MATHEMATICS TEACHER QUESTIONING: COGNITIVE AND AFFECTIVE DOMAINS TO NURTURE STUDENTS' MATHEMATICAL IDENTITY FORMATION

Ana-Maria Haiduc
Purdue University
ahaiduc@purdue.edu

Elizabeth Suazo-Flores
University of North Dakota
elizabeth.suazo@und.edu

Signe E. Kastberg
Purdue University
skastber@purdue.edu

Rachael Kenney
Purdue University
rhkenney@purdue.edu

Sarah E. Leach
Purdue University
leachs@purdue.edu

Andrea F. Barber-Dansby
Purdue University
abarberd@purdue.edu

Mathematics teacher questioning plays a significant role in students' learning. Research has persisted in analyzing teacher questioning from a cognitive perspective. Considering teaching as a relational practice, we explored teacher questioning from cognitive and affective perspectives. Data comprises 14 video-audio recordings, field notes, teacher's reflections, and pictures of a mathematics high school teacher while he taught linear functions. We analyzed the teacher's questioning using Bloom's cognitive and affective domains. We then characterized the type of questions concerning students' mathematical identity formation. We found that the teacher questioning techniques consisted of shifts between cognitive levels domains and added affective connotations with influence on students' mathematical identity. Using questions to promote students' cognition and mathematical identity formation is a form of caring teaching practice.

Keywords: teachers' questioning, caring teaching, students' identity, cognition, affect.

Teacher questioning during mathematics classwork plays a significant role in guiding students' thinking and gathering information about their ways of knowing mathematics. This study aims to reveal one high school mathematics teacher's questioning, the intentions of his questions, and ways questioning was used to support students' mathematical identity formation. We aimed to answer the question: How does teacher questioning offer insights about teaching practices that promote students' mathematical identity formation?

Theoretical Framework

Students' mathematical identity is shaped by their stories of learning mathematics (Sfard & Prusak, 2005a, 2005b). These stories include narratives about students' relationships with teachers (Noddings, 1994, 2013, 2017) and mathematics (Ingram, 2015), stories about who they are (actual identities), and what they can become (designated identities) (Sfard & Prusak, 2005a, 2005b), and stories about how their peers perceive them as mathematics doers (Ingram, 2015). When students construct knowledge, their stories change (Sfard & Prusak, 2005a, 2005b; Sfard, 2008). Thus, identities are dynamic. Sfard and Prusak (2005a) draw the concept of identity from human communication, asserting that "learning to think mathematically is tantamount to being initiated into a special form of discourse, known as mathematical." (p. 41 - 42). One form of communication in schools is in the form of questions and answers. Dillon (1981) pointed out that "the teacher typically speaks in questions. Students speak in answer" (p. 51). Teacher questioning maintains the classroom discourse and represents perhaps a central part of students' mathematical identity formation. That is because teachers get to know their students through questioning, which also informs their design of subsequent lessons. In addition, questioning allows students to share their ideas and be exposed to their peers' ideas.

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2). University of Nevada, Reno.

In care theory, teaching is a relational practice (Noddings, 2013). Teachers who get to know students as learners and individuals become carers. Teachers as carers are receptive listeners and respond to the students' needs. Students' recognition of their care completes a cycle that defines caring relations (Noddings, 2012). In a caring relationship, "teacher [the one caring] and student [the cared for] become partners in fostering the student's growth" (Noddings, 2017, p. 224). Developing caring relationships facilitates mathematics learning. Knowing the students as learners means understanding their mathematical cognitive processes. Knowing the students as individuals means having a "sympathetic understanding of individuals as individuals which gives [the teacher] an idea of what is actually going on in the minds of those who are learning" (Dewey, 1988, p. 33). In learning mathematics, students use cognitive functions and experience feelings that correspond to the affective domain: anxiety, arousal, attitude, attributions, beliefs and opinions, confidence, the expectancy of success, interests, motivational level, motives, perceived relevance, satisfaction, self-efficacy, and values (Bohlin, 1998).

A caring teacher gives voice to all students regardless of their proficiency, so they feel their contribution is valued (Davis, 1997; Louie, 2017). Teachers who evaluate students' various abilities inform students about the skills they might have. Boaler (2010) called this process *assigning competence,* and it takes place in a classroom where teachers value many dimensions of mathematical work. Boaler (2010) observed that to be able to value multiple skills, teachers adapted the "problems from different curriculum to make them group-worthy" (p. 42). Such problems were "open-ended problems that illustrated important mathematical concepts, allowed for multiple representations, and had several possible solution paths" (Boaler, 2010, p. 42). In addition, these problems facilitated discussions, engaging students in collaboration and sharing ideas. The teacher praised students' ideas publicly, raising the students' statuses which helped to build their mathematical identity.

Additional studies center on teacher questioning as a relational practice that frames students' mathematical ability (Battey, 2012). Battey showed that teachers' questioning and teacher-student micro-interaction facilitate students' engagement in mathematical work. Battey identified four relational practices through which teachers mediate students' access to mathematics: "addressing behavior, framing mathematics ability, acknowledging student contributions, and attending to culture and language" (p.125). Dillon (1981) explained that "discussion is a process of engaging students in some activity other than transmitting or exchanging knowledge of a factual or conventional type. The material for discussion is not factual knowledge itself, but its applications, implications, interpretations, and the like." (p. 53). In addition, Dillon (1981) highlighted that "affective processes are involved either directly or by implication, as when material bears on attitudes, beliefs, opinions, personal experiences and the like." (p. 53-54). These two statements potentially illustrate that cognitive and affective processes evolve simultaneously during relational interactions.

Literature Review

Ways of Categorizing Teachers' Questions

Bloom et al.'s (1956) cognitive domain taxonomy provides a hierarchical classification of students' cognitive processes when learning consisting of knowledge, comprehension, application, analysis, synthesis, and evaluation of learners' cognition. These levels serve as a framework for instructors "to construct and revise assessments so that they are consistent with what has been taught" (Kastberg, 2003, p.1). Besides written assessments, teachers assess students' learning in the classroom through questioning. Bloom et al.'s (1956) cognitive domain taxonomy served many mathematics researchers as a tool to analyze the type and pattern of the

questions. A common classification of teacher questioning focuses on students' cognition and uses two phases, such as lower-order and higher-order (e.g., DeJarnette et al., 2020; Drageset, 2014; Kaya & Ceviz, 2017). Lower-order questions elicit learners' memorized information, and higher-order questions elicit students' critical thinking. Studying teachers' fluctuations and adjustments of questions across the hierarchy of cognitive levels, Nathan and Kim (2009) described four levels of elicitation in teacher questioning: choice (requiring a yes or no decision), product (request for factual knowledge), process (probe for explanations and interpretations), and meta-process (requires a justification for a response earlier provided). *Choice* and *product* are considered lower-order questions, and *process* and *meta-process* are considered higher-order questions. Other types of teacher questioning include: "correcting questions (redirecting), closed progress detail questions to clarify steps of a process or procedure (progressing), to open progress questions with more than one possible answer (focusing)" (DeJarnette et al., 2020, p. 4). In addition, they mention elaboration questions and open-ended questions.

The Relationship Between Cognition and Affect

The affective domain describes ways someone reacts to other people's emotions. Bloom's et al. (1956) original taxonomy of the affective domain was revised by Krathwohl et al., (1964) to comprise: receiving the phenomenon, responding to the phenomenon, valuing, organizing, and internalizing values. Research on the relationship between affect and cognition in mathematics education has focused mostly on anxiety and attitude toward mathematics, concluding with negative results such as "anxiety inhibits cognitive processes, e.g., recall of prior learning, reducing performance" (Zan et al., 2006, p. 113). Zan et al. (2006) stated that "affect has generally been seen as 'other' than mathematical thinking, as just not part of it." (p. 113). That is because there is a belief that reasoning must overcome emotions. However, other research shows that when instruction is based solely on the cognitive domain, students struggle to value the content they learn (Bolin et al., 2005), asking the teacher, " 'Why do we have to learn this?' 'When will we ever need this?' and 'Will this information be on the test?' " (p. 154). A reconciliation between affective and cognitive domains is emphasized by Leder (2005), who pointed out a series of affective factors which pair up with the cognitive domain enabling the understanding of teachers' instructional practices. Students need the teacher's affective support to increase their motivation and interest. Affective support helps students develop a positive attitude and behavior that shapes their mathematical identity formation.

Methods

Data was comprised of 14 video-audio recordings, field notes, teacher's reflections, and pictures of slides and student work from a mathematics high school teacher while he taught the concept of linear functions. The study took place in a Midwestern high school over a period of five weeks during the fall of 2021. The school focused on project-based learning. The teacher we observed worked with students in mathematics-specific sessions to strengthen their skills and their identities as learners of mathematics. The teacher taught with student-centered activities that used estimations, science contexts, and real-life applications. Initially, 20 days of instruction were planned. However, due to the COVID-19 pandemic, a few lessons were canceled, thus instruction was reduced to 14 days. All sessions were video-audio recorded and transcribed using a machine transcription. We analyzed all the teachers' questions in two rounds. In the first round, we categorized the questions based on how they linked to the cognitive domain of Bloom's taxonomy. Because some of the questions had an affective connotation where we observed the teacher verifying students' feelings when doing mathematics, we found it useful to use the affective domain of Bloom's taxonomy for an additional interpretation of the questions.

Consequently, we used the different levels of Bloom's cognitive and affective domain to interpret the teacher's questioning practice.

Results

The teacher in this research was flexible when applying questioning techniques during instruction. The analysis showed that the teacher's questions had an affective dimension in relation to the learning event. These learning events corresponded to the cognitive phases described by Bloom et al. (1956), namely, knowledge, comprehension, application, analysis, synthesis, and evaluation. These are considered hierarchical levels. However, the teacher's questions did not follow this hierarchical order because the teacher shifted strategically between different levels during his interactions with the students. We situated the teacher's questioning within Bloom's taxonomy, characterizing the teacher's questions according to how he responded to students' reasoning. We thus considered both the cognitive and affective domains of Bloom's taxonomy. The affective dimension in the teacher's questions encouraged all students to speak, promoted students' mathematical identity formation, and demonstrated the teacher's openness to students' struggle: "Why is this [problem] throwing you off?" "You feel good about what's being asked?" "What information could you get from me that I might have that could help you?" This resulted in our organization of findings into three themes related to the teacher's questioning: Identity Forming, Strategy-Sharing, and Unpacking Students' Mathematics.

Identity Forming

Questions grouped under this theme illustrated the teacher's practice of giving all students an opportunity to freely communicate their struggle with mathematics reasoning. The teacher identified himself with the students by using the pronoun "we." The researchers identified four categories of questions that supported students' mathematical identity formation. In most of the lessons, the teacher posed questions that we categorized as *caring questions*. Through these questions, the teacher tested the students' stimulation or depletion level (Hackenberg, 2005), linked to their work or to their comprehension of the context. Examples of such questions were: "You feel good about what's being asked?" or "Were you starting to stumble on something?" The teacher broke the students' silence with questions when the students could not recall knowledge. Since the teacher was checking students' understanding of and feelings towards ideas using questioning, we linked the questions to the comprehension level of the cognitive domain and to the receiving level of the affective domain of Bloom's taxonomy. Another category of questions that we included under this theme was *Checking for mathematical confidence*. These questions tested students' self-confidence in their own way of reasoning; for example, "Who knows that their estimate is closer to correct?" The teacher invited students to judge the value of their work and decide the best-suited solution. Thus, we linked the synthesis cognitive domain and the evaluation affective domain. The *Revoicing with questions about the students' reasoning* category was comprised of questions that helped students see the value of their contributions, such as "X was saying we need to know like, how long does it last?" Through such questions, the teacher invited students to analyze a problem and to value the knowledge that they acquired from others, thus valuing others' thinking. This links the analysis level of the cognitive domain with valuing level of the affective domain in Bloom's taxonomy. We included the category *Questions to test students' attention/motivation* under this theme because such questions encourage students to participate actively in the lesson. For example, questions like "What is our task right now?" encourage students to provide responses even when they are unsure of a mathematics approach to take to the posed task. Student responses inform

the teacher about their attentiveness. Knowledge in the cognitive domain and responding in the affective domain are thus linked.

Table 1: Analyzing Questions in Relation to Identity-Forming Theme

Categories	Definition of the Category	Example Questions	Bloom's Taxonomy Cognitive/Affective
Caring questions	The teacher feels the students' struggle (he is putting himself in the students' shoes).	"You feel good about what's being asked?" "Were you starting to stumble on something?"	Comprehension/ Receiving
Checking for mathematical confidence	The teacher is testing students' self-esteem in their ways of thinking.	"Who knows that their estimate is closer to correct?"	Evaluation/Valuing
Revoicing with questions about the students' reasoning	The teacher uses students' words/responses to explain the solution.	"X was saying we need to know like, how long does it last?"	Analysis/Organizing
Questions to test students' attention/ motivation	Checking students' interest in being active (including being prepared for the lesson)	"What is our task right now?" "Anyone need something to write with?"	Knowledge/Receiving

Strategy-Sharing

This theme includes four groups of questions that invited students to justify their work or to explain what they understood when listening to others. We named one group of questions *Asking about the mathematical process.* These questions aimed at eliciting students' reasoning. For example, questions like "What's that based on?" or "[say] why you drew it the way you drew it?" invited students to analyze and make inferences while responding to a stimulus that required students to react with an explanation. Therefore, these questions linked analysis in the cognitive domain with responses in the affective domain. The category of questions, *Reminding what needs to be found*, asked students to make connections between the part of the work done and the learning goal. An example of such questions was "So now am I able to answer the question, is it a good deal?" The students demonstrated that they organized their ideas and thus comprehended and gave value to the knowledge they acquired. The third category of questions, *Guidance with hints/extra examples,* provided support by diverting students' thinking away from struggling and towards obtaining clarification of ideas. Examples of such question include: "What about this starting point?" and "If I said it's negative seven degrees, and it gets three degrees colder, so it's like more negative, right?" The hints and extra examples provided a bridge between students' thinking about their prior knowledge and the stimulus supplied by the teacher. Hence,

application in the cognitive domain and responding in the affective domain were linked to these questions. The fourth category of questions under this theme was *Guidance to think/to find strategies for solutions*. The teacher collaborated with the students to find solutions. Questions such as "How much [have] you spent in a week? How long is it going to be worth until it's going to be worth it? How much would 100 trips cost? How many weeks would it take them before buying the $139 mug as a better deal?" involved the teacher and students in collaboration through sharing problem-solving strategies to make sense and decide. Thus, such questions link the synthesis cognitive domain and organizing affective domain.

Table 2: Analyzing Questions in Relation to Strategy-Sharing Theme

Categories	Definition of the Category	Example Questions	Bloom's Taxonomy Cognitive/Affective
Asking about the mathematical process	Eliciting student's reasoning	"What's that based on?" "Why you drew it the way you drew it?"	Analysis/Responding
Reminding what needs to be found.	Linking part of the work done with the goal	"So now, am I able to answer the question, is it a good deal?"	Comprehension/ Valuing
Guidance with hints/extra example	Questioning in parallel with other examples less sophisticated or simplified	"What about this starting point?" "If I said it's negative seven degrees, and it gets three degrees colder, so it's like more negative, right?"	Application/ Responding
Guidance to think/ find strategies for solutions.	Teacher-student collaboration to find solutions.	"How much you spent in a week? How long is it going to be worth until it's going to be worth it? How much would 100 trips cost? How many weeks would it take them before buying the $139 mug is a better deal?"	Create/Receiving

Unpacking Students' Mathematics: Linking to Students' Ways of Knowing

Questions under this theme focused on students' ways of making sense and confidence in their knowledge development processes. This theme included 7 categories of questions. The first category, *Checking mathematical noticing and its use in solutions,* includes questions that ask students to intuit a response. Intuition is the most subtle form of intelligence. Here students analyze contexts and try to locate or identify helpful information to obtain a logical statement quickly. Since there might be a difference in what every student observes, these kinds of questions invite students to share and listen respectfully to their peers, which demonstrates the

connection between the analysis cognitive domain and receiving affective domain. An example of such a question is: "How many movies if you're just thinking about it, and just looking at that…?" The second category, *Initiating guidance,* guides students in identifying one helpful piece of information to start solving a problem: "Which picture do you want to use?" Such questions also show the connection between the analysis and receiving domains. The third category, *Searching questions,* helps arouse students' curiosity to explore ideas: "What would you need to be able to figure it out?" Searching questions are important because they help students to be creative when solving problems, bringing to bear facts that they consider useful. Such questions connect the application and valuing domains. The fourth category, *Checking mathematical concepts/notations,* requires students to recall facts that they have learnt; students are passive in such a situation: "Do you remember, the rate for slope ever being talked about as rise over run?" A question like this connects knowledge and receiving. The fifth category, *Checking for understanding/ validity/ sense-making,* requires students to analyze their work and make connections, and look for what is reasonable. Therefore, the link between comprehension and organizing is evident. An example of such questions is: "Do you think you could have done something wrong?" The sixth category, *Questions for different methods/to verify the results,* requires students to apply/analyze and respond accordingly. Examples of such questions are: "Can somebody improve that?" and "Who's got other strategies?" The seventh category, *Sense-making in real life,* encourages students to make connections with real life, for example, "Does that affect how much goes into the landfill?" A reasonable student response would be to evaluate their own work.

Table 3: Analyzing Questions in Relation to Unpacking Students' Mathematics

Categories	Definition of the Category	Example Questions	Bloom's Taxonomy Cognitive/Affective
Checking mathematical noticing and its use in solutions	What students intuit logically	"How many movies if you just think about it, and just looking at that you don't have to calculate anything necessarily, how many movies it's going to take to make it worth it?"	Analysis/Receiving
Initiating guidance	Identifying what can be used to start the solution	"Which picture do you want to use?" "So what's the thing that I don't know?"	Analysis/Receiving
Searches questions	Instilling curiosity for the exploration of ideas	"What would you need to be able to figure it out?"	Application/Valuing
	The teacher checks students'		Knowledge/

Checking mathematical concepts/ notations	mathematical foundation The teacher asks to analyze the	"Do you remember, the rate for slope ever being talked about as rise over run?"	Receiving Comprehension/
Checking for understanding/ validity/ sense-making	work, make connections, and look for what is reasonable. The teacher looks for different ways	"Do you think you could have done something wrong?"	Organizing Application/
Questions for different methods/verify the results	of thinking, asking students to observe and interpret the results. Connection with real life.	"Can somebody improve that?" "Who's got other strategies?"	Responding Evaluation/
Sense-making in real life.		"Does that affect how much goes into the landfill? "	Responding

Discussions and Limitations

We aimed to answer the question: How does teacher questioning offer insights about teaching practices that promote students' mathematical identity formation? Our analysis of one teacher's questioning illuminates how questions can have cognitive intentions, focusing on students' mathematics, and affective intentions, focusing on students' feelings when doing mathematics. The teacher's use of questions promoted students' cognition and affect while positioning them as thinking persons in mathematics class. The teacher asked questions we described as *identity formation, strategy-sharing,* and *unpacking students' ways of knowing mathematics*.

This teacher's questions contributed to his intentional goal of building students' mathematical identities and skills as doers of mathematics. He provided opportunities for students to share strategies, which created space for students to feel that they were listened to, their ideas were valued, and encouraged them to value others' ideas. There was evidence in our analysis of the questions that the teacher demonstrated great care for the students and wished to support confidence building in their mathematical identity. One limitation of our study is the absence of sufficient student data or reflections on their perceived mathematical identity. Future work is needed in this area. However, by attending to the teacher's intended actions, we recognize his use of questions to promote students' cognition and mathematical identity formation as an effective, caring teaching practice that can be modeled and understood by others.

References

Battey, D. (2013). "Good" mathematics teaching for students of color and those in poverty: The importance of relational interactions within instruction. Educational Studies in Mathematics, 82(1), 125-144. https://doi.org/10.1007/s10649-012-9412-z

Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed--

ability approach. British Educational Research Journal, 34(2), 167-194.
Bloom, B. S., Engelhart, M.D., Hill, W.L, Furst, E.J., and Krathwohl, D.R. (1956). Taxonomy of educational objectives: Cognitive domain. Longman.
Bohlin R. M. (1998). The affective domain: A model of learner-instruction interactions. In M.R. Simonson & D.J. Seepersaud (Eds), Proceedings of Selected Research and Development Presentations at the National Convention of the Association for Educational Communications and Technology (pp. 38–44). https://files.eric.ed.gov/fulltext/ED423824.pdf
Bolin, A. U., Khramtsova, I., & Saarnio, D. (2005). Using student journals to stimulate authentic learning: Balancing Bloom's cognitive and affective domains. Teaching of Psychology, 32(3), 154–159. https://doi.org/10.1207/s15328023top3203_3
DeJarnette, A. F., Wilke, E., & Hord, C. (2020). Categorizing mathematics teachers' questioning: The demands and contributions of teachers' questions. International Journal of Educational Research, 104, 101690.. https://doi.org/10.1016/j.ijer.2020.101690
Dewey, J. (1998). Experience and education (60th-anniversary ed.). Kappa Delta Pi.
Dillon, J. T. (1981). To question and not to question during discussion: 1. Questioning and discussion. Journal of Teacher Education, 32(5), 51–55. https://doi.org/10.1177/002248718103200512
Drageset, O. (2014). Redirecting, progressing, and focusing actions—A framework for describing how teachers use students' comments to work with mathematics. Educational Studies in Mathematics, 85(2), 281–304. https://doi.org/10.1007/s10649-013-9515-1
Ingram, N. (2015). Students' relationships with mathematics: Affect and identity. In M. Marshman, V. Geiger, & A. Bennison (Eds.), Mathematics education in the margins. Proceedings of the 38th Annual Conference of the Mathematics Education Research Group of Australasia (pp. 301–308). MERGA.
Kastberg, S. E. (2003). Using Bloom's Taxonomy as a Framework for Classroom Assessment. The Mathematics Teacher, 96(6), 402–405. https://doi.org/10.5951/MT.96.6.0402
Kaya, S., & Ceviz, A. E. (2017). Pre-service teachers' use of dynamic discourse variables during classroom teaching. Journal of Education and Practice, 8(12), 82–89. https://eric.ed.gov/?id=EJ1140633
Krathwohl, D. R., Bloom, B. S., & Masia, B. B. (1964). Taxonomy of educational objectives: The classification of educational goals. Handbook II: Affective domain. David McKay Company.
Leder, G. C. (1993). Reconciling affective and cognitive aspects of mathematical learning: Reality or a pious hope? In I. Hirabayashi, N. Nohda, K. Shigemathu, & F.-L. Lin (Eds.), Proceedings of the Seventeenth PME Conference (pp. 46–65). Tsukuba, Ibaraki, Japan: University of Tsukuba.
Nathan, M. J., & Kim, S. (2009). Regulation of teacher elicitations in the mathematics classroom. Cognition and Instruction, 27(2), 91-120. https://doi.org/10.1080/07370000902797304
Noddings, N. (2013). Caring: A relational approach to ethics and moral education. University of California Press.
Noddings, N. (2017). Care ethics and education. In N. Aloni & L. Weintrob (Eds.), Beyond bystanders: Moral development and citizenship education (pp. 183–190). Sense Publishers. doi:10.1007/978-94-6351-026-4_14.
Sfard, A. (2008). Thinking as communicating: Human development, and the growth of discourses, and mathematizing. Cambridge University Press.
Sfard, A., & Prusak, A. (2005a). Identity that makes a difference: Substantial learning as closing the gap between actual and designated identities. In Chick, H. L., & Vincent, J. L. (Eds.). (2005). Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education. Vol. 1, (pp. 37-52). Melbourne: PME.
Sfard, A., & Prusak, A. (2005b). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. Educational Researcher, 34(4), 14-22. https://doi.org/10.3102/0013189X03400401
Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. Educational Studies in Mathematics, 63(2), 113–121. https://doi.org/10.1007/s10649-006-9028-2