SCHEMA REVISION ON MONTY HALL PROBLEM

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The importance of statistical and mathematical literacy has been argued for a long time from social justice and equity perspectives. In this study, the famous brain teaser in math literacy, the Monty Hall Problem is used to investigate if schema revision is possible with a minimal intervention. The participants are 55 undergraduate students. Three participants' schema revision was observed during the assessment with a questionnaire focused on reversible thinking, i.e., what would have caused the favorable future outcome. The result could lead to reform in teaching and learning on reversible thinking in counterintuitive statistical and mathematical problems.

Keywords: cognition, probability, problem solving, mathematical literacy

The importance of statistical and mathematical literacy has been argued for a long time. Statistical literacy is crucial for individuals to be able to read, evaluate, and make informed decisions based on the statistical arguments they are bombarded with on a daily basis (Weiland, 2017).

Monty Hall hosted a game show on American television called "Let's Make a Deal" from 1963 for nearly 30 years. Contestants on this show were often faced with a dilemma in which they had to decide whether to stick with an initial choice or switch to an alternative. What contestants should do in this situation sparked a heated debate in 1991 among academics as well as lay people. Many researchers (Miller & Sanjurjo, 2019; Hirano et al., 2017; De Neys & Verschueren, 2006; Krauss & Wang, 2003; Tubau & Alonso, 2003; Ichikawa & Shimojo, 1989; Verschueren, 2006; Lucas et al., 2009; Gilovich et al., 1995; Franco-Watkins et al., 2003; D'Ariano et al., 2002) in many disciplines, for example, mathematics, statistics, psychology and social science have studied the problem for the last 30 years. Ichikawa and Shimojo (1989) pointed out the importance of understanding the Monty Hall Problem from the public mathematics/statistics literacy perspective.

The question known today as the Monty Hall Problem (MHP), the Monty Hall Dilemma, or the Three Door Problem is the following:

> Suppose you are on a game show, and you are given the choice of three doors. Behind one door is a car and behind the others are goats. Assume that the car is equally likely in any three doors. You pick a door, say, Number 1, and the host, who knows what's behind the doors, opens another door, say, Number 3, which has a goat. He then says to you, "Do you want to switch to door Number 2". Is it to your advantage to switch?

While the intuitively compelling answer is that either of the two remaining doors leads to the same chance of winning the car, actually, their chance doubles if the contestant switches from door Number 1 to Number 2. People, even mathematically inclined academics, typically get this problem wrong at first, and furthermore even many mathematicians have expressed disbelief when told the correct answer (Vazsonyi, 1999). It is not only difficult to find the correct solution to the problem, but it is more difficult to make people accept the solution.

Several assumptions are needed to solve this problem.

- A car is equally likely to be placed behind any door at the beginning.
- Monty, the game show host, knows where the car is. Therefore, he always opens the door which has a goat after the contestant picks one door initially.
- If the contestant initially picks the door which has a car, the host randomly chooses one of the two remaining doors.

There are two main approaches to understand the MHP. The first one is to list every possible outcome if the contestant initially picks door Number 1. There are exactly 4 scenarios. Out of which the probability of winning by switching is 2/3 (Figure 1*)*.

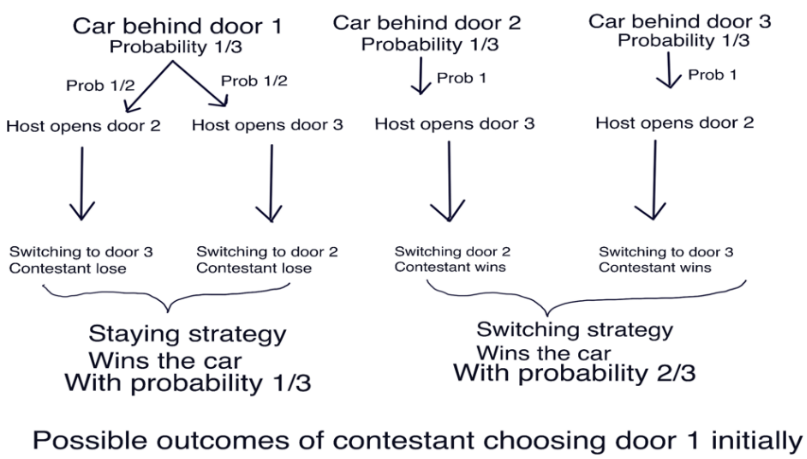

Figure 1: All 4 possible outcomes of contestant choosing door Number 1 initially

The other solution is calculating the conditional probability by Bayes' Theorem. Many studies from many perspectives have been done on MHP to examine the dilemma/counterintuitive-ness and look for ways to help people understand the correct solution. However, almost no study has been done in mathematics education community on MHP and no study has focused on *reversible thinking*, which means focusing on what would have caused the desirable outcome in the future. In this study, Cognitive Schema Theory (CST) is used to identify specific cognitive mechanisms that underlie schema construction and revision during working on MHP. What is very interesting, and fascinating is that most people, including many professional mathematicians, could not accept the correct solution of MHP even after thinking it over carefully for a while. This study intends to examine students' schema revision on MHP. The participants of this study are 55 undergraduate students. The research question is the following:

- Can we observe participants' schema revision (changing their opinions) when they are given a questionnaire focused on *reversible thinking*, without training or explaining the answer?

This question will contribute to pedagogical reform on how to help students solve counterintuitive problems in mathematics as well as in public statistical literacy. Many studies were conducted from psychological perspectives on why people resist switching to the alternative, and why we find correct solutions counterintuitive on the MHP. Also, much research was done on people's performance after providing explanations or training on the MHP game, however, no study has been done on people's performance by aiming to trigger their appropriate cognitive field.

Literature Review

Why does the correct solution of the MHP feel counterintuitive? The MHP became very famous and controversial for the column in *Parade* magazine of Marilyn Vos Savant. In three of her weekly columns, Vos Savant (1990a, 1990b, 1991) explained the correct solution of the MHP, and she received numerous passionate responses in disagreement. Granberg and Brown (1995) conducted experimental study on MHP. In the study, they asked participants if they would stick to door number 1 or switch to door number 2. The study found that 13% indicated to switch to door number 2. People's tendency to stick to their initial pick has been reported despite the fact they believe the chance is 50/50 to win the car either for stick or switch. Granberg and Brown (1995) also found that participants would feel worse if they switched from a door with the car behind it than if they stick to a door with a goat behind it. This may be due to the status quo bias which is an emotional bias of people's preference to stick with the choice of door they have initially made. Gilovich et al. (1995) asked if people reduce dissonance more for their errors of commission than their errors of omission, and argued that all other things being equal, people prefer to make errors through inaction (staying) as opposed to action (switching).

Some studies (Franco-Watkins et al., 2003; Hirano et al., 2016; Tubau & Alonso, 2013; Granberg & Brown, 1995) conducted experiments using computer programs or card games to let participants play the game or watch the simulation numerous times to experience the odd in the MHP. Result of such experiments increased the proportion of switching responses but did not change erroneous intuitions or did not increase understanding of the mathematical reasons why switching (to door number 2) increase their chance of winning the car. This suggests that there is dissociation between implicit knowledge gained from the task and explicit understanding as to why switching was the best strategy (Franco-Watkins et al., 2003).

De Neys and Verschueren (2006) presented an empirical test of the involvement of working memory resources in MHP reasoning. Working memory component is a system of human cognitive capacity, and they found that participants who gave the correct switching response had a significantly large working memory capacity than participants who reasoned erroneously.

Can people gain an understanding of the probabilistic structure of the MHP? Is it possible to develop appropriate ways to present and explain the MHP such as to eliminate the typical resistance to the switch decision? Krauss and Wang (2003) identified correct reasonings to solve the MHP and conducted a study to answer these questions. In the study, they provided the Bayes's formula sheet, pictures of all possible scenarios of locations of a prize in the MHP game, a chart of mental models based on the frequency simulation and wrote the problem from the perspective of the game show host, to train the participants to evaluate their learning. They reported that before training, none of the 110 participants solved the MHP with a correct justification for the switch choice. However, after giving them training, they found that the frequency simulation and perspective change training only improve participants' performance if presented in combination. The result was that 10~ 46% of the participants improved their performance on related or similar problems.

Tubau and Alonso (2003) also conducted a study on how to improve people's performance on the MHP. They investigated the effects of practicing the simulation game, making explicitly describing the structure, or enhancing the representation of the different possibilities of the MHP, on reaching and stating the correct answer. They found that representing the MHP as a game between two adversaries (decision maker and informant perspectives) significantly improved the participants' reasoning, that is 25% of them stated a correct reasoning. Moreover, by showing the participants the explicit all possible outcomes of the game with the adversary perspective improved the participants' performance more significantly, to 60%. They noted that the direct game practicing of the MHP simulation did not improve their performance on reasoning a correct answer. Therefore, the literatures confirmed that it is possible, although it is still difficult, to learn and understand a correct reasoning to solve the MHP even with training and showing the explicit all possible outcomes.

Intuition

The commonly accepted definition of intuition is the ability to understand something immediately, self-evidently without the need for conscious reasoning. Fischbein (1999) claimed that intuitions are very sensitive to the influences of a representation of the context, especially because no logical, formal support intervenes. Moreover, intuitions are very resistant to change because intuitions are related to well-structured systems of our cognitive-behavioral, adaptive activity. This implies that intuitions change together with the entire adaptive system to which they belong, namely cognitive field currently activated. Intuitions are profoundly related to cognitive field. Intuitions are the cognitive counterpart of some structural schemata, and they may be manipulated by non-adequate schemata (Fischbein, 1999). Since for most of us, the correct solutions of the MHP feel counterintuitive, with his assumptions, it may be that the original presentation of the MHP triggers non-adequate schema, a cognitive field, for most of us. Then, is it possible for students to activate a propriate cognitive field to understand the probability structure of the MHP so that they can solve it intuitively, without training or without showing all the explicit outcomes, even for those who think that there is 50/50 chance in winning the car by staying?

Theoretical Framework: Cognitive Schema Theory (CST)

Derry (1996) explained CST as an important theoretical perspective that has significant potential for building conceptual bridges among cognitive constructivism viewpoints. The purpose of CST is to identify specific cognitive mechanism that underlie schema construction and revision. A schema is a program which enables the individual to a) record, process, control and mentally integrate information, and b) to react meaningfully and efficiently to the environmental stimuli (Fischbein, 1999). From this definition, one may see that a schema is similar to a computer software, for example, schema construction is considered as download a programing software. The theory assumes that long-term memories store previously learned schemas and working memories represent a person's span of immediate attention. Cognitive activities, like thinking and learning, take place in working memory, where prior knowledge schemas are activated in response to environmental input, providing context for interpreting experience and assimilating new knowledge (Derry, 1996). According to CST, learning involves constructing three types of schemas that interact during the learning process. They are *memory objects*, *mental models*, and *cognitive fields*. Memory-object represents the permanent results of learning stored in memory and thus constitute the collection of all knowledge and preconceptions that one might use to interpret any situation. For example, "the total probability of a set of related events sums up to one, i.e., 1/2: 1/2, or 1/3: 2/3." is a memory object. Mental modeling can be viewed as a process of constructing a mental representation of a situation. For example, a process of thinking that 'the probability of one box contains a prize is ½ if there are two boxes and one prize is randomly placed in one of them' is a mental model. The goal of mental modeling is to form an understanding of a phenomenon. Cognitive field is a mental program which activates relevant preconceptions in response to a particular event, such as a problem posed, that makes certain memory objects more available for use than others. Cognitive field mediates experience and learning. Starting to think a posed problem triggers activation of a cognitive field, then it set out the memory objects which are readily available for modeling the problem. The cognitive field determines what interpretations and understandings of the problem are probable. The MHP asks us if we want to stay or switch. This wording may trigger a cognitive field of choosing one out of two, namely, the 50/50 schema. Therefore, we could say, an inappropriate cognitive field gets activated when the MHP is posed for the first time for most people.

Radical constructivists believe that students must actively reflect on the fit of new information or experience to currently activated logical schema in order to update or revise their cognitive structure. On the contrary, CST says that schema revision does not always require deep reflection, direct contradiction to prior beliefs, or feelings of disequilibrium. Schema revision requires the activation of relevant appropriate cognitive fields with introduction of useful, understandable information at critical times during the model construction process. Therefore, it may be possible for students to activate an appropriate cognitive field for MHP by providing them useful, understandable instruction. Moreover, it may be possible to promote schema revision of students who thought that the remaining doors have the same chances of winning the car by re-presenting the problem in a way to trigger appropriate cognitive field.

Method

So far, the most successful mental model to increase people's understanding of the mathematical structure of the MHP was the model used by Krauss and Wang (2003). They formulated the MHP using manipulations in natural frequencies and perspective change. They asked the participants guiding frequency questions to motivate them to construct the mental model. The 1/3: 2/3 model used in this study was modified from their model (Figure 2). The objective of this study is to see if the participants who first triggered inappropriate cognitive field could activate the 1/3: 2/3 mental model by a questionnaire focused on reversible thinking. The participants of this study are 55 undergraduate students in precalculus and in business calculus. They are first-year students. Three written assessments are provided and given to the students each consecutive week for 3 weeks as bonus quizzes. The first assessment includes the original MHP and some extra questions to evaluate student's memory objects on probability. This assessment is to evaluate if they have basic probability schema. Also, the students are asked if they knew of a similar problem already. Students are asked to explain (in writing) their answers.

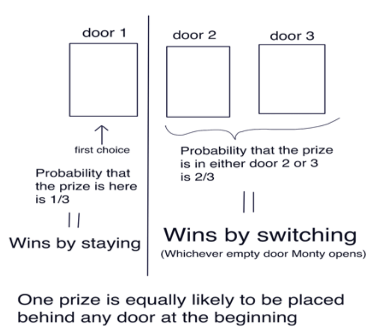

Figure 2: The 1/3: 2/3 Mental Model

Second Assessments

The second assessment is the following: Instead of a car, a diamond is used.

1. What is the probability that door number 1 has the diamond?
2. What is the probability that the contestant didn't choose the door with the diamond?

Let's imagine a line between door number 1 and the other doors. Now there are two sections, the right side has two doors (2 & 3) and the left side has door 1. (Figure 3)

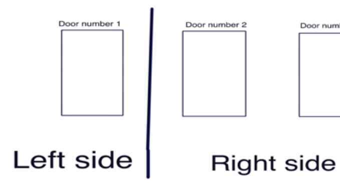

Figure 3: Three doors with a line

3. If YOU can bet on the two doors on the right side (2 & 3) for the diamond, or one door (number 1) on the left side, which would you choose? Circle one.
   Right side   Left side   It doesn't matter   I don't know
4. What is the probability that the diamond is on the right side?
5. What is the probability that the diamond is in one of the doors, number 2 or 3?

At least, one of the door number 2 or 3 is empty, right? (Because there is only one diamond)

Then, the host reveals that door number 3 is empty.

6. Now the host gives the contestant another chance. Which door should the contestant choose?
7. Based on your answer in 6, suppose the contestant followed your advice, what is the probability that the contestant wins the prize?
8. Please explain your answer.

Third Assessment

The third assessment is the following:

Suppose you won the diamond by switching to door number 2. Do you agree/disagree with the following statements: Circle one.

1. This means that you picked an empty door at first. Agree, Disagree, Don't know
2. Winning by switching = Picking an empty door at first. Agree, Disagree, Don't know
3. The probability of winning by switching = The probability of picking an empty door at first. Agree, Disagree, Don't know
4. What is the probability of picking an empty door at first?
5. What is the probability of winning the diamond by switching?

Finally, let me ask again. (The original MHP is asked again here) Would you switch to door number 2? Please explain your answer.

Analysis

The first assessment determines if a participant knows the problem already and knows the correct answer (to switch). Also, it determines if a participant has basic probability memory object, which means that the probability of door number 1 containing the prize is 1/3 if the prize is randomly placed, and the probability of door number 1 not containing the prize is 2/3. The second assessment is given to determine if providing Figure 3, the line between door number 1 and 2, helps them launch the correct, or inappropriate cognitive field. The third assessment is to see if reversible thinking in the 'winning by switching' case helps them launch the right cognitive field.

Results

On the first assessment, out of 55 participants, 4 participants indicated that they knew similar problems and knew that they should switch. 2 participants who didn't know the MHP still answered both the second and the third assessments correctly. Most of the remaining 49 participants answered problem 1, 2 and 3 on the second assessment correctly. Out of these 49, 17 participants thought the odds were 50/50 incorrectly, and 28 participants correctly, on problem 4 in the second assessment. However, all these 28 participants indicated that the odds were 50/50 after the host revealed door number 3 was empty. One participant answered correctly on both assessments but didn't explain the reasoning in detail to evaluate further. However, 3 participants indicated that they didn't know the MHP or similar problems before, they stated that this is a 50/50 situation for the first and second assessments but answered correctly with the correct reasoning on the third assessment.

The Three Participants (*Table 1*)

1. Participant α stated that they didn't know the problem, and "it doesn't matter if you switch or stay because it is a 50/50 situation" on the first assessment. On the second assessment, they answered problem 1 – 5 correctly to indicate that α thinks that the probability that the prize is in one of the two doors, number 2 or 3 is 2/3, but after the host revealed that door number 3 is empty, α stated that it is still a 50/50 situation. Then, on the third assessment, α answered correctly on all the questions. α changed their mind to "switch" from "it doesn't matter, it's 50/50". α wrote that "because you first picked a door on a 1/3 chance, and then switch (to door number 2) after an empty door is shown, boost switching to the other increase the odds".
2. Participant β indicated the situation is a 50/50 on the first assessment. On the second assessment, β answered problem 1- 4 correctly. However, on problem 5, β stated that the probability that the prize is in one of doors number 2 or 3, is ½ and explained that "it is ½, or 50/50, because door 3 is empty which leaves 2 doors and a 50% chance to win the diamond". However, on the third assessment, β answered all questions correctly and explained that the probability of winning the diamond by switching is 2/3 "because switching after he (the host) has opened one of the doors actually gives me a higher chance of winning the diamond".
3. Participant γ first indicated that γ would stay with door number 1 because the host knows where the prize is and hoping to trick the contestant. γ indicated that the probability that the prize is on the right side of the line is ½ on the second assessment. γ wrote that there is ½ probability that the diamond is in either door 1 or door 2. However, on the third assessment, γ answered all problem correctly, and wrote that the reason for switching is "there is a 2/3 probability of picking an empty door first".

On the first and second assessments, participants α and β launched wrong cognitive field of 'the situation is 50/50' after the host opened door number 3. Participant γ on the other hand, launched the wrong cognitive field after the line was shown between door number 1 and 2. However, all three participants' activating the appropriate cognitive field was observed on the third assessment. This may be evidence that the questionnaire in the third assessment helped them launch the correct cognitive field.

Table 1: Analysis of the three participants α, β and γ

	Assessment 1	Assessment 2	Assessment 3

	Memory Object	Cognitive Field	Mental Model	Cognitive Field	Cognitive Field
Participant $\boldsymbol{\alpha}$	50/50 1/3:2/3 confirmed	Activated 50/50 schema	Line between Door 1 and 2&3 1/3:2/3 model	Door 3 revealed empty activated 50/50 schema	Reversible Thinking activated 1/3:2/3 schema
Participant $\boldsymbol{\beta}$	50/50 1/3:2/3 confirmed	Activated 50/50 schema	Line separates right and left 50/50 model	Activated 50/50 schema	Reversible Thinking activated 1/3:2/3 schema
Participant $\boldsymbol{\gamma}$	50/50 1/3:2/3 confirmed	Activated Tricky Host schema	Line separates right and left 50/50 model	Activated 50/50 schema	Reversible Thinking activated 1/3:2/3 schema

Discussion

The Monty Hall Problem is notorious for being mistaken for a 50/50 situation and people have difficulty accepting the solution even after being shown the correct answer. In terms of Cognitive Schema theory, most of the participants launched an inappropriate cognitive field after the host reveals one empty door, or for some, after the line is introduced to separates 3 doors into two sections, right side, and left side. Either way, the wordings of "switch or stay", "right side or left side", or "door 1 or 2" seem to trigger the 50/50 situation schema. In this study, three participants' schema revision was observed during the third assessment. The third assessment focused on reversible thinking about the 'winning by switching' case. The questions were chosen to encourage participants to focus on what would have caused (in the past) the favorable outcome in the future. This perspective may have activated the appropriate cognitive field to these 3 participants to revise their inappropriate schema (50/50 situation) to the correct schema.

This study focused on schema revision with only a written questionnaire on reversible thinking, a minimum intervention. Students' difficulty and importance in reversible thinking in mathematics has been documented (Tall & Razali, 1993; Hirashima & Kurayama, 2011; Warren, Cooper & Lamb, 2006). However, future research is needed to further evaluate the role of reversible thinking in people's schema revision. The result of this study could lead to reform in teaching and learning counterintuitive statistical or mathematical problems, and on public statistical literacy.

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