SUPPORTING PRESERVICE SECONDARY MATHEMATICS TEACHERS TO DEVELOP THEIR TPACK KNOWLEDGE

Enrique Galindo
Indiana University
egalindo@iu.edu

Mehmet Fatih Öçal
Ağrı İbrahim Çeçen University
fatihocal@gmail.com

Selim Yavuz
Indiana University
syavuz@iu.edu

Secondary mathematics preservice teachers enrolled in a methods class at a large Midwestern University were supported to develop their TPACK knowledge. Instructional tasks included engaging with explorations using technology to support reasoning and sense making, creating entries for a technology portfolio, studying theoretical constructs about different ways to use technology, and teaching mathematics lessons that integrate uses of technology. Portfolio entries were analyzed to look for the types of technology selected as well as for the ways in which technology was used. Preservice teachers included a variety of digital technologies in their portfolio entries, but most of the uses described aligned with ways to use technology to support reasoning and sense making. Findings suggest that the course helped preservice teachers develop their TPACK knowledge.

Keywords: Technology; Preservice Teacher Education

The use of technology in the teaching and learning of mathematics has been considered essential for many years (NCTM, 2000). Recommendations for the preparation of mathematics teachers call for coursework that provides opportunities for preservice teachers (PSTs) to effectively use technology to engage in mathematics and statistics concepts, deepen their understanding of mathematics, and apply mathematical ideas (NCTM & CAEP, 2012; NCTM & CAEP, 2020). And that these opportunities should be present in all content areas as well as in the Mathematical Practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Furthermore, PSTs should experience coursework that helps them to become proficient with tools and technology designed to support mathematical reasoning and sense making in their future teaching (AMTE, 2017). This flexible knowledge that is needed to effectively teach with technology is known as Technological Pedagogical Content Knowledge or TPACK (Mishra & Koehler, 2005; 2006). This report describes an experience in which secondary mathematics preservice teachers were supported to develop their TPACK knowledge in the context of a secondary mathematics course.

Theoretical Perspectives

Knowing that teaching is a complex task, and that teachers and educators need guidance on how to integrate technology into the teaching of mathematics, a teacher knowledge framework for technology integration known as TPACK or Technology Pedagogy and Content Knowledge (Koehler & Mishra, 2013; Mishra & Koehler, 2005; 2006) was developed. The framework posits that effective teaching with technology requires teachers to go beyond knowledge of each of the three components of content, pedagogy, and technology, to develop a new kind of flexible knowledge that also considers the ways in which these domains interrelate (Koehler & Mishra, 2013).In order to have indicators of how the PSTs in this study were developing their TPACK knowledge, we documented the types of tools they selected and the ways in which they were using these tools as PSTs were exploring or teaching mathematics with technology. Tools and their use were classified according to two frameworks described below.

Researchers have classified uses of technology in different ways (Dick & Hollebrands; 2011; Hoyles & Noss, 2003; 2009; Taylor, 1980). From the perspective of planning to teach mathematics we found Dick and Hollebrands' classification relevant. They classify technologies in two categories;(Koehler et al., 2013) (i) tools for the transmission/presentation of mathematical knowledge that they call *conveyance tools/technologies*) and (ii) tools for doing mathematics called *mathematical action tools*. Teachers planning to teach with technology would benefit from choosing mathematical action technologies in ways that support reasoning and sense making in mathematics class. Teachers would also benefit from using conveyance technologies in ways that support reflection and collaboration as well as monitoring and formative assessment of students thinking in mathematics.

Another classification was proposed by Pea. Pea (1985; 1987) classified the use of mathematical action or cognitive tools in two ways; (i) amplifier and (ii) reorganizer. A tool is considered an amplifier when it helps conduct a task in a faster or more efficient way, compared to doing the same task with paper and pencil only. On the other hand, the purpose of reorganizers for in-class activities is to develop students deeper understanding of mathematical concepts which cannot be done or is difficult to do without using technology. With reorganizers, exploration of interrelations among concepts (e.g., representations of functions with dynamic environments) can be carried out more meaningfully and in a brief time (Pea, 1985). We selected these two frameworks to examine the portfolio entries submitted by the PSTs.

Purpose

The following research questions guided this study.

1. What is the secondary mathematics preservice teachers' knowledge about the digital tools they choose and plan to use?
2. What kind of digital tools do secondary mathematics preservice teachers choose to explore mathematical concepts? Why do they choose these technologies/tools?

Methods

Participants

Participants were 15 PSTs enrolled in a secondary mathematics methods course; a course focusing on teaching mathematics in high school taken during the first semester of their Senior year. Prior to this semester they took a first methods course that focused on teaching mathematics in the middle school, they also took most of their mathematics content courses and some pedagogy and content courses.

Data Sources

Data sources were technology portfolios that the PSTs completed throughout the semester. In the portfolio entries PSTs provide evidence of their knowledge of the uses of technology and their experience using technology for mathematics learning and teaching. Entries in the portfolio can come from work they have done for their mathematics courses, their mathematics methods courses, or their field experiences. Portfolio entries can be from 1 to several pages long. They generally dedicate one section to discuss specific items, such as describing a mathematical exploration or lesson integrating the use of technology, or a technology resource that can be used to explore mathematics. Another section is used for reflecting on or discussing how the item showcases achievement of the goals for the portfolio entry. For instance, if an entry focuses on an exploration utilizing an applet, one section describes the key features of the applet and displays a screenshot of the applet . On another section PSTs provide a description of the exploration, the learning goal, and an analysis of how exploring with the applet supported the attainment of the learning goal.

For their portfolios, PSTs needed to include entries in four categories, namely: 1) Explore mathematics with digital technologies; 2) Knowledge of resources for teaching and learning mathematics with technology; 3) Integrate technology into mathematics learning; and 4) Reflecting on the effectiveness of technology in teaching and learning mathematics. A minimum of 9 entries were required with three entries for the first category and 2 entries for each of the remaining three categories.

For this report, we analyze entries from the first two categories which can be a total of up to six entries. In the first category they had to include at least three entries about exploring mathematics concepts with technology. These entries need to span at least three of four content areas, namely: geometry, algebra, calculus, and probability and statistics. In each entry, they needed to adrees the following: (a) a description of the exploration task, (b) the learning goal of the exploration, (c) a description of the tool used in the exploration (applet, software, graphing calculator), (d) the way in which the tool supported their mathematical learning describing the understandings gained while exploring with the tool, and (e) compare and contrast with a similar task that could be done without the use of technology addressing what is gained with the use of technology and what is lost. In the second category they showcased their understanding of mathematical technology tools and software that are available to both students and teachers.

They demonstrate their capacity to make thoughtful decisions regarding the selection of technological resources to enhance mathematics education. Their entries can be annotated lists of resources or a critical evaluation of one technology resource, the latter being similar to a technology section of a journal or the software review section of a magazine. One entry of each type.

Data Analysis

Portfolio entries were analyzed using the two theoretical perspectives described above. First it was noted if they were conveyance or *mathematical action* tools (Dick & Hollebrands, 2011). And then for mathematical action tools whether they were used as an *amplifier* or a *reorganizer* (Pea, 1985; 1987). For example, if the PST chose a tool to present a mathematical idea by means of a slide show, this was considered a conveyance tool. On the other hand, if the focus was to explore mathematics using a tool that provides interactive feedback based on mathematics, such as with dynamic geometry software, the digital tool was classified as a mathematical action tool. In what follows we provide some spefic examples derived from PSTs entries.

In PST06's entry for exploring concepts in geometry, the participant used Microsoft PowerPoint software to present some information related to the content taught in the classroom. Since the main purpose of using this software was to present the content, the authors referred to Dick and Hollebrands' (2011) framework and the entry was coded as a conveyance tool. On the other hand, some participants considered various technological dynamic tools including Geogebra (n.d.), CODAP (2014) and Desmos (2023) in their entries. If the main aim was to do mathematics, perform the mathematical operations in a faster way, and explore the mathematical concepts and procedures in-depth, entries were coded as mathematical action tools based on the same framework. Once tools used in the portfolio entries were coded as conveyance or mathematical action tools, the authors used Pea's (1985;1987) framework to make differentiation in the usage of mathematical action tools. Technological tools could be coded as amplifiers and/or reorganizers in accordance with the purpose of use.

For example, PST03 and PST05 used CODAP to explore concepts in probability. The purpose in PST05's entry was to see different samples to calculate probabilities and see the visual results, but in a more effective and faster way. On the other hand, PST03 used CODAP to make students understand how to calculate probabilities and explore what the probability means behind the observed number. The purpose was much deeper than just using it to do mathematics in a faster and more efficient way. Therefore, this use of CODAP was coded as amplifier in PST05's entry and as reorganizer in PST03's entry.

Each entry was analyzed by two of the authors. After agreeing upon the analysis criteria and coding a few portfolio entries together to develop shared meanings, the researchers independently analyzed the remaining entries. Inter-rater reliability was calculated and found to be 94%, which is higher than the acceptable level (Marques & McCall, 2005). The codes on which the researchers disagreed were discussed to arrive at a joint code.

Results

Tables 1 and 2 summarize the selection and use of digital technologies for each entry in the first category Explore Mathematics with Technology. PSTs included a variety of tools for their entries. The total submissions for each entry for Algebra, Geometry, Calculus, and Probability and Statistics were 13, 14, 8 and 12, respectively. Deadlines for submissions spanned throughout the semester. PSTs generally preferred to submit first for Geometry topics while they submitted Probability and Statistics topics last. An examination of the types of tools included reveals that they generally preferred online interactive platforms or websites for Algebra topics. For example, a Fractions Bar applet from Cool Math 4 Kids. The second most preferred tool was Desmos where they generally used pre-prepared online activities. For the Geometry topic, the general trend was to use dynamic geometry environments such as GeoGebra, Desmos and Geometer's Sketchpad. The Desmos software was dominant in the entry for Calculus. For Probability and Statistics, they mainly selected explorations using CODAP, an online simulation and data visualization software that includes various online pre-prepared activities for both teachers and students.

Table 1: Entries for Explore Mathematics with Technology

Entries / Participants	Algebra	Geometry	Calculus	Probability & Statistics
PST01*	Fraction Bars - Mathematics Learning Center Website	Geogebra	WolframAlpha	Excel/Google Sheets
PST02	GSP	Geogebra	NA	CODAP
PST03	Desmos	GeoGebra	NA	CODAP
PST04	Math Open References Website	Geogebra	NA	NCTM Illuminations
PST05	NA**	Math Learning Center	Desmos	CODAP

PST06	Fraction Bars - Mathematics Learning Center Website	Powerpoint & MathLearning Center Website	Desmos	CODAP
PST07	NA	Desmos	Desmos	NCTM Illuminations
PST08	NA	Interactive Pythogoras Website	Desmos	Microsoft Excel
PST09	Desmos	NA	Desmos	Matlab
PST10	Number-pieces-Mathematics Learning Center Website	Geometer's SketchPad	NA	NA
PST11	Fraction Bars	Geometer's SketchPad	NA	CODAP
PST12	Cool Math 4 Kids	Desmos	Desmos	Shorodor Software
PST13	Desmos	GeoGebra	Desmos	CODAP
PST14	Algebra	GeoGebra	Desmos	NA
PST15	Desmos	GeoGebra	NA	NA

NA: Did not submit.

Table 2 presents the distribution of the digital technologies according to the constructs used for data analysis. Almost all digital technologies used for exploring mathematical concepts were mathematical action tools. PSTs relied on amplifiers in exploring mathematical concepts in Geometry and Calculus. On the other hand, there was a balanced distribution of digital technologies chosen by PSTs in exploring mathematical concepts in Algebra, and Probability and Statistics entries. Interestingly, there was no single reorganizer digital tool in the geometry topic. However, there were three digital tools considered both amplifier and reorganizer. From this selection, it could be interpreted that PSTs decided to use digital tools to save time and make quick calculations for exploring Geometry concepts. In addition, there were both deep explorations and easier uses of digital technologies to teach and explore intended concepts in Algebra, and Probability and Statistics topics. Considering the comparison of amplifier and reorganizer tools used in different concepts of mathematical topics, there is a variation of concepts investigated. Similar concepts in different topics were explored either with amplifiers or reorganizers with no apparent pattern.

Table 2: Classification of Entries According to the Constructs Used for Data Analysis

Entries for Exploring Mathematics Concepts / Selection and Use of Digital Technologies		Algebra (n=13) F (%)	Geometry (n=15) F (%)	Calculus (n=8) F (%)	Probability & Statistics (n=12) F (%)
Conveyance Tools		-	1 (6.67)	-	-
Mathematical Action Tools	Amplifier	6 (50)	11 (73.33)	4 (44.44)	6 (50)
	Reorganizer	5 (41.66)	-	3 (33.33)	6 (50)
	Amplifier & Reorganizer	1 (8.33)	3 (20)	2 (22.22)	-

Table 3 shows frequency and percentages for the types of digital tools that PSTs included in their first entry for the second category *Knowledge of Resources for Teaching and Learning Mathematics with Technology.* PSTs provided descriptions for 61 digital technologies in their annotated lists (average of approximately 4.07). With 19 being conveyance tools while 35 of them were mathematical action tools. furthermore, 7 digital technologies were both conveyance and mathematical action tools due to the intention of using such technologies. Overall, the number of mathematical action tools in the annotated lists surpassed the number of conveyance tools. As far as the second entry, critical review of a digital tool, 4 out of 14 PSTs evaluated digital tools that can be considered as conveyance ones, while 10 PSTs evaluated various kinds of mathematical action tools. One PST did not submit.

Table 3: Types of Entries Included in the Category Knowledge of Resources

Classification of the Digital Tools	Digital Technologies in the Annotated List (n=61) F (%)	Critically Evaluated Digital Technology (n=15) F (%)
Conveyance Tools	19 (31.15)	4 (26.67)
Mathematical Action Tools	35 (57.38)	10 (66.67)
Both Conveyance and Mathematical Action Tools	7 (11.48)	-
Empty	-	1 (6.67)

Overall, the leading digital mathematical action tool found in these entries was Desmos, an online dynamic environment that includes a graphing tool, a geometry tool, and some computer algebra system features. There were also some other dynamic environments in their entries such as GeoGebra, Geometer's Sketchpad, and GeometryPad. There were also various online interactive websites, software, simulation tools and platforms that were designed for both teachers' and students' use in mathematics. Some of the examples are Illuminations, Math PlayGround, MathWay and the National Library of Virtual Manipulatives. These were all described to be used as mathematical action tools. As conveyance tools, the leading tool was Khan Academy, an online platform with resources, videos, presentations, and worksheets. There were also some other online conveyance tools including IXL Learning, TeacherTube, and Better Lesson.

Conclusion

PSTs' entries regarding exploring mathematical concepts in Algebra, Geometry, Calculus, and Probability and Statistics indicated that all chosen digital technologies were mathematical action tools. Although the concepts intended to be explored vary, the fact that PSTs did not prefer to choose any conveyance tools gave evidence that there was a gradual development in their TPACK knowledge. The integration of such digital technologies also varied in exploring the intended concepts. The number of reorganizer tools were limited in such entries. For example, almost all entries in Geometry were amplifiers, while half of the total submissions in Algebra, and Probability and Statistics entries were amplifiers. The only exception where the activities included more reorganizer tools was in exploring Calculus concepts.

PSTs placed both conveyance and mathematical action tools as digital tools to teach and learn mathematics in their annotated lists of digital tools. In their annotated list of digital technologies, some PSTs mainly mentioned conveyance tools which were generally used for presentation of the documents and mathematical facts, and providing resources of assignments, questions, and fact sheets. Although some PSTs critically evaluate conveyance tools in teaching and learning mathematics, they focused more on mathematical action tools in such entries. These results suggest that it is possible to help PSTs develop their TPACK knowledge during their teacher preparation.

Acknowledgement

This study is supported in part by the Scientific and Technological Research Council (STRC) of Türkiye, project number 1059B19210110. The opinions, findings, conclusions, and recommendations in this paper are those of the authors and do not necessarily reflect the views of the STRC.

References

Association of Mathematics Teacher Educators (AMTE). (2017). Standards for preparing teachers of mathematics. Author.

Common Online Data Analysis Platform [Computer software]. (2014). Concord, MA: The Concord Consortium. Desmos Studio PBC (2023). Desmos. https://www.desmos.com

Dick, T. P. & Hollebrands, K. F. (2011). Focus in high school mathematics: Technology to support reasoning and sense making. Reston, VA: National Council of Teachers of Mathematics.

Geogebra GmbH (n.d,). Geogebra. https://www.geogebra.org

Koehler, M., Mishra, P., & Cain, W. (2013). What is Technological Pedagogical Content Knowledge (TPACK)? Journal of Education, 193, 13-19. https://doi.org/10.1177/002205741319300303

Marques, J. F. & McCall, C. (2005). The application of interrater reliability as a solidification instrument in a phenomenological study. The Qualitative Report, 10(3), 439-462. http://www.nova.edu/ssss/QR/QR10-3/marques.pdf

Mishra, P. & Koehler, M. J. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. Journal of Educational Computing Research, 32(2), 131-152.

Mishra, P. & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. Teachers College Record, 108(6), 1017-1054.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics and the Council for the Accreditation of Educator Preparation (NCTM & CAEP). (2012). NCTM CAEP standards—middle grades (Initial preparation).

National Council of Teachers of Mathematics and the Council for the Accreditation of Educator Preparation (NCTM & CAEP). (2020). Standards for the preparation of secondary mathematics teachers.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, DC: Authors.

Pea, R. D. (1985). Beyond amplification: Using the computer to reorganize mental functioning. 185, Educational Psychologist, 20(4), 167-182. https://doi.org/10.1207/s15326985ep2004_2
Pea, R. D. (1987). Cognitive technologies in mathematics education. A. H. Schoenfeld (Ed.), Cognitive science and mathematics education içinde (s. 89-122). Hilldale, NJ: Erlbaum.