OPPORTUNITIES FOR INSTRUMENTAL GENESIS IN MATHEMATICS LESSONS

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As digital technologies become a commonplace in mathematics classrooms, they transform the ways students and teachers interact with mathematics. Students learn to use a technological tool for solving mathematical tasks through a process known as instrumental genesis. Teachers' support for this process is known as instrumental orchestration. In this paper we explore how both peers and teachers provide opportunities to influence the instrumental genesis process with group, one-on-one, and full class discourse. Through analyzing videos of three lessons from professional development websites we found evidence of six literature-based categories of instrumental orchestration, as well as five new categories expanding the instrumental orchestration framework. The study outcomes can be used to advance research on integration of digital technology in classrooms to promote rich mathematics learning for all students.

Keywords: Learning theory, technology, instrumental genesis, instrumental orchestration

The potential of digital technologies to transform teaching and learning in mathematics classrooms has long been recognized (Bray & Tangney, 2017; Clark-Wilson et al, 2020). In recent years, there have been several developments with respect to digital technology that support its growing integration in mathematics classrooms. One such development has been the advancement of free digital technology tools for learning mathematics, such as *Desmos* and *GeoGebra*, which are designed to support user interaction with the tool in exploratory ways. As opposed, for example, to scientific calculators, whose primary role is to simplify calculations, digital tools like *GeoGebra* allow for exploring properties of mathematical objects and relationships among them (e.g., how changing the value of parameters in an equation affects the graph of a function corresponding to this equation). The second development is students' increased access to and familiarity with hand-held devices such as smartphones and laptops, making the students "Digital Natives" (Francis & Hoefel, 2018). Concurrently, there is an increased number of mathematics applications and digital tools available on these devices, as well as the growing number of ready-available digital tasks and activities, sometimes called "techtivities" (e.g., Johnson et al., 2021). In addition, new technologies like projectors, screen share, and communication devices are opening new doors for discussing mathematical ideas.

To integrate technology into teaching mathematics, teachers need to adapt their methods of instruction, modify the organization of their classrooms and the nature of mathematical tasks (Bozkurt & Uygan, 2020). As students interact with a technological tool, the tool gradually becomes an *instrument* supporting students' mathematical thinking. This process is called *instrumental genesis*, and the process that supports students' instrumental genesis is called *instrumental orchestration* (we elaborate on these in the theoretical perspectives section) (Drijvers et al., 2010; Maschietto & Trouche, 2010; Trouche, 2004; Trouche & Drijvers, 2010). Researchers identified six categories of instrumental orchestration, which arose from characterizing teacher work in leading whole class discussions in technology rich lessons, with teachers at the center and with students contributing to the whole class discussion (Buteau et al., 2020; Drijvers et al., 2010). However, with students being digital natives it is reasonable to expect that their technological competence would affect the nature of classroom interactions surrounding instrumental genesis as well as teacher instrumental orchestration. Additionally, there has been little research on the impact of peers, lesson types, and varied modes of discourse on instrumental genesis.

Research Aims and Questions

As technology is becoming more ubiquitous in mathematics classrooms, it is expected that the repertoire of instrumental orchestration types will evolve and grow (Buteau et al., 2020). This study expands upon the existing research by examining instrumental orchestration in three technology rich lessons in which both teachers and peers could influence instrumental genesis. Through this exploration, we attempted to confirm the presence of literature-based categories for instrumental orchestration (e.g., Drijvers et al., 2010) while also documenting the potential emergence of additional categories of peer influence on instrumental genesis. The study was guided by the following research questions.

1. What are the didactical configurations (the setting and the artefacts) for each lesson?
2. How do teachers enact instrumental orchestration in various modes of discourse?
3. How do peers influence each other's instrumental genesis?

While most studies on instrumental orchestration utilized real-time observations, this study seeks to examine whether the types of instrumental orchestration can be observed in video recordings of classroom teaching, as a way to expand methodological practices for studying instrumental genesis and instrumental orchestration.

Theoretical Perspectives and Background

An *artefact* is a material object, e.g., a calculator, or abstract object, e.g., quadratic formula, whose aim is to assist in future mathematics activity or learning (Maschietto & Trouche, 2010). What gives an artefact, either material or abstract, its meaning are cognitive structures, or schemes. Once a student puts a meaning to the artefact and uses it as a tool for performing conceptual or physical tasks, the tool becomes an *instrument.* The process of users, or learners, turning an artefact into an instrument is called *instrumental genesis* (Clark-Wilson et al., 2020). Teachers support students' instrumental genesis through the process of *instrumental orchestration* by carefully planning and organizing lessons that engage students with technology (Clark-Wilson et al., 2020). The orchestration metaphor is used because of many moving parts, intentions, and instruments needed for learning and instrumental genesis to occur in harmony.

Instrumental orchestration and instrumental genesis are frameworks for investigating how teachers and students integrate technological tools in mathematics classroom. The frameworks are grounded in cognitive theories of learning and have been developed empirically and shaped over the years (Artigue, 2002; Drijvers et al., 2010; Trouche & Drijvers, 2010; Trouche, 2004).

Scholars (Drijvers et al., 2010; Trouche, 2004) describe three components of instrumental orchestration: didactical configuration, exploitation mode, and didactical performance. *Didactical configuration* refers to the setting and artefacts for the lesson. How is the room organized? What artefacts are available? How easily can students access them? The *exploitation mode* involves planning mathematical tasks that support instrumental genesis. Teachers decide what tasks to use and how technology is integrated in the task. Lastly, the *didactical performance* can be observed during the lesson in teacher's actions, both pre-planned as well as ad hoc, or in the moment, decisions. The exploitation mode and the didactical performance may vary considerably, even when teachers use the same instructional task. Pierce et al. (2010) identified great variety in how CAS technology was used, and the amount of responsibility for independent learning provided by

12 mathematics teachers implementing the same lesson plan on quadratic relationships. In addition, Bozkurt and Ruthven (2018) observed that novice and experienced teachers' use of dynamic geometry software in their classrooms differed at all levels of instrumental orchestration: the lesson organization, the choice of the task and the level of integration of the tool into mathematics teaching.

Drijvers et al. (2010) identified six types of instrumental orchestration in technology-rich classrooms: technical demo, explain-the-screen, link-screen-board, discuss-the-screen, spot-and-show, and sherpa-at-work. The first three orchestration types are teacher centered. With *technical demo* a teacher demonstrates the technical features of a tool and how to use it in mathematical tasks. This can occur with a small group of students or with the whole class. In e*xplain-the-screen* a teacher explains a projected screen to the entire class. It can be an example problem or a technical component. With *link-screen-board* the teacher makes connection between the technological representations and the paper, pencil, or board representations of mathematics. The next three orchestration types are more student centered. With *discuss-the- screen* the whole class engages in collaborative conversation about a (projected) computer screen. In *spot-and-show* student reasoning is discussed amongst a group or whole class. Lastly, *sherpa-at-work* is a technique where one student uses technology to represent his or her work to the entire class.

Methods

Three videos were chosen for this investigation. All videos are freely available online. The lessons were videorecorded in United States classrooms; each using a different technological tool and classroom setting. The variety of lesson types intended to reveal a variety of activities, modes of discourse and contributions to instrumental genesis, both through teachers and peers.

Lesson 1 video[6] (20 min) showed an Algebra 2 class with about 20-25 students working in groups on an activity called "Modeling with Polynomials". Ms. Burke, the teacher, had 13 years of teaching experience at the time. Students had to determine the maximum volume of an open top box made from cutting grid paper with 1-centimeter squares. Students modeled polynomials using *Desmos*, working in groups of 4-5 students and sharing *Chromebooks*.

The lesson 2 video[7] (10 min) featured an Algebra 1 classroom, similarly, with about 20-25 students, who worked more independently, led by Mr. Kwon, a second-year teacher. Students used *Desmos* and *code.org* to investigate rates of change of a rocket, utilizing multiple representations afforded by *Desmos*, to make connections between graphs, tables, and functions. Every student had a desktop computer and appeared to be working independently.

The video for lesson 3[8] (4 min) also showed students using *Desmos* in a virtual learning environment. Ms. Saltz, who had around 13 years of teaching experience, taught an Algebra 1 lesson on *Zoom* with a group of about 20 students. The lesson was about the roots of polynomials.

Analytic Framework

The videos were fully transcribed for the analysis. In addition, the first author of the paper wrote analytic memos, and summarized data on classroom environment, lesson format, and technological resources provided to students. The analysis of video, notes, and transcriptions was used to respond to research question one - the didactical configuration of each lesson, which was identified by examining questions such as: How were the classrooms arranged? What resources were provided to students for instrumental genesis? With what modality was the lesson being

[6] https://www.insidemathematics.org/classroom-videos/public-lessons/11th-12th-grade-algebra-ii-modeling-polynomials
[7] https://learn.teachingchannel.com/video/code-in-hs-math/?1664818181743
[8] https://www.philasd.org/etvl/2021/04/08/using-desmos-in-math-virtual-lessons/

taught? Same data sources, as well as worksheets accompanying the video, were analyzed for identifying instances of instrumental genesis, when students used technology to solve a mathematical task.

To answer research questions 2 and 3, we coded each lesson by identifying instances of the six types of instrumental orchestration according to Drijvers et al.'s (2010) framework. In addition, we captured instances where students supported each other's instrumental genesis and described them as new categories. Since the term instrumental orchestration usually refers only to teacher actions (Clark-Wilson et al., 2020), we use the term "influence on instrumental genesis" to describe both teacher and peer actions. Additionally, each instance of influence of instrumental genesis was coded for the mode of discourse, i.e., who is involved in the discourse: group, teacher-group, teacher-student, and whole class. Group was coded any time discourse took place between more than two students; teacher-student was coded when a teacher spoke to a student one-on-one.

Results

Lesson 1

Didactical Configuration. Mrs. Burke's class was organized into groups, with four students per group, seated at desks that face each other. Some of the group members were assigned roles, like facilitator, recorder, and resource manager. For example, Mrs. Burke said, "Facilitators, will you make sure that you're moving your group into step 4?" In the first part of the "Cutting Corners" activity, each group received a piece of graph paper, and was guided to fold it into an open box by cutting four squares at the corners of the sheet. Each group calculated the volume of the resulting box, and the whole class completed a table on the board recording the dimensions and the volume of various boxes. Next, each group received two *Chromebooks* to enter the data from the table into *Desmos* software, graph it and fit a polynomial function though the plotted points. The worksheet guidelines assisted students with using *Desmos*, for example, "Use the regression feature to find a function to model the data."

Modes of Discourse and Instrumental Genesis Influence. This lesson contained four modes of discourse: group, teacher-group, teacher-student, and whole class. There were 38 transcript lines that contained evidence of influence on instrumental genesis. Each line was coded for the type(s) of instrumental genesis influence present and who was involved in the discourse. For example, within one group one student performed a technical demonstration through *Desmos* for another student, saying "You see how I did it?". This was coded as a technical demo in group discourse. Table 1 shows the modes of discourse in the rows, and the type of instrumental genesis influence in the columns. Percentages were calculated out of 38 lines.

Table 1: Lesson 1 Modes of Discourse and Influence on Instrumental Genesis

Discourse Mode	Technical Demo	Explain the Screen	Link-Screen-Board	Discuss the Screen	Sherpa at Work	Trouble-shooting Tech*	Written Directions*	Total
Group	8%	3%	18%	13%	0	5%	0	47%
Teacher- Group	0	0	16%	11%	3%	3%	0	32%
Teacher-Student	0	0	5%	0	3%	0	0	8%
Whole Class	0	0	0	3%	3%	0	8%	13%
Total	8%	3%	39%	26%	8%	8%	8%	

New Categories: Written Directions and Troubleshooting Technology. As seen in Table 1, the five out of six categories of instrumental orchestration framework (expect for spot-and-show) appeared in multiple areas of discourse, beyond whole class discussions. In fact, 79% of all influences on instrumental genesis occurred within the groups or between the teacher and the groups. Additionally, two new categories of influence on instrumental genesis arose: written directions and troubleshooting technology. Some of the worksheets Mrs. Burke gave provided written directions for students to make links between mathematics and the *Desmos* software. For example, “Use your model to find the maximum volume of the box.” Since the directions were from the teacher and to the whole class, they were coded as “whole class” discourse.

In addition, Mrs. Burke and the students had to use their knowledge of both technology and mathematics to troubleshoot technological issues that arose. For example, when one group was creating a scatterplot in *Desmos* their graph showed all points on the y-axis (Figure 1). They questioned, “why is it like that?”, knowing something must be wrong, and worked together to fix the technological issue. This instance was coded as group troubleshooting technology. Through this process students can learn more about how mathematics content interacts with technology.

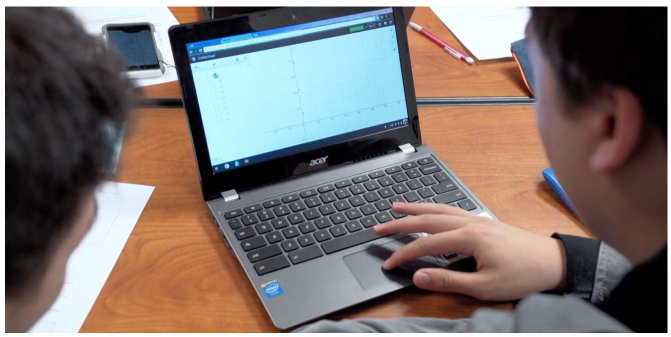

Figure 1: Group “troubleshooting technology” when graph has all outputs on y axis

Lesson 2

Didactical Configuration. Mr. Kwon’s class was in a computer lab with one-to-one desktops. The students sat in rows facing the front of the room. In the mathematical task, the students first watched a *Desmos* animation of a rocket and had to change the parameters of the animation to make the rocket go at different speeds and initial height values. Students also created various representations of the flight of the rocket using tables, graphs, and equations. Then students had to switch to a different software, *Code.org,* to match the code that animates the rocket to a mathematical representation of its movement. Students experimented with both constant and varied rate of change of the rocket’s velocity. The students seemed to have a basic understanding of the computer programs since they appeared to work on their own with minimal guidance. Students worked independently for most of the lesson, occasionally asking the teacher questions.

Modes of Discourse and Instrumental Genesis Influence. This lesson contained three modes of discourse: group, teacher-student, and whole class. There were 27 transcript lines that were coded for influencing student’s instrumental genesis. Five out of six original instrumental orchestration categories (all except *link-screen-board*) were detected, as well as a new category: *link-screen-screen* which described instances when a teacher was explicating connections between two or more software programs. For example, Mr. Kwon said, “At each part of the lesson I had students ask me to come over and check to see if their table and the graph and their equation matches the animation” (Figure 2).

New Categories: Written Directions, Troubleshooting Technology, and Link-Screen-Screen. The two new categories detected in lesson 1: written directions and troubleshooting technology were also present in this lesson (Table 2). Another new category, labled *link-screen-screen* arose since students used two different mathematics programs *Desmos*, and *Code.org*. to work on the mathematical task. *Link-screen-screen* describes teacher (or peer) actions supporting students' coordination of representations across various computer screens and mathematical task.

Figure 2: Teacher-Student discourse. Link-screen-screen.

Table 2: Lesson 2 Modes of Discourse and Influence on Instrumental Genesis

Discourse Mode	Explain the Screen	Link-Screen-Board	Discuss the Screen	Sherpa at Work	Spot and Show	Link-Screen-Screen*	Trouble-shooting Tech*	Written Directions *	Total
Group	0	0	0	4%	0	0	4%	0	7%
Teacher-Student	11%	33%	0	0	0	19%	7%	0	70%
Whole Class	0	7%	4%	0	7%	0	0	4%	23%
Total	11%	41%	4%	4%	7%	19%	11%	4%	

Lesson 3

Didactical Configurations. Ms. Saltz taught her lesson via *Zoom* video conferencing. Her screen was shared so students could see her actions on *Desmos*. All students attending the lesson and the teacher were logged into the same *Desmos* activity. Ms. Saltz had the ability to pace students' *Desmos* screens and display student work anonymously. The lesson involved finding solutions to polynomial equations using both algebraic and graphical representations.

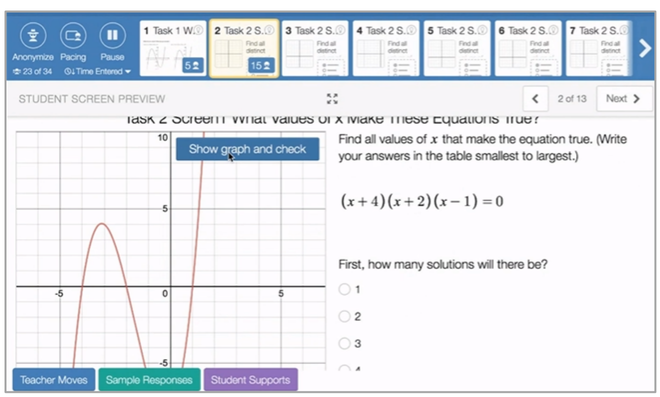

Figure 3: Teacher's shared *Desmos* screen during the virtual lesson

Modes of Discourse and Instrumental Genesis Influence. This lesson only had whole class discourse and contained eight influences on instrumental genesis. Two new categories arose: *anonymous spot-and-show* and *verbal directions*. Table 3 shows the distributions.

Table 3: Lesson 3 Modes of Discourse and Influence on Instrumental Genesis

Discourse Mode	Technical Demo	Spot and Show (Anonymous)*	Sherpa at Work*	Verbal Directions*	Total
Whole Class	38%	25%	12%	25%	100%
Total	38%	25%	12%	25%	

New Categories: Anonymous Spot-and-Show and Verbal Directions. *Verbal directions* were similar to written directions in the previous lessons. Ms. Saltz gave students directions on how to use their technology for mathematics without showing it first, since all students were following on simultaneously on their screens. For example, she gave students verbal directions on what to click to get to the correct page or screen, "I want you to click on the link-I want you to go to our Desmos for the day". Additionally, she used the *spot-and-show* technique to discuss student work with the whole class, but did it anonymously, without disclosing the name of the student. This way the teacher avoided putting social pressure on students to show their work to the whole class, but student work was still utilized to advance the mathematical goal of the lesson.

Summary

Regarding research question 1, the didactical configurations of each lesson afforded different teaching methods and modes of discourse. In lesson 1 the room arrangement of grouped desks allowed for easy communication amongst students. The groups sharing *Chromebooks* enabled students to discuss the technology as it related to the mathematics they were studying. In lesson 2 all seats faced the front of the classroom making it challenging for students to talk to each other. Additionally, each student had their own desktop, so they rarely looked at each other's screens. This didactical configuration seemed to encourage single student-to-teacher discourse rather than discourse among students or the whole class. Lesson 3 was fully remote. Although the task had a potential to elicit student participation, the teacher lectured most of the time, without providing students opportunities to discuss mathematics in groups or with each other.

Research question 2 asked how teachers enact modes of instrumental orchestration during different modes of discourse. In lesson 1 the teacher interacted with groups, individual students, and the whole class. Teacher-group interactions had the highest frequency, with 32% of the total. The results showed that *link-screen-board*, *discuss-the-screen*, and *sherpa-at-work* orchestration strategies appeared in all different modes of discourse. The new categories of *troubleshooting technology* and *written directions* appeared during group-teacher interactions and whole class interactions. In lesson 2 the teacher's discourse modes included whole class and teacher-student interactions, the latter accounted for 70% of the total interactions. The orchestration categories: *explain-the-screen, link-screen-board, discuss-the-screen, and sherpa-at-work* were detected, as well as new categories of *troubleshooting technology*, *written directions*, and *link-screen-screen*. Lesson 3 only had whole class interactions in a lecture style. Orchestration categories included *technical demo*, *spot-and-show*, and *sherpa-at-work*, with a new category of *verbal directions*.

Expending the scope of examination of modes of discourse beyond whole class discussion allowed for identifying opportunities for instrumental genesis between peers (research question 3). The most common mode of discourse for lesson 1, accounting for 47% of interactions, was within groups. The categories of *technical demo, explain-the-screen, link-screen-board*, and *discuss-the-screen* were all transferrable into group interactions. Additionally, students worked together to *troubleshoot technology*. Lesson 2 only had two instances (7%) of peer interaction for instrumental genesis: one *sherpa-at-work* and one *troubleshooting technology*. Lesson 3 had no evidence of peer interactions, thus, no peer support for instrumental genesis could be detected.

In summary, different didactical configurations and lesson types can impact modes of discourse and opportunities for instrumental genesis. The six instrumental orchestration categories from Drijvers et al. (2010) could be applied in a variety of discourse modes. The group work heavy lesson, lesson 1, had the most opportunities for instrumental genesis through peer interactions in groups. Lesson 2 with one-to-one devices allowed for minimal peer interactions and minimal peer influence on instrumental genesis. However, there were many teacher-student opportunities for instrumental genesis. The remote lecture style lesson only had whole class discourse opportunities for instrumental genesis. Different discourse modes allowed for the emergence of new categories of instrumental genesis influence: *written and verbal directions, troubleshooting technology, link-screen-screen,* and *anonymous spot-and- show.*

Discussion and Implications for Education

Our study attempted to extend the scope of examination of instrumental orchestration beyond whole class discussions in real classrooms (e.g., Bozkurt & Uygan, 2020; Drijvers et al., 2010), to video recordings featuring various parts of technology-integrated lessons. In addition, we used video footage that is freely available online, taken from websites geared toward teachers and teacher educators. Since these videos are intended for use in professional development, or to showcase "exemplary teaching" (e.g., see description link to video 3), it is important to understand what types of opportunities for instrumental orchestration they contain.

While our methodological approach appeared to yield fruitful results, it has clear limitations. The main one being the lack of access to the raw footage of the whole lesson but only to selected clips uploaded to the web. For example, the lesson 1 video featured various groups of students, not allowing for prolonged observation of a single group's dynamic. Lessons 2 and 3 were represented by even shorter, edited down clips. While the types of didactic configurations, discourses, and influences on instrumental genesis observed in the video may be representative of the other parts of the lesson, there is no evidence in the video to support this assumption. Importantly, our goal in this paper was not to characterize the lessons themselves, but to characterize the types of supports for instrumental genesis that are visible in the existing data. Thus, we make no claims about the types of support for instrumental orchestration in these lessons in general.

Our study was exploratory. Methodologically, we attempted to provide proof of concept about the utility of using publicly available video of teaching and learning with technology to study instrumental genesis. The results show that the literature-based categories of instrumental orchestration appeared in all the examined videos, supporting our methodological assumption. In addition, our study revealed novel categories of influence on instrumental genesis in the actions of teachers and, importantly, peers.

The results of this study can be useful to teacher educators. With the growing integration of technology in teaching mathematics, future teachers can benefit from applying the categories of instrumental orchestration to analyze instructional video. This can potentially help teachers to enrich their repertoire of instrumental orchestration strategies, and to gain appreciation for how different lesson configurations afford different opportunities for instrumental genesis and instrumental orchestration.

Our study also points to the need to continue examining teacher and peer support for instrumental genesis beyond the existing frameworks. The modes of instruction and technological tools are constantly evolving, and with them the pedagogical practices of teachers (Bray & Tangney, 2017). Continuing to examine instrumental orchestration is important to advance our understanding of student mathematics learning in technology-rich environments.

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