THE LEARNING THROUGH ACTIVITY DESIGN FRAMEWORK: THE FRAMEWORK IN ACTION

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In this theoretical paper, I use an empirically based example to illustrate particular design features of the Learning Through Activity (LTA) design framework and examine the impact of particular design principles. The LTA design framework is based on our elaboration of Piaget's construct of reflective abstraction. The example discussed here, involving the learning of a fraction concept, contains both an unsuccessful attempt, not based on the LTA framework, and a subsequent successful attempt, based on the framework. I use this contrast to make theoretical distinctions with regards to designing for the learning of mathematical concepts.

Learning Theory, Learning Trajectories and Progressions, Instructional Activities and Practices,

In this report, I use data from a teaching experiment to demonstrate the unique features of the Learning Through Activity (LTA, Simon, Kara et al., 2018) design framework. The data segment presented here shows two attempts to teach a particular fraction concept. Only the second attempt was informed by the LTA design framework, so the contrast presents a useful context for explication of the design theory. This is a theoretical report; the goal is not to report on data, but to use them to elucidate particular affordances of the LTA design framework.

Development of theoretical frameworks for instructional design is critical to significant improvement in mathematics instruction. Koedinger (2002) commented on the need to develop theories of mathematics learning and instructional design that build on more general learning theories.

> There is a significant gap between theories of general psychological functions on one hand … and theories of mathematical content knowledge on the other …. To better guide the design of groundbreaking and demonstrably better mathematics instruction, we need instructional principles and associated design methods to fill this gap in a way that is not only consistent with psychological and content theories but prompts and guides us beyond what those theories can do. (p. 21)

The LTA research program focuses on filling the theoretical gap indicated by Koedinger. Whereas the construct of reflective abstraction has been around for decades, design frameworks grounded in reflective abstraction are generally lacking. DiSessa and Cobb (2004) wrote, "Piaget's theory is powerful and continues to be an important source of insight. However, it was not developed with the intention of informing design and is inadequate, by itself, to do so deeply and effectively" (p. 81). LTA researchers have elaborated Piaget's (2001) construct of reflective abstraction to explain mathematics conceptual learning and developed instructional design principles based on the elaborated construct (Simon, Kara, et al., 2018).

Why This Research Report

Although the LTA framework has been reported on previously, this research report can serve a particular function. In laying out the framework (Simon, Kara, et al., 2018), our emphasis was on specifying the design principles. In this report, I focus on the impact of the design framework and how the design principles are specifically implicated in that impact. A secondary function of this report is that it can offer an image of reflective abstraction (as opposed to just a claim that reflective abstraction took place). Because the LTA design framework can promote the process of reflective abstraction, not just occasion it, there is greater access to observing the process.

Theoretical Framework

A principal goal of LTA has been to improve capacity to design for *guided reinvention* of mathematical concepts, an idea developed by Freudenthal (1991). In Simon, Kara et al. (2018), we elaborated the construct and explained that "reinvention" refers to the opportunity for students to come to mathematical ideas through their mathematical work and "guided" indicates a well-planned sequence of tasks to support the reinvention.

To explain conceptual learning, we have built on and further elaborated Piaget's (1980) construct of reflective abstraction. Piaget characterized reflective abstraction as a *coordination of actions* (p. 90). We have explained the coordination of actions that produces mathematical concepts as follows. The process begins with students solving tasks that they can solve with their available concepts. For a given task, they set a goal and call on a sequence of available actions to achieve their goal. This goal and sequence of actions is what we refer to as an "activity." An activity is the precursor to a new concept. Reflective abstraction is the process by which the actions that make up the activity (or a subset of those actions) are coordinated into a single higher-level action. The result of this coordination is that the learners no longer need to go through the original sequence of actions. The abstraction allows the learners to anticipate the result of the previously enacted activity. (This is a brief and incomplete discussion of our elaboration of reflective abstraction. Readers are referred to Simon et al., 2016 and Simon, Kara et al., 2018.) This process will become clearer in the illustration provided by the data excerpt discussed below.

The five-step LTA design framework is based on this elaboration of reflective abstraction. Steps 1, 2, and 5 are common to many design approaches. Steps 3 and 4 represent the unique aspects of the LTA design framework.

Step 1: Specify prior knowledge of the (actual or hypothetical) students.

Step 2: Identify specific conceptual learning goals for the students.

Step 3: Specify an activity (sequence of actions) available to the students that could serve as the raw material for the intended abstraction.

Step 4: Generate a sequence of tasks designed to elicit the activity specified in Step 3 and promote reflective abstraction. Sometimes the tasks that elicit the activity are sufficient to promote the abstraction. In some cases, subsequent tasks are created that restrict the student's ability to carry out the sequence of actions in the activity – prompting the students to use developing anticipations.

Step 5: Symbolize, introduce vocabulary, and/or promote institutionalization of ideas.

Note that a review of prior work on reflective abstraction is not included in this report for two reasons. First, the focus of this article is not the LTA learning theory, but rather the design framework and its impact. Second, a review of that literature is available in the article in which the theory was elaborated (Simon, Kara, et al., 2018).

The Research-Based Example

The research-based example presented here focuses on what is commonly referred to as "fraction of a set" when the quantities in question are discrete quantities. In the teaching experiment from which this example derives, our goal was not restricted to discrete quantities and discrete quantities were not employed in the task sequences. Our goal was to promote abstraction of the *process of taking a fraction of a whole number for tasks in which the result is a whole number*. This is a modest instructional goal. I have selected the example because it allows us to examine carefully a single theoretically-based instructional design, which would not be a possibility with a more complex goal.

Although this is a theoretical article, I offer some brief comments on the teaching-experiment context from which the example derived. The research discussed here was part of the Measurement Approach to Rational Numbers (MARN) Project, which used LTA's adaptation of the single-subject teaching experiment (Simon et al., 2010, Simon, 2018) to promote a range of fraction concepts. Because the research focused on students' development of concepts *through their mathematical activity*, we used one-on-one teaching experiments, to avoid the confounding factor of input from other students. Also, the researcher conducting the teaching experiment avoided using demonstrations, suggestions, hints, and leading questions. These were research-methodological decisions.[3] Line-by-line analysis was conducted to build up models of the students' mathematical understandings and how the understandings changed (described in depth in Simon, 2018). The data discussed here are from our 2-year teaching experiment with Kylie and are from her 5th grade year (10 years of age), her second academic year in the project.

THE EXAMPLE[4]

Prior to the work discussed here, Kylie had developed a concept of fraction-as-measure (Simon, Placa, Avitzur, & Kara, 2018), which included an understanding of improper fractions. That is, she could think about a unit fraction as a partial unit that measures the unit a particular number of times, and a non-unit fraction as a quantity created by iterating a unit fraction.

Preassessment related to the example presented here showed that she was unable to solve tasks such as ¾ of 12. We originally expected that Kylie's learning to conceptualize this type of task would be a trivial first step towards the more challenging goal of reinventing fraction multiplication. Because of this expectation, we initially approached this part of our teaching experiment in a fairly typical way, uninformed by the LTA design theory.

We created a simple task sequence in which Kylie first found a unit fraction of a whole number (e.g., 1/3 of 15). She demonstrated the ability to do such tasks the first time she encountered them. We followed these tasks with tasks that asked her to find a non-unit fraction of a whole number, which she was generally able to do. Given Kylie's success with tasks involving non-unit-fraction of a whole number when they immediately followed tasks involving a unit-fraction of a whole number, we needed to determine if learning had occurred. To assess learning, we posed tasks in subsequent sessions involving a non-unit-fraction of a whole number, this time *not* preceded by tasks involving a unit-fraction of a whole number. Kylie showed no ability to solve these tasks. We repeated the task sequence, but subsequent assessment again revealed that we had failed to promote the intended learning.

At this point, we began to take more seriously the challenge of promoting the concept in question and applied the LTA design framework described above. This resulted in creation of a new hypothetical learning trajectory (HLT, Simon, 1995). The HLT included use of the computer application, JavaBars (Biddlecomb & Olive, 2000), which we had adapted and used with Kylie in developing prior concepts. In our version of JavaBars, students can create a bar of any length, equipartition the bar, pull out a part (disembed), and iterate a bar or a part of a bar to make a composite bar. Kylie's use of the Java Bars application is demonstrated in Figure 1.

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[3] Some of what we omit from our teaching experiments would be important in classroom instruction (e.g., group work, class discussions).

[4] The empirical results were briefly discussed in Simon, Placa, Kara, & Avitzur (2018).

Consistent with the LTA design framework, the initial tasks were designed so that Kylie would be able to solve them *using her available concepts*. We began with the following task (each task presented here is representative of a small set of similar tasks that was employed):

> Here's a bar. It's 10 units long. Show me three-fifths of the bar. How many units long is three-fifths of the bar?

The task design was based on our expectation that Kylie, using JavaBars could create 3/5 of a bar calling on concepts she had previously constructed and demonstrated. To do so, she would partition the unit bar into five parts, pull out one part and iterate that part three times. (In JavaBars, the 5-part bar remains on the screen after a part is pulled out.) She could then figure out how many units in each of the five parts using whole-number division. Finally, she could evaluate the size of the new (3/5) bar using whole-number multiplication. (She had previously demonstrated such competence with whole-number multiplication and division.) This is in fact what happened as evidenced in the transcript that follows (see Figure 1 for Kylie's work with JavaBars).

K: [Cuts the bar into five parts, pulls out a part, and iterates the part three times]
R: How long is the new bar?
K: It's six units long.
R: Explain that.
K: Two times five is ten and then if you repeat two –
R: Tell me about the bars.
K: I cut it up into fifths, and if you want to know how much each piece equals – two units.
R: How do you know?
K: Two times five is ten.
R: What is it about the bar?
K: Ten units.
R: Why do you say, "Two times five?"
K: It's easier than saying, "Five plus five."
R: What is the five?
K: How many pieces I cut the unit into.
R: What is the two?
K: The number of units in each piece.
R: How did you get six?
K: I looked at each of the pieces and counted by twos or two times three is six.

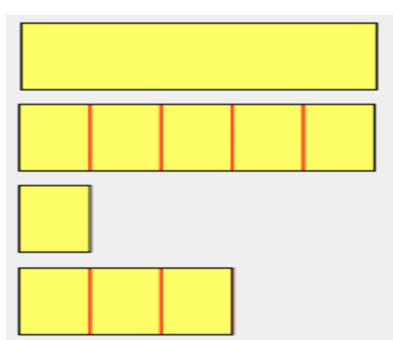

Figure 1: Kylie's use of JavaBars for 3/5 of 10.

Initially, when she was working with the bars, Kylie was solely focused on creating the 3/5 bar from the unit bar. She performed the computational actions once the work with the bars had been completed.

However, this sequence of actions began to change, and, for subsequent tasks, Kylie began to anticipate the calculations without using the bars.

Here's a bar. It's 18 units long. How many units long is 2/3 of the bar?

K: [long pause] Twelve units.
R: How did you get that?
K: Six times three –
R: [Interrupting] Where did the six times three come from?
K: Six comes from the third that I cut the eighteen up into. I know that six times three is eighteen. So, I figured six plus six equals twelve and since two-thirds and you wanted to know how many units was in two-thirds – well six plus six units.

After tasks of this type, Kylie was given tasks without reference to the bars:

What is 5/6 of 12?

She responded, "Ten units." Whereas the HLT seemed to be producing positive results, the real test (as it was in our earlier attempts to foster the concept) was whether she could solve computational tasks (with understanding), such as the latter task, when they were posed without the earlier tasks preceding them (i.e., those involving manipulating bars). In a subsequent session, Kylie was given the following as the first task of the session:

What is 2/3 of 15?

K: 10 units.
R: How did you get it?
K: … you are talking about fifteen units. So, two-thirds of fifteen units is ten. Since there are – Three-thirds are a whole. So that's fifteen, and five times three – So the five are the thirds, and there are two of the fives, so ten.

The learning evidenced by this task proved to be stable as Kylie continued to solve similar assessment tasks in subsequent sessions.

Analysis of Kylie's Learning

Here I summarize the results of our analysis of these data. Initially, Kylie solved the tasks using an available activity. It is characterized in the following four actions. (Numbers reflect the first task in the transcript above (3/5 of a 10-unit bar).

3. Partition the bar and pull out a part (partition into 5 parts and pull out 1 part).
4. Iterate the part to create the fractional bar (iterate the part 3 times to make 3/5 of the bar).
5. Use whole-number multiplication/division to find the length of each part (10 units divided by 5 parts).
6. Use whole-number multiplication to determine the length of the new bar (2 parts/unit times 3 parts).

In the problem "2/3 of an 18-unit bar", Kylie began to coordinate pairs of actions from the activity above–Actions 1 and 3 and Actions 2 and 4. The coordination of each pair of actions resulted in a single higher-level action. Kylie's statement, "Six comes from the third that I cut the eighteen up into," suggests the merging of Steps 1 and 3. She was beginning to think about partitioning the bar and dividing the quantity as a single action; she was coming to see that she was partitioning the 18-unit bar into three parts of length six units. Similarly, she was seeing the iteration and multiplication as a single step, iterating a six-unit part two times creating a 12-unit bar. The coordination of these two pairs or actions led to Kylie being able to anticipate the solution to such tasks as resulting from two calculation steps, the abstraction we were trying to foster.

The positive results with respect to Kylie's learning indicated that this second HLT had the intended impact. This HLT was a three-part process represented by the three tasks discussed above.

Conclusions

The example presented here illustrates application of the Learning Through Activity (LTA) design framework, the process of Kylie's learning, and the nature of the resulting abstraction.[5] In this section, I examine how the design principles contributed to Kylie's learning. I also use the comparison with our first, unsuccessful task sequence to further clarify important theoretical distinctions about designing for conceptual learning.

The successful second HLT described above and the results of its use with Kylie demonstrated application of the LTA theoretical framework, a framework composed of an elaboration of reflective abstraction and a set of design steps based on that elaboration. In this example we see the following:

- The goal Kylie set (related to solving the first task) and the activity she called upon to solve that task were available to Kylie given her knowledge at the outset of this instructional unit. Further, Kylie's learning did not necessitate Kylie setting a goal of finding/learning a method of calculation, nor was it necessary for the researcher/teacher to announce such a goal. Her learned anticipation (abstraction) emerged through her work with the task sequence without her having intended to achieve that outcome.
- The coordination of actions that led to the abstraction was a product of Kylie's activity. Note, Kylie was readily able to solve the tasks (using JavaBars) from the outset. What changed was her insight that allowed her to avoid going through the original sequence of actions, replacing them with an abstracted two-step computation method, a new activity composed of higher-level actions.
- Although, by the end, Kylie could quickly compute the result, she did so with knowledge of the logical necessity (Simon, 2017) of what she was doing. When asked, she demonstrated the ability to explain the quantities involved and why her computation made sense.
- As stated earlier, the unique aspects of the LTA design framework are Steps 3 and 4. We can see that Step 3 was executed successfully in the design of this HLT. Step 3, "Specify an activity (sequence of actions) available to the students that could serve as the raw material for the intended abstraction," has two parts. We have already concluded that the activity was *available* to Kylie. The activity also proved to be the raw material for the abstraction. Pairs of actions in the four-step action sequence were coordinated to produce the goal reasoning (concept). Thus, the activity was the raw material for the coordination of actions (reflective abstraction).
- Design Step 4, "Generate a sequence of tasks designed to elicit the activity specified in Step 3 and promote reflective abstraction," was also executed successfully. We can see that the

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[5] The data do not allow for a claim about the effectiveness of the approach to a classroom of diverse students, which requires further study.

initial tasks elicited the expected activity. Further, the modified tasks (represented by the second and third task in the example) promoted the coordination of actions.

A key question raised by the example presented is. why did the second HLT work and the first did not? Our original unsuccessful approach to promoting the concept involved beginning with unit fractions of a whole number and then moving to non-unit fractions of a whole number. Although Kylie was able to solve the tasks when the latter tasks followed the former, she did not demonstrate a sustained learning effect.

The HLT developed subsequently contrasted with the original approach in an important way. Instruction did not begin with unit-fraction tasks. Rather the HLT began with JavaBars activity involving non-unit fraction tasks–tasks which Kylie could do prior to any learning taking place. By engaging in several of these tasks and the modified tasks represented by the second and third tasks above, Kylie began to coordinate pairs of actions. At a certain point, she no longer needed the bars and could compute the result quickly. Thus, the second HLT began with tasks similar to the target tasks (i.e., non-unit fractions). Whereas the first HLT proceeded from a less difficult to a more difficult computational task, the second HLT proceeded from a less to a more sophisticated solution to similar tasks. As explained, this change in the sophistication of the solution was the result of a coordination of actions. So why is this distinction important?

Having contrasted the two approaches, I answer this question by returning to explanation of the lack of success of the first and the success of the second. In the first approach (in hindsight), the posing of tasks involving unit fractions immediately prior to posing tasks involving non-unit fractions served to break down the requisite two-step process into two one-step processes. Another way of saying this is that the task sequence allowed Kylie to associate the non-unit -fraction tasks with the unit-fraction tasks. However, the results proved that she did not learn to anticipate the two-step process. When given just the non-unit-fraction tasks, she did not think to associate them with the unit-fraction tasks.

The second approach fostered coordination of actions, actions that were part of the activity that Kylie called on in the initial non-unit-fractions tasks done with the Java Bars application. This coordination of actions resulted in anticipation of the calculation process–a learned anticipation that proved to be stable in assessments in subsequent sessions. That is, the coordination of actions (reflective abstraction) changed her assimilation of the tasks–she viewed the tasks differently. This proved more powerful and more lasting than needing to recall an association between problem types.

The example that I have presented and analyzed demonstrates the value of fostering abstraction through coordination of actions. Further it demonstrates how the LTA design framework, in particular its unique aspects represented by Steps 3 and 4, can inform the generation of HLTs that foster particular coordination of actions.

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