RELATIONSHIPS BETWEEN STUDENTS' MICRO-IDENTITIES AND STUDENTS' MATHEMATICS

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The purpose of this paper is to investigate the relationships between adult students' mathematical micro-identities (informed by positioning theory) and their mathematics (informed by radical constructivism). I used positioning theory to conduct a detailed turn-by-turn analysis of students' micro-identities, and to inform my models of students' mathematics. Results showed how positive micro-identities can promote persistence, vulnerability, visibility, or flourishment in students' mathematics. Implications for research and practice are discussed. The latter connects to the PMENA conference theme by highlighting the type of classroom environment teachers can foster to engage their students and maximize opportunities for learning.

Keywords: Cognition, Calculus, Classroom Discourse

Steffe and Thompson (2000) used the term "students' mathematics" to refer to the mathematical realities of students. Radical constructivist research programs have utilized models of students' mathematics to better understand students' knowledge of mathematical concepts including, but not limited to, fractions (e.g. Steffe, 2001), graphing (e.g. Moore & Thompson, 2015), speed and time (e.g. Thompson, 1994), and variation (e.g. Jones & Kuster, 2021; Saldanha & Thompson, 1998). While I acknowledge the importance of this work to the field of mathematics education, one limitation is that it does not attend to who students are, specifically in micro-level moments of interaction. I argue that attending to the micro-level identities that students enact[1] as they engage with their mathematics in moments of interaction can add valuable information to researchers' models of students' mathematics.

Conceptualizing identity at the micro-level is ideal for investigating momentary shifts in who people are (Wood, 2013). I adopt Wood's (2013) definition of *micro-identity* as "the position of a person in a moment of time" (p. 780), as well as her definition of *mathematical micro-identities* as "the subset of micro-identities that position the person relative to mathematics and mathematical activity" (p. 780). Defining micro-identities as positions is useful for investigating how micro-identities shift from moment-to-moment: say from mathematical student to menial worker (Wood, 2013), novice to expert (Esmonde, 2009), or evaluator to leader (Evans et al., 2006). The purpose of this paper is to investigate how the micro-identities that adult students enact during an online synchronous lesson are related to their mathematics. Thus, my research question is as follows: "What are the relationships between adult students' mathematical micro-identities and their mathematics?"

Theoretical Framework

Students' Mathematics and Researcher Positionality

Steffe and Thompson (2000) distinguish between students' mathematics and mathematics of students. They define *students' mathematics* as "whatever might constitute students' mathematical realities" (p. 268), and *mathematics of students* as "our interpretations of students' mathematics" (p. 268). Mathematics of students refers to the models of students' mathematics that researchers

[1] Similar to Bishop (2012), I use the term "enact" to highlight that identity is a dynamic construct.

construct by analyzing mathematical *discourse*, which I define as any interaction that utilizes written or spoken words, gestures, and representations to convey meaning (cf. Bishop, 2012). Models of students' mathematics can never be said to depict students' actual mathematical realities since it is impossible for researchers to fully separate from their own experiences, biases, and interpretations. Thus, it is important for researchers to share their positionality so that readers can then make their own inferences about how compatible these models are to students' mathematical realities.

My focus on students' mathematics is connected to my view of knowledge, which I frame from the perspective of radical constructivism (von Glasersfeld, 1995). A central tenet of radical constructivism is that "knowledge is not passively received", but rather, "is actively built up by the cognizing subject" (von Glasersfeld, 1995, p. 51). Thus, in my view, knowledge is not static; it is dynamic and constantly developing from moment-to-moment. It is this "moment-to-moment" view of knowledge that motivates my decision to construct models of students moment-to-moment mathematical micro-identities, as well as models of students' mathematics.

From a radical constructivist perspective, models of students' mathematics should include descriptions of the mental operations that underpin students' mathematical explanations and strategies (Steffe & Thompson, 2000). These mental operations are discovered by researchers as they interact directly with students via short-term clinical interviews (Goldin, 2000) or long-term teaching experiments (Steffe & Thompson, 2000). For the sake of modeling students' mathematical micro-identities in a more natural setting, I chose to interact with students indirectly in the current study by observing them as they engaged with their mathematics during a single class lesson. This made it difficult for me to make any concrete claims about students' mental operations. As a result, the models of students' mathematics that I describe are focused on descriptions of students' strategies rather than their mental operations.

Connecting Positioning Theory and Students' Mathematics

This study utilizes positioning theory—a theory that applies to discourse—to operationalize *micro-identity* as "the position of a person in a moment of time" (Wood, 2013, p. 780). Many researchers who use positioning theory draw from the work of Davies and Harré (1999) and van Langenhove and Harré (1999). These scholars noted that there are three constructs that are central to positioning theory: speech acts, positions, and storylines.

Herbel-Eisenmann et al. (2015) refined Davies and Harré's (1999) speech act construct. They used the term *communication act* to refer to the meaning that spoken words, as well as gestures, physical positions, and stances have for participants in discourse. Harré (2012) defined a *position* as "a cluster of short-term disputable rights, obligations, and duties" (p. 193). He defined *positioning* as "a process by which rights and duties are assigned, ascribed, or appropriated and resisted, rejected, or repudiated" (p. 196). Here, *rights* refer to "what you (or they) must do for me" and *duties* refer to "what I must do for you (them)" (p. 197).

Storylines have been given minimal attention in the literature compared to positioning (Herbel-Eisenmann et al., 2015), and have been conceptualized vaguely (Kayi-Aydar & Miller, 2018). Part of the complication with the conceptualization of storylines is that at any given moment, "multiple storylines exist and may even be embedded in broader sets of discursive conventions" (Herbel-Eisenmann et al., 2015, p. 191). Thus, it is important for researchers to clarify the scale at which they are conceptualizing storylines (Herbel-Eisenmann et al., 2015). In the current study, I am focused on storylines that occur at the micro-level of classroom discourse.

Given my positionality as a radical constructivist researcher, I view a *storyline* as a model of the story that is being constructed by participants in discourse as they engage with positioning and communication acts. The term *model* is used to highlight the fact that storylines are based on the experiences, biases, and interpretations of the researcher. The storylines in the current study are constructed specifically from mathematical discourse, and hence, they contain my models of students' mathematics and mathematical micro-identities. To illustrate my conceptualization of the relationship between communication acts, positioning, storylines, and students' mathematics, I extend Herbel-Eisenmann et al.'s (2015) dynamic positioning theory framework (see Figure 1).

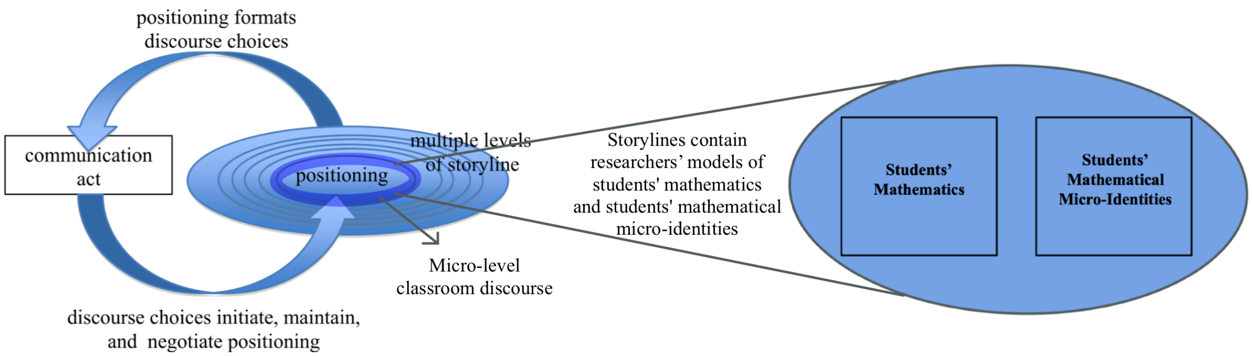

Figure 1: Extension of Herbel-Eisenmann et al.'s (2015) Positioning Framework (p. 194)

Methods

The discourse analyzed in this study involved four students who were enrolled in a 10-week mathematics course titled "Concepts of Calculus for Middle School Teachers" at a public university in the United States. I attended the course as a graduate student mentor, whose responsibilities included assisting the instructor with lesson planning, facilitating discussion during group work, and teaching one lesson independently. Due to the lingering effects of the Covid-19 pandemic, the instructor taught the course remotely through Zoom in an online synchronous format. A major goal of the course was to improve students' covariational reasoning abilities (Carlson et al., 2002).

There were eight students enrolled in the course: six white women and two white men. Three women—Sam, Kelly, and Eden—and one man, James, were selected to participate in this study (pseudonyms were used). Selection was based on convenience as these were the only students who consented to being video recorded for research purposes. Kelly and James were both in-service teachers who had prior exposure to calculus. Eden and Sam were both prospective teachers who had not previously completed calculus courses.

The data for this study was made up of one 67-minute video recorded lesson that included the four participating students. The recorded lesson was chosen because it involved a variety of five different math tasks. The primary focus was on the four students' work within a Zoom breakout room. In this paper I focus on modeling the mathematics and micro-identities of one student, Sam, as I inferred the most growth in her mathematics during the lesson.

Analysis

I created a transcript that was separated by speaker turns (Wood, 2013), and then split the transcript into five episodes (Erickson, 2006). Each episode corresponded to one of the five math tasks in the lesson. I focused my analysis on three of the five episodes (2, 3, and 5) because they involved graphical tasks with potential for engendering students' covariational reasoning.

I conducted a turn-by-turn analysis of each episode by placing each turn in its own row of a single column in a Google spreadsheet, and then adding on additional columns for separate components of my analysis (e.g. gestures, communication acts, micro-identities, etc.). Similar to Wood (2013), I marked a turn as relevant if it involved mathematical discussion. For each relevant turn, I then engaged in four phases of analysis (see Figure 2). First, I replayed the video recording several times to document the verbal and nonverbal behaviors that were exhibited by each of the four participating students. Second, I documented my interpretations of the communication acts that stemmed from these behaviors. Third, I used the information from the first two phases to document my interpretations of the mathematical micro-identities that were being assigned by the speaker, both to themself and to the other students. Fourth, I documented my interpretation of the storyline that was being constructed by the participating students.

The storylines contained my models (i.e. interpretations and descriptions) of students' mathematical micro-identities and students' mathematics; these models were informed by my interpretations of the verbal and nonverbal behaviors, and communication acts. Although my analysis of storylines occurred turn by turn, I also combined storylines across multiple turns for the sake of modeling more detailed stories. Lastly, after completing the turn-by-turn analyses, I scanned each of the analyzed episodes holistically by looking at how the storyline was constructed across all of the turns. This allowed me to hone in on Sam in order to highlight important relationships between Sam's mathematical micro-identities and Sam's mathematics.

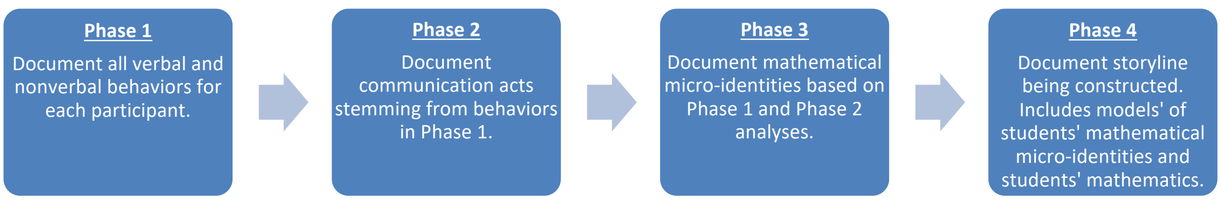

Figure 2: Four-Phase Analysis for Relevant Speaker Turns

Results

In the following sections, I highlight three transcript excerpts that correspond to one episode of the lesson. I focus on one episode for the sake of space, and because this episode was critical for Sam in that it promoted the flourishment of her mathematics. Each turn in the lesson transcript was numbered, resulting in a total of 291 turns for the entire lesson. The focal episode

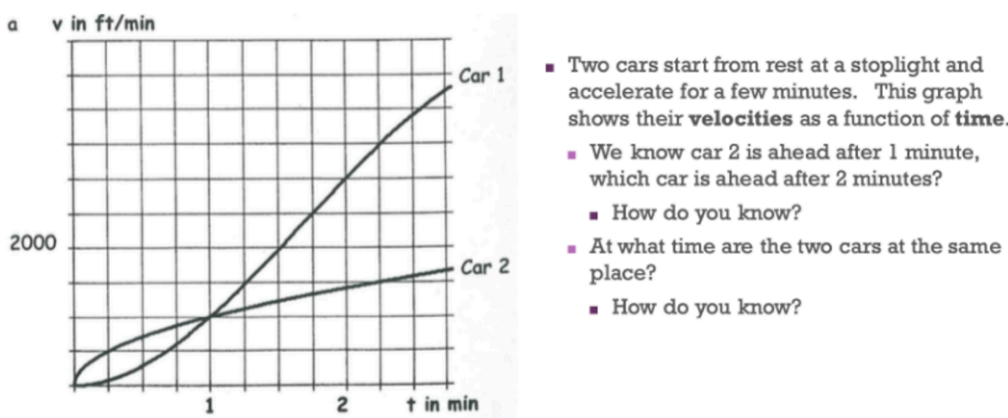

Figure 3: Part 2 of "Truckin'…" Task

began approximately halfway into the lesson at Turn 137 and ended approximately two-thirds of the way into the lesson at Turn 198. For context, the students were working primarily on the second part of the "Truckin'…" task throughout the episode (see Figure 3).

Micro-Identities of Capability In Mathematics

The students started this episode by discussing part one of the "Truckin'…" task, which involved determining which car would be ahead after one minute. Then, the instructor (who had joined their breakout room) challenged them to move on to the second part of the "Truckin'…" task and think about which car would be ahead after two minutes. Kelly, James, and Eden were attempting to arrive at a solution to the problem but were having little success. During this time (Turns 156-171), Sam was not speaking. She appeared to be thinking, however, as she was writing on something that was not visible on the Zoom camera, and also shifting back and forth between her computer screen and her written work. Sam shared her thinking with the rest of the group, but only after Kelly and Eden had given up (see Table 1).

Table 1: Transcript Excerpt 1: Sam Shares her Preliminary Thoughts

Turn	Speaker	Transcript[2]
169	Eden	Also, this is such a wild time for a class too, cause I feel so brain dead, and I feel like I could have half of a thought, and my mind is just like "boom", it's like elevator music like when you click a link and it makes that sound .. I:, Yeah, I think my .. we're only an hour in and / / it's fading fast ....
170	Kelly	Yeah I feel like I just hit a wall with this question / / and my brain just sort of shut off.
171	Eden	It was like we stopped talking for like 30 seconds, and my brain was like, well that's it, you've had all the thoughts you're gonna have today (giggles).
172	Sam	I don't know if this is the right way to go about it but / / right now I'm like estimating .. like where it's at on the y-axis, and honestly just adding it together one by one? Which I don't know .... / / if that's the way to go about this. But that's, currently what I am doing right now.

Eden's mental exhaustion ("I feel so brain dead…") contributed to her enacting a micro-identity (position) of incapable math student (Turn 169) as she was unwilling to persist in thinking about the problem (i.e. which car would be ahead after two minutes). Kelly accepted this micro-identity of incapable math student (Turn 170) by also declining to persist with the problem. Sam, on the other hand, was unwilling to give up on thinking about the problem (Turn 172). By continuing to persist, Sam rejected the micro-identity of incapable math student that was initially offered by Eden, and instead enacted a micro-identity of capable math student.

Storyline: Persistence and vulnerability in Sam's mathematics. The negative micro-identity of incapable math student that Eden and Kelly enacted was associated with a lack of persistence.

[2] Transcript conventions taken from Temple and Wright (2015):

..	noticeable pause, less than 0.5 seconds
…	half-second pause; each extra dot represents additional half-second pause
:	lengthened sound (extra colons represent extra lengthening)
/ /	slash marks indicate uncertain transcription or speaker overlap
[XX] [XX]	overlapping brackets indicate two speakers talking at the same time

Eden felt “brain dead” (Turn 169) and Kelly felt like her “brain just sort of shut off” (Turn 170). This caused them to disengage entirely with their mathematics. Rather than following suit, Sam enacted the positive micro-identity of capable math student and persisted with the problem by sharing her thinking with the other students.

Sam’s strategy involved estimating points on the y-axis, and then adding them together. At this point in the episode, it was not clear whether Sam was aware that the y-axis represented velocity, nor was it clear how she was coordinating her y-axis estimates with their corresponding values on the x-axis (time). It was clear, however, that Sam was not confident in her strategy as she admitted that she was not sure whether her strategy was the correct approach (Turn 172). Thus, persisting with the problem meant that Sam had to allow herself to be vulnerable to critique. This indicates that the micro-identity of capable math student is associated with both persistence and vulnerability in one’s mathematics.

Supportive Mathematical Micro-Identity

A few turns after Sam shared her preliminary thoughts on the problem (Turn 172 in Table 1), Kelly asked Sam if she had ever taken calculus (Turn 175 in Table 2), thus positioning her as a calculus student. Sam rejected the micro-identity of calculus student (“I am doing this without a calculus mindset”), and instead enacted a micro-identity of precalculus student (Turn 176). Kelly then said that she was “very excited” (Turn 177), presumably for Sam to share her completed strategy with the group. The encouraging nature behind the excitement that Kelly exhibited towards Sam’s strategy suggested that she was enacting a micro-identity of supportive math student. Sam began to doubt herself (“I don’t know whether that’s a good thing or a bad thing”), and enacted a micro-identity of unconfident math student (Turn 178). Kelly, however, eased Sam’s doubt by reassuring her that “it’s awesome” and encouraging her to continue, thus maintaining the micro-identity of supportive math student (Turn 179).

Table 2: Transcript Excerpt 2: Kayla Supports Sam’s Thinking

Turn	Speaker	Transcript
175	Kelly	Okay Sam, just like a super quick question. I don't want to interrupt you, I'm just very curious. Have you taken calculus before? ...
176	Sam	Umm::, I took Precalculus in high school, [and that is as far as I've gotten. So I am doing this without a calculus mindset.
177	Kelly	[Okay .... Okay cool ...... Cool. Okay, I'm very excited. That's all, I'm done.
178	Sam	I don't know whether that's a good thing or a bad thing, but / /
179	Kelly	No it's awesome, it's awesome. Keep going, please.

Storyline: Supporting Sam’s mathematics. When Kelly assigned the position of calculus student to Sam (Turn 175), it was not clear whether she intended this to support Sam’s mathematics. In fact, after Sam rejected the micro-identity of calculus student (Turn 176) she began to doubt herself (Turn 178), which led her to enact a micro-identity of unconfident math student. Kelly, however, reassured Sam that her non-calculus background was “awesome” (Turn 179), making it evident that she had initially positioned Sam as a calculus student because she valued her knowledge, and thus, was supporting her thought process. This indicates that Kelly was positioning herself as a supportive math student from the outset of this excerpt. Although Sam did not explicitly share her mathematics in this excerpt, the micro-identity of supportive math student that Kelly enacted allowed Sam to continue working on her solution strategy, albeit out of view of the others. After Kelly told Sam to "Keep going" (Turn 179), Sam said "Okay" and then spent a little over two minutes moving her gaze back and forth between her computer screen and her written work (outside the view of the camera). Thus, the micro-identity of supportive math student can contribute to the flourishment of students' mathematics.

Micro-Identity Of Mathematical Explainer

After working intently on the problem for over two minutes, Sam told the other students that Car 2 was ahead of Car 1 by fifty yards at the two minute mark. But Sam did not have time to explain her thought process to her group members because the instructor had closed the breakout rooms. She was, however, selected by the instructor to share her solution strategy with the entire class. Table 3 depicts the final turn in the episode (Turn 198) where Sam shared her computer screen through Zoom, and then enacted a micro-identity of mathematical explainer by explaining her completed solution strategy to the whole class.

Table 3: Transcript Excerpt 3: Sam Explains her Solution Strategy

Turn	Speaker	Transcript
198	Sam	Okay .... Umm:, so I .... went to here (pointing to the quarter minute mark on the t-axis) and then, like for car:: 2 for example, right here (pointing to the output of car 2 at a quarter minute), I knew that the: velocity was uhh:, 500 .. umm: because each of these (pointing to the tick marks on the v-axis) represents 500 feet per minute. And umm:, I went through and like added 500 (pointing to the output of car 2 at a quarter minute), plus umm:, 525 (pointing to the output of car 2 at a half minute), plus 575 (pointing to the output of car 2 at three-quarters of a minute), umm: plus right here is 1000 (pointing to the output of car 2 at one minute), and so on up to 2 minutes. And then I did the same thing for car 1 up to 2 minutes (slides pointer along the graph of car 1 until she reaches the output at 2 minutes). And umm:, these are all estimates cause I can't see the exact points. But, through doing that, I got that they were only 50 yards apart after two... (gets interrupted by Instructor)

Storyline: Sam explains her mathematics. By enacting a micro-identity of mathematical explainer (Turn 198), Sam was able to explicitly reveal the thought process that was directly linked to her mathematics. In other words, she was able to make her mathematics visible to the whole class. This indicates that the micro-identity of mathematical explainer is important in making students' mathematics visible.

Turning now to Sam's solution strategy (Turn 198), she began by estimating the distance traveled by Car 2 after two minutes. To do this, she estimated the velocity of Car 2 that corresponds to each tick mark (up to two minutes) on the t-axis: 500 corresponds to the first tick mark, 525 corresponds to the second tick mark, 575 corresponds to the third tick mark, 1000 corresponds to the fourth tick mark, etc. As Sam was estimating these values, she was also

simultaneously adding them together one by one. She then did the same thing for Car 1 and somehow determined that the two cars would be 50 yards apart after two minutes. It was not clear what Sam did with the velocities that she added together for each car to obtain her final result, nor was it clear what she interpreted the quantities of these summed velocities to be. In addition, it was not clear whether Sam envisioned the cars taking on velocities in between values that correspond to the tick marks on the t-axis. The thinking she displayed was primarily discrete and did not contain evidence of continuity across the quantities' values.

There are two important things to note about Sam's explanation. First, she did not attend to the fact that the t-axis was partitioned into intervals of quarter minute length. Doing so would have required her to divide each estimated velocity for Car 2 (500, 525, 575, 1000, etc.) by four as these velocities correspond to each minute on the t-axis, rather than each quarter minute. Second, Sam's final solution of 50 yards did not have the correct units of feet. Since one of the earlier tasks the students worked with in this lesson involved units of yards, it is possible that Sam conflated the units in the "Truckin'…" task with the units of this earlier task. Ultimately, however, the structure of Sam's solution strategy was correct. Had she attended to the units in the problem, and to the way she was coordinating her velocity and time values, her final answer would have been "correct" by normative standards.

Discussion

This study focused on investigating the relationships between the mathematical micro-identities and mathematics of one student, Sam, during a single online synchronous mathematics lesson. Thus, the answer to my research question—"What are the relationships between adult students' mathematical micro-identities and their mathematics?"—is based on this context. The results showed three important relationships between the mathematical micro-identities enacted throughout the episode and Sam's mathematics.

First, the micro-identity of capable math student allowed Sam to continue persisting with the problem (i.e. which car was ahead after two minutes) even though (1) the other students had given up on the problem and (2) she was not confident in her solution strategy. This suggests that capable mathematical micro-identities can help students persist with their mathematics, and also exhibit vulnerability in their mathematics. The latter relationship is interesting as it is more common for students to associate capability in mathematics with confidence and correct solutions, rather than vulnerability. Second, the micro-identity of supportive math student that was enacted by Kelly motivated Sam to continue working on and refining her solution strategy, suggesting that supportive mathematical micro-identities can promote the flourishment of students' mathematics. Third, the micro-identity of mathematical explainer allowed Sam to share her thinking with the rest of the class, suggesting that mathematical micro-identities involving explanations and thought processes make students' mathematics visible.

The results of this study have two important implications: one for researchers and one for teachers that connects to the PMENA conference theme. First, teacher-researchers that conduct future teaching experiments (Steffe & Thompson, 2000) should consider the ways they are positioning students, as well as the way students are positioning each other, as these positions can give rise to micro-identities that can add more nuanced descriptions to their models of students' mathematics. Second, mathematics teachers should consider fostering a classroom environment that (1) positions all students as capable, (2) provides a safe space for students to exhibit vulnerability in their mathematics, (3) encourages students to support one another, and (4) allows students the opportunity to share their explanations and thought processes. This implication connects to the PMENA conference theme because teachers who foster such classroom environments can promote the enactment of positive micro-identities that can engage students with their mathematics in a variety of ways: say by persisting with their mathematics, exhibiting vulnerability toward their mathematics, or explaining their mathematics to others. Engaging students with their mathematics, in turn, maximizes the opportunities for learning.

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