GEOGEBRA APPLETS FOR FOSTERING CONCEPTUAL UNDERSTANDING IN ALGEBRA

Ma. Louise Antonette N. De Las Peñas[1], Mark Anthony C. Tolentino[1], Maria Alva Q. Aberin[1], Agnes D. Garciano[1], Juan Carlo F. Mallari[1], Jumela F. Sarmiento[†,1] and Debbie Marie B. Verzosa[2]
[1]*Department of Mathematics, Ateneo De Manila University, Philippines*
[2]*Department of Mathematics and Statistics, University of Southern Mindanao, Philippines*

ABSTRACT

This paper discusses two *GeoGebra* applets, *Radical* and *Parabola*, that are designed to strengthen the conceptual understanding of specific topics in algebra. The design and pedagogical basis of the applets are presented. The integration of the applets in teaching Grade 9 mathematics in a partner high school in the Philippines is then discussed. Finally, we report feedback gathered from students and teachers during this integration. Their feedback indicates the potential of these applets for improving students' learning of algebra.

KEYWORDS

Mobile Technology in Teaching Mathematics, Mathematical Applets, *Geogebra* Applets, Algebra Applets

1. INTRODUCTION

In the time of the COVID-19 pandemic in the Philippines, the Department of Education (DepEd) has called for the blended learning modality in schools (DepEd Memo 012, 2020). This introduced several challenges in the teaching and learning of mathematics. Due to the limited time for DepEd to prepare printed materials in lieu of face-to-face classes, there was a need to prepare more instructional materials to help teachers address the Most Essential Learning Competencies (MELCs) prescribed by DepEd's curriculum (DepEd 2020). Under projects funded by the Philippine government (De Las Peñas et al, 2023, De Las Peñas et al 2022), the authors collaborated with particular DepEd School Division Offices to design and create mathematical applications (apps) accompanied with teaching guides and instructional videos to provide teachers with alternative means to teach mathematics. These apps are also being recommended for use in the new normal under the DepEd's *Basic Education Learning Recovery Plan* in which schools are directed towards addressing learning gaps brought about by the pandemic (DepEd Memo 664, 2022).

This short paper contributes to the literature on mathematical learning and the role of technology during the time of COVID-19 and the new normal. In particular, we present two mathematical applets, namely *Radical* and *Parabola*, that were created under the aforementioned government projects and that were designed for Grade 9 students in algebra. We focus on these two applets that give students opportunities to interact with mathematical concepts in a dynamic geometry environment (DGE). These were created using the open-access software *GeoGebra* (https://www.geogebra.org/), making these easily accessible through contemporary technologies such as tablets and mobile phones.

The applets highlight different instructional design features that can be provided with digital tools to support the teaching and learning of mathematics. The *Radical* applet employs the immediate feedback feature that supports practice, where students get the opportunity to strengthen content knowledge previously acquired by practicing at their own pace, and repeatedly, as often as needed (Hillmayr et al., 2020). The *Parabola* applet, on the other hand, makes use of the dragging mode of a DGE to arrive at dynamic families of graphs and to observe relationships among various elements such as the graphs and their corresponding equations. This makes the applet a potential tool for students to engage in conjecture making (Baccaglini-Frank & Marrioti, 2011). Through these features, the two applets aim to add to the large repository of applets that are available online and demonstrate how educators can enhance existing applets with additional instructional design features. For example, while the GeoGebra repository contains many applets on radicals, few if any offer

ISBN: 978-989-8704-55-9 © 2024

prompts or hints. Similarly, it includes many applets on parabolas, but there appears to be no applet that simultaneously displays the vertex and standard form of a parabola together with its discriminant.

The interested reader can access the applets discussed in this paper and the other mathematical apps of the aforementioned projects from the website https://mathplusresources.wordpress.com/.

2. DESCRIPTION OF THE APPLETS

Two of the mathematical apps that we have designed are applets created with the algebra/geometry software *GeoGebra*. Existing studies around the world point towards the affordances in using *GeoGebra* in mathematics education (Antonini & Lisarelli, 2021; Azizah et al., 2021; Tong et al, 2021; Mudaly et al., 2019; Wassie & Zergaw, 2019; Shadaan et al., 2013). In our case, we found that *GeoGebra* applets are appropriate for the Philippine setting as they can provide wider accessibility for teachers and students since these can run on mobile technologies (both Android and iOS). Moreover, *GeoGebra* allows for the creation of content that can be exported as interactive and dynamic worksheets published as applets. This made it easy for us to create instructional materials on topics that were needed immediately based on the requests of teachers in our partner schools during the time of COVID-19. The applets are straightforward to use via *GeoGebra*'s tools that are intuitive and easy to manipulate. Advantageous to teachers was *GeoGebra*'s "construction protocol bar," through which they gathered insights on the processes used to create the applets. This allowed them to create their own applets for other topics and was helpful given their limited time for teaching preparations.

2.1 The *Radicals* Applet

The *Radicals* applet is designed to address one of the learning competencies for Grade 9 mathematics under the content strand algebra, which is "simplifying radical expressions using the law of radicals" (DepEd, 2016). The objective of the app is to strengthen the student's conceptual understanding of simplifying radicals, particularly breaking down the radicand into perfect and non-perfect *n*th powers and applying the property $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$. The students are guided to solve each question provided by the applet (Figure 1(a)). Students can verify their answer using the applet and get appropriate feedback (Figure 1(a)-(b)). Afterwards, the students proceed to the next question. Note that the applet randomizes the questions, so the questions vary per student.

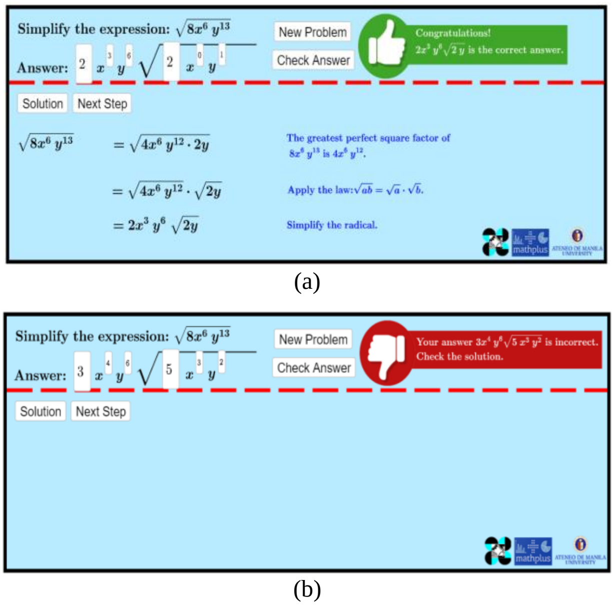


Figure 1. Screenshots from the *Radical* app showing: (a) step-by-step solution and feedback on a correct answer; and (b) feedback on a wrong answer

2.2 The *Parabola* Applet

The *Parabola* applet addresses the following Grade 9 learning competencies: graphs a quadratic function of the form $y = a(x - h)^2 + k$ (domain, range, intercepts, axis of symmetry, vertex, direction of opening of the parabola). The applet allows the student to analyze the effects of changing the values of a, h, and k in the quadratic function $y = a(x - h)^2 + k$ on its graph. Sliders are provided so students can experiment on the values of a, h, k (See Fig. 2). It also gives the value of the discriminant, so students can explore its connection to other properties of its graph, such as its x-intercepts.

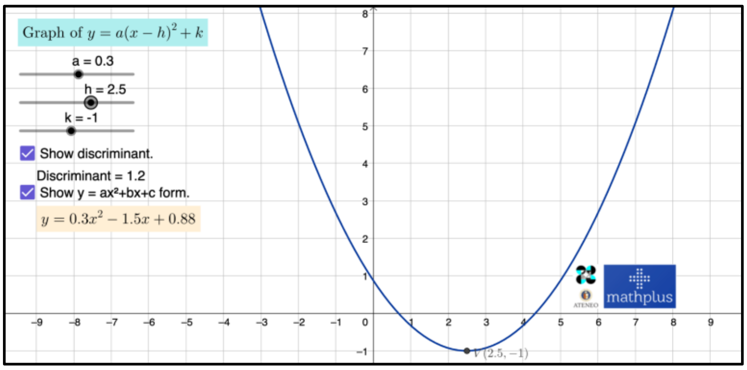


Figure 2. A screenshot of the *Parabola* applet showing the sliders on the values a, h, and k

3. PEDAGOGICAL BASIS OF THE APPLETS

The *Radicals* applet is an example of the use of technology to support "practicing," where repeated use of technology deepens and develops skills that foster understanding of mathematical concepts and strengthens content knowledge (Thurm & Barzel, 2021; Hillmayr et al., 2020). The repetitive feature of the applet allows for deliberate practice, which can modify and enhance previously learned skills and build advanced skills in addition to pre-existing ones (Ericsson, 2016; Garciano et al., 2023). The design of incorporating a feedback mechanism in the applet is crucial so that students engage in a process of practice when working independently without the aid of the teacher (Bokhove, 2011).

The *Parabola* applet is anchored on the framework that technology helps in the teaching and learning of geometry through dynamic interactive graphical representations (Laborde et al., 2006). For example, exploring the graphs of $y = a(x - h)^2 + k$ and using sliders for a, h, and k gives students the opportunity to test, conjecture, and generalize effects of the changing values for a, h, and k. The sliders serve as a dragging mechanism. The dragging feature in a dynamic geometry environment (DGE) illustrates the transformation technology can bring in the representations of mathematical objects, the family of graphs in this case, representing invariant relationships with each of a, h, k (Laborde & Laborde, 2014). The graphs are no longer static objects and are now dynamic objects.

4. INTEGRATION AND USE OF THE APPLETS

In this section we discuss how the *Radical* and *Parabola* applets were used in the teaching and learning of Grade 9 mathematics in a high school in Quezon City, Philippines in school years 2021-2022 and 2022-2023.

For 2021-2022, DepEd implemented a blended-modality form of instruction, so there were synchronous and asynchronous classes. Modules and instructional materials, prescribed by DepEd for the students, were distributed by the school administrators to the parents. During this time, teachers met their students at least once a week through Google Classroom. Written works were required from the students on a regular basis, which were to be done asynchronously. In the synchronous classes, the applets were used to introduce the lesson (Figure 3(a)-(b)). Follow-through sessions were done asynchronously by asking the students to use the applets for exploration and to answer exercises (Figure 4(a)-(b)).

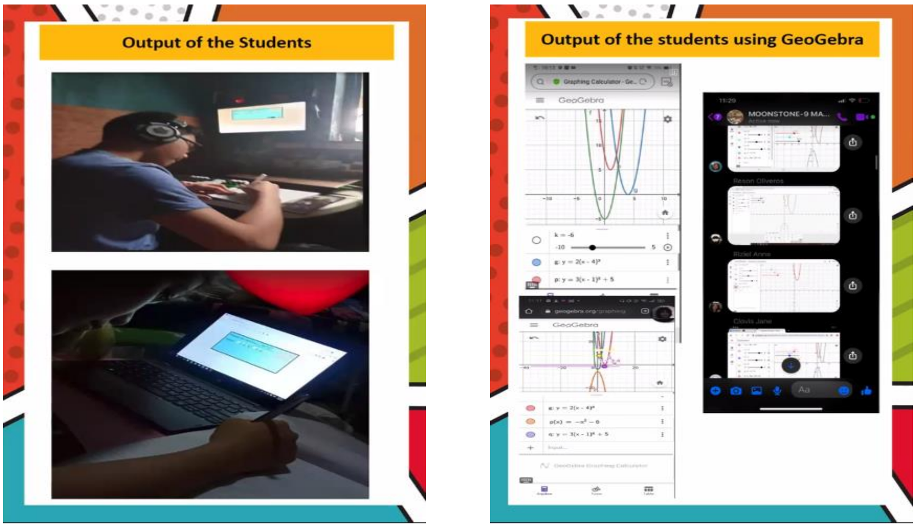
ISBN: 978-989-8704-55-9 © 2024

For 2022-2023, DepEd resumed face-to-face classes in schools. The applets were used in the classroom for the lessons, and for assigned homework.

We were able to gather feedback from teachers and students based on the integration of the two applets in school years 2021-2022 and 2022-2023. We discuss these feedback below.

(a) (b)

Figure 3. Screenshots of the use of the a) *Radical* applet and b) *Parabola* applet in Google classroom



(a) (b)

Figure 4. Screenshots of the work of the students carried out asynchronously

a) *Radical* applet: We were able to get qualitative feedback from teachers and students on the use of the applet during the blended modality of instruction in 2021-2022. First, specific instructions were given by the teachers to the students on how to use the *GeoGebra* applets, including downloading instructions (Figure 5(a)). The applet was used in the synchronous sessions for teaching the concept. Then students were asked to answer exercises given by the applet asynchronously. For each radical expression, they were first asked to record their solutions in their notebooks. Then afterwards, they were asked to check their solutions versus the detailed solution provided by the applet. They submitted snapshots of their work through Messenger to their teachers (Figure 5(b)).

(a) (b)

Figure 5. (a) Instructions on the *GeoGebra* files; (b) screenshots of sample solutions on simplifying radicals sent through messenger app to the algebra teacher

During an online Program Implementation Review (PIR) a few months after the use of the *Radical* applet, the teachers reported the experiences in their classes. Parents also gave comments with regards to the experiences of their children. In general, the applet helped the students understand the simplification of radical expressions through constant practice. It was useful that the applet was able to generate various types of expressions, and the students did not only rely on the exercises given by the teacher or those provided in the modules. This was also useful for the teachers, because of the limited opportunities they had to create as many exercises as possible for the students. The teachers shared the following during the PIR: The students remarked that being able to see the detailed solutions with the reasoning of each step from the applet was helpful. They liked that they were able to identify their mistakes from the solution provided by the applet since they were not always with their teachers. Working with the applet kept them interested in doing the exercises.

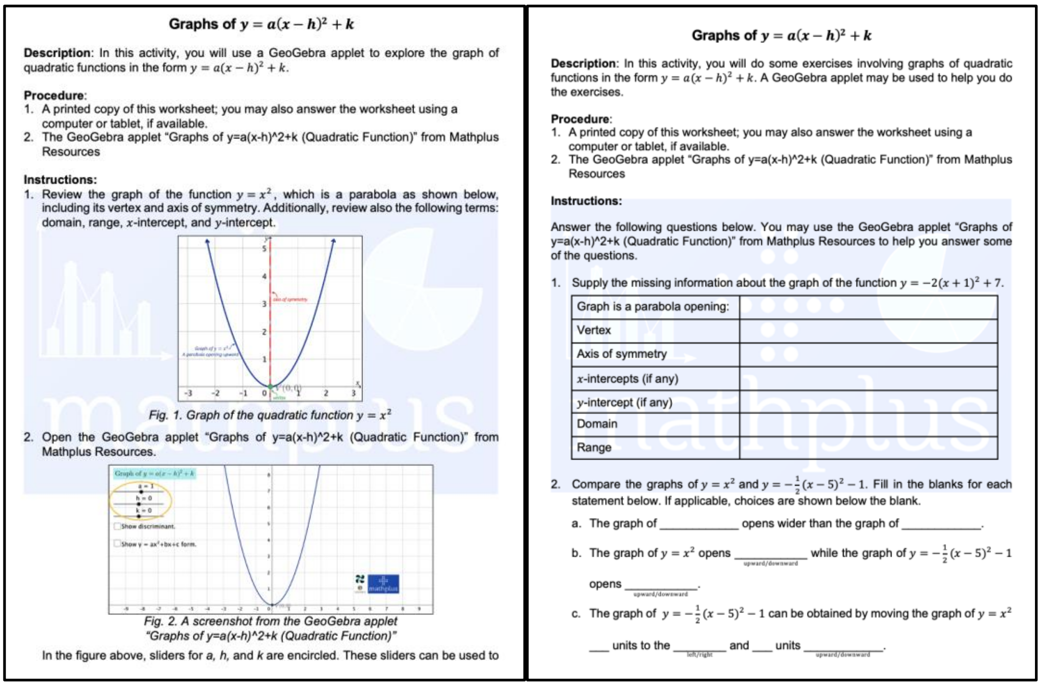
b) *Parabola* applet: In 2022-2023, during the first year of implementation of face-to face classes after the pandemic, we were able to obtain an initial quantitative assessment on the use of the applet. While teaching the lesson on quadratic functions, four Grade 9 sections were randomly selected to form two pairs of experimental and control classes in such a way that each pair (experimental and control) of classes are taught by the same teacher; thus, two teachers are involved. Pretests and posttests were administered to the four classes.

The experimental classes used the *Parabola* applet. Each student had a tablet where the applet was installed. The worksheets (Figure. 6(a)-(b)) were answered by the students individually on different days. One worksheet (Figure. 6(a)) dealt with exploring the graph $y = a(x - h)^2 + k$ and using sliders for a, h, and k. The students were first asked to explore the effects of changing the values of a, h, and k on the graphs and write their observations. Then they were asked to make conjectures on properties of the graphs such as the coordinates of the vertex, equation of the axis of symmetry, opening of the graph, and domain and range of $f(x) = a(x - h)^2 + k$. For the second worksheet (Figure. 6(b)), one type of questioning involved giving specific equations (e.g., $y = -2(x + 1)^2 +7$), and students were asked to provide the vertex, x- and y-intercepts, domain, and range. Another type of question involved giving properties of the graph of the parabola and the student was asked to give a possible equation for the parabola that satisfied the properties.

The control sections employed the traditional approach, where the teachers taught the lesson without the *Parabola* applet and the worksheets.

ISBN: 978-989-8704-55-9 © 2024

Based on the data shown in the table below, for one pair of experimental and control classes, the experimental class showed a higher level of improvement from their pretest to posttest scores when compared to the control. On the other hand, the other pair of classes only exhibited little difference in their levels of improvement. Thus, these results indicate that the use of the *Parabola* applet and other related resources (teaching guide, student worksheets) may have the potential to affect students' learning positively.



Graphs of $y = a(x-h)^2 + k$

Description: In this activity, you will use a GeoGebra applet to explore the graph of quadratic functions in the form $y = a(x-h)^2 + k$.

Procedure:
1. A printed copy of this worksheet; you may also answer the worksheet using a computer or tablet, if available.
2. The GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources

Instructions:
1. Review the graph of the function $y = x^2$, which is a parabola as shown below, including its vertex and axis of symmetry. Additionally, review also the following terms: domain, range, x-intercept, and y-intercept.

Fig. 1. Graph of the quadratic function $y = x^2$

2. Open the GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources.

Fig. 2. A screenshot from the GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)"

In the figure above, sliders for *a*, *h*, and *k* are encircled. These sliders can be used to

Graphs of $y = a(x-h)^2 + k$

Description: In this activity, you will do some exercises involving graphs of quadratic functions in the form $y = a(x-h)^2 + k$. A GeoGebra applet may be used to help you do the exercises.

Procedure:
1. A printed copy of this worksheet; you may also answer the worksheet using a computer or tablet, if available.
2. The GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources

Instructions:

Answer the following questions below. You may use the GeoGebra applet "Graphs of y=a(x-h)^2+k (Quadratic Function)" from Mathplus Resources to help you answer some of the questions.

1. Supply the missing information about the graph of the function $y = -2(x+1)^2 + 7$.

Graph is a parabola opening:	
Vertex	
Axis of symmetry	
x-intercepts (if any)	
y-intercept (if any)	
Domain	
Range	

2. Compare the graphs of $y = x^2$ and $y = -\frac{1}{2}(x-5)^2 - 1$. Fill in the blanks for each statement below. If applicable, choices are shown below the blank.
   a. The graph of ____________ opens wider than the graph of ____________.
   b. The graph of $y = x^2$ opens ____________ (upward/downward) while the graph of $y = -\frac{1}{2}(x-5)^2 - 1$ opens ____________ (upward/downward).
   c. The graph of $y = -\frac{1}{2}(x-5)^2 - 1$ can be obtained by moving the graph of $y = x^2$ ___ units to the ________ (left/right) and ___ units ____________ (upward/downward).

(a) (b)

Figure 6. Worksheets for the experimental class

Table 1. Mean pretest, gain, and posttest scores for the two pairs of experimental and control classes

	Teacher A		Teacher B	
	Experimental A	Control A	Experimental B	Control B
Number of Students	36	28	31	31
Mean Pretest Scores	20.24%	22.70%	22.58%	22.81%
Mean Gain Scores	31.15%	11.73%	13.36%	15.21%
Mean Posttest Scores	51.39%	34.44%	35.94%	38.02%

However, assessing the effectiveness of the applet and complementary resources more accurately will require further studies. As evident in Table 1, all four classes involved in the study had relatively similar mean pretest scores. The mean gain scores then indicate that all four classes actually received low mean posttest scores, with only Experimental A receiving a mean posttest score (i.e., 51.39%) above 50%. The performance of these classes indicates that there has been a *learning gap* in mathematics that might have transpired even before (aside from during) the integration of the resources. Considering this, further studies on assessing the effectiveness of the aforementioned resources will need to carefully consider the possible presence of such learning gaps.

6. CONCLUSION AND OUTLOOK

In this paper, we discussed two *GeoGebra* applets *Radicals* and *Parabola* that were designed and developed to support the teaching and learning of algebra, especially in the context of the COVID-19 pandemic and the new normal in the Philippines. Capitalizing on sound pedagogy and the affordances of mobile technology, the two applets demonstrate how educators can enhance existing apps with additional instructional design features. These two applets were integrated into Grade 9 classes in one of our partner high schools in the Philippines. Based on feedback gathered from teachers and students during this integration, the applets, as well as some complementary resources, can ease teachers' work and have the potential to improve students' learning of algebra topics.

However, various factors such as learning gaps in mathematics or given the possibly extraordinary education situation brought about by the pandemic, further studies are needed to accurately assess the effectiveness of these applets. In the near future, additional experimental studies employing a pretest-posttest design will be conducted. This will involve the inclusion of more classes, thereby expanding the overall sample size for a comprehensive analysis. Given their potential, however, it is also important to continue the development and improvement of such resources.

ACKNOWLEDGEMENT

This paper is one of the outputs of the Ateneo Mathplus Resources Laboratory housed at the Department of Mathematics, School of Science and Engineering, Ateneo de Manila University. The authors thank the Department of Science and Technology-Philippine Council for Industry, Energy, and Emerging Technology Research and Development (DOST-PCIEERD) and the University Research Council (URC), Ateneo de Manila University for the support of the development of the mathematical resources for Grades 1 to 11 Mathematics. Much appreciation is extended to our DepEd partners particularly DepEd School Division Quezon City, Dr Joel Feliciano, Ms. Gina Acosta and Ms. Nora Villalobos and the other teachers and administrators of our partner schools.

REFERENCES

Antonini, S. and Lisarelli, G. (2021). Designing tasks for introducing functions and graphs within dynamic interactive environments. *Mathematics*, Vol. 9, No. 5, 572.

Azizah, A. N. et al. (2021). The effectiveness of GeoGebra to improve visual representation ability. *Journal of Physics: Conference Series*, Vol. 1808, 012059.

Baccaglini-Frank, A. and Mariotti, M. A. (2011). Conjecture-generation through dragging and abduction in dynamic geometry. In Méndez-Vila, A., (Ed.), *Education in a Technological World: Communicating Current and Emerging Research and Technological Efforts* (pp. 100-107).

Bokhove, C.,9 (2011). *Use of ICT for acquiring, practicing and assessing algebraic expertise*, Dissertation, Utrecht University.

De Las Peñas, M. L. A. N. et al. (2022). *Technology Interventions for Mathematical Reasoning, Statistical Thinking and Assessment, DOST-PCIEERD Terminal Report*.

De Las Peñas, M. L. A. N. et al. (2023). *Mathematical Resources for Distance Learning Utilizing Community LTE Networks and Television Frequencies DOST-PCIEERD Terminal Report*.

Department of Education (DepEd), (2016). *K to 12 Curriculum Guide: Mathematics*. Retrieved September 17, 2023, from https://www.deped.gov.ph/wp-content/uploads/2019/01/Math-CG_with-tagged-math-equipment.pdf

Department of Education (DepEd), (2020). *K to 12 Most Essential Learning Competencies with Corresponding CG Codes*. Retrieved September 17, 2023, from https://commons.deped.gov.ph/K-to-12-MELCS-with-CG-Codes.pdf

Department of Education (DepEd), *DepEd Order No. O12, s.(2020): Adoption of the Basic Education Learning Continuity Plan for SY 2020-2021 in light of the COVID 19 Public Health Emergency.* Retrieved September 19, 2023, from https://authdocs.deped.gov.ph/deped-order/do_s2020_012-adoption-of-the-be-lcp-sy-2020-2021/

Department of Education (DepEd) Region I, (2022). *Regional Memorandum No. 664, s. 2022: Basic Education Learning Recovery Plan in light of the COVID 19 Pandemic*. Retrieved September 19, 2023, from https://depedro1.com/ wp-content/uploads/2022/06/ rm0664s2022.pdf

Ericsson, K.A. (2016). Summing up hours of any type of practice versus identifying optimal practice activities: commentary on Macnamara, Moreau, & Hambric. *Perspectives on Psychological Science*, Vol. 11, No. 3, pp. 351-354.

Garciano, A.D. et al. (2023). Practice through play using mobile technology. In P. Kommers, I. Arnedillo Sánchez, & P. Isaías (Eds.), *Proceedings of the International Conferences on E-society 2023 and Mobile Learning 2023* (pp. 351-358). International Association for the Development of the Information Society.

Hillmayr, D. et al. (2020). The potential of digital tools to enhance mathematics and science learning in secondary schools: a context-specific meta-analysis. *Computers & Education*, Vol. 153, 103897.

Laborde, C. and Laborde, J.M. (2014). Dynamic and tangible representations in mathematics education, In S. Rezat, M. Hattermann, & A. Peter-Koop (Eds.), *Transformation – A Fundamental Idea of Mathematics Education: A New Approach* (pp. 187-202).

Laborde, C. et al. (2006). Teaching and learning geometry with technology, In A. Gutiérrez & P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future* (pp. 275-304).

Mudaly, V. and Fletcher, T. (2019). The effectiveness of GeoGebra when teaching linear functions using the iPad. *Problems of Education in the 21st Century*, Vol. 77, No. 1, pp. 55-81.

Shadaan, P. and Leong, K. E. (2013). Effectiveness of using GeoGebra on students' understanding in learning circles. *Online Journal of Educational Technology*, Vol. 1, No. 4, pp. 1-11.

Thurm, D. and Barzel, B. (2022). Teaching mathematics with technology: a multidimensional analysis of teacher beliefs. *Educational Studies in Mathematics*, Vol. 109, pp. 41-63.

Tong, D. H. et al. (2021). The effectiveness of using GeoGebra software in mathematics classrooms: a case study of teaching continuous functions in high schools. *Journal of Hunan University*, Vol. 48, No. 9, pp. 256-268.

Wassie, Y. S. and Zergaw, G. A. (2019). Some of the potential affordances, challenges and limitations of using GeoGebra in mathematics education. *EURASIA Journal of Mathematics, Science and Technology Education*, Vol. 15, No. 8, em1734.