which is much bigger than typical parking spaces (approximately 2.5m by 5.5m). The diagram on the left is students' completed work and the diagram on the right is an illustration of students' work. The blue lines represent students' parking lot design.

DISCUSSION AND CONCLUSION

In my use of *rudimentary mathematics complex tasks* (RMCT) to tip the balance between modelling and mathematics, I was able to slow down the modelling process for students to focus on their EMK, and in turn for me to investigate the strategies students used to produce a real model based on their interpretation and understanding of the real situation. I found that their strategies include reducing reality and reducing complexity. My analysis of the reasons that contribute to these decisions highlight the challenges they experienced during the modelling processes.

Reducing reality and reducing complexity are strategies that come naturally to students during their modelling processes. They act as ways for students to simplify a problem situation and to proceed especially when they are stuck. While reducing reality allows students to simplify and solve the problem from a real-world perspective, reducing complexity changes the nature of the problem, is not what the problem intended, and is likely unavoidable. Identifying and understanding these strategies allow us to pay attention to students' challenges experienced during their modelling processes, including their insufficient EMK and their difficulties to interpret the problems from a real-world perspective (real-world problems vs. problems in mathematics class and reality vs. fantasy), and to help students remain in flow. It also allows us to highlight the richness of these real-world problems, prevent students from developing into "train spotters", and to promote students' rich mathematics learning experiences so that mathematics goes beyond the mathematical skills students learn inside the classroom.

REFERENCES

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, *1*(1), 45–58.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM: The International Journal on Mathematics Education*, *38*(2), 86–95.

Borromeo Ferri, R. (2007). Personal experiences and extra-mathematical knowledge as an influence factor on modelling routes of pupils. *CERME 5: Fifth Conference of the European Society for Research in Mathematics Education* (pp. 2080–2089). Larnaca.

Borromeo Ferri, R. (2010). On the influence of mathematical thinking styles on learners' modeling behavior. *JMD*, 31, 99–118.

Csíkszentmihályi, M. (1990). *Flow: the psychology of optimal experience*. Harper and Row.

Hewitt, D. (1992, September). Train spotter's paradise. *The Association of Teachers of Mathematics*, 6–8.

Liljedahl, P. (2010, September). *Numeracy tasks*. http://www.peterliljedahl.com/teachers/numeracy-tasks

Liljedahl, P. (2018). On the edges of flow: Student problem solving behavior. In S. Carreira, N. Amado, & K. Jones (Eds.), *Broadening the scope of research on mathematical problem solving: a focus on technology, creativity and affect* (pp. 505–524). Springer.

Liu, M., & Liljedahl, P. (2019). Flow and modelling. In S. Chamberlin & B. Sriraman (Eds.), *Affect in mathematical modelling* (pp. 273–296). Springer.

Maaβ, K. (2006). What are modelling competencies? *ZDM*, *38*(2), 113–142.

Steen, L. A. (Ed.). (2001). *Mathematics and democracy: The case for quantitative literacy*. The Woodrow Wilson National Fellowship Foundation.

TOWARDS RECURSIVE MATHEMATICS CURRICULA: A COMPLEXIFIED HERMENEUTIC JOURNEY

Lixin Luo
University of Alberta

ABSTRACT

The present study re-conceptualizes mathematics curriculum as recursive through the lens of complexity thinking, which studies fractal-like complex systems (i.e., cognition, knowledge, learners, and etc.) that co-evolve with their environments recursively. Recursive curricula are reinterpreted as a process-oriented interplay of part and whole. Recursion is reinterpreted as re-encountering: a play with contingent equivalency that has four forms (re-languaging, re-imaging, re-inbodying, and re-storying). This study is informed by lived re-viewing experiences, in which a learner learned something new through encountering what they had encountered before. Such texts of experiences are generated through autobiographical reflections, teaching documents analysis, conversations with experienced high school mathematics teachers, and iterative reinterpretations of them. This study has led to a metaphorical and iconic image of recursive curricula that represents abundant mathematics curriculum possibilities rather than a fixed one. This model has implications in visualizing and engendering recursive curricula in both mathematics education and mathematics teacher education.

THE ORIGIN OF THE STUDY AND THEORETICAL FRAMEWORK (WHY BOTHER?)

The present study is rooted in a world view supported by complexity thinking (Davis & Sumara, 2006), and it is an effort to actualize Doll's (1993) influential vision of recursive curriculum. Complexity thinking is a school of thought that assumes a complex universe (Davis & Sumara, 2006). Many natural and social phenomena, such as anthills, climate, ecosystems, cognition, and knowledge, are considered as complex systems that can learn and adapt. Complex systems evolve through self-organization. New order emerges, and it cannot be caused. So the future of complex systems is not predictable. Complex systems are fractal-like phenomena formed by recursion, which is a reflexive and iterative process. A complex system has holistic features that are not owned by any of its parts. Meanwhile the whole of a complex system can be reflected in any of its parts across scales. This feature of self-similarity is captured well by William Blake's poetic language "to see a world in a grain of sand" (Blake, 1988, p. 490).

From the perspective of complexity thinking, both learners and school subjects are complex systems with a biological structure (Davis & Sumara, 2002) that emerges. Knowledge is no more a static structure made of fixed parts and inert modules. Rather knowledge is considered as a living system that evolves. It is dynamic and organic. It does not have a fixed division of parts, and a change of a part can trigger a transformation of the whole. Learning for a complex system is an unpredictable self-organizing process through which a learner and her environment co-evolve (Davis & Sumara, 2002; Davis et al., 2008). Learning is not an accumulative linear assembly line like process, where parts are put together in a sequence to build a whole. But it goes in cycles as recursive elaboration (Davis & Sumara, 2002; Davis et al., 2008). In each cycle, a learner's previous understanding exists as a whole and gets elaborated. From one cycle to another, a transformation happens. Such change is holistic; it is a change in kind, not in degree.

The above view of learner, knowledge, and learning demands a recursive curriculum that emphasizes revisiting and renewing the past and changes along its formation. The significance of recursion in curriculum has been recognized by various authors (e.g., Doll, 1993; Davis & Sumara, 2002; Davis et al., 2008; Thom, 2012) and several mathematics education researchers have studied the growth of mathematical understanding as a recursive phenomenon (e.g., Sawada & Pothier, 1993; Sfard, 1991; Pirie & Kieren, 1989, 1994), yet the study in recursion and recursive curriculum is insufficient. A noticeable gap exists in our (i.e., mathematics educators and mathematics education researchers) understanding of recursive mathematics curricula and their implication for teaching and learning. It is still rather common for mathematics curricula in practice to be designed as a fixed and linear sequence towards predictable outcomes. The present study is a response to the call for recursion. What recursive mathematics curricula might be like, particularly at the high school level, in theory and practice, is my research focus. Please note that part of this research report, particularly the section on re-encountering, has been published in Luo (2020).

METHODOLOGY AND RESEARCH PROCESS (HOW DID I RESEARCH?)

Given the abstract nature of the concepts of recursion and recursive curriculum, I chose reviewing as an entry point for my study. In theory, reviewing has a looping back structure. Hence it can be considered as a form of recursion and consequently a central process in recursive curricula. In practice, reviewing is a pervasive teaching and learning practice in mathematics classes. So a study of reviewing situates me in both theory and practice and helps me to understand both recursion and recursive curricula. It allows me to talk about recursion with research participants in a sensible way. Therefore I asked what a process of reviewing might be like in a recursive mathematics curriculum, or more practically, "How can we help mathematics learners to learn something new from what they have encountered before?

Aiming for reinterpretation, and noticing its potential to embody complexity thinking, I chose the tradition of hermeneutics (Gadamer, 1989/2013) as the research methodology. Hermeneutics attends to language. It emphasizes emergent holistic understanding brought forth through iterative loops of interpretations and interplays of parts and whole. The interpretations in this research are informed by three kinds of entry texts: my personal reflections about recursive curriculum, teaching documents (i.e., programs of studies and textbooks) analysis, and conversations with teachers. They serve to provoke my thinking and generate further reflection subjected to new rounds of interpretations. Several high school mathematics teaching documents from Canada and China were examined to see in what ways a planned curriculum might afford recursion. Conversations with eleven experienced high school mathematics teachers were conducted in professional development workshops and/or individual meetings. Teachers were invited to reflect on their learning and teaching experiences, comment on several teaching and learning practices (e.g., reviewing) that are orientated towards helping students to learn something new from what they have encountered before, and work with me to revise or generate curriculum materials to promote these practices.

Consistent with a hermeneutic attitude of letting (Jardine, 2006) and a hermeneutic inquiry being a response to what addresses the inquirer instead of a procedure to follow (Jardine, 2015), my text generation unfolded while I was walking on the research journey. The generation of an entry text and its interpretation affected the next text generation and interpretation. The iterative loops of interpretation and reinterpretation in a hermeneutic inquiry brought forth unexpected re-viewing experiences in which I learned something new about a familiar mathematics idea and mathematics in general through revisiting the idea. These experiences, along with other lived re-viewing experiences of others' and mine, rose as attractors for my reinterpretation.

My interpretations are supported largely by complexity thinking and sign theory. I used key ideas in complexity thinking, such as self-organization, emergence, fractal-like self-similarity, and so on, to frame reinterpretations of learning, learner, knowledge, school subject, and curriculum. Noting the significance of semiosis for mathematics as Presmeg et al. (2016) do, I enlist their interpretation of Peirce's sign theory to provide my interpretation of reviewing with necessary terms and needed assumptions about signs in mathematics.

REINTERPRETATION (WHAT DID I FIGURE OUT?)

REVIEWING AS RE-ENCOUNTERING

As a form of recursion, reviewing is understood anew as re-encountering in four forms (re-languaging, re-imaging, re-inbodying, and re-storying). It is not seeing something again and seeing it better, but to encounter it again and perceive it differently. It is no more a reproductive process to make the familiar more familiar, but a generative process that makes the familiar strange. Basically, re-encountering is a process of attending to, utilizing, or generating equivalent expressions of the same thing using the same or different mediums (i.e., a signifier/representation and process). What counts as equivalent is a contingent individual observation obtained more in the sense of “A is like B” instead of “A is B”. The fluidity of equivalency allows various modes of knowing (e.g., logical analytical, kinesthetic, tacit, narrative, and analogical) and ways of learning (e.g., formal, informal, conscious, unconscious, planned, serendipitous, etc.) to come into play. Equivalency is a key concept in the theorization of recursion. It will be further elaborated in the next section.

In a re-encountering process, one works with the medium of thought to afford re-interpreting and re-experiencing. Different forms of re-encountering engage different types of mediums: language, image, body, story (or worded differently, linguistic, visual-spatial, physical, and narrative representation), and the process of expressing, representing, and working with the medium itself. Each type of medium enables one to express and/or engage with ideas in a certain way, invoking certain ways of thinking and eliciting different modes of knowing. Since in all four forms of re-encountering, learners work with equivalent ideas/texts, which could be different ideas/texts about the same thing or same idea/text encountered at different times, they are subjected to the influence of the different affordances and limitations of the different mediums. New understanding often emerges when a learner establishes or revises a sense of equivalency by seeing different objects as the same, seeing the same or equivalent objects differently, or generating something equivalent. In the language of sign theory, all four forms use a sign vehicle (i.e., a wording, an image, a physical engagement, and a story respectively) for meaning, and use the means and process to inform mathematical meaning making. They are a process of making a sign vehicle more mathematically meaningful, either through seeing/sensing it anew or changing between equivalent ones.

The focus of a re-encountering process, though, is not at the end of the process. It is not to close sense making by finding the “best” sign vehicle once for all. Rather, in line with Sfard’s (2010) conceptualization of learning mathematics as “changing forms of communication” (p. 217), re-encountering is a process of learning through change and difference. The harmony or tension that arises from the comparison of forms is the source of insights in re-encountering. The space for growth is in between.

Each of the forms of re-encountering is informed by related lived re-viewing experiences. For example, the theorization of re-inbodying drew inspirations from my mathematical and pedagogical growth that is benefited from my experience of (re)enacting a previously learned idea in novel ways. Through paralleling my physical enactments of $\mathbf{-6-2=-8}$ in two representation systems (i.e., number line and integer token), I renewed my understanding of zero and negative numbers, and found that two problem-solving strategies frequently used in higher grades (i.e., introducing needed symbols through making 1 or 0, simplifying mathematical expressions before calculations) have already appeared in lower grades through the use of negative tokens.

RECURSION AS A PLAY WITH CONTINGENT EQUIVALENCY

Based on my interpretation of re-encountering, recursion is understood as a play with contingent equivalency. Equivalency is defined as a perceived condition of two things being the same, similar, associating, or resonating. Equivalency is primarily established through partial similarity or correspondence, in other words, analogy. It is affected by both what it is expressed (i.e. the message) and how it is expressed (i.e., the medium). Take the sign of half, i.e., ½, as an example. Led by its meaning, we might say that it is the same as 1 divided by 2, similar to any fraction, or an image showing one thing as half of the other. We might say it is associated with experience of doubling or it is resonating with experience involving balancing two sides and making them equal. Look at $^1/_2$ again. Led by its appearance, we might say that it is the same as one over two, similar to an image of one thing over the other, or associated with $^2/_1$ because $^1/_2$ and $^2/_1$ play with the same numbers. We might also say that they are resonating with negativity because our experience with things looking like $^1/_2$ has been frustrating. Essentially, equivalency is an individual, contingent, heuristic, and playful interpretation.

The contingency of equivalency is not a disadvantage in recursion. Rather it is what makes play and growth possible. A recursive process, such as re-encountering, acknowledges that equivalency is always set up tentatively and conditionally; hence, revision is always possible and beneficial for mathematical growth. Therefore, in a recursive process, a learner is invited to keep her sense making going with a good-enough-for-now perception of equivalency and see what it leads to and come back to revise it if necessary. A hermeneutic attitude of letting (Jardine, 2006) is at play. The engagement with two or more equivalent expressions of the same thing in a recursive process is a deliberate setup to destabilize the learner's previously established sense of equivalency. It is when this (un)conscious sense becomes strangely noticeable or uncomfortable in a new context that reflection and revision become necessary.

This is exemplified in my aforementioned experience of re-inbodying $-6 - 2 = -8$, in which I needed to physically enact the equation on the number line and parallel my actions with a teacher participant's operations on integer tokens. Led by the affordance of the number line, I intuitively represented both minus sign and negative sign as walking right at first. This led to dead ends. I had to revise this equivalency and use two different actions to represent these two signs. While doing that, I had a moment of "unclogging" (Arcavi & Nachmias, 1989, p. 81): I regained access to the source of insight needed for understanding negative numbers yet no longer accessible for me due to years of automation. The renewal of my mathematics understanding also benefited from a previously unconsciously established sense of equivalency of mine being interrupted. Being new to integer tokens, I was intrigued by my bodily discomfort towards negative tokens. My reflection on it revealed my unconsciously associating negative numbers with not having something. Without knowing, I considered negative numbers equivalent to something that does not exist. This contradicts what negative tokens afford: not having something can be represented as having something. To resolve this conflict, I had to renew my understanding of negative numbers and zero, and consequently mathematics in general.

RECURSIVE CURRICULUM AS A PROCESS-ORIENTED INTERPLAY OF PART AND WHOLE

The theorization of recursion as working with equivalent things makes comparison essential for recursive curricula. A comparison of comparisons in different textbooks, as part of the teaching document analysis conducted in this study, brought forth a promising curriculum design model. This design utilizes multiple levels of similarity to trigger comparisons across scales and afford recursive movement.

Figures 1 and 2 show a curriculum text that embodies this design. The text is taken from a Grade 9 Chinese textbook (e.g., Department of Secondary Mathematics of the People's Education Press, 2001). Figure 1 includes four comparisons in two lessons in the chapter of linear functions. To focus on the form of comparison, only information related to the content and layout of the comparison (i.e., what the two items for comparison are and how they are displayed) is presented. For the purpose of comparison across lessons, the wordings of the questions cited are simplified. For example, the example in Comparison 1 "Graph $y = x + 0.5$" was changed from "画出函数$y = x + 0.5$的图像 (literally translated as: sketch a graph of the function $y = x + 0.5$)".

Comparison 1	Comparison 2	Comparison 3	Comparison 4
Example: Graph $y = x + 0.5$ *Exercise:* Graph $y = 0.5x$, each in a different Cartesian plane.	*Example:* Graph two linear functions ($y = 0.5x$, $y = -0.5x$) each in a different Cartesian plane.	*Example:* Graph two linear functions ($y = 2x + 1$, $y = -2x + 1$), in the same Cartesian plane.	*Exercise:* Graph four linear functions ($y = 3x$, $y = -3x$, $y = 3x + 3$, $y = -3x + 3$), in the same Cartesian plane.
from Lesson 13.3, p. 91	from Lesson 13.5, p. 99	from Lesson 13.5, p. 100	from Lesson 13.5, p. 101

Figure 1. Four equivalent comparisons.

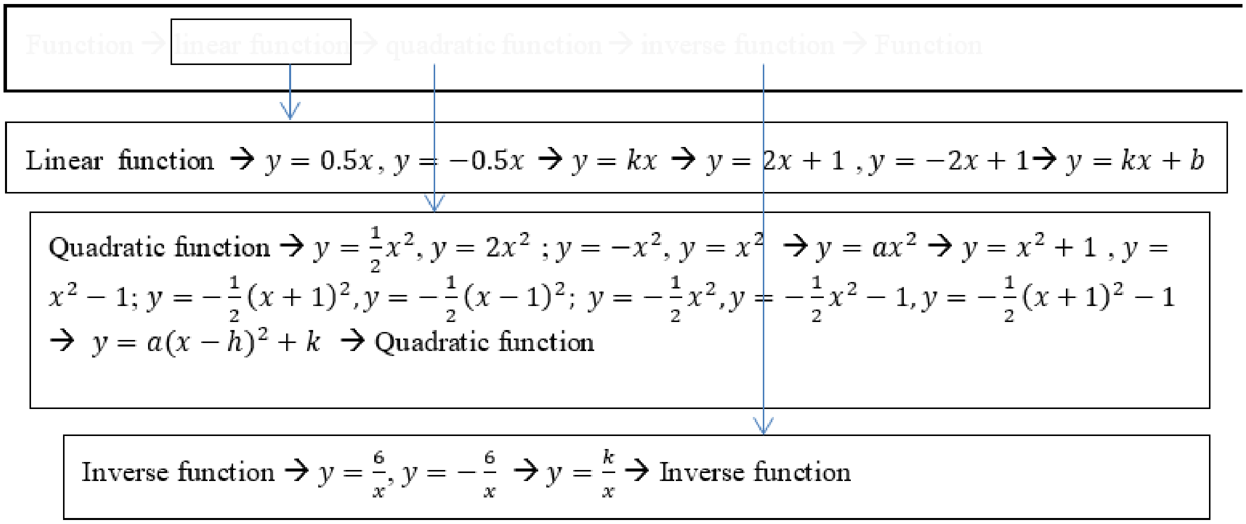
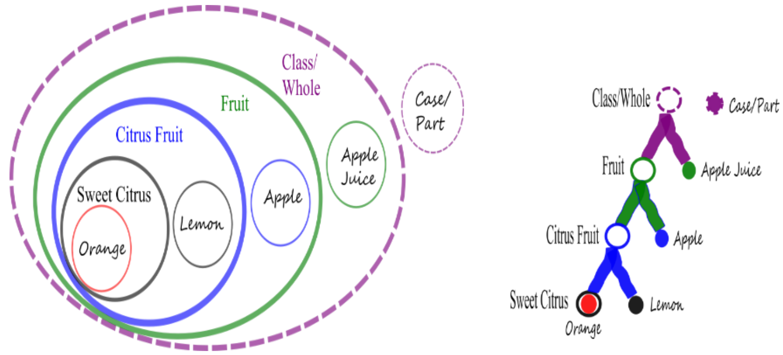


Figure 2. A visualization of the Functions and Their Graphs unit.

This text shows a consistent way to organize content across scales. The functions in each comparison use symbolic expressions that closely resemble each other, yet their categories can differ in kinds (e.g., linear functions with a non-zero y-intercept vs. linear functions with a zero y-intercept in Comparison 1). This subtle yet salient difference can also be observed among comparisons. Each new comparison resembles its immediate past or the past as a whole. It is also different enough to urge one to think about categories as each new addition can bring forth new whole(s). For example, when a learner encounters a comparison between $y = 0.5x$ and $y = -0.5x$, they might (un)consciously view them as different cases rather than classes/categories. But when they move to a comparison between $y = 2x + 1$ and $y = -2x + 1$, they might start to see the previous pair more alike and form two different wholes (e.g., $y = ax$ and $y = ax + b$, or, linear functions with zero y-intercept and linear functions with non-zero y-intercept). They might also notice how part of the new pair is similar to part of the old pair and consequently form other wholes (e.g., linear functions with positive slope and linear functions with negative slope). Their (un)conscious understanding of the class of linear functions as a whole might be renewed as well.

This use of multiple levels of similarity is further observed at a larger scale. Figure 2 shows a simplified outline of the unit in which the aforementioned linear function lessons are situated along with two other chapters. Each new chapter introduces a new kind of function using a similar plot design (i.e., each kind of function goes through a similar sequence of transformations: vertical stretch/compression/reflection → vertical shift) and organizes comparisons in a similar way. Hence, each chapter becomes a review for the previous one(s) yet brings forth something significantly different. A comparison in the new chapter becomes a review for the previous comparison(s) in the same chapter, the others in previous chapter(s), and the previous chapter(s) as a whole. This use of multiple levels of similarity helps to remind a learner frequently of the past at part and whole levels, making a renewal of her understanding of the past possible.

This design can be better understood using an analogy and its equivalent representation, i.e., a binary tree (see Figure 3).



a. An analogy of the curriculum design of using multiple levels of similarity.

b. A binary tree.

Figure 3. A recursive curriculum design model.

This analogy gives an example of forming a recursive curriculum with emergent wholes through utilizing multiple levels of similarity. Starting from showing you an orange, if I put a lemon beside it, you can tell that they are different. You might form two categories/classes (un)consciously (e.g., Sweet vs. Sour, Orange vs. Yellow, etc.). By the time I add an apple, you might notice that orange and lemon are more alike than you thought before. You might change your understanding of the past at part and whole levels (un)consciously and create new classes such as Sweet Citrus Fruit, Citrus Fruit, and Non-citrus Fruit. Now if I add a bottle of apple juice, you would most likely see the three objects shown earlier the same (e.g., as cases of fruits). Meanwhile, a new class/whole that includes all the four objects is emerging and will not be actualized until a new object arrives. This actualization might trigger you to re-understand the four objects and consequently re-categorize them.

The above example illustrates a design in which each new addition resembles its immediate past or the past as a whole yet it introduces a difference in kind. Through resembling and differing from the past at both part and whole levels, each new addition invites learners to re-encounter the previously established whole and re-understand it as a whole and as parts. When all the new additions at different scales resemble each other in some ways yet differ significantly, this design has potential to afford a fractal-like or self-similar curriculum in which a part at any scale represents the whole, hence promoting comparison across scales and recursion. Such a recursive curriculum has a constant part-whole interplay.

A planned curriculum, though, no matter how thoroughly designed, is always a map, not the territory. As the aforementioned analogy shows, what a learner understands as a whole is emerging when she walks on a recursive journey. It is inseparable with what she has encountered and what she will encounter next. It is not predictable, and it is the result of the learner's self-organization. Recognizing this, a recursive curriculum keeps the future open. It welcomes a novel theme that presents itself through the recursive process and uses it to inform what might be added next as an equivalent entity of the existing whole. This addition in turn can transform the existing theme. This openness is supported by the fluidity of equivalency as one can always generate equivalent contents for an existing one as long as one is willing to break existing categorization and generate novel and tentative categories. In this way, recursive curricula are process-oriented.

IMPLICATIONS (SO WHAT?)

This study has implications in both theory and practice. It extends Davis et al.'s (2008) work that provides provocative recursive curriculum design advice for particularly collective learning settings. It offers finer details of recursive elaborations and broadens the possible starting points of a recursive curriculum for both individual and collective learning settings. The theorization of elaborative processes as re-encountering and the proposal of four possible forms (re-languaging, re-imaging, re-inbodying, and re-storying) allows practical questions, such as "What might it be worded like? What might it be imagined like? What might it be embodied/enacted/perceived like? What might a story about it be like?", or general prompts, such as "It is like__" and "It reminds me of __", where the word "it" refers to the mathematics idea that one reviews and/or the medium used to express the idea, to be used to occasion recursion in class or guide design towards recursive curricula. The key here is, not about interpreting something as what it is, but what it might be like. Re-encountering enlists the narrative, artistic, metaphorical, and intuitive modes of knowing, beyond the analytic and logical ones. Such a mix is what Doll (1993), following Bruner, advocates in post-modern curriculum design and believes "would encourage us to think of knowledge in a new light" (p. 124).

This study brings forth a design model that can guide recursive curriculum experiment and development, and afford fluid mathematical understanding and novel curriculum design. This model offers workable principles (e.g., enlisting subtle yet salient changes, using rapid equivalent learning cycles, utilizing nested categories and recategorization to maintain equivalency across scales) to facilitate recursive curricula generation. This model, blended with two other recursive curriculum models, i.e., Bruner's (1962) spiral curriculum and Davis et al.'s (2008) fractal tree, has been transformed into an iconic and metaphorical visualization of abundant recursive curricula (see Figure 4). This visualization has shown promise in facilitating recursive curriculum visualization and formation (see Luo, 2019 for examples).

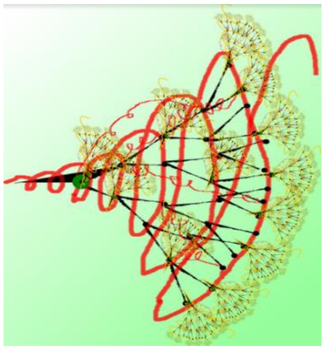


Figure 4. A fractal-like representation of recursive curricula.

Designing towards a recursive curriculum is not really about generating a specific sequence of tasks to be delivered or executed. Rather, it is a reflective and creative thought experiment to inquire and generate fractal-like contents and tasks, and to enact, experience, and engender a recursive learning process. It aims for the process's generativity for rich possibilities in both teaching and learning. Such generativity has been continuously exemplified after my PhD through my experiments with designing recursive mathematics stories for children (e.g., Luo, 2021) and recursive assignments for teacher education. Further inquiry and practice of recursive curricula in both mathematics education and teacher education is worth trying.

REFERENCES

Arcavi, A., & Nachmias, R. (1989). Re-exploring familiar concepts with a new representation. *Proceedings of 13th conference of the International Group for the Psychology of Mathematics Education, France* (Vol. 1, pp. 77–84). PME.

Blake, W. (1988). In D. V. Erdman (Ed.). *The complete poetry and prose* (Newly revised ed., p. 490). Anchor Books.

Bruner, J. S. (1962). *The process of education*. Harvard University Press.

Davis, B., & Sumara, D. (2002). Constructivist discourses and the field of education: Problems and possibilities. *Educational Theory*, *52*(4), 409–428.

Davis, B., & Sumara, D. (2006). *Complexity and education: Inquiries into learning, teaching, and research*. Lawrence Erlbaum Associates.

Davis, B., Sumara, D. J., & Luce-Kapler, R. (2008). *Engaging minds: Changing teaching in complex times* (2nd ed.). Routledge.

Department of Secondary Mathematics of the People's Education Press (Ed.) (2001). *Dai shu (di san ce)*代数(第三册) [Algebra (Vol. 3)]. People's Education Press.

Doll, W. E., Jr. (1993). *A post-modern perspective on curriculum*. Teachers College Press.

Jardine, D. W. (2006). “Let Eric’s age be ‘x’”: A brief mathematical phenomenology. In D. W. Jardine, S. Friesen, & P. Clifford (Eds.). *Curriculum in abundance* (pp. 61–67). Lawrence Erlbaum Associates.

Jardine, D. W. (2015). On hermeneutics: “Over and above our wanting and doing”. In K. Tobin & S. R. Steinberg (Eds.), *Doing educational research* (2nd ed., pp. 235–254). Sense Publishers.

Gadamer, H. G. (2013). *Truth and method* (Rev. 2nd ed., J. Weinsheimer & D. G. Marshall, Rev. Trans.). Bloomsbury Academic. (Original work published 1989)

Luo, L. (2019). *Towards recursive mathematics curricula: A complexified hermeneutic journey* [Unpublished doctoral dissertation]. University of Alberta.

Luo, L. (2020). Reviewing, re-viewing, and re-encountering. *Mathematics Teaching-Research Journal*, *12*(2).

Luo, L. (2021, March 12). Shui duo shui shao bu zhonng yao 谁多谁少不重要 [It doesn’t matter who gets more or less]. **深圳青少年**报（启蒙周刊）[Shenzhen Youth and Children’s Newspaper].

Pirie, S., & Kieren, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics*, *9*(3), 7–11.

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? In P. Cobb (Ed.), *Learning mathematics: Constructivist and interactionist theories of mathematical development* (pp. 61–86). Kluwer Academic Publishers.

Presmeg, N. C., Radford, L., Roth, W-M., & Kadunz, G., (2016). *Semiotics in mathematics education*. Springer. doi: 10.1007/978-3-319-31370-2

Sawada, D., & Pothier, Y. (1993). Mathematical imagination through recursion. *Mathematics in School*, *22*(5), 14–19.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, *22*(1), 1–36. doi:10.1007/BF00302715

Sfard, A. (2010). Challenges of researching mathematics as discourse. In M. Pinto & T. Kawasaki (Eds.), *Proceedings of the 34th conference of the international group for the psychology of mathematics education* (Vol. 1, pp. 217–218). Belo Horizonte, Brazil: PME.

Thom, J. S. (2012). *Re-rooting the learning space: Minding where children’s mathematics grow*. Sense.

AFFORDANCES OF CULTURALLY RESPONSIVE TEACHING THROUGH THE ADINKRA SYMBOLS ON THE EMERGENCE OF STUDENTS' MATHEMATICS PROFICIENCY

Mavis Okyere
University of Alberta

INTRODUCTION

It has been noted that a mismatch between school culture and the culture of the students creates the potential for misunderstanding of actions and misinterpretation of communication between teachers and students. This misunderstanding and miscommunication or lack of cultural harmonization increase the possibility of failure for students who lack the cultural knowledge to navigate the unstated culture and norms of the school (Delpit, 1995). Those students who become successful in school bring to school values that the school considers appropriate. Irvine (2003) states that those students who fail to assimilate or switch to the dominant culture of the school are at a greater risk of failing.

Gay (2010) also shares a similar sentiment by noting that cultures of schools and different ethnic groups do not always synchronize, and this disconnection can interfere with students' academic achievement in part because how some ethnically diverse individuals usually engage in intellectual processing, and self-presentation, and task performance is different from the processes used in schools. Teachers, therefore, need to understand different cultural intersections and incompatibilities, minimize the tensions, and bridge the gaps among different cultural systems. This study, therefore, investigated how bridging the gap between the student's home culture and the culture of the Ghanaian mathematics classroom influences students' learning.

THEORETICAL FRAMEWORK

The theory of culturally responsive pedagogy was used to inform the design of the classroom intervention. This theory postulates that the discontinuity between the school culture and the home and community cultures of students is an important factor that affects their academic achievement. Bridging the gap between students' home cultures and school culture, by reflecting and drawing on the students' cultures, will consequently increase the academic achievement of these students (Gay, 2010; Ladson-Billings, 1994).

Gay (2002) proposed some elements that need to be considered when preparing for culturally responsive teaching, and these elements were used to guide the planning and the implementation of the intervention in this study. The elements are developing a cultural diversity knowledge base, designing a culturally relevant curriculum, using cultural communications, and ensuring cultural congruity in classroom instruction. Decisions for the mathematics lessons led to culturally responsive strategies that could promote students' mathematics proficiency and were implemented in the classrooms. The decisions are shown in Figure 1.

Figure 1 illustrates how the aims of the mathematics curriculum relate to the content, and the objectives for the content (in other words, each informs the other). These content objectives were restated to reflect the cultural aspect intended for the lessons (hence the restated objectives might correlate with the aims of the curriculum). Hence, two-directional arrows were used to connect the aims, the content objectives, and the culturally responsive content objectives. Again, the stated culturally responsive objectives informed the culturally responsive teaching that, in return, reflected the achievement of the culturally responsive objectives as shown by a two-directional arrow. Lastly, the success of the culturally responsive intervention was dependent on five main elements: Adinkra symbols as the context of activities, use of local language, Adinkra-related activities, teachers' facilitation, and cooperative groups. Based on the belief that culturally responsive teaching has a positive impact on students' mathematics learning (Gay, 2002, 2010; Mukhopadhyay et al., 2009), the five strands of mathematics proficiency as identified by Kilpatrick et al., (2001) namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition were used to interpret the kinds of students' learning that occurred in the culturally responsive intervention.

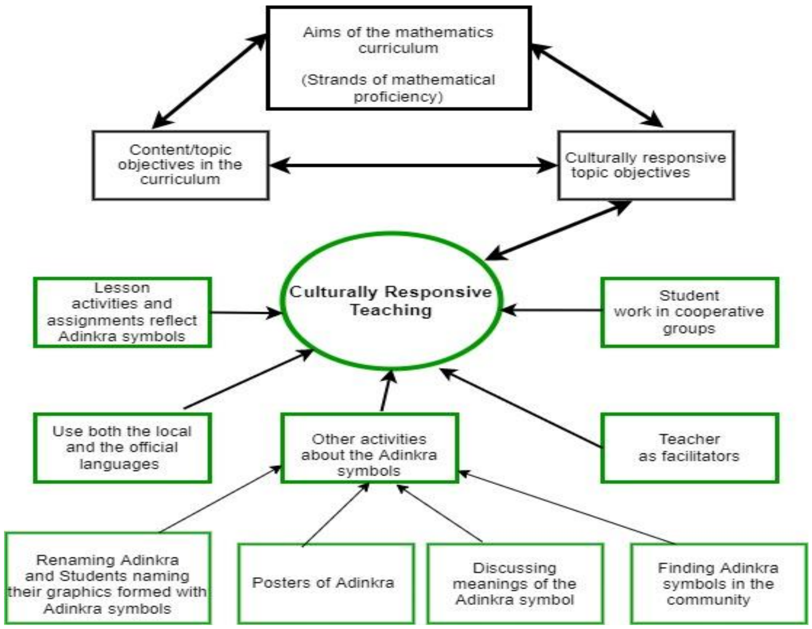


Figure 1. Components of the Model for Culturally Responsive Teaching in The Study.

METHODOLOGY

My aim in conducting this study was to explore the kinds of students' learning defined in terms of the strands of mathematics proficiency that could emerge in a culturally responsive intervention using the Adinkra symbols of Ghana. To achieve this aim, I employed a design-based research methodology. Design-based research emphasizes the need to develop a theory and design principles that guide, inform, and improve practice and research in educational contexts (Anderson & Shattuck, 2012; Cobb et al., 2016). I began the study with five mathematics teachers [three Junior High School (JHS, ages 12 to 15 years or grades 7 to 9) and two Senior High School (SHS, ages 15 to 18 years or grades 10 to 12), to find related mathematics concepts of Adinkra symbols to be used as mediating tools in the design of lesson activities. The teachers and I came together and studied images of Adinkra symbols for their related mathematics concepts and observed a craftsman creating the selected symbols to infer mathematics concepts employed in the creation processes. After we identified the mathematics concepts in the Adinkra symbols, JHS and SHS teachers each selected four concepts for lessons to be designed using the Adinkra symbols for teaching. Together, the teachers and I designed the activities to include the Adinkra symbols related to the concepts for implementation in the classrooms. The lessons designed were implemented in one SHS (Grade 11) and one JHS class (Grade 8). Due to the paucity of space, in this paper, I report on only the affordances of the culturally responsive teaching strategies employed in the intervention for the emergence of students' proficiency.

The classroom data were gathered through video recordings, observational notes, and reflective journals. Briefing and debriefing sessions were held before and after each lesson to reflect on lessons. These sessions were also audio recorded. After the intervention, the teachers were interviewed for their views on the challenges and affordances of the intervention. The interview was also audio recorded. The data sets obtained from the study were analyzed chronologically. The analysis followed Roper and Shapira's (2000) data analysis steps of (a) coding for descriptions, (b) sorting for patterns, (c) identification of outliers or negative cases, (d) generalizing (in the context of the study) with constructs and theories, and (e) memoing-including reflective remarks. To identify the affordances of the culturally responsive intervention, the interpretations made about the strands of mathematics proficiency that emerged were examined against the working papers, noting how each implemented feature of the intervention contributed to the emergence of the strands of mathematical proficiency. First, audio recordings of the briefing and debriefing meetings during the implementation stage, and the teachers' interviews and video recordings of lessons, were transcribed verbatim. The transcribed data, together with the field notes and the reflective journals, were analyzed for relationships and explanations of the affordances of the intervention.

FINDINGS AND DISCUSSIONS

Of all the decisions taken and implemented in the classroom to enhance students' learning, the use of the Ghanaian language (Twi) facilitated access to all students' learning that occurred in the lessons through group conversations, whole-class discussions, and group presentations. The affordances of each of the implemented ideas are presented in Table 1. The aim is not to highlight them as the best culturally responsive teaching practices; however, the study suggests they are promising practices for a teacher who wishes to use an artefact, like Adinkra symbols, for mathematics teaching.

Culturally Responsive Strategy	Its Affordances	Mathematics Proficiency
The use of the local language	enhanced engagement with tasks enhanced interaction with peers Enhanced thought process.	All five strands: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition
Adinkra symbols as the context of tasks/problems	Connection between math and culture Relational learning Increased engagement and interest in the tasks	All five strands: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition
Discussing the meaning of the Adinkra symbols	Increased interest in the lessons Learning moral values	Productive disposition
Renaming Adinkra Symbols	Developed a sense of agency Self-reliance	Conceptual understanding Adaptive reasoning Strategic competence Productive disposition
Images of the symbols as teaching/learning materials	Models of the concepts Mediating tools	All five strands: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition
Teachers as facilitators of lessons	enhanced engagement with tasks enhanced interaction with peers enhance self-reliance	All five strands: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition
The use of the small cooperative groups	enhanced engagement with tasks enhanced interaction with peers enhance self-reliance	All five strands: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition

Table 1. Affordances of the Culturally Responsive Strategies on the Emergence of Mathematics Proficiency.

THE USE OF THE GHANAIAN LANGUAGE

Other studies, on the promotion of the strands of mathematics proficiency in classrooms, have revealed that mathematics teachers tend to emphasize procedural fluency at the expense of the other four strands (Ally, 2011; Schoenfeld, 2007). However, in the case of this study, it was observed that all five strands emerged. I believe that I was able to access all the strands due to the use of the local language in the lessons. Students could express their mathematical ideas and reasoning in the lessons through their local language, Twi. This is an indication that the language barrier could be a contributing factor to why mathematics teachers focus on procedural fluency. English, which is the official language of Ghana, is the second language of Ghanaian teachers and students, therefore, the teachers and students may have deficits in the English language, hence, using it for instruction becomes difficult, and may cause teachers to emphasize procedural knowledge in the mathematics classroom at the expense of the other strands of proficiency. Similar observations were made by Erling et al. (2017) that 1) the language of instruction can constitute a barrier to good pedagogy, 2) teachers' competency in the language of instruction is an important factor in teaching, and 3) students' deficit in the language of instruction can limit opportunities for communication.

The level of participation and students' engagement in the lesson activities in this intervention could be attributed to the use of the local language. It can therefore be said that students may become passive in the mathematics classroom and show a lack of interest in the subject partly because the local language is not used. It was observed that students spoke in Twi and inserted English mathematical terms mid sentences. A teacher in his reflective journal said, "*Students made use of mathematical terms in English even when they were speaking Twi, because they are mostly taught mathematics in English and that the only name that they know to represent the concept is the English name*." I also indicated in my field notes that, in my experience as a native speaker of the Twi language, this could be attributed to the fact that mathematical terms are hardly expressed in single words in the Twi language. Students involved in this intervention were more relaxed in the lessons and contributed to building on each other's ideas. An argument in favor of the use of students' first language in education has been on the learning benefits that all the students may get from having the opportunity to use the language they know and understand in the development and communication of their mathematical thinking (Planas & Civil, 2013), and this finding supports this argument.

THE USE OF THE ADINKRA SYMBOLS

In this intervention, the Adinkra symbols were used in four main ways: 1) using images of the symbols as teaching/learning material, 2) as a context of students' mathematical tasks/problems, 3) discussing the values of the Adinkra symbols, and 4) renaming of the symbols. The images of the Adinkra symbols, posted in the classroom and those handed to students during class activities, served as models for students to refer to when necessary. Using the images as examples of the mathematics concepts for students enhanced their conceptual understanding. The images of the Adinkra symbols, which were found to be associated with the concepts, were used as mediating tools (Hassan, 2002; Vygotsky, 1978) that mediated students' learning of mathematics concepts. This enabled students to make sense of the mathematics problems/tasks, and they served as representations of the mathematics concepts students learned. It was observed that students referred to the images mostly in their initial attempts to make sense of the problems and after creating Adinkra symbols using mathematics ideas to compare their creations with the images. Through this act of comparing and talking about the graphics they created compared to the images, students strengthen their mental representation of the mathematics concepts they used to create the graphics (Adinkra symbols). Again, it was observed in the transformation activities (both the SHS and JHS), that students' mathematical ideas resulted from studying the images and recreating them on the Cartesian plane. The use of the images, as learning materials for students, therefore, promoted their conceptual understanding, adaptive reasoning, strategic competence, and productive disposition. The cultural interpretation of the Zone of Proximal Development (Davydov & Markova, 1983 cited in Daniels, 2017) and the concept of horizontal mathematization (Freudenthal,1991), all indicate that the context of the knowledge we want students to learn must be related to their experiences; hence, the Adinkra symbols related to the mathematics concepts used in the lessons were used as context to design students' classroom activities as well as the evaluation exercises. That is, just as teachers found themselves in the curriculum they developed, students also found themselves in the curriculum. Students found an element of their culture (and an element of their identity), which is the Adinkra symbols that give livelihood to their community, in their classroom mathematics activities. Finding their identity in the mathematics classroom motivated them to fully engage with the mathematics tasks they were assigned. Students, being fully engaged in mathematics tasks, promoted the emergence of all the strands of mathematics proficiency. The students reasoned, based on the context, and developed strategies that were consistent with the problems at hand; and using their understanding of relevant mathematics ideas and procedures, they solved the problems. Researchers have observed that one advantage of using students' cultural artifacts for mathematics learning is that students find connections to the curriculum through their culture (Rosa & Orey, 2011; Sharma & Orey, 2017).

The use of the Adinkra symbols, as context for mathematics tasks, revealed to students the application of mathematics in their environment. Students realized how different mathematics ideas are related to different Adinkra symbols, and they used mathematics ideas to create some of the Adinkra symbols. The use of the Adinkra symbols to design students' intervention activities made students begin to realize mathematics as sensible and worthwhile. Other researchers had similar observations about the impact that the use of students' cultural experiences in the mathematics classroom has on their learning (Rosa & Orey, 2011; Sharma & Orey, 2017; Weldeana, 2016).

The teachers' decision to discuss the meanings and the values of the Adinkra symbols as part of the culturally responsive intervention was meant to help students develop an interest in their cultural values and to learn social values from the symbols. The students' classroom work and conversations revealed that the students connected to the meanings of some of the symbols in their interaction with the activities on the Adinkra symbols. In particular, the students connected to the meanings of two Adinkra symbols: *ɔwɔ foro adobe* which means "persistence" and *Adinkrahene* which also means "creativity," in mathematically productive ways. One student said "yes, we are drawing the *ɔwɔ foro adobe*, which means we should continue to try no matter the difficulty" and another said, "*The Adinkrahene means creativity, we are, therefore, creative because we have created Adinkrahene with mathematics*." Thus, students engaging in discussing the meanings of the Adinkra symbols also contributed to the emergence of their productive disposition.

Another important observation that was made from the students' activity with the Adinkra symbols is the fact that students made use of their understanding of mathematics concepts to rename Adinkra symbols and to name their created graphics. Before the students renamed an Adinkra symbol, they related the appearance of the symbol to mathematics concepts. For example, in the SHS students' activity with the *Mate Masie* symbol, which signifies knowledge, the students renamed the symbol by first noting that the symbol is made up of the concept of circles, and relating the circles to the human head said, "*the four circles represent human head because knowledge is kept in the head*" and again by their understanding of the mathematics concept of four as a quantity being three more than one, said, "*I think because of the four circles we can say that knowledge is acquired best in groups just as we are doing now*." Likewise, in the JHS students naming of the graphic they obtained by enlarging the *Akoma* symbol with different scale factors, from the students' abstraction of small and big, they said, "*since it is like the heart is growing from small to big, let's call it 'grow in love and patience*.'" This is a different use of mathematics concepts in real life. Here, apart from students using mathematics ideas to create the designs, they also referred to mathematics concepts to name the designs. That is, they made use of their conceptual understanding and adaptive reasoning about the appearance of the symbols to give them names.

The teachers' decision to ask students to rename the Adinkra symbols used in their activities, and to name their created graphics and give their meaning, was one of the attempts to encourage students to know that they too can develop knowledge that could be meaningful and acceptable. It was a form of helping students to have a sense of self-confidence, knowing that they can also come out with names and meanings of symbols that could be reasonable to the members of the classroom community. This aim was achieved as demonstrated in the confidence with which students called out their given names and their meanings. In that attempt, students also demonstrated ownership of their created knowledge (names and meanings of graphics). Other researchers have also observed that culturally responsive teaching and ethnomathematics promote students' self-confidence in mathematics (e.g., Weldeana, 2016).

Another important observation that was made about the names students gave to the Adinkra symbols and their created graphics, and their meanings are that the names and meanings signified social values. That is, students' class activities were also linked to the learning of social values through discussing the already known meanings of the Adinkra symbols, and through their attempts to name and give meanings to their graphics. Students gave names like "*Adwene ntoatoa*—putting ideas together," "grow in love and patience," these names signify social values that are encouraged in different ethnic groups. We need such values to sustain the world. Learning these values is equipping them for social life in the present and in adulthood. Weldeana (2016) noted that incorporating students' cultural backgrounds into the mathematics curriculum offers opportunities for all students to learn and achieve, as well as promoting their spiritual, moral, social, and cultural development and preparing them for opportunities, responsibilities, and experiences of adult life. Renaming Adinkra symbols and naming their created graphics was an intellectual activity for students that enhanced their reasoning and use of their mathematics knowledge; it also helped them to have a sense of agency.

THE USE OF SMALL COOPERATIVE GROUPS

Communal living is a way of life in Ghana, especially in semi-urban towns and villages; hence, we decided to make use of cooperative groups in the lessons. The small cooperative groups also enhanced students' engagement and interaction with peers; the students became independent of the teacher and relied on each other for support. This enhanced their mathematics proficiency at the group level, before the individual level, as they learned from each other's ideas and built on each other's ideas. Mr. Oti in the interview on the affordances of the intervention mentioned, "*I did not have to respond to many questions from the students because they had colleagues in the groups to respond to their questions*." Since the students became independent of the teacher, they changed their identity as doers and learners. That is, this culturally responsive strategy fostered students' self-reliance. They were able to successfully claim ownership of the mathematics knowledge and graphics they created during the intervention. The small group work increased students' confidence in their mathematical ability and creativity. The collective generation of mathematical ideas by the students in the intervention prompted the emergence of all five strands of mathematics proficiency.

TEACHERS FACILITATING STUDENTS' LEARNING

Unlike the usual Ghanaian mathematics classroom where the teacher stands in front of the class and gives information to the students, and the students look up to the teacher for knowledge (Watson, 2008), in this study, teachers gave students the autonomy to make decisions about how they tackled mathematics problems/tasks. Teachers allowed students to choose their procedures and methods. This independence made students dependent on their peers in the group for answers to their questions. Through this experience, interaction with peers and engagement with tasks were enhanced. Butakor et al. (2017) identified the lack of engagement of students in mathematics lessons in Ghanaian classrooms as one of the causes of the low performance of students in TIMSS, 2011. The high level of engagement and interaction among peers, which was enhanced by the fact that the teachers were only facilitators of the lessons, contributed to the students' ability to develop ideas that were beyond what was planned for them to achieve. In a debriefing meeting, Mr. Oteng said, "*students understood what they were doing, and it also shows they have knowledge and abilities beyond what we teach them, but they do not demonstrate them because we don't give them the chance*." It can also be said that the decision of teachers to give autonomy to students was enhanced by the small collaborative groups that were employed in the intervention and made it possible for students to collectively contribute their mathematics proficiency in solving the tasks.

CONCLUSION

Based on these findings, relating to how each of the implemented strategies contributed to the emerged students' proficiency, I speculate that without implementing the elements of the culture for culturally responsive teaching, the observed mathematics proficiency would not have occurred. Having knowledge of mathematics in the cultural artefact and using that knowledge is not enough, it is also important to incorporate knowledge about the values, learning approaches, interaction patterns, and language of the ethnic group of the students in the lessons. Using the traditional approach of mathematics teaching described by Watson (2008), which is common in Ghanaian secondary schools, to implement mathematics ideas in the culture of the students might not have much impact on students' learning. My speculation, as stated above, is aligned with Bishop (1994) who hypothesized that any formal mathematical education is a process of cultural interaction and that every child experiences some degree of cultural conflict in his/her learning process.

The conflicts Bishop identified included language, geometric concepts, calculation procedures, symbolic representations, logical reasoning, attitudes, objectives, cognitive preferences, values, and beliefs. It can be said that in this design-based research, most of the conflicts, including language, geometric concepts, cognitive preferences, and values, were resolved through the incorporation of knowledge about the cultural practices of the community in the classrooms. Mukhopadhyay et al. (2009) stated that ignoring student culture demonstrates a lack of respect for students and means that students' learning should be treated independently of their role as citizens, in which they must contribute to society using the knowledge gained in education. It can be said that respect for students was high in this culturally responsive intervention, as students learned mathematics through their cultural way of being from their cultural elements; something that provides livelihood to most members of their community.

REFERENCES

Ally, N. (2011). *The promotion of mathematical proficiency in Grade 6 mathematics classes from the Umgungundlovu district in KwaZulu-Natal* [Master's thesis] University of KwaZulu-Natal. http://hdl.handle.net/10413/5791

Anderson, T., & Shattuck, J. (2012). Design-based research: A decade of progress in education research? *Educational Researcher*, *41*(1), 16–25. https://doi.org/10.3102%2F0013189X11428813

Bishop, A. J. (1994). Cultural conflicts in mathematics education: Developing a research agenda. *For the Learning of Mathematics*, *14*(2), 15–18.

Butakor, P. k., Ampadu, E., & Cole, Y. (2017). Ghanaian students in TIMSS 2011: Relationship between contextual factors and mathematics performance. *African Journal of Research in Mathematics, Science and Technology Education*, *21*(3), 316–326. https://doi.org/10.1080/18117295.2017.1379281

Cobb, P., Jackson, K., & Dunlap, C. (2016). Design research: An analysis and critique. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed., pp. 481–503). Routledge.

Daniels, H. (2017). *Introduction to Vygotsky* (3rd ed.). Routledge.

Delpit, L. (1995). *Other people's children: Cultural conflict in the classroom*. The New Press.

Erling, E. J., Adinolfi, L., & Hultgren, A. K. (2017). *Multilingual classrooms: Opportunities and challenges for English medium instruction in low and middle income contexts*. Education Development Trust.

Freudenthal, H. (1991). *Revisiting mathematics education*. *China Lectures*. Kluwer

Gay, G. (2002). Preparing for culturally responsive teaching. *Journal of Teacher Education*, *53*(2), 106–116. https://doi.org/10.1177%2F0022487102053002003

Gay, G. (2010). *Culturally responsive teaching: Theory, research and practice* (2nd ed.). Teachers College Press.

Hasan, R. (2002). *Semiotic mediation, language and society: Three exotripic theories—Vygotsky, Halliday and Bernstein*. The Laboratory of Comparative Human Cognition. https://lchc.ucsd.edu/mca/Mail/xmcamail.2005_06.dir/att-0012/01-hasan_on_SEMIOTIC_MEDIATION.doc

Irvine, J. J. (2003). *Educating teachers for diversity: Seeing with a cultural eye*. Teachers College Press.

Kilpatrick, J., Swafford, J. & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics. Mathematics Learning Study Committee. Center for Education*. Division of Behavioral and Social Sciences and Education. National Academy Press.

Ladson-Billings, G. (1994). *The dream keepers: Successful teachers of African American children*. Jossey-Bass.

Mukhopadhyay, S., Powell, A. B., & Frankenstein, M. (2009). An ethnomathematical perspective on culturally responsive mathematics education. In B. Greer & S. Mukhopadhyay (Eds.), *Culturally responsive mathematics education* (pp. 65–84), Roultledge.

Planas, N., & Civil, M. (2013). Language-as-resource and language-as-political: Tensions in the bilingual mathematics classroom. *Mathematics Education Research Journal*, *25*(3), 361–378. http://doi.org/10.1007/s13394-013-0075-6

Roper, J. M., & Shapira, J. (2000). *Ethnography in nursing research*. Sage Publications.

Rosa, M., & Orey, D. C. (2011). Ethnomathematics: The cultural aspects of mathematics. *Revista Latinoamericana de Etnomatemática*, *4*(2). 32–54.

Sharma, T., & Orey, D. C. (2017). Meaningful mathematics through the use of cultural artifacts. In M. Rosa, L. Shirley, M. E. Gavarrete, & W. V. Alangui (Eds.), *Ethnomathematics and its diverse approaches for mathematics education* (pp.153–179). ICME-13 Monographs, Springer. http://doi.org/10.1007/978-3-319-59220-6_7

Schoenfeld, A. H. (2007). What is mathematical proficiency and how can it be assessed? *Assessing Mathematical Proficiency*, *53*(1), 59–73. https://doi.org/10.1017/cbo9780511755378.008

Vygotsky, L. S. (1978). *Mind and society: The development of higher psychological processes*. Harvard University Press.

Watson, A. (2008). School mathematics as a special kind of mathematics. *For the Learning of Mathematics*, *28*(3), 3–7.

Weldeana, H. N. (2016). Ethnomathematics in Ethiopia: Futile or fertile for mathematics education? *Momona Ethiopian Journal of Science* (MEJS), *8*(2), 146–167. http://doi.org/10.4314/mejs.v8i2.4

APERÇU D'UNE RECHERCHE-FORMATION À L'ENSEIGNEMENT DES PROBABILITÉS DU SECONDAIRE AVEC DES OUTILS TECHNOLOGIQUES

Mathieu Thibault
Université du Québec en Outaouais

RÉSUMÉ

Ce rapport reprend les idées de ma présentation à la conférence du *Groupe canadien d'étude en didactique des mathématiques (GCEDM)*, dans laquelle j'ai cherché à dépeindre l'essence de ma thèse de doctorat (Thibault, 2021). Cette recherche doctorale vise à documenter les éléments importants à considérer dans une formation à l'enseignement des probabilités du secondaire avec des outils technologiques. À partir d'un problème de recherche se résumant au fait que nous connaissons peu les enjeux de formation à ce sujet, le recours à trois modèles (TPACK, Laborde et Tapan) permet des angles d'entrée complémentaires. Afin de décrire et comprendre des enjeux de formation, la méthode de recherche-formation permet au chercheur-formateur de documenter une formation continue de l'intérieur. Cinq séances de formation (sous forme de rencontres de travail) ont été expérimentées avec cinq enseignants et deux conseillers pédagogiques en mathématiques au secondaire ayant des expertises complémentaires. Une analyse thématique a permis de dégager trois thèmes et onze sous-thèmes. Une analyse à l'aide des catégories conceptualisantes a permis de faire ressortir dix-sept enjeux de formation, en guise de résultats. Pour cette communication, un aperçu de la recherche sera présenté, en exemplifiant l'un des enjeux de formation : *Juger de la pertinence d'un outil technologique pour soutenir l'enseignement des probabilités.*

PROBLÉMATIQUE

Cette thèse se penche plus particulièrement sur l'ensemble de préoccupations concernant la formation à l'enseignement des probabilités du secondaire avec des outils technologiques. On y retrouve des préoccupations scientifiques et sociales, mais de rares travaux ciblent ce sujet. Pour enseigner les probabilités, les simulateurs d'expériences aléatoires s'avèrent riches, mais Theis et Savard (2010) suggèrent que les enseignant·e·s pourraient bénéficier d'un accompagnement plus structuré dans leur utilisation d'un tel outil technologique pour l'enseignement. Au Québec, les résultats d'une enquête font ressortir que 74 % des enseignant·e·s de mathématiques du secondaire (n=397) disent utiliser rarement ou jamais des outils technologiques pour enseigner les probabilités (Martin et Thibault, 2017). Des analyses statistiques inférentielles ont ensuite fait ressortir un sentiment de compétence pour l'enseignement des probabilités significativement plus faible que pour l'enseignement des autres domaines mathématiques (Martin et al., 2021). De plus, la formation en didactique des probabilités demande une préparation particulière puisque le raisonnement probabiliste est distinct d'autres raisonnements en mathématiques et est contre-intuitif (Batanero, 2014). Par contre, une telle préparation s'avère limitée pour les personnes enseignantes au secondaire (Gattuso et Vermette, 2013), autant en formation initiale qu'en formation continue (Martin et al., 2019). Elles pourraient ainsi bénéficier d'une telle formation, en particulier avec des outils technologiques qui permettent de réaliser rapidement un grand nombre d'essais (Stohl, 2005).

Le problème de recherche se résume au fait qu'on en sait peu sur ce qui doit être pris en compte dans la formation : ce sur quoi il faut insister, sur les approches à favoriser, sur les difficultés à anticiper, sur les tensions qui peuvent surgir entre le formateur, les formés et les savoirs, etc. La question de recherche pour cette thèse est donc formulée ainsi : Quels enjeux de formation émergent d'une formation continue à l'enseignement des probabilités du secondaire avec des outils technologiques ?

CADRE CONCEPTUEL

Pour situer les assises théoriques de cette recherche doctorale, je vais définir les concepts d'enjeu de formation et d'outils technologiques, puis prendre appui sur trois modèles complémentaires.

DÉFINITION DE CONCEPTS

Il est à noter que le terme « enjeu » est utilisé plus largement dans de nombreux textes scientifiques sans toutefois y être défini. La thèse porte donc un éclairage nouveau sur ce qu'est un enjeu de formation, à partir d'autres concepts scientifiques qui s'y apparentent : les concepts organisateurs (Pastré et al., 2006) qui amènent à comprendre l'activité d'enseignement de l'intérieur par son organisation interne, le « professional noticing » (Jacobs et al., 2010) qui considère l'expertise professionnelle par rapport à ce que remarque l'enseignant·e au sujet du raisonnement mathématique de ses élèves ainsi que la double approche didactique et ergonomique (Robert et Rogalski, 2002) qui permet de mieux comprendre pourquoi la personne enseignante agit comme elle agit, à partir des marges de manœuvre qui lui permettent d'optimiser son action selon les contraintes et les objectifs reliés à son enseignement. Ces trois concepts, considérés sous l'angle de la formation au lieu de l'enseignement, soulèvent des éléments qui ont permis de décrire et comprendre des enjeux de formation. Ainsi, dans cette recherche, le concept d'enjeu de formation est défini comme un élément important à considérer pour organiser la formation des enseignant·e·s (Pastré et al., 2006), à partir de ce que la personne formatrice remarque et identifie comme important pour le développement professionnel des formé·e·s (Jacobs et al., 2010), ce qui prend alors la forme de repères sous diverses dimensions qui tiennent compte des contraintes et marges de manœuvre de la formation (Robert et Rogalski, 2002).

Concernant le concept d'outil technologique, d'autres formulations sont aussi utilisées en lien avec le concept plus large de technologie, par exemple TIC, TICE ou NTIC. Puisqu'une distinction doit être apportée entre le contenant (appareil numérique) et le contenu (ses outils), cette thèse cible l'outil technologique, qui amène à se pencher sur des usages plus spécifiques pour faire des mathématiques. Plus précisément, un outil technologique est considéré comme un artefact (Rabardel, 1995) des technologies numériques éducatives qui permet d'accomplir des fonctions particulières (Gouvernement du Québec, 2019), plus précisément au sens où il sous-tend un potentiel pour devenir un instrument (Rabardel, 1995) pour l'apprentissage ou l'enseignement des probabilités du secondaire.

TROIS MODÈLES

Trois modèles sont choisis pour la complémentarité des angles d'entrée qu'ils offrent (voir Figure 1). Le modèle TPACK de Koehler et Mishra (2009) permet de se pencher sur les connaissances et les compétences des enseignants. Le modèle de Laborde (2001) propose une classification des niveaux d'utilisation d'outils technologiques pour l'enseignement. Quant au modèle de Tapan (2006), il concerne plutôt des dynamiques d'interactions en contexte de formation. Ces modèles ne constituent pas un cadre d'analyse, mais servent plutôt à dégager des angles complémentaires pouvant permettre d'éclairer des enjeux de formation.

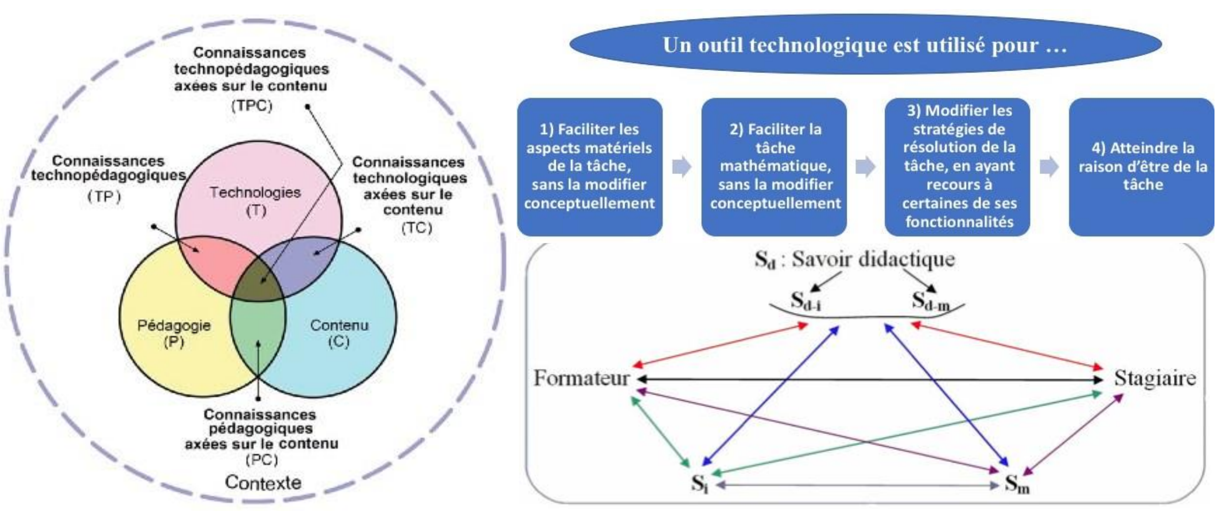


Figure 1. a) Modèle TPACK de Koehler et Mishra (2009), b) modèle de Laborde (2001), c) modèle de Tapan (2006).

Cette recherche doctorale vise donc à décrire et comprendre des enjeux de formation, orientés par les angles d'entrée des trois modèles ciblés, dans le contexte particulier de l'enseignement des probabilités du secondaire avec des outils technologiques.

MÉTHODOLOGIE

En guise de considérations méthodologiques de cette recherche, je vais expliquer brièvement quelques choix méthodologiques pour donner les lignes directrices de la façon dont cette recherche a été menée. Puis, je vais expliquer comment j'ai procédé pour analyser les données recueillies.

CHOIX MÉTHODOLOGIQUES

Cette recherche se situe dans le courant de la recherche qualitative et exploratoire. Aussi, puisque les préoccupations à l'origine de cette étude touchent à la fois la formation et la recherche, la méthode de recherche-formation s'avère pertinente. En effet, cette méthode amène à percevoir la recherche et la formation comme deux pôles « en interrelation, conservant chacune leur spécificité tout en s'enrichissant réciproquement » (Marcel, 1999, p. 91). De plus, une telle « recherche imbriquée dans l'action de formation conduit à une meilleure compréhension des situations de formation, elle permet de re-penser ces situations, de formuler de nouvelles pistes pour l'action du formateur, etc., venant donc alimenter en retour la formation elle-même dans un va-et-vient entre formation et recherche » (Bednarz et Proulx, 2010, p. 25). Dans une recherche-formation, la finalité est donc de documenter, de l'intérieur même d'une formation continue, des contenus professionnels par, pour et chez des enseignant·e·s (Bednarz et Proulx, 2010). La richesse du double rôle du chercheur-formateur m'a d'ailleurs permis d'y parvenir, en m'intégrant dans le processus même de la formation. Cinq enseignant·e·s et deux conseiller·ère·s pédagogiques ont été recruté·e·s pour la diversité et la richesse de leur expertise.

Pour favoriser le mouvement itératif de va-et-vient entre formation et recherche qui s'installe au fil des séances (Bednarz et Proulx, 2010), j'ai planifié cinq séances de trois heures espacées d'au moins un mois. Je passais alors, d'une séance à l'autre, d'une phase d'*action* en formation à une phase de *réflexion* en recherche. En d'autres mots, il y avait un cycle articulé entre la réflexion pour planifier une séance de formation, suivie de l'action de la séance, puis de la réflexion pour analyser cette séance. Je pouvais ensuite ajuster la planification pour la séance suivante. Cinq situations probabilistes ont été expérimentées pendant les séances pour amorcer les discussions avec les participant·e·s sur les plans mathématique et didactique.

ANALYSE DES DONNÉES

Lors de l'analyse préliminaire après chaque séance, j'ai fait ressortir quelques (pistes d') enjeux de formation, mais la plupart des enjeux de formation étaient implicites et ont donc été dégagés par la suite, lors des analyses plus détaillées dans une démarche d'analyse inductive. Souhaitant « être à l'écoute des données » pour bien analyser le phénomène dans toute sa complexité (Paillé et Mucchielli, 2016), j'ai procédé à analyse en reprenant les comptes rendus et les récits de formation des cinq séances pour faire ressortir ce qui semble le plus important parmi environ 21 heures d'observations filmées. Ce repérage a permis de cibler 16 moments (allant d'une durée de 4 à 40 minutes) jugés intéressants par la richesse des propos, qui ont ensuite été retranscrits et analysés. À la manière de Paillé et Mucchielli (2016), j'ai alors eu recours à une analyse thématique (en notant les thèmes retenus, puis en procédant à des fusions, subdivisions, regroupements et hiérarchisations des thèmes et sous-thèmes) ainsi qu'à une analyse à l'aide des catégories conceptualisantes (en adoptant une perspective théorisante, qui ne fait que très peu appel à des concepts ou catégories déjà nettement formulés).

RÉSULTATS

DIX-SEPT ENJEUX DE FORMATION

L'analyse thématique m'a permis de dégager trois thèmes qui émergent ainsi que onze sous-thèmes. À partir de ces sous-thèmes, l'analyse à l'aide des catégories conceptualisantes a fait ressortir dix-sept enjeux de formation. Plus particulièrement, au-delà du libellé de chaque enjeu de formation, ce sont les extraits analysés qui ont permis de décrire, mais surtout de comprendre des enjeux de formation en les situant dans le processus de la formation, par des exemples d'échanges qui font ressortir des bons coups, des difficultés et des besoins qui émergent chez les participant·e·s. Ces enjeux de formation concernent à la fois le formateur et les formé·e·s.

L'analyse des données a aussi permis de dégager des enjeux de formation qui prennent différentes formes. En premier lieu, un enjeu de formation a parfois émergé en lien avec un évènement important qui est survenu dans les séances et qui constituait un moment clé de la recherche-formation. Dans ce cas, le travail du formateur et des formé·e·s a été analysé dans ce sens. Il s'agit d'un évènement important, car il se répercutait de manière importante sur les séances dans les gestes et les paroles du formateur et des formé·e·s. En deuxième lieu, un enjeu de formation a parfois pris la forme d'un élément important à comprendre ou à faire, évoqué par le formateur ou par des formé·e·s lors des séances. Il s'agit alors de quelque chose qu'on souhaiterait que la personne formée connaisse ou fasse en lien avec son enseignement. Il est ainsi question d'intentions ou encore d'un idéal que l'on voudrait atteindre. En troisième et dernier lieu, un enjeu de formation était parfois lié à un problème qui survenait dans les séances, faisant ou non l'objet de discussions explicites. Cet élément important à considérer pour la formation était d'ailleurs parfois perçu comme un problème par des formé·e·s (parfois la majorité, ce qui semble saillant, mais parfois un·e seul·e, ce qui semble un cas unique intéressant à documenter) ou par le formateur. Dans ce cas, il peut s'agir de quelque chose de difficile à faire pour le formateur ou la personne formée, en raison d'un obstacle ou d'une contrainte à franchir.

APERÇU D'UN ENJEU DE FORMATION

L'ensemble des dix-sept enjeux de formation sont expliqués de façon détaillée dans la thèse (Thibault, 2021). Pour ce texte, je présente un aperçu de l'enjeu de formation ***Juger de la pertinence d'un outil technologique pour soutenir l'enseignement des probabilités*** (voir l'enjeu de formation #13 dans la Figure 2), issu du thème *Appropriation d'un outil technologique dans une situation probabiliste* et plus particulièrement du sous-thème *L'exploration d'outils technologiques en probabilités*.

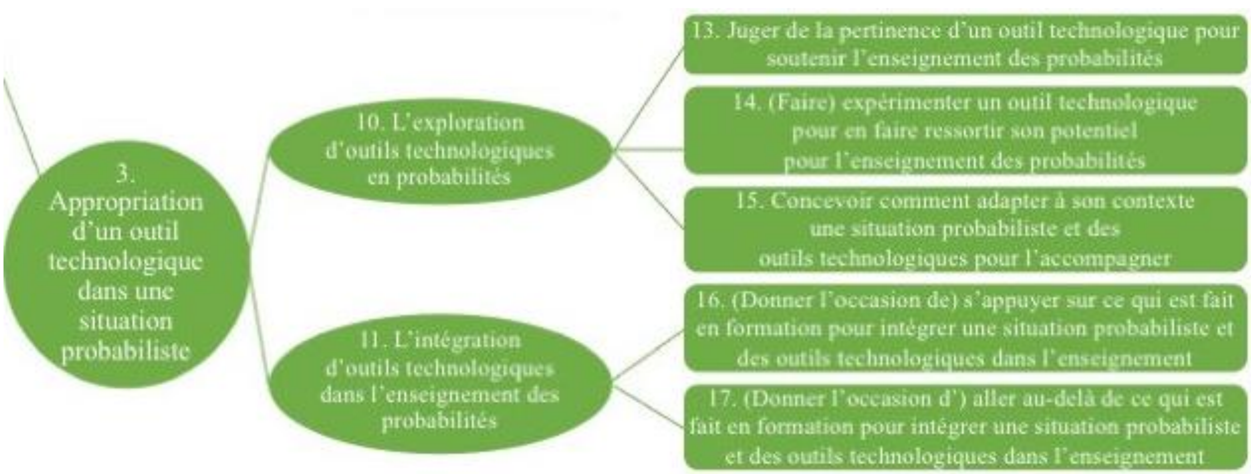


Figure 2. Exemple d'enjeux de formation découlant d'un thème et de sous-thèmes.

Cet enjeu de formation est lié à un enjeu d'enseignement où l'enseignant fait face à une multitude de facteurs qui l'influencent à juger de la pertinence d'un outil technologique, ce qui peut le mener ou non à l'utiliser pour enseigner les probabilités. D'une part, cet enjeu de formation a pris la forme de trois problèmes lors des séances de formation, alors que 1) un simulateur n'était pas jugé convivial pour toutes les personnes formées, 2) une capsule vidéo proposée par le formateur a été jugée non pertinente par l'ensemble des formé·e·s, puis 3) la programmation a soulevé des difficultés non négligeables qui pourraient influencer la pertinence qu'on peut lui accorder. D'autre part, l'enjeu de formation a pris la forme d'un élément important à comprendre, évoqué par le formateur, concernant la façon dont un simulateur génère des données pseudo-aléatoires.

Revenons d'abord sur le premier problème. En raison de leur engagement avec un simulateur, facilité par sa convivialité, les personnes participantes qui ont exploré un tel outil technologique dans les séances de formation décortiquaient les fonctionnalités qui l'accompagnent et parvenaient parfois à se l'approprier pour elles-mêmes si elles le considéraient pertinent. L'attitude positive des personnes participantes envers les simulateurs explorés a témoigné de la pertinence qu'elles leur accordent pour l'enseignement des probabilités. Le fait qu'elles se soient engagées dans l'exploration des simulateurs, qu'elles aient dévoilé leur intérêt, qu'elles en aient apprécié la convivialité et qu'elles aient pris le temps d'en décortiquer les fonctionnalités signale une pertinence pour elles. Toutefois, un problème est survenu lors de l'exploration d'un simulateur dans une séance de formation : une participante a éprouvé des difficultés d'utilisation de ce simulateur, ébranlant ainsi la pertinence à lui accorder. Il est à noter que, puisque c'est la seule participante qui a éprouvé de telles difficultés, cela illustre comment un outil technologique peut être convivial pour un·e participant·e mais pas pour un·e autre.

Le deuxième problème est survenu lorsque le formateur a proposé une capsule vidéo qui aborde le concept d'expérience pseudo-aléatoire, mais celle-ci a été jugée non pertinente par l'ensemble des formé·e·s. Le premier réflexe des personnes formées a été de signaler que la capsule vidéo ne serait pas une bonne ressource pour des élèves, en raison d'éléments considérés distrayants. On sent implicitement avec ceci un lien fort entre leur propre intérêt pour un outil et la pertinence qu'elles lui attribuent à l'utiliser en classe. Après avoir questionné les participant·e·s à savoir si cette capsule vidéo peut leur être utile pour faire évoluer leur propre pratique (sans y avoir recours avec les élèves), la distinction entre aléatoire, pseudo-aléatoire et déterminisme n'a pas été jugée importante à développer, pas plus chez eux·elles que chez les élèves.

Il s'agissait alors d'un enjeu de formation, car en plus du problème soulevé par les participant·e·s, les idées véhiculées dans cette capsule vidéo représentaient un élément important à comprendre pour le formateur. En effet, en tant que chercheur-formateur, je juge qu'il est pertinent de se questionner sur la façon dont les simulateurs génèrent des résultats, par la modélisation d'une expérience aléatoire, avant de leur attribuer notre confiance. En ce sens, il semble pertinent de connaître les fondements de ce qui est aléatoire, en comparant avec la façon dont les outils technologiques génèrent des résultats pseudo-aléatoires. Cette prise de conscience du fonctionnement d'un générateur de résultats pseudo-aléatoires pourrait permettre de renforcer le jugement critique envers les outils technologiques. En effet, faire confiance aveuglément aux résultats générés par un simulateur sans les questionner pourrait révéler une limite au niveau de la compréhension de l'aléatoire. Cette tension entre ce que les participant·e·s jugent pertinent et ce que le formateur juge pertinent soulève un élément qui semble important à considérer dans la formation.

Le troisième problème est survenu lors du recours à la programmation dans une séance de formation, ce qui a soulevé des difficultés non négligeables qui pourraient influencer la pertinence qu'on peut lui accorder. Plus particulièrement, des difficultés liées au langage de programmation Scratch ont apporté certaines résistances de la part des participant·e·s, ce qui a rendu l'appropriation plus ardue. Pour surmonter quelques difficultés et favoriser l'engagement des personnes formées lorsqu'elles explorent un outil technologique qui est jugé pertinent par le formateur, l'accompagnement de celui-ci est important. Du point de vue du formateur, il s'agit d'un enjeu, car il doit choisir un nombre limité d'outils technologiques à explorer, sachant qu'on ne peut pas tous les retenir. Le formateur peut ainsi tenter d'anticiper ce qui serait pertinent pour les formé·e·s en choisissant les outils qu'il fait explorer, mais il peut se tromper (comme ça a été le cas avec la capsule vidéo), ce qui peut amener des tensions. Le formateur peut aussi renforcer le jugement critique des formé·e·s lorsqu'ils·elles jugent de la pertinence d'un outil technologique.

DISCUSSION

Les résultats de cette recherche font ressortir un lien fort entre les notions d'enjeu *d'enseignement* et d'enjeu *de formation*. Donc, si l'analyse des propos des enseignant·e·s a fait ressortir des enjeux d'enseignement, ces enjeux peuvent être pris en compte de manière à être considérés dans la formation (initiale et continue). Autrement dit : si un élément est important à considérer pour organiser l'enseignement, il pourrait naturellement s'agir d'un élément important à aborder en formation. En ce sens, la formation peut permettre aux personnes participantes de travailler sur des enjeux d'enseignement, en provoquant des discussions pour qu'elles partagent leurs bons coups, difficultés et besoins liés à l'enseignement.

Cette thèse permet de mieux comprendre des enjeux de formation à l'enseignement des probabilités du secondaire avec des outils technologiques, mais il subsiste encore plusieurs zones d'ombre. Une piste intéressante serait de se pencher sur la programmation de simulateurs pour faire des probabilités, ce qui pourrait faire émerger des enjeux de formation spécifiques et adaptés au fait que la programmation semble prendre de l'importance dans l'enseignement. Il y a certainement encore beaucoup à explorer afin de mieux comprendre ce thème fascinant.

REMERCIEMENTS

Je tiens à remercier mon comité de recherche, Caroline Lajoie et Jean-François Maheux, pour leur soutien constant. Je remercie aussi les membres du jury, Simon Collin, Bernard Parzysz et Miranda Rioux, qui ont évalué la thèse avec attention. Il convient également de souligner le soutien financer du Conseil de Recherche en Sciences Humaines du Canada pour la bourse d'études doctorales. Finalement, je remercie les sept participant·e·s de cette recherche-formation, pour leur engagement remarquable qui m'a beaucoup inspiré pour la these !

RÉFÉRENCES

Batanero, C. (2014). Probability teaching and learning. In S. Lerman (Ed.), Encyclopedia of mathematics education (p. 491–496). Springer.

Bednarz, N., & Proulx, J. (2010). Développement professionnel des enseignants en mathématiques. *Éducation & formation*, *293*, 21–36.

Gattuso, L., & Vermette, S. (2013). L'enseignement de statistiques et probabilités au Canada et en Italie. *Recherches et perspectives - Statistique et Enseignement*, *4*(1), 107–129.

Gouvernement du Québec. (2019). *Cadre de référence de la compétence numérique*. Ministère de l'Éducation et de l'Enseignement supérieur.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Koehler, M. J., & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60–70.

Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, *6*(3), 283–317.

Marcel, J.-F. (1999). La démarche de recherche-formation : propositions pour un trait d'union entre la recherche et la formation dans le cadre de la formation continue des enseignants. *Recherche et formation, 32*, 89–100.

Martin, V., & Thibault, M. (2017). Enquête sur les pratiques déclarées d'enseignement des probabilités au primaire et au secondaire au Québec : esquisse d'un portrait statistique. Dans A. Adihou, J. Giroux, A, Savard, K. Mai Huy, & S. Mathieu-Soucy (Éds.), *Données, variabilité et tendances vers le futur : Actes du Colloque du Groupe de didactique des mathématiques du Québec 2017* (p. 179–195). GDM.

Martin, V., Thibault, M., & Theis, L. (2019). *Enseigner les premiers concepts de probabilités : un monde de possibilités !* Presses de l'Université du Québec.

Martin, V., Thibault, M., & Roy, N. (2021). Pratiques déclarées d'enseignement des probabilités : enquête auprès de personnes enseignantes du primaire et secondaire au Québec. *Canadian Journal of Science, Mathematics and Technology Education*, *21*(3), 596–624.

Paillé, P., & Mucchielli, A. (2016). *L'analyse qualitative en sciences humaines et sociales* (4e éd.). Armand Colin.

Pastré, P., Mayen, P., & Vergnaud, G. (2006). La didactique professionnelle. *Revue française de pédagogie, 154*, 145–198.

Rabardel, P. (1995). *Les hommes et les technologies : une approche cognitive des instruments contemporains*. Armand Colin.

Robert, A., & Rogalski, J. (2002). Le système complexe et cohérent des pratiques des enseignants de mathématiques : une double approche. *Canadian Journal of Science, Mathematics and Technology Education*, *2*(4), 505–528.

Stohl, H. (2005). Probability in teacher education and development. Dans G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (p. 345–366). Springer US.

Tapan, M. S. (2006). Différents types de savoirs mis en œuvre dans la formation initiale d'enseignants de mathématiques à l'intégration de technologies de géométrie dynamique [Thèse de doctorat]. Université Joseph Fourier. https://theses.hal.science/tel-00133569/document

Theis, L., & Savard, A. (2010). Recours à un simulateur pour enseigner les probabilités : quels défis et occasions pour des enseignants du début du secondaire ? Dans V. Freiman, A. Roy, et L. Theis (Éds.), *L'enseignement des mathématiques dans et à travers des contextes particuliers : quel support didactique privilégier ? Actes de colloque annuel du Groupe des Didacticiens des Mathématiques du Québec* (pp. 263–272). GDM.

Thibault, M. (2021). *Recherche-formation sur l'enseignement des probabilités du secondaire avec des outils technologiques : enjeux de formation* [Thèse de doctorat]. Université du Québec à Montréal. https://archipel.uqam.ca/14804/

UNDERSTANDING AFRICAN IMMIGRANT FAMILIES' SUPPORT FOR THEIR CHILDREN'S MATHEMATICS LEARNING IN CANADA

Kwesi Yaro
University of British Columbia, Vancouver

ABSTRACT

This study reports an aspect of a qualitative case study's findings which employed Yosso's (2005) community wealth drawn from Bourdieu's cultural capital theory and Afrocentric worldviews to investigate the experiences of African immigrant families in supporting their children's (10- to 15-year-olds) mathematics learning in Canada. The study revealed that African immigrant families leverage various forms of capital (often taken for granted by the dominant group) to support their children's mathematics learning: cultural, linguistic, technological/digital, and navigational capital. *These forms of capital challenge the narrowly conceived definition of capital as accumulated wealth, resources, and experiences of white middle-class families. The findings provide much-needed insights into ways to build culturally responsive mathematics education informed by African students' and families' funds of knowledge.*

BACKGROUND AND LITERATURE REVIEW

Global statistics indicate that approximately 273 million people have emigrated from countries with emerging economies, primarily from Africa, South and East Asia, Latin America, and Eastern Europe, to developed economies such as Europe and North America (McAuliffe & Khadria, 2019). Since the early 1990s, the number of landed immigrants in Canada has remained relatively high, with an average of approximately 235,000 new immigrants per year (Statistics Canada, 2019). The immigration trend highlights Canada's commitment to welcoming people from diverse cultures, with many African immigrant populations in Ontario, Quebec, Alberta, and British Columba (Statistics Canada, 2019). The growing trend in immigration and its consequent cultural diversity in Canada further places a responsibility on schoolteachers and curriculum developers to attend to issues of culture, multi/bilingualism in the school context. In addition, recent mathematics education reform documents such as the National Council of Teachers of Mathematics (2020) and the new British Columbia curriculum (2016) both acknowledged families' experiences or homes as a significant hub of knowledge that teachers could harness to support children's learning in various school subjects including mathematics (British Columbia Ministry of Education, 2016). The accumulated experiences and knowledge that families might draw on supporting their children's learning is what Bourdieu described as "cultural capital" (Bourdieu, 1990). While every family possesses some form of cultural capital, I argue that non-western families like African immigrants may have different experiences, beliefs, and means of supporting their children's learning in an out-of-school setting which may not necessarily parallel what educators or researchers may assume based on normative middle-class parents. While different, these out-of-school experiences likely contribute to the overall mathematics learning of the children from these non-western families.

In the context of this study, I investigated how African immigrant families draw on their cultural experiences in supporting their children's (10- to 15-year-olds) mathematics learning in Canada.

PARENTAL INVOLVEMENT PRACTICES AMONG NON-DOMINANT FAMILIES

Previous studies suggest that there are different ways parents and families from different cultures provide mathematics learning support for their children across grade levels. For instance, Anderson et al. (2016) worked with 32 parent-child dyads from multilingual, mixed socioeconomic backgrounds in early-years children's mathematics learning. The study focused on three South Asian dyads who played a mathematical board game for the first time. The researchers concluded that these three children paid close attention to their parents' nonverbal interactions as parents "enacted executive autonomy" by demonstrating how to play the game and what they needed to do. The practices of the parent participants appeared to be a culturally appropriate way of learning among these South Asian families and are likely shaped by parents' experiences (Anderson et al., 2016) based on cultural norms that appear to value respect through actively listening to knowledgeable adults.

Prior studies have also focused on out-of-school involvement practices with attention to elementary, upper, and middle school years. For instance, Civil (2007) worked with elementary (Grades 1-5) teachers to conduct ethnographic household visits to learn about daily lived experiences of their students' household chores and play in which families engage children within the home domain. Through such visits and interactions, teachers realized that many of these families have deep knowledge about construction, repairs, carpentry, household management, folk medicine, and farming, all of which involve the application of mathematical knowledge and skills. These everyday contextualized mathematics practices were consistent with later studies (e.g., Goldman & Booker, 2009; Williams et al., 2020). In fact, González (1996) earlier argued for "re-conceptualizing households, not as the source of barriers to educational attainment, but as repositories of resources that could be used strategically for the betterment of children's learning" (p. 3). These studies argue that the need for teachers to incorporate students' and families' embodied culture and home experiences into mathematics lessons can enable students to succeed in their mathematics learning (Jorgensen, 2018). Despite the affordance of using cultural and home activities in teaching mathematics, it is still unclear whether African immigrant families bring with them similar cultural practices (or otherwise) to the host country or not. Are these cultural practices adapted or discontinued, and what new practices may arise when situated in the Canadian context?

THEORETICAL PERSPECTIVES

This study draws on Bourdieu's (1990) theoretical concepts of capital and Afrocentric worldviews (Asante, 1990) to investigate how African immigrants and refugee parents support their children's mathematics learning in Canada. Bourdieu (1990) defined capital as the valued resources underlying actors' interactions and relationships with society. Bourdieu's concept of capital helps us to interrogate issues of power in that some knowledge 'counts' more highly than others, and some dialects are 'better' than others. Of course, what counts and what is better is determined by people who have more power in society.

In extending Bourdieu's work on cultural capital, Yosso (2005, 2017) proposed what he referred to as "community cultural wealth." She argued that people of colour possess cultural experiences that are not adequately acknowledged by Bourdieu's cultural capital theory; aspirational, navigational, social, linguistic, familial, and resistant capital.

Although different forms of capital proposed by Yosso (2005, 2017) are relevant in offering an interpretation of the experiences of people of colour and marginalized communities, I argue that such interpretations need to be grounded within African-centred worldviews. Therefore, to better situate this study, I also employed Afrocentric worldviews (Asante, 1990) to interpret the data to account for the unique lived experiences of Africans and how they support their children's mathematics learning in Canada. Afrocentric worldviews deal with issues of reality and creation in consonance with the African people's culture, values, society, and history (Asante, 1990). Therefore, to better understand the practices and experiences of African immigrant families, it is important to situate this study and its interpretations within African worldviews and values (e.g., reciprocity, respect, relationship building, the value of home language as a cultural asset).

METHODOLOGY AND DATA ANALYSIS

To better understand how African immigrants and refugee parents from Sub-Saharan Africa support their children's (10- to 15-year-olds) mathematics learning, a case study design (Yin, 2003) employs individual semi-structured interviews and home visits observation over 12 weeks. Data were collected from six African immigrant and refugee parents and their children (10- to 15-year-olds) from Sub-Saharan Africa living in the Greater Vancouver area, Canada. Parents' and children's separate individual interview data sets and observational field notes from eight home visits over 12 weeks were examined for common patterns and possible nuances. The interpretative nature of qualitative case studies legitimizes the centrality of African epistemologies and cosmologies as a valid frame of reference in the data collection and analysis processes (Reviere, 2001).

RESULTS AND DISCUSSION

The analysis of the data corpus suggests at least four forms of capital that these African immigrant families use in supporting their children's mathematics learning; cultural capital (material resources such as textbooks), technological capital, linguistic capital and navigational capital.

Cultural capital: Mathematics education resources are one form of capital families in the study draw on to support their children's mathematics learning in a Canadian context. For instance, five African immigrant families shared various cultural resources they brought with them to Canada, including textbooks, worksheets, and other materials that they believe are relevant to their children's mathematics learning in Canada. These educational resources reflect the cultural model of mathematics education parents themselves experienced back in their home countries. As one parent noted:

> *You can see we bought a lot of books here, mathematic books, and we're trying different curricula materials as well.* (Joseph—Jones' father)

As observed during my home visits, families (e.g., Joseph, Mashy and Kata's family) consulted these textbooks for either a mathematical formula or for alternative algorithms to explain a particular mathematics concept to their children. I conceptualize the educational resources from Africa that parents alluded to as cultural capital they draw upon to support their children's mathematics learning in Canada, even though their resources might not necessarily have the same meaning or be given the same value in the educational context of the dominant culture (Canada). Afrocentricity legitimatizes and reinforces the centrality of cultural resources from Africa as valid and sound (Asante, 1990), as opposed to being deficit.

Linguistic capital refers to accumulated communication experiences in more than one language/style (Yosso, 2005). In this study, linguistic capital broadly defines families' ability to draw on their bilingual and multilingual repertoire (First language: L1 and Second Language: L2)—verbal, visual, print, or digital—supporting their children's mathematics learning (Barwell, 2018). From an Afrocentric lens, home language (L1) is intrinsically connected to culture, history, ability to relate to and portray one's identity (Wa Thiong'o, 1992). Parents in the study continue using L1 with their children after several years of living in Canada points to the significance they attach to their home language, both for cultural relevance and as a medium for providing mathematics support to their children. As one parent noted:

> *I studied mathematics in my language [Arabic], everything in my language…Because I have no idea about the English terminology they use in mathematics. I have to translate in my head to English from Arabic and back to English…Sometimes through "YouTube videos" to understand the math in English.* (Mashy—Hakeem's Mother)

Parents saw their local language as an asset in the host country, where English is the dominant language. A 'deficit discourse' could often characterize home language use in children's mathematics and learning in general, especially in the context where the language of instruction is different from the child's mother tongue (Barwell, 2012, 2018). However, this finding points to families' bilingualism as linguistic capital (or resource) in fostering their children's mathematics learning in Canada.

Technological Capital: Technology use is another form of capital that families deploy to support their children's mathematics. I broadly define technological capital as the ability to use digital tools (navigate through websites and applications), decipher and critique those digital tools (Teichert & Anderson, 2014). Specific to their children's mathematics learning, parents in the study used technological/digital resources for 1) internalizing mathematics concepts and relaying mathematics support to their children (e.g., YouTube videos); 2) monitoring their children's mathematics progress (e.g., Khan Academy, Mathletics); 3) as a shared medium for communicating between their children and their children's teachers (e.g., FreshGrade).

While parents and their children acknowledged the affordance of technology and digital resources in supporting their children's math learning, they were also critical in pointing out some constraints. For instance, one parent her frustration:

> *I do not like the Mathletics because it is just not very well designed. It just asks the answer and doesn't mention, for example, "round to the nearest tens or 100." It [is] exhausting, and it makes me angry. it says "Wrong, wrong," but we are right! I tell her [daughter], "Forget about mathletics, do it on the paper."* (Malee—Mother)

African immigrant families' use of these technological/digital resources is indicative of capital because they had enough content knowledge and technological skills to understand, critique, or identify that some web platforms were, indeed, problematic. Admittedly, I have no data to ascertain the nature or validity of mathematics content in the YouTube video resources or websites parents were using, how they were internalising those resources, and how they were using them with their children in the home contexts. However, I am convinced that since parents themselves expressed their continued usage of such resources, it means they appear relevant and valuable to them.

Navigational Capital: African immigrant families demonstrated their ability to navigate through an education system not designed with the cultures and experiences of communities of colour in mind (Yosso, 2005, 2017). For instance, African immigrant families grappled with what could pass as appropriate involvement practices in the Canadian context. As one parent noted:

> *I wish I can observe what goes on in their classroom, but I don't know if this is allowed …I don't want to bring conflict between myself and the teacher.* (Kata—Amy's mother)

Similarly, parents continue to negotiate what they believe counts as legitimate mathematics and what does not count in the Canadian context. As one parent noted:

> *Sometimes she [daughter] will tell me, "Mom, we don't do real maths."* (Malee—Zahara's mother)
>
> *What is real math?* (Kwesi—Interviewer)
>
> *Yes, we are talking about polynomials, equations, addition. These are the real math, NOT learning about ancient number.* (Malee—Zahara's mother)

Kata's excerpt indicates the challenge in navigating the school's expectations as a parent. For instance, she seems unsure how schoolteachers will perceive her visits to her daughter's mathematics class. However, she would like to learn more about the new teaching and learning environment her child is experiencing in Canada. I could sense that Kata's perspective, which was also shared by four other parents, on the role of the school and the nature of relationship and interaction between the teachers and school, appears to be partly informed by Africa's core value of respect for teachers and school authorities. From an African perspective, teachers are mostly seen as parents of the children when they are away from home, and there is trust that teachers will offer the best care and teachings; for that matter, THE parents' role is only to provide the necessities such as food, shelter, books and so on, needed for the day-to-day child-rearing. However, in the North American context, teachers may interpret this non-interference as parents being disinterested in their children's mathematics learning (Civil et al., 2005; de Abreu & Cline, 2005; Yaro et al., 2020). Despite the contextual differences and the accompanying school expectations, immigrant families demonstrated their ability to support their children to meet the mathematics content expectations of the two curricula worlds (their home and host country), as indicated in Malee's excerpt. For example, parents were able to support their children to master mathematics content areas they deem useful or valorize (de Abreu et al., 2002), while ensuring children excel in mathematics content taught in the Canadian school system.

CONCLUSIONS AND IMPLICATIONS

The findings of this study indicate that African immigrant families leverage different forms of capital in support of their children's mathematics learning. In Figure 1, I have revisited Yosso's (2005) community cultural wealth (cultural capital, linguistic capital, navigational capital) to account for one more form of capital (technology), which was originally not part of Yosso's (2005) work.

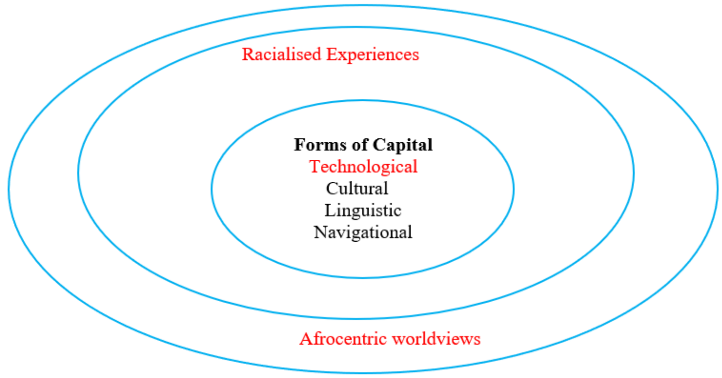


Figure 1. An extended model of different forms of capital (community cultural wealth).

I end this paper by drawing the following conclusions: first, immigrant families inevitably encounter obstacles due to being in a new cultural context. I argue that their ability to continuously manage dilemmas and other home-school discontinuities in quest of mathematics success for their children is an asset or capital families possess. Second, the forms of capital as captured in Figure 1 are informed by families' unique cultural experiences as African immigrants living in a racialized environment, and how they view the world (from an African-centered perspective). Following Yosso (2005), I argue that the various forms of capital are not mutually exclusive and that they are deployed concomitantly. For instance, families' bilingual identity served as a capital for reading/communicating and internalising mathematics educational resources from home and host countries and discussing their mathematics ideas.

African immigrant participants' demonstration and use of various forms of capital (often taken for granted) seem to challenge the narrowly conceived definition of capital as accumulated wealth, resources, and experiences of the dominant culture (white middle class). This study, therefore, reiterates the call for culturally sustaining education practices that acknowledges the cultural repertoires of students' communities/families as well as multifaceted approaches to parental involvement among different ethnicities and cultures.

ACKNOWLEDGEMENT/DISCLOSURE

This paper was also presented at the 2022 AERA conference.

REFERENCES

Asante, M. K. (1990). *Kemet, Afrocentricity, and knowledge*. Africa Research and Publications

Anderson, A., Kim, J. E., & McLellan, S. (2016). Culture and mathematical learning: A case study of South Asian parents and children playing a board game. In A. Anderson, J. Anderson, J. Hare, & M. McTavish (Eds.), *Language, learning, and culture in early childhood. Home, school, and community contexts* (pp.142–167). Routledge.

Barwell, R. (2012). Discursive demands and equity in second language mathematics classrooms. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education* (pp. 147–163). Springer, Dordrecht.

Barwell, R. (2018). From language as a resource to sources of meaning in multilingual mathematics classrooms. *The Journal of Mathematical Behavior*, *50*, 155–168. https://doi-org.ezproxy.library.ubc.ca/10.1016/j.jmathb.2018.02.007

Bourdieu, P. (1990). *The logic of practice*. Stanford University Press.

British Columbia Curriculum (2016, November). *Curriculum overview*. https://www.curriculum.gov.bc.ca/curriculum/overview

Civil, M. (2007). Building on community knowledge: An avenue to equity in mathematics education. In N. Nassir & P. Cobb (Eds.), *Improving access to mathematics: Diversity and equity in the classroom* (pp. 105–117). Teachers College Press.

Civil, M. (2012). Mathematics teaching and learning of immigrant students. In O. Skovsmose & B. Greer (Eds.), *Opening the cage: Critique and politics of mathematics education* (pp. 127–142). Rotterdam: Sense.

Civil, M., Planas, N., & Quintos, B. (2005). Immigrant parents' perspectives on their children's mathematics education. *ZDM*, *37*(2), 81–89.

de Abreu, G., Bishop, A., & Presmeg, N. (2002). Mathematics learners in transition. In G. Abreu & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 7–21). Springer, Dordrecht

de Abreu, G., & Cline, T. (2005). Parents' representations of their children's mathematics learning in multiethnic primary schools. *British Educational Research Journal*, *31*(6), 697–722.

Goldman, S., & Booker, A. (2009). Making math a definition of the situation: Families as sites for mathematical practices. *Anthropology & Education Quarterly*, *40*(4), 369–387.

González, N. (1996). Applied anthropology as educational innovation: Learning from households. In L. Moll, N. González, & M. Civil (Eds.), *Funds of knowledge for teaching. Final Report* (pp. 12–26). Tucson, AZ: The University of Arizona.

Gonzalez, N., Andrade, R., Civil, M., & Moll. L. (2005). Funds of distributed knowledge. In N. Gonzalez, L. Moll, & C. Amanti, (Eds.), *Funds of knowledge: Theorizing practices in households, communities, and classrooms* (pp. 257–274). London: Routledge.

Jorgensen (Zevenbergen), R. (2016). Early years swimming: A way of supporting school transitions? *Early Child Development and Care*, *186*(9), 1429–1437. https://doi.org/10.1080/03004430.2015.1096785

Jorgensen, R. (2018). Language resources to scaffold mathematical learning for remote Indigenous learners. In R. Hunter (Ed.), M*athematical discourse that breaks barriers and creates space for marginalized learners* (pp. 235–255). Brill Sense. https://doi.org/10.1163/9789004378735_012

McAuliffe, M., & Khadria, B. (2019). Report overview: Providing perspective on migration and mobility in increasingly uncertain times. *World Migration Report*, *2020*(1), 1–14. https://doi.org/10.18356/9c7a02cd-en

National Council of Teachers of Mathematics (NTCM). (2020). *Executive summary: Principles and standards for school mathematics*. NCTM

Reviere, R. (2001). Toward an Afrocentric research methodology. *Journal of Black Studies*, *31*(6), 709–728.

Statistics Canada. (2019, April 18). *Focus on geography series, 2016 census*. Statistics Canada. https://www12.statcan.gc.ca/census-recensement/2016/as-sa/fogs-spg/Facts-can-eng.cfm?Lang=Eng&GK=CAN&GC=01&TOPIC=7

Wa Thiong'o, N. (1992). *Decolonising the mind: The politics of language in African literature*. East African Publishers.

Williams, J. J., Tunks, J., Gonzalez-Carriedo, R., Faulkenberry, E., & Middlemiss, W. (2020). Supporting mathematics understanding through funds of knowledge. *Urban Education*, *55*(3), 476–502.

Yaro, K., Amoah, E., & Wagner, D. (2020). Situated perspectives on creating mathematics tasks for peace and sustainability. *Canadian Journal of Science, Mathematics and Technology Education*, *20*(2), 1–12.

Yin, R. K. (2009). *Case study research: Design and methods* (4th ed.). Sage.

Yosso, T. J. (2005). Whose culture has capital? A critical race theory discussion of community cultural wealth. *Race ethnicity and education*, *8*(1), 69–91. https://doi.org/10.1080/1361332052000341006

Yosso, T. J. (2017). Whose culture has capital?: A critical race theory discussion of community cultural wealth. In A. D. Dixson, C. K. Rousseau Anderson, & J. K. Donnor (Eds.), *Critical race theory in education* (1st ed., pp. 113–136). Routledge.

Appendices

Annexes

Appendix A / Annexe A

WORKING GROUPS AT EACH ANNUAL MEETING / GROUPES DE TRAVAIL DES RENCONTRES ANNUELLES

1977 *Queen's University, Kingston, Ontario*

- Teacher education programmes
- Undergraduate mathematics programmes and prospective teachers
- Research and mathematics education
- Learning and teaching mathematics

1978 *Queen's University, Kingston, Ontario*

- Mathematics courses for prospective elementary teachers
- Mathematization
- Research in mathematics education

1979 *Queen's University, Kingston, Ontario*

- Ratio and proportion: a study of a mathematical concept
- Minicalculators in the mathematics classroom
- Is there a mathematical method?
- Topics suitable for mathematics courses for elementary teachers

1980 *Université Laval, Québec, Québec*

- The teaching of calculus and analysis
- Applications of mathematics for high school students
- Geometry in the elementary and junior high school curriculum
- The diagnosis and remediation of common mathematical errors

1981 *University of Alberta, Edmonton, Alberta*

- Research and the classroom
- Computer education for teachers
- Issues in the teaching of calculus
- Revitalising mathematics in teacher education courses

1982 *Queen's University, Kingston, Ontario*

- The influence of computer science on undergraduate mathematics education
- Applications of research in mathematics education to teacher training programmes
- Problem solving in the curriculum

1983 *University of British Columbia, Vancouver, British Columbia*

- Developing statistical thinking
- Training in diagnosis and remediation of teachers
- Mathematics and language
- The influence of computer science on the mathematics curriculum

1984 *University of Waterloo, Waterloo, Ontario*

- Logo and the mathematics curriculum
- The impact of research and technology on school algebra
- Epistemology and mathematics
- Visual thinking in mathematics

1985 *Université Laval, Québec, Québec*

- Lessons from research about students' errors
- Logo activities for the high school
- Impact of symbolic manipulation software on the teaching of calculus

1986 *Memorial University of Newfoundland, St. John's, Newfoundland*

- The role of feelings in mathematics
- The problem of rigour in mathematics teaching
- Microcomputers in teacher education
- The role of microcomputers in developing statistical thinking

1987 *Queen's University, Kingston, Ontario*

- Methods courses for secondary teacher education
- The problem of formal reasoning in undergraduate programmes
- Small group work in the mathematics classroom

1988 *University of Manitoba, Winnipeg, Manitoba*

- Teacher education: what could it be?
- Natural learning and mathematics
- Using software for geometrical investigations
- A study of the remedial teaching of mathematics

1989 *Brock University, St. Catharines, Ontario*

- Using computers to investigate work with teachers
- Computers in the undergraduate mathematics curriculum
- Natural language and mathematical language
- Research strategies for pupils' conceptions in mathematics

1990 *Simon Fraser University, Vancouver, British Columbia*

· Reading and writing in the mathematics classroom
· The NCTM "Standards" and Canadian reality
· Explanatory models of children's mathematics
· Chaos and fractal geometry for high school students

1991 *University of New Brunswick, Fredericton, New Brunswick*

· Fractal geometry in the curriculum
· Socio-cultural aspects of mathematics
· Technology and understanding mathematics
· Constructivism: implications for teacher education in mathematics

1992 *ICME–7, Université Laval, Québec, Québec*

1993 *York University, Toronto, Ontario*

· Research in undergraduate teaching and learning of mathematics
· New ideas in assessment
· Computers in the classroom: mathematical and social implications
· Gender and mathematics
· Training pre-service teachers for creating mathematical communities in the classroom

1994 *University of Regina, Regina, Saskatchewan*

· Theories of mathematics education
· Pre-service mathematics teachers as purposeful learners: issues of enculturation
· Popularizing mathematics

1995 *University of Western Ontario, London, Ontario*

· Autonomy and authority in the design and conduct of learning activity
· Expanding the conversation: trying to talk about what our theories don't talk about
· Factors affecting the transition from high school to university mathematics
· Geometric proofs and knowledge without axioms

1996 *Mount Saint Vincent University, Halifax, Nova Scotia*

· Teacher education: challenges, opportunities and innovations
· Formation à l'enseignement des mathématiques au secondaire: nouvelles perspectives et défis
· What is dynamic algebra?
· The role of proof in post-secondary education

1997 *Lakehead University, Thunder Bay, Ontario*

· Awareness and expression of generality in teaching mathematics
· Communicating mathematics
· The crisis in school mathematics content

1998 *University of British Columbia, Vancouver, British Columbia*

· Assessing mathematical thinking
· From theory to observational data (and back again)
· Bringing Ethnomathematics into the classroom in a meaningful way
· Mathematical software for the undergraduate curriculum

1999 *Brock University, St. Catharines, Ontario*

· Information technology and mathematics education: What's out there and how can we use it?
· Applied mathematics in the secondary school curriculum
· Elementary mathematics
· Teaching practices and teacher education

2000 *Université du Québec à Montréal, Montréal, Québec*

· Des cours de mathématiques pour les futurs enseignants et enseignantes du primaire/Mathematics courses for prospective elementary teachers
· Crafting an algebraic mind: Intersections from history and the contemporary mathematics classroom
· Mathematics education et didactique des mathématiques : y a-t-il une raison pour vivre des vies séparées?/Mathematics education et didactique des mathématiques: Is there a reason for living separate lives?
· Teachers, technologies, and productive pedagogy

2001 *University of Alberta, Edmonton, Alberta*

· Considering how linear algebra is taught and learned
· Children's proving
· Inservice mathematics teacher education
· Where is the mathematics?

2002 *Queen's University, Kingston, Ontario*

· Mathematics and the arts
· Philosophy for children on mathematics
· The arithmetic/algebra interface: Implications for primary and secondary mathematics / Articulation arithmétique/algèbre: Implications pour l'enseignement des mathématiques au primaire et au secondaire
· Mathematics, the written and the drawn
· Des cours de mathématiques pour les futurs (et actuels) maîtres au secondaire / Types and characteristics desired of courses in mathematics programs for future (and in-service) teachers

2003 *Acadia University, Wolfville, Nova Scotia*

· L'histoire des mathématiques en tant que levier pédagogique au primaire et au secondaire / The history of mathematics as a pedagogic tool in Grades K–12
· Teacher research: An empowering practice?
· Images of undergraduate mathematics
· A mathematics curriculum manifesto

2004 *Université Laval, Québec, Québec*

- Learner generated examples as space for mathematical learning
- Transition to university mathematics
- Integrating applications and modeling in secondary and post secondary mathematics
- Elementary teacher education – Defining the crucial experiences
- A critical look at the language and practice of mathematics education technology

2005 *University of Ottawa, Ottawa, Ontario*

- Mathematics, education, society, and peace
- Learning mathematics in the early years (pre-K – 3)
- Discrete mathematics in secondary school curriculum
- Socio-cultural dimensions of mathematics learning

2006 *University of Calgary, Calgary, Alberta*

- Secondary mathematics teacher development
- Developing links between statistical and probabilistic thinking in school mathematics education
- Developing trust and respect when working with teachers of mathematics
- The body, the sense, and mathematics learning

2007 *University of New Brunswick, New Brunswick*

- Outreach in mathematics – Activities, engagement, & reflection
- Geometry, space, and technology: challenges for teachers and students
- The design and implementation of learning situations
- The multifaceted role of feedback in the teaching and learning of mathematics

2008 *Université de Sherbrooke, Sherbrooke, Québec*

- Mathematical reasoning of young children
- Mathematics-in-and-for-teaching (MifT): the case of algebra
- Mathematics and human alienation
- Communication and mathematical technology use throughout the post-secondary curriculum / Utilisation de technologies dans l'enseignement mathématique postsecondaire
- Cultures of generality and their associated pedagogies

2009 *York University, Toronto, Ontario*

- Mathematically gifted students / Les élèves doués et talentueux en mathématiques
- Mathematics and the life sciences
- Les méthodologies de recherches actuelles et émergentes en didactique des mathématiques / Contemporary and emergent research methodologies in mathematics education
- Reframing learning (mathematics) as collective action
- Étude des pratiques d'enseignement
- Mathematics as social (in)justice / Mathématiques citoyennes face à l'(in)justice sociale

2010 *Simon Fraser University, Burnaby, British Columbia*

- Teaching mathematics to special needs students: Who is at-risk?
- Attending to data analysis and visualizing data
- Recruitment, attrition, and retention in post-secondary mathematics
  Can we be thankful for mathematics? Mathematical thinking and aboriginal peoples
- Beauty in applied mathematics
- Noticing and engaging the mathematicians in our classrooms

2011 *Memorial University of Newfoundland, St. John's, Newfoundland*

- Mathematics teaching and climate change
- Meaningful procedural knowledge in mathematics learning
- Emergent methods for mathematics education research: Using data to develop theory / Méthodes émergentes pour les recherches en didactique des mathématiques: partir des données pour développer des théories
- Using simulation to develop students' mathematical competencies – Post secondary and teacher education
- Making art, doing mathematics / Créer de l'art; faire des maths
- Selecting tasks for future teachers in mathematics education

2012 *Université Laval, Québec City, Québec*

- Numeracy: Goals, affordances, and challenges
- Diversities in mathematics and their relation to equity
- Technology and mathematics teachers (K-16) / La technologie et l'enseignant mathématique (K-16)
- La preuve en mathématiques et en classe / Proof in mathematics and in schools
- The role of text/books in the mathematics classroom / Le rôle des manuels scolaires dans la classe de mathématiques
- Preparing teachers for the development of algebraic thinking at elementary and secondary levels / Préparer les enseignants au développement de la pensée algébrique au primaire et au secondaire

2013 *Brock University, St. Catharines, Ontario*

- MOOCs and online mathematics teaching and learning
- Exploring creativity: From the mathematics classroom to the mathematicians' mind / Explorer la créativité : de la classe de mathématiques à l'esprit des mathématiciens
- Mathematics of Planet Earth 2013: Education and communication / Mathématiques de la planète Terre 2013 : formation et communication (K-16)
- What does it mean to understand multiplicative ideas and processes? Designing strategies for teaching and learning
- Mathematics curriculum re-conceptualisation

2014 *University of Alberta, Edmonton, Alberta*

- Mathematical habits of mind / Modes de pensée mathématiques
- Formative assessment in mathematics: Developing understandings, sharing practice, and confronting dilemmas
- Texter mathematique / Texting mathematics
- Complex dynamical systems
- Role-playing and script-writing in mathematics education practice and research

2015 *Université de Moncton, Moncton, New Brunswick*

- Task design and problem posing
- Indigenous ways of knowing in mathematics
- Theoretical frameworks in mathematics education research / Les cadres théoriques dans la recherche en didactictique des mathématiques
- Early years teaching, learning and research: Tensions in adult-child interactions around mathematics
- Innovations in tertiary mathematics teaching, learning and research / Innovations au post-secondaire pour l'enseignement, l'apprentissage et la recherche

2016 Queen's University, Kingston, Ontario

- Computational thinking and mathematics curriculum
- Mathematics in teacher education: What, how… and why / Les mathématiques dans la formation des enseignants : quoi, comment… et pourquoi
- Problem solving: Definition, role, and pedagogy / Résolution de problèmes : définition, rôle, et pédagogie associée
- Mathematics education and social justice: Learning to meet the others in the classroom / Éducation mathématique et justice sociale : apprendre à rencontrer les autres dans las classe
- Role of spatial reasoning in mathematics
- The public discourse about mathematics and mathematics education / Le discours public sur les mathématiques et l'enseignement des mathématiques

2017 McGill University, Montréal, Québec

- Teaching first year mathematics courses in transition from secondary to tertiary
- L'anxiété mathématique chez les futurs enseignants du primaire : à la recherche de nouvelles réponses à des enjeux qui perdurent / Elementary preservice teachers and mathematics anxiety: Searching for new responses to enduring issues
- Social media and mathematics education
- Quantitative reasoning in the early years / Le raisonnement quantitatif dans les premières années du parcours scolaire
- Social, cultural, historical and philosophical perspectives on tools for mathematics
- Compréhension approfondie des mathématiques scolaires / Deep understanding of school mathematics

2018 Quest University, Squamish, British Columbia

- The 21st century secondary school mathematics classroom
- Confronting colonialism / Affronter le Colonialisme
- Playing with mathematics / Learning mathematics through play
- Robotics in mathematics education
- Relation, ritual and romance: Rethinking interest in mathematics learning

2019 St. Francis Xavier University, Antigonish, Nova Scotia

- Problem-based learning in postsecondary mathematics / L'apprentissage par problems en mathématiques au niveau postsecondaire
- Teaching primary school mathematics…what mathematics? What avenues for teacher training? / Enseigner les premiers concepts mathématiques à l'école primaire…quelles mathématiques? Quelles avenues pour la formation à l'enseighnement?
- Humanizing data / Humaniser les données
- Research and practice: Learning through collaboration / Recherche et pratique : apprendre en collaborant
- Interdisciplinarity with mathematics: Middle school and beyond
  Capturing chaos? Ways into the mathematics classroom / Capturer le chaos ? Entrées sur la classe de mathématiques

2021 Online (Virtual)

- Learning Theories / Théories (de l') apprenant
- Pour ou contre les tests : est-ce la bonne question ? / To test or not to test: Is this the question?
- The rewards and challenges of video in the field of mathematics education: Looking back in order to prepare for the future / Les apports et défis de la vidéo pour (la formation à) l'enseignement-apprentissage des mathématiques : regard du passé pour préparer le futur
- How can we be creative with large classes? / Comment composer avec les grands groupes ?
- Returning to our roots: Exploring collaborative possibilities for research and teaching in mathematics and mathematics education

2022 Online (Virtual)

- Contenu et pratiques pour la formation initiale et continue des enseignants : Un regard plus approfondi sur les potentiels, les défis, les pièges et les perspectives / Content and practices for pre-service and in-service teacher education: A deeper look into the potentials, challenges, pitfalls, and prospects
- Critical mathematics working group: Changing mathematics to fit our whole selves / Les mathématiques critiques : On change les mathématiques pour s'adapter à nous-mêmes
- Weaving identity in mathematics education: Fads, fictions, fibers, and freedoms / Le tissage d'une identité dans l'enseignement des mathématiques : les modes, les histoires, les ficelles et les libertés
- Assessment in undergraduate mathematics / Évaluation en mathématiques au postsecondaire
- Matériel de manipulation dans l'apprentissage et l'enseignement des mathématiques au primaire / Manipulatives in elementary mathematics teaching and learning
- Facilitating learning mathematics online / Favoriser l'apprentissage des mathématiques en ligne

Appendix B / Annexe B

PLENARY LECTURES AT EACH ANNUAL MEETING / CONFÉRENCES PLÉNIÈRES DES RENCONTRES ANNUELLES

1977	A.J. COLEMAN	The objectives of mathematics education
	C. GAULIN	Innovations in teacher education programmes
	T.E. KIEREN	The state of research in mathematics education
1978	G.R. RISING	The mathematician's contribution to curriculum development
	A.I. WEINZWEIG	The mathematician's contribution to pedagogy
1979	J. AGASSI	The Lakatosian revolution
	J.A. EASLEY	Formal and informal research methods and the cultural status of school mathematics
1980	C. GATTEGNO	Reflections on forty years of thinking about the teaching of mathematics
	D. HAWKINS	Understanding understanding mathematics
1981	K. IVERSON	Mathematics and computers
	J. KILPATRICK	The reasonable effectiveness of research in mathematics education
1982	P.J. DAVIS	Towards a philosophy of computation
	G. VERGNAUD	Cognitive and developmental psychology and research in mathematics education
1983	S.I. BROWN	The nature of problem generation and the mathematics curriculum
	P.J. HILTON	The nature of mathematics today and implications for mathematics teaching
1984	A.J. BISHOP	The social construction of meaning: A significant development for mathematics education?
	L. HENKIN	Linguistic aspects of mathematics and mathematics instruction
1985	H. BAUERSFELD	Contributions to a fundamental theory of mathematics learning and teaching
	H.O. POLLAK	On the relation between the applications of mathematics and the teaching of mathematics
1986	R. FINNEY	Professional applications of undergraduate mathematics
	A.H. SCHOENFELD	Confessions of an accidental theorist
1987	P. NESHER	Formulating instructional theory: the role of students' misconceptions
	H.S. WILF	The calculator with a college education

1988	C. KEITEL	Mathematics education and technology
	L.A. STEEN	All one system
1989	N. BALACHEFF	Teaching mathematical proof: The relevance and complexity of a social approach
	D. SCHATTSNEIDER	Geometry is alive and well
1990	U. D'AMBROSIO	Values in mathematics education
	A. SIERPINSKA	On understanding mathematics
1991	J .J. KAPUT	Mathematics and technology: Multiple visions of multiple futures
	C. LABORDE	Approches théoriques et méthodologiques des recherches françaises en didactique des mathématiques
1992	ICME-7	
1993	G.G. JOSEPH	What is a square root? A study of geometrical representation in different mathematical traditions
	J CONFREY	Forging a revised theory of intellectual development: Piaget, Vygotsky and beyond
1994	A. SFARD	Understanding = Doing + Seeing ?
	K. DEVLIN	Mathematics for the twenty-first century
1995	M. ARTIGUE	The role of epistemological analysis in a didactic approach to the phenomenon of mathematics learning and teaching
	K. MILLETT	Teaching and making certain it counts
1996	C. HOYLES	Beyond the classroom: The curriculum as a key factor in students' approaches to proof
	D. HENDERSON	Alive mathematical reasoning
1997	R. BORASSI	What does it really mean to teach mathematics through inquiry?
	P. TAYLOR	The high school math curriculum
	T. KIEREN	Triple embodiment: Studies of mathematical understanding-in-interaction in my work and in the work of CMESG/GCEDM
1998	J. MASON	Structure of attention in teaching mathematics
	K. HEINRICH	Communicating mathematics or mathematics storytelling
1999	J. BORWEIN	The impact of technology on the doing of mathematics
	W. WHITELEY	The decline and rise of geometry in 20^{th} century North America
	W. LANGFORD	Industrial mathematics for the 21^{st} century
	J. ADLER	Learning to understand mathematics teacher development and change: Researching resource availability and use in the context of formalised INSET in South Africa
	B. BARTON	An archaeology of mathematical concepts: Sifting languages for mathematical meanings
2000	G. LABELLE	Manipulating combinatorial structures
	M. B. BUSSI	The theoretical dimension of mathematics: A challenge for didacticians

2001	O. SKOVSMOSE	Mathematics in action: A challenge for social theorising
	C. ROUSSEAU	Mathematics, a living discipline within science and technology
2002	D. BALL & H. BASS	Toward a practice-based theory of mathematical knowledge for teaching
	J. BORWEIN	The experimental mathematician: The pleasure of discovery and the role of proof
2003	T. ARCHIBALD	Using history of mathematics in the classroom: Prospects and problems
	A. SIERPINSKA	Research in mathematics education through a keyhole
2004	C. MARGOLINAS	La situation du professeur et les connaissances en jeu au cours de l'activité mathématique en classe
	N. BOULEAU	La personnalité d'Evariste Galois: le contexte psychologique d'un goût prononcé pour les mathématique abstraites
2005	S. LERMAN	Learning as developing identity in the mathematics classroom
	J. TAYLOR	Soap bubbles and crystals
2006	B. JAWORSKI	Developmental research in mathematics teaching and learning: Developing learning communities based on inquiry and design
	E. DOOLITTLE	Mathematics as medicine
2007	R. NÚÑEZ	Understanding abstraction in mathematics education: Meaning, language, gesture, and the human brain
	T. C. STEVENS	Mathematics departments, new faculty, and the future of collegiate mathematics
2008	A. DJEBBAR	Art, culture et mathématiques en pays d'Islam (IX^{e}-XV^{e} s.)
	A. WATSON	Adolescent learning and secondary mathematics
2009	M. BORBA	Humans-with-media and the production of mathematical knowledge in online environments
	G. de VRIES	Mathematical biology: A case study in interdisciplinarity
2010	W. BYERS	Ambiguity and mathematical thinking
	M. CIVIL	Learning from and with parents: Resources for equity in mathematics education
	B. HODGSON	Collaboration et échanges internationaux en éduction mathématique dans le cadre de la CIEM : regards selon une perspective canadienne / ICMI as a space for international collaboration and exchange in mathematics education: Some views from a Canadian perspective
	S. DAWSON	My journey across, through, over, and around academia: "...a path laid while walking..."
2011	C. K. PALMER	Pattern composition: Beyond the basics
	P. TSAMIR & D. TIROSH	The Pair-Dialogue approach in mathematics teacher education
2012	P. GERDES	Old and new mathematical ideas from Africa: Challenges for reflection
	M. WALSHAW	Towards an understanding of ethical practical action in mathematics education: Insights from contemporary inquiries
	W. HIGGINSON	Cooda, wooda, didda, shooda: Time series reflections on CMESG/GCEDM

2013	R. LEIKIN	On the relationships between mathematical creativity, excellence and giftedness
	B. RALPH	Are we teaching Roman numerals in a digital age?
	E. MULLER	Through a CMESG looking glass
2014	D. HEWITT	The economic use of time and effort in the teaching and learning of mathematics
	N. NIGAM	Mathematics in industry, mathematics in the classroom: Analogy and metaphor
2015	É. RODITI	Diversité, variabilité et convergence des pratiques enseignantes / Diversity, variability, and commonalities among teaching practices
	D. HUGHES HALLET	Connections: Mathematical, interdisciplinary, electronic, and personal
2016	B. R. HODGSON	Apport des mathématiciens à la formation des enseignants du primaire : regards sur le « modèle Laval »
	C. KIERAN	Task design in mathematics education: Frameworks and exemplars
	E. MULLER	A third pillar of scientific inquiry of complex systems—Some implications for mathematics education in Canada
	P. TAYLOR	Structure—An allegory
2017	Y. SAINT-AUBIN	The most unglamorous job of all: Writing exercises
	A. SELDEN	40+ years of teaching and thinking about university mathematics students, proofs, and proving: An abbreviated academic memoir
2018	D. VIOLETTE	Et si on enseignait la passion?
	M. GOOS	Making connections across disciplinary boundaries in preservice mathematics teacher education
2019	J-M. DE KONINCK	Découvrir les mathématiques ensemble avec les étudiants
	R. GUTIERREZ	Mathematics as dispossession: Reclaiming mental sovereignty by living mathematx
2021	S. MAYES-TANG	Teaching on empty: Trauma, achievement, and what's next in our math education community
2022	E. PETITFOUR	Quel enseignement de la géométrie pour les élèves dyspraxiques ?

Appendix C / Annexe C

PROCEEDINGS OF ANNUAL MEETINGS / ACTES DES RENCONTRES ANNUELLES

Past proceedings of CMESG/GCEDM annual meetings have been deposited in the ERIC documentation system with call numbers as follows:

Proceedings of the 1980 Annual Meeting . . . . ED 204120

Proceedings of the 1981 Annual Meeting . . . . ED 234988

Proceedings of the 1982 Annual Meeting . . . . ED 234989

Proceedings of the 1983 Annual Meeting . . . . ED 243653

Proceedings of the 1984 Annual Meeting . . . . ED 257640

Proceedings of the 1985 Annual Meeting . . . . ED 277573

Proceedings of the 1986 Annual Meeting . . . . ED 297966

Proceedings of the 1987 Annual Meeting . . . . ED 295842

Proceedings of the 1988 Annual Meeting . . . . ED 306259

Proceedings of the 1989 Annual Meeting . . . . ED 319606

Proceedings of the 1990 Annual Meeting . . . . ED 344746

Proceedings of the 1991 Annual Meeting . . . . ED 350161

Proceedings of the 1993 Annual Meeting . . . . ED 407243

Proceedings of the 1994 Annual Meeting . . . . ED 407242

Proceedings of the 1995 Annual Meeting . . . . ED 407241

Proceedings of the 1996 Annual Meeting . . . . ED 425054

Proceedings of the 1997 Annual Meeting . . . . ED 423116

Proceedings of the 1998 Annual Meeting . . . . ED 431624

Proceedings of the 1999 Annual Meeting . . . . ED 445894

Proceedings of the 2000 Annual Meeting . . . . ED 472094

Proceedings of the 2001 Annual Meeting . . . . ED 472091

Proceedings of the 2002 Annual Meeting . . . . ED 529557

Proceedings of the 2003 Annual Meeting . . . . ED 529558

Proceedings of the 2004 Annual Meeting . . . . ED 529563

Proceedings of the 2005 Annual Meeting . . . . ED 529560

Proceedings of the 2006 Annual Meeting . . . . ED 529562

Proceedings of the 2007 Annual Meeting . . . . ED 529556

Proceedings of the 2008 Annual Meeting . . . . ED 529561

Proceedings of the 2009 Annual Meeting . . . . ED 529559

Proceedings of the 2010 Annual Meeting . . . . ED 529564

Proceedings of the 2011 Annual Meeting . . . . ED 547245

Proceedings of the 2012 Annual Meeting . . . . ED 547246

Proceedings of the 2013 Annual Meeting . . . . ED 547247

Proceedings of the 2014 Annual Meeting . . . . ED 581042

Proceedings of the 2015 Annual Meeting . . . . ED 581044

Proceedings of the 2016 Annual Meeting . . . . ED 581045

Proceedings of the 2017 Annual Meeting . . . . ED 589990

Proceedings of the 2018 Annual Meeting . . . . ED 595075

Proceedings of the 2019 Annual Meeting . . . . ED 610111

Proceedings of the 2021 Annual Meeting . . . . ED 620362

NOTES

-There was no Annual Meeting in 1992 because Canada hosted the Seventh International Conference on Mathematical Education that year.
-There was no Annual Meeting in 2020 due to COVID-19.