MATHEMATIZING FAIRNESS: HOW ELEMENTARY STUDENTS DRAW ON FUNDS OF KNOWLEDGE WHILE SOLVING A MODELING TASK

Amy Been Bennett
University of Nebraska-Lincoln
abennett14@unl.edu

Julia Aguirre
University of Washington
jaguirre@uw.edu

Erin E. Turner
University of Arizona
eturner@arizona.edu

Elzena McVicar
University of Washington
elzenam@uw.edu

Erin Carll
University of Washington
ecarll@uw.edu

Mathematical modeling is a process in which students investigate authentic problems and everyday situations using mathematics. In doing so, they bring their multiple mathematical knowledge bases and cultural funds of knowledge into their solution strategies. During a task called "Abuelo's Birthday", 297 students in grades 3-5 decided how to split the costs of a gift "fairly" and justified their work using early notions of ratio and proportional reasoning. We argue that these young students were successful with a complex task due to the way the realistic context connected to students' lived experiences and funds of knowledge related to home and family life. We share rich examples of how students included elements outside of the task to justify and enhance their mathematical models and conclude with implications for the importance of modeling and non-routine tasks in elementary classrooms.

Keywords: Elementary School Education, Modeling, Rational Numbers & Proportional Reasoning, Ethnomathematics

Conceptual Framework

Mathematical modeling is internationally regarded as a beneficial mathematical practice for students across all grade levels (Schukajlow et al., 2018; Sriraman & English, 2010; Verschaffel & De Corte, 1997). In the U.S., the Common Core State Standards (2010) describe the mathematical practice of modeling as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (p.72), which is intended for grades K-12. At the elementary level, open-ended mathematics tasks that align with the modeling process can connect to children's mathematical thinking in a variety of content areas, such as ratio and proportional reasoning. Moreover, the real-world contexts of modeling tasks can connect to disciplines and situations outside of the mathematics classroom. In this paper, we explore how the mathematical modeling process, students' funds of knowledge, and proportional reasoning strategies intersect in one rich task.

Mathematical Modeling

Mathematical modeling is defined in multiple ways in research literature and education standards. Modeling is a process for connecting the real world to the world of mathematics (Blum & Borromeo Ferri, 2009)**,** or "a process in which students consider and make sense of an everyday situation that will be analyzed using mathematics for the purpose of understanding, explaining, or predicting something" (Anhalt et al., 2018, p. 202). During the modeling process, authentic situations are made sense of, simplified, modeled, analyzed, interpreted, and generalized. Mathematical modeling benefits students in multiple ways, including fostering creativity, problem solving, and communication (e.g., Chamberlin et al., 2022; Tidwell et al., 2021). Modeling tasks are open-ended tasks that foreground diverse solution strategies and connections to multiple mathematical knowledge bases (Aguirre et al., 2013; Turner et al., 2012).

Cultural Funds of Knowledge

Mathematics education scholars have found that students learn mathematics best when they work collaboratively on challenging tasks, draw on background knowledge and experiences, and connect mathematics activities to their everyday lives (e.g., Turner et al., 2012; Civil, 2002). Civil (2002) explored how children's mathematics learning and engagement can be better understood through the lens of *funds of knowledge* (Moll et al., 1995), as well as how mathematics activities can draw on students' cultural knowledge and experiences to help them connect school mathematics to authentic situations. This work emphasized that there are benefits to culturally responsive, community-based approaches to teaching mathematics, particularly for students from underrepresented groups in STEM fields.

Mathematical modeling tasks bring these features together in interdisciplinary tasks that connect mathematics content to real-world phenomena, such as using proportional reasoning to share the cost of a ride fairly (Sawatzki et al., 2019), using unit rates to upcycle plastic bags into jump ropes for a school (Turner et al., 2021), or addressing the clean water crisis in Flint, Michigan (Aguirre et al., 2019). At the elementary level, research has shown that mathematical modeling is accessible to young children and to students from a diverse range of mathematical and cultural backgrounds (e.g., English & Watters, 2005; Sawatzki et al., 2019; Verschaffel & De Corte, 1997). In this way, mathematical modeling tasks present an opportunity to draw on students' cultural funds of knowledge in the mathematics classroom.

Ratio and Proportional Reasoning

Proportional reasoning is typically taught in middle (6-8) grades, after elementary students have a deeper understanding of multiplicative reasoning. Research literature documents elementary and middle grades students' struggles with ratio and proportional reasoning problems, illustrating how students sometimes rely on additive rather than multiplicative reasoning (e.g., Lo & Watanabe, 1997; Steinthorsdottir & Sriraman, 2009). Recent studies have demonstrated that younger students (grades 3-6) can successfully solve tasks related to real-world situations using multiplicative reasoning (e.g., Sawatzki et al., 2019).

Researchers have explored how children as young as first grade (about age 6) can understand some aspects of proportional reasoning, such as the Fittingness Principle and Covariation (Resnick & Singer, 1993). Fittingness refers to the notion that two (or more) things go together because their sizes or amounts are appropriate for one other. Covariation means when one quantity exists in an ordered series, it will covary, directly or inversely, with the other ordered quantity. This early reasoning is sometimes called *protoquantitative*, because it describes qualities or sets of objects, rather than precisely determining equivalent ratios with numeric values. Thus, Resnick and Singer (1993) suggested considering different approaches to help children develop flexible command of multiplicative structures, such as discussing multiple solutions to story problems. For instance, in fair sharing contexts, students may consider nonmathematical aspects of the situation to make sense of the quantities and their relations.

To this end, we posed the following two research questions:

- How do elementary students use proportional reasoning strategies to justify their notion of fairness in the context of solving a modeling task?
- How do elementary students' funds of knowledge and lived experiences shape their mathematical strategies and solutions?

Methods

Setting and Participants

The research presented here is part of a larger, multi-institutional project Mathematical Modeling with Cultural and Community Contexts (M2C3) in two diverse regions of the U.S. During the three years of this project, we worked with teachers and students at elementary schools that served multi-racial, multilingual, multicultural, and working-class communities. This project was motivated by prior research claiming that mathematics interest and learning improves when students can draw on their experiences and connect classroom mathematics to their everyday lives (e.g., Achmetli et al., 2019; Turner et al., 2012). We drew on elementary students' cultural funds of knowledge to create modeling tasks that fostered their mathematical thinking and their community-based ways of learning (e.g., Aguirre et al., 2013; Civil, 2002).

Teachers participated in a week-long summer workshop and ongoing monthly professional development sessions over two one-year cycles. During workshops, teachers engaged in solving and discussing modeling tasks; the Abuelo's Birthday task was one task out of several that teachers were introduced to and encouraged to use in their classrooms. In this task, students must create a fair plan for four grandchildren to share the costs of a birthday gift for their grandfather. The children are of different ages and have different weekly earnings, which challenges students to formulate models that differ from dividing the total cost into four equal parts (see Figure 1).

Abuelo's Birthday Task (adapted from Aguirre & del Rosario Zavala, 2013)
It is Señor Aguirre's 70th birthday. Four of his grandchildren want to buy him a photo printer so he can print photos of his family members. They found the printer on sale for $120.

- Alex, a 9th grader, earns $15 each week from babysitting.
- Sam, a 6th grader, earns $10 each week taking care of his neighbor's pets.
- Elena, a 4th grader, earns $5 each week doing jobs for an aunt.
- Jaden, a 1st grader, has no weekly job but has saved $8 in her piggy bank.

One of the grandchildren says they should split the cost of the printer between them and each pay the same amount. Another grandchild says that is not fair and they should each pay different amounts. What do you think? What is *fair* in this situation?

Figure 1. The task prompt for the Abuelo's Birthday Task

The Abuelo's Birthday task is a complex task for elementary students because it relies on proportional reasoning, which they likely have not experienced in formal classroom settings. The Abuelo's Birthday task is not a traditional missing value task, where students need to set up and solve a single proportion; rather, it involves multiple relationships between quantities that covary. This structure requires students to create composite units, or "units of units" (Lamon, 1993), and decide how to iterate them for an unspecified amount of time. The openness of this task adds to its complexity, as students must make assumptions and decisions about how much each child should contribute and how long they will earn money toward the gift. The flexibility of strategies and solutions encourages students to engage in the modeling process, constantly making sense of, formulating, analyzing, and interpreting their model for "fairness".

Data Collection and Analysis

For this paper, we focus on the dataset of classroom artifacts from the Abuelo's Birthday task, which consists of individual and group posters from student work during class. We collected artifacts from 297 students in grades 3-5 across 14 classrooms in two diverse regions of the U.S. We analyzed 119 student work samples, often in the form of group posters, and excluded instances where teachers altered the original task for their classrooms, or the student work was illegible. We drew on Hatch's (2002) notion of artifact analysis to engage in multiple rounds of coding student work for their mathematical strategies and other salient features of their solutions, such as explanations of "fairness". We drew on the work from Aguirre et al. (2013) to justify our coding of students' references to their cultural funds of knowledge. We initially used Lamon's (1993) framework as a guide for classifying proportional strategies, but after iterative coding and discussions, we adapted our categories to more accurately describe the strategies we observed. For instance, we added the category Roughly Proportional to document students' consistent use of protoquantitative reasoning. Table 1 first presents the five proportional strategies and then the five categories that describe other solution characteristics.

Table 1: Codebook for classifying proportional strategies and other solution characteristics

Code	Description
All Equal	The three older kids or all four kids contribute the same amount, with no connection to earnings; this can include Jaden or not.
All Equal with Adjustments	Initially used strategy "All equal" where everyone paid the same amount. Then, they realized that this will not work and made some adjustments to the existing strategy; they do not just start over with a new strategy (e.g., they see that Jaden cannot pay her $30 and have Alex "cover" it).
Roughly Proportional	Must abide by protoquantitative principles, where children's contributions are in the order of their earnings. May or may not include the number of weeks it takes to earn the money; includes solutions that start out using some proportional thinking, but final solution is not proportional due to major adjustments.
Proportional with Adjustments	Initially used a Proportional strategy, then adjusted plan (in a minor way) to make numbers friendly or sum to $120 (e.g., all 3 older kids contribute for 3 weeks, and then Elena doesn't contribute for week 4). Or, used a Proportional strategy but then re-distributed the excess earnings back to the grandchildren disproportionally.
Proportional	The three older kids' contributions must be proportional to their earnings. Can use addition or multiplication and any representation; this can include Jaden or not. Solution should not include errors in the setup or selection of amounts.
Over $120?	Does total earned exceed $120?
Explained Excess	Did students show or describe what to do with money earned over $120? If so, how did they use the excess money?
Role of Jaden	Does Jaden contribute money (in the final solution)? This includes paying for things besides the printer. If yes, how much did Jaden contribute?
Cultural Funds of Knowledge	Do students directly use reasoning or bring up aspects outside of the task prompt that may connect to home, family, or community?
Statements of Fairness	Do students explicitly say, "it is fair that...", or, "it is not fair because..."? They may not use the word "fair" but imply it through statements like "older kids pay more, younger kids pay less".

Findings

We found that 90 of the 119 student work samples (76%) showed evidence of protoquantitative reasoning, attending to both the Fittingness Principle and Covariation (Resnick & Singer, 1993). This includes solutions classified as Proportional, Proportional with

Adjustments, and Roughly Proportional. As described above, the Abuelo's Birthday task is a complex proportional reasoning task; thus, it is significant that the majority of elementary students in our study were successful with protoquantitative strategies. Table 2 presents the distribution of strategies across grades 3-5. It is noteworthy that students in all grade levels used proportional strategies.

Table 2: Frequency of Math Strategies across all Student Work

	Grade 3	*Grade 4*	*Grade 5*	***Total***
Proportional	10	15	12	**37**
Proportional w/ Adjustments	9	15	1	**25**
Roughly Proportional	18	10	0	**28**
All Equal w/ Adjustments	0	2	2	**4**
All Equal	1	10	1	**12**
Other	4	7	2	**13**
Total	**42**	**59**	**18**	**119**

Ultimately, we were interested in the evidence for how students drew on their cultural funds of knowledge, which we defined as explicit connections to home, family, and community that went beyond general notions of fairness. However, there were other characteristics of mathematical solutions that indicated students were drawing on outside knowledge and experiences (see Table 3). Many student solutions (53 of 119) across all mathematics strategies exceeded $120 in their plan for buying Abuelo's gift, and most of these solutions (30 of 53) explained what would be done with the extra funds, such as pay for a birthday card or cake, cover the sales tax on the gift, or save the money for another relative's gift in the future.

Of the 119 solutions, 84 of them (71%) decided that Jaden should contribute to the gift. However, about half (44 of 84) decided that Jaden should only contribute a partial amount of her $8. This is noteworthy since almost all of the 119 solutions showed the three older children contributing all of their earnings each week. This suggests that students differentiated between a constant rate of earnings for the three older children and the non-increasing amount in Jaden's piggybank and demonstrated this distinction through their models.

Table 3: Characteristics of Solutions and Fairness by Math Strategy

	Exceeded $120	*Explained Excess*	*Included Jaden*	*Direct Statement about Fairness*
Proportional	30	20	29	18
Proportional w Adjustments	9	3	23	18
Roughly Proportional	9	2	20	22
All Equal & All Equal w/ Adjustments	5	5	12	11
Total	**53**	**30**	**84**	**69**

In the following subsections, we describe select student work samples, organized by their math strategy. These samples were chosen as representatives of the ways students drew on their funds of knowledge and mathematized fairness. An important assumption of this work is that students are always drawing on their experiences as they learn, but in some cases, such as these posters of student work, we have explicit evidence.

Proportional Strategies

One key variation among student solutions was how they decided to include Jaden or not. In an example from a group of third graders (see the left image in Figure 2), Jaden's money was not included in the plan ("Jaden will not pay."), and so the older three children contributed proportional to their earnings over four weeks to earn exactly $120.

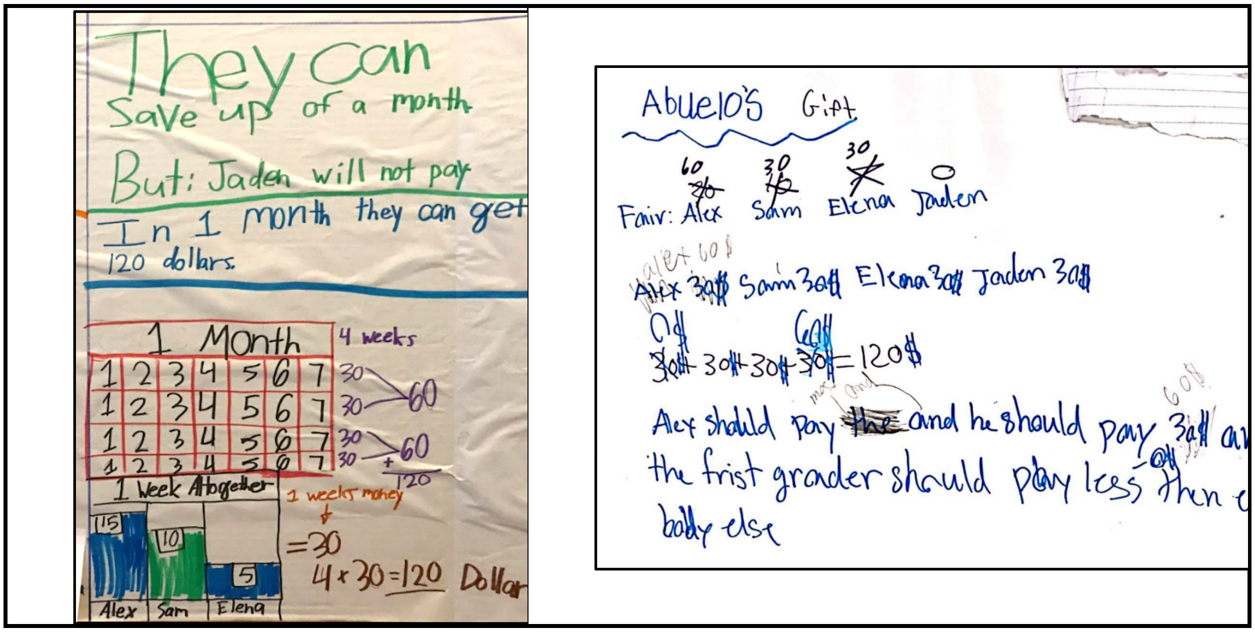

Figure 2. A Proportional strategy and an All Equal with Adjustments strategy

In contrast to the solution in Figure 2, a group of grade 4 students were purposeful to include Jaden in their proportional strategy, even in a small way. The older three children contributed all of their earnings over five weeks, but Jaden only gave $2 (25%) of her savings. This suggests that these students drew on their experiences with siblings and family dynamics to decide what a young child with no weekly income should contribute. The total earnings for five weeks summed to $192, which surpassed $120 by $72. The students decided that the grandchildren "can use [the extra] for tax or buy a cake". These students meaningfully made sense of the remainder of money in the context of buying a gift. Their realistic considerations connected to their funds of knowledge, which led to incorporating additional elements into the task (i.e., sales tax and a birthday cake).

In another proportional solution, a group of third graders wrote about how "Jaden can get money cleaning her room" if she needs to contribute more. They included an equation with parentheses to demonstrate how the three older children contributed at a constant rate over four weeks and deemed this "fair". Then, this group suggested how their plan could be applied to similar sharing situations, like sharing the cost of a room or buying "pizza, drinks, and Hershey's". They wrote, "Our plan can work in other situations because they can work the same amount and probably have enough money for room service."

Multiple groups expressed how their plan could inform other cost sharing situations between children, particularly for paying for some kind of lodging or "renting a beach house". Although we only have their final written work, it appears that these students shared about family vacations, paying for rent, or other experiences where family members had to share the cost of a place to live. There were no references to traveling with family or paying for lodging in the task sheet, which indicates that students made (mathematical) connections between a grandparent's birthday and other family events.

Proportional with Adjustments Strategies

In the Proportional with Adjustments category, students started with a proportional approach and then adjusted in a minor way, often referring to the realistic elements of the task context. One common strategy in this category involved decreasing the amount that Elena (the second youngest child) contributed so that Jaden could be included. One group of grade 4 students clearly wrote, "Fair: Little kids pay less. Big kids pay more," which emphasizes their understanding of protoquantitative fittingness and how it relates to fairness. In their model, Alex and Sam contributed earnings for four weeks ($60 and $40, respectively), but Elena only contributed for three weeks ($15) to allow Jaden to contribute $5 of her savings. This group made the assumption that it is important for all grandchildren to contribute, even if this makes the plan no longer proportional.

Two other groups of grade 4 students planned for one of the three older siblings to pay slightly less (than what is proportional) so that Jaden could contribute to the gift. Their solutions included several assumptions about the situation, including: that Abuelo will like the gift and will keep it (possibly thinking about gifts their own grandparents might like); that Abuelo lives in a different city and the children will need to ship the gift (connecting to experiences with mailing gifts and shipping costs); that they have ample time to earn the money; that the older siblings will be able to keep working and earn the money back that they contributed (possibly drawing on experiences with jobs); and that any extra earnings will cover sales tax. These assumptions influenced their mathematical models and demonstrated how students drew on their funds of knowledge about the gift-giving context.

In another solution, students made an initial assumption that Abuelo's birthday was in three weeks. They wrote equations to show each of the older children's weekly earnings multiplied by three, which gave a total of only $90. The students realized that they still needed to account for $30; they decided that Jaden would give her $8, which left $22 for the older children to cover. They wrote the assumption "that the older kids already had saved some money" so that they did not need to earn for additional weeks. From their savings, Alex would give $9, Sam $7, and Elena $6. Rather than focusing on the lack of proportionality, we noted the attention to the Fittingness Principle and to their notion of fairness (i.e., that all children contribute, and the older children take on more financial responsibility), likely influenced by family experiences with siblings and cousins.

All Equal with Adjustments Strategies

This strategy assumes a different notion of "fairness" than the previous two, since in an All Equal Strategy, all children contribute the same amount of money toward the gift. There were only a few examples of the All Equal with Adjustments strategy, but all of them made connections to students' funds of knowledge. In general, students initially used an All Equal strategy; then, they realized that this was not feasible or "not fair" and adjusted, usually reducing the amount that Jaden needed to contribute. In the image on the right of Figure 2, the students decided "Alex should pay most and he should pay $60 and the first grader should pay less than everybody else." In their work, it is clear that they revised their initial plan by crossing out the $30 for Alex and Jaden and writing the new amounts of $60 and 0, respectively. In other words, they revised their model so that Jaden did not have to pay any of her savings and instead her older sibling (or cousin) Alex paid for her part. In this example, when Jaden did not have enough money to pay her share, it became the responsibility of the oldest grandchild to cover it, rather than splitting her share among all three older children.

In another All Equal with Adjustments solution, each of the four children initially paid $30; the group wrote a thorough description of how Jaden can earn the additional $22:

> "Each pay thirty dollars and little Jaden can get some money from her parents or can start a bake sale of box cookies or homemade cookies or maybe pie or lemonade to get thirty dollars. Or if Jaden can't get a job she can get money from her college account because she still has several years until college."

This excerpt is a rich example of how students related to Jaden and connected their own family experiences to hers. However, the students later made adjustments so that the older two children each paid $50 and the younger two each paid $10. Ultimately, everyone contributed, but more responsibility was given to the two older siblings.

Discussion and Conclusion

We reiterate our finding that 90 of the 119 student work artifacts (76%) showed evidence of protoquantitative reasoning, attending to both the Fittingness Principle and Covariation (Resnick & Singer, 1993). This demonstrates that many young children enter the mathematics classroom with an understanding of quantity and covariation of units, which lays the foundation for multiplicative reasoning and proportional strategies. The Abuelo's Birthday task structure of listing grandchildren of different ages may have strengthened the notion of (direct) covariation, since the order of their ages aligned with the order of their earnings. Due to the open-ended nature of the Abuelo's Birthday task, students were able to draw on multiple strategies, representations, and prior mathematics concepts while attending to a realistic situation. This highlights the ability of tasks that are aligned with the modeling process to assess and revisit previous concepts but also introduce and build intuitive understanding of new math concepts. Our task, rooted in the authentic situation of sharing costs fairly, elicited informal notions of proportional reasoning that students had not experienced in formal classroom instruction (see Sawatzki et al., 2019). This is consistent with prior research claiming that mathematics problems involving realistic contexts help students develop deeper and stronger mathematical understandings (Lo & Watanabe, 1997; Verschaffel, & De Corte, 1997).

Students clearly communicated their model and justifications in the form of assumptions or sense-making about remainders (see Verschaffel, & De Corte, 1997). In our analysis of different strategies, we highlighted the strengths of students' work, rather than concluding that students were not demonstrating (formal) proportional reasoning. While students' mathematical solutions were justified by multiplicative and proportional strategies, their rationales were not exclusively mathematics-based, suggesting they were drawing on other mathematical knowledge bases (Turner et al., 2012) and their cultural funds of knowledge (Aguirre et al., 2013). Most of the student artifacts that exhibited evidence of students' funds of knowledge also included Jaden in their plans. This suggests that the non-routine structure of the Abuelo's Birthday task (i.e., including Jaden as a non-proportional component) added complexity to the task in a way that students had to draw on their experiences to make sense of. Our study provides evidence that young children across diverse cultural and linguistic backgrounds are capable of sophisticated mathematical reasoning while solving complex modeling tasks, and we encourage the use of similar tasks in elementary education settings.

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