ESTABLISHING A MATHEMATICAL BELONINGNESS CONSTRUCT: EXPLORATORY FACTOR ANALYSIS OF IES'S HIGH SCHOOL LONGITUDINAL STUDY 2009

Lybrya Kebreab
Saint Louis University
lybrya.kebreab@slu.edu

Sarah B. Bush
University of Central Florida
Sarah.Bush@ucf.edu

Debbie Hahs-Vaughn
University of Central Florida
debbie@ucf.edu

Farshid Safi
University of Central Florida
Farshid.Safi@ucf.edu

Janet Andreasen
The Geneva School
Jbandreasen@genevaschool.org

Christa Jackson
Saint Louis University
Christa.jackson@slu.edu

This investigation utilized publicly available data from the High School Longitudinal Study 2009 (HSLS:09) by the National Center for Educational Statistics (NCES) to examine any latent structures among variables which may empirically support the validity and reliability of a mathematical sense of belonging (MSB) construct. Using the nationally representative survey data in the HSLS:09, the proposed study performed an exploratory factor analysis (EFA) to create a complex construct of students' MSB or mathematical belongingness. A weighted sample of over 13,000 (N=13,354) high school students was used to conduct the EFA, which was then mapped onto Mahar and colleagues' (2012) intersecting, transdisciplinary themes of belonging. Possible implications for preservice teachers, professional development for practicing teachers, as well as future research directions are discussed.

Keywords: Affect, Emotion, Beliefs, and Attitudes; Teacher Educators; Professional Development; High School Education

Introduction

Research is scarce on empirical, quantitative studies of variables related to secondary students' mathematical sense of belonging, especially based on data from a nationally representative sample. The field could benefit from a deeper understanding of high school students' perceptions of their mathematical belongingness, based on the five themes of belongingness that have been found across disciplines (Mahar et al., 2012). Consequently, this investigation seeks to utilize the publicly available data from the High School Longitudinal Study 2009 (HSLS:09) by the National Center for Educational Statistics (NCES) to examine any latent structures among variables which may empirically support the validity and reliability of a mathematical sense of belonging (MSB) construct.

Using nationally representative survey data in the HSLS:09, the proposed study performed exploratory factor analyses to parsimoniously explain the shared variance among a plethora of variables related to a complex construct of MSB or mathematical belongingness (hereafter used interchangeably). Experts in exploratory factor analysis (EFA) support these parsimonious solutions as they are "generally considered to have greater external validity and, as such, are more likely to replicate" (Henson & Roberts, 2006, p. 394). Importantly, this research extended and shed new light on over two decades of previous studies on belonging with respect to not only school sense of belonging, but also domain-specific belongingness. Simultaneously, the study addressed contemporary needs to elucidate factors which may be important for increasing equity and access in mathematics education as it a well-known gatekeeper for preparation and belongingness in the STEM fields (U.S. Department of Education, 2022).

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2). University of Nevada, Reno.

Purpose and Research Questions

Belongingness is basic human need which can impact several different areas of a student's school experiences as well as life and career choices (U.S. Department of Education, 2022). However, studies suggest upwards of 84% of students (Holian et al., 2020; NCTM, 2014) retain no interest (nor intention to remain) in mathematics/STEM related careers after they finish their compulsory, high school education. There is an ongoing debate regarding which current mathematics classroom norms (e.g., mathematics as a gift not a learned skill; too much focus on testing and answer-getting; lack of academic safety; little to no high-quality studies of wonder, beauty and joy) are unproductive and may impede a mathematical sense of belonging for many students (NCTM, 2018).

Despite this unfortunate state of affairs, there is still hope that we can change this situation through research which provides a deeper understanding of students' affective states, i.e., mathematical belongingness. If students' motivation and achievement in mathematics tends to wane as they enter adolescence (Wang & Pomerantz, 2009), it is important to empirically define affective variables, i.e., MSB, in secondary education to begin to understand why. Using common themes from and alignment to extant literature, students' MSB will be operationally defined, then empirically examined through a composition of variables in the HSLS:09, a large-scale, publicly available data set. Hence, the purpose of this study is to refine the literature on mathematical belongingness and its importance as an area of study in mathematics education. To this end, the following questions were researched:

1. Which factors have been found to contribute to various disciplinary theories of belongingness irrespective of the academic discipline?
2. What factor(s) can be empirically researched to establish a construct for a discipline-specific mathematical sense of belonging (MSB) within existing theoretical frameworks in the research?

Literature Review

Extant Literature on Belonging

Researchers have long found a sense of belonging to be a broadly established (Lerner et al. 2005) basic human need (Maslow, 1943; Schunk, 2012). Belongingness in school has been theorized by Baumeister and Leary (1995) as a prerequisite for overall school functioning. Further, multiple studies have found a sense of belonging at school to positively relate to students' motivation, self-esteem, classroom behavior and academic achievement, while negatively relating to school dropout (Korpershoek et al., 2020).

Teachers are empowered to and responsible for establishing optimal learning environments for students in which their cognitive and affective needs can be met (Graesser & D'Mello, 2011). Graesser and D'Mello (2011) contend that deep learning occurs when the learning environment responds to learners in ways that are sensitive to the learners' affective as well as their cognitive states. Therefore, a holistic approach to learning, where both academic and socioemotional skill development are addressed simultaneously, is necessary for student success in mathematics (de Royston et al., 2020). Secondary mathematics preservice and practicing teachers experience professional development related to honing flexible and effective pedagogical strategies, which can be described as pedagogical fluency (Kebreab et al., 2021), to address cognitive aspects of the classroom (e.g., content standards, curriculum, formative and summative assessments). However, it can be challenging to overcome so-called traditional beliefs and practices (Anderson et al., 2018) in order to cultivate pedagogical fluency related to the affective domain (e.g., students' MSB, intrinsic motivation) (Hannula et al., 2019). Teachers may not be cognizant of their own unproductive beliefs and practices (Jackson & Delaney, 2017; Schoenfeld, 2015) which may also impede their students' MSB. In the affective domain, belongingness is a vital component of learning that cannot be emphasized enough nor overstated (Arends & Visser, 2019).

Although numerous studies have examined the impact of belongingness on *general* academic and social outcomes in schools, research literature is scarce on students' sense of belonging specifically in the mathematics community (Barbieri & Miller-Cotto, 2021). It is crucial for mathematics education researchers and practitioners to develop productive beliefs and effective practices (Kilpatrick et al., 2001; NCTM, 2014) that attend to the ways in which belongingness and learning are associated in mathematics (Barbieri & Booth, 2016; Barbieri & Miller-Cotto, 2021; Good et al., 2012; OECD, 2017) in order to cultivate students' genius across academic disciplines (Muhammad, 2022) and focus on their brilliance (Leonard & Martin, 2013). Additionally, the need for mathematics education has evolved from economic and national defense motives (Tate, 2013) to becoming a necessity for understanding the world and fully engaging in a democratic society (NCTM/NCSM, 2019). Furthermore, Moses and Cobb (2001) argue "[i]n today's world, economic access and full citizenship depend crucially on math and science literacy," and that "math literacy…is the key to the future of disenfranchised communities" (Moses & Cobb 2001, pp. 4-5). For these reasons, belongingness in mathematics, as a precursor for learning in which successful achievement provides access to developing mathematical literacy and prepares students as informed members of a democratic society, can also be considered a 21st-century civil rights issue (Moses & Cobb, 2001).

At the time of this writing, there is a dearth in the research regarding empirical studies specifically focused on belongingness in secondary mathematics classrooms and its impact on student learning and achievement outcomes. Importantly, there are few studies which explicitly investigate connections between students' MSB to mathematics learning and/or academic performance (Barbieri & Booth, 2016; Barbieri & Miller-Cotto, 2021; Good et al., 2012). Although limited in quantity, the high *quality* of the insights in these studies provided important frames of reference and evidence of the field's need for this study.

Defining Belonging

As a foundation for the operationalized definition of belongingness used in this study, a brief summary of how researchers have previously conceptualized sense of belonging is provided. Theoretical frameworks on sense of belonging are commonly based on the belongingness hypothesis from Baumeister and Leary (1995). The belongingness hypothesis contends that people have "a pervasive drive to form and maintain…lasting, positive, and significant interpersonal relationships" (Baumeister & Leary 1995, p. 497). Additionally, many studies cite the participation-identification model (Finn, 1989) which suggests only students who identify with their school develop a sense of belonging to it. Importantly, Mahar and colleagues (2012) conducted a scoping, transdisciplinary literature review of previous research on belongingness. They used the key search term "sense of belonging", and reviewed referent lists of relevant papers. After reaching theoretical saturation at 40 papers, including 22 qualitative works, the researchers uncovered five intersecting themes: subjectivity, groundedness to an external referent; reciprocity; dynamism; and self-determination (Mahar et al., 2012). The final product of their review resulted in defining "sense of belonging" as follows:

> [A] subjective feeling of value and respect derived from a reciprocal relationship to an external referent that is built on a foundation of shared experiences, beliefs, or personal characteristics. These feelings of external connectedness are grounded to the context or referent group, to

whom one chooses, wants, and feels permission to belong. This dynamic phenomenon may be either hindered or promoted by complex interactions between environmental and personal factors. (Mahar et al., 2012, p.1031)

To address the first research question, Table 1 defines the five intersecting, transdisciplinary themes and connects them with the related research on sense of belonging in secondary mathematics education research (Mahar et al., 2012).

Table 1: Connecting 5 Themes and Mathematics Education

5 Intersecting, Transdisciplinary Themes of Belongingness (Mahar et al., 2012)	Mathematics Education Research Connections
Subjectivity-a perception that is unique to each individual and centers on feelings of being valued, respected and fitting in	Students need to develop a positive mathematics identity (Aguirre et al. 2013). Students' mathematical skills in the affective domain are as important as those in the cognitive domain for deep, lasting learning (Graesser & D'Mello, 2011; Hannula et al., 2019).
Groundedness to an External Referent-required referent group to anchor the subjective feeling of belongingness. One must belong to something	The field of mathematics education must broaden the purposes of learning mathematics, dispel the myth of a math person, and dismantle structural inequities (Ball & Moses, 2009; NCTM 2014; NCTM, 2018; NCTM, 2020).
Reciprocity-a sense of relatedness or connectedness that is shared by the person and the external referent	Teachers must attend to their own mathematical identities, adopt evidence-based productive beliefs, equitably apply effective mathematics teaching practices, and be culturally relevant/responsive to students' identities (Aguirre et al., 2013; Gay, 2000; Ladson-Billings, 1995; NCTM, 2014; Thomas & Berry III, 2019).
Dynamism-refers to dynamic physical and social environments which may contribute to or detract from one's sense of belonging	All stakeholders in mathematics education must be committed to lifelong learning, e.g., continuous development of mathematical habits of mind in service of the 5 strands of proficiency and the standards for mathematical practice, where learning and teaching are done with social others in context (DeRoyston et al., 2020; Kilpatrick et al., 2001; National Governors Association, 2010).
Self-Determination-respects a person's right to decide whether to interact with referents and their perceived power in the interaction	Students need to be given multiple, high-quality options, which do not lead to dead-end pathways, regarding which courses to take after completing essential concepts in high school (NCTM, 2018).

For the purposes of this study, mathematical sense of belonging (MSB) is defined as human beings' feelings of rightful presence (Isler, et al., 2021) in the broader mathematics community and/or its specialized communities of practice (e.g., mathematical artists, researchers, data analysts, educators, etc.), in which people choose to identify, participate, and persist in engaging with any dimension of mathematics (i.e., applied, pure, art, physics, data science, modeling, learning and teaching, etc.). This definition of MSB *intentionally centers the humanity and identity of the learner*, as well as an inclusive perspective on the definition of mathematics.

Extant Analyses of the HSLS:09

Although there have been several studies which utilize the data from the HSLS:09, their foci varied greatly. Some studies analyzed students' self-efficacy, identity and achievement, but were centered on select subgroups of students, i.e., Black girls (Joseph et al., 2020), African American males (Briggs, 2014), or students with visual impairments and the deaf (Lund, 2020). Another study focused on these variables but only compared male and female students during their 9th (2009) and 11th grade (first follow-up) years (Bonitto, 2020). For studies which took a more holistic approach to analyzing high school students' post-secondary education goals, researchers report students' college application trends (U.S. Department of Education, 2017) or their trajectories of academic achievement (Alhadabi & Li, 2020; Sharpe & Marsh, 2021) but did not use the most recent data on students' actualized career paths thus far. Finally, one finding closely related to the focus of this study centered on students' readiness and intentions to continue their education within Science, Technology, Engineering and Mathematics (STEM) fields, but did not explore their actualized STEM college and career accomplishments (Kurban & Cabrera, 2020).

Methodology

Composite index creation of a theory-based construct which cannot be directly observed or measured is common practice in many research fields (i.e., social science, psychology, and political science). For instance, the NASDAQ, a composite index used to measure the "health" of the National Association of State Departments of Agriculture (Chakrabartty, 2017), was founded in 1916 on the New York Stock Exchange. Chakrabartty and colleagues (2017) also note the well-known Human Development Index (HDI) as another example of an overall index combining indicators of health, education and income. Validation of new constructs resulting from the composite indices of related measures often involves advanced statistical methods, i.e., exploratory factor analysis (EFA).

To address the second research question, an EFA was conducted to create a construct of students' perceived mathematical sense of belonging (MSB) using Likert-scale and binary affective variables. EFA affords researchers the opportunity to work with several variables and concurrently reveal patterns in their underlying data structure. Therefore, EFA is often used as evidence of construct validity (Hahs-Vaughn, 2016). As mathematics-specific sense of belonging has limited theoretical and empirical research, an EFA was used as a "data-driven approach" (Brown, 2006, p.14) to establish an MSB construct. Details regarding the design of the EFA are discussed in the next section.

Delimiting the Sample

The publicly available HSLS:09 included 21,444 unweighted cases. The initial unweighted sample ($N = 21{,}444$) was first restricted to students for whom there was existing data on the base-year weight, which resulted in the sample size $N = 15{,}558$. The independent variables included in the EFA were delimited using listwise deletion as they met the threshold of less than 10% of the sample. Missing data on the affective mathematics questions from the survey were removed individually in sequential order. These variables were used in the EFA to create an MSB construct based on the theory of the transdisciplinary, scoping review of belongingness in education (Mahar et al., 2012). The variables used to construct the MSB subscales had missing that ranged from a low of $n = 59$, 0.4% for *C01A (9th grader sees himself/herself as a math person)* to a high of $n = 350$, 2.2% for *C07C (9th grader thinks fall 2009 math course is useful for future career)*. After removing this missing data, the final analytic sample size was over 13,000 high school students ($N = 13{,}534$).

Addressing EFA Assumptions

To create a mathematical belongingness construct, the data were screened to determine the extent to which the assumptions associated with EFA were met. The assumptions were generated in SPSS with base weights applied only and may contain minimal bias. Nonetheless, assumptions of a conventional EFA, including (a) independence, (b) linearity, (c) absence of outliers (both univariate and multivariate), and (d) lack of extreme multicollinearity and singularity (Hahs-Vaughn, 2016), were met for this analysis. Having addressed the assumptions, there will now be a discussion of the results of the EFA used to create a mathematical sense of belonging (MSB) construct.

Results

Evidence for construct validity for the MSB construct was obtained using an EFA. The initial factorability criteria included examination of the following: (1) bivariate correlations, (2) Kaiser-Meyer-Olkin measure of sampling adequacy (overall and individual), (3) Bartlett's test of sphericity, and (4) communalities. Then, the proceeding EFA results were presented in two parts due to the complex (ordinal and binary components) sample design. These proceeding items were chosen in that they either captured the definitive letter (parallel syntax) or the spirit (approximate semantics) of the five transdisciplinary themes of belonging (Table 1). The first is based on 17 Likert-scale items mapped onto four of the five themes, namely subjectivity, groundedness to an external referent, reciprocity, and dynamism (Mahar et al., 2012). The second used 9 binary items to theoretically map onto self-determination, the fifth and final theme. As the binary items were not included in the EFA, their relationships are based on theory not statistical analysis.

Using a Spearman's correlation coefficient because the data are ordinal, each of the items correlated at least .30 with at least one other item. The overall Kaiser-Meyer-Olkin measure of sampling adequacy was .90, larger than the recommended value of .50. In addition, the measure of sampling adequacy values for the individual items were all greater than the recommended value of .50. Next, the uniqueness (computed as $1 - communality$) exported from R (R Core Team, 2020) using the complex survey design (with base and replicate weights) was assessed. Generally, the uniqueness values are below .30 and indicate a large proportion of the variance of the item being accounted for by the factor.

All Likert-scale items contributed to a simple factor structure and had a primary factor loading above the recommended .30. Therefore, each Likert-scaled item was retained in the EFA. Although three items are the widely accepted minimum number of factors to index a subscale, the fourth factor only has two items. Nonetheless, it was retained as the two items combined were already a composite scale, named mathematics identity (coefficient of reliability $\alpha = 0.84$), by the survey designers (Ingels et al., 2011). Mathematical identity is an established, well-researched construct in extant mathematics literature (Aguirre et al., 2013). Similarly, the third factor also mirrored the results of the original survey in a composite scale named mathematics utility with a coefficient of reliability $\alpha = 0.78$ (Ingels et al., 2011). Given the other criteria for determining factorability were met, it was reasonable to proceed with determining the factor structure of the four items mapped onto the first four transdisciplinary themes of belonging (Mahar et al., 2012).

Principal component analysis (PCA) was conducted with the complex survey design in R, meaning both base and replicate weights were applied. The PCA was conducted first with Promax (an oblique rotation because the data are related in the survey), then varimax (to clarify the relationships between the variables) rotations were used to extract the factors from the data in R. Parallel analysis was used to determine the number of factors to retain. The results suggested a four-factor model was appropriate (i.e., the first four raw data eigenvalues were greater than the random and permutated mean and 95th percentile eigenvalues; all other raw data eigenvalues were less in value). Although a more subjective tool for determining the number of factors, the scree plot (Figure 1) indicated the eigenvalues leveled off after four factors, again supporting a four-factor solution. The interpretation of a four-factor solution was also plausible and was a consideration in retaining two factors. All but one component variable, *S1 C11F 9th grader's fall 2009 math teacher treats some kids better than others*, were retained before testing for internal consistency and reliability. Post extraction, the four-factor solution represented about 59% of the variance explained.

Scree plots are a graphic tool that can be used to decide on the number of factors to retain when conducting an EFA. In interpreting the scree plot in Figure 1, researchers examine where the line goes from being diagonal to being horizontal. There is some subjectivity with the decision to determine the four factors to retain in this graph, but scree plots generally perform better than the eigenvalue greater than one rule (Hahs-Vaughn, 2016).

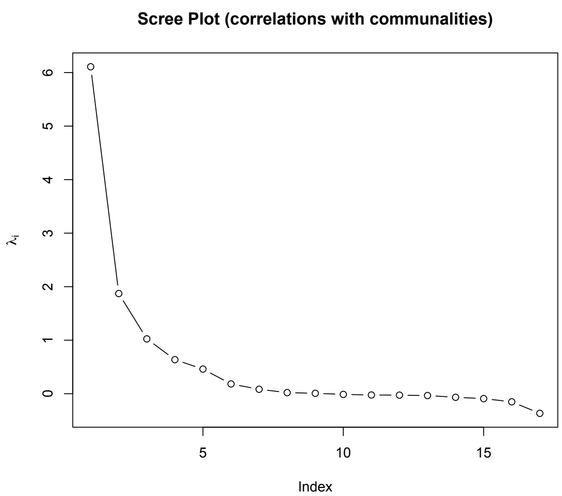

Figure 1: EFA Scree Plot

Table 2 provides the factor loading pattern matrix for the final solution. The names of the four sub-scaled factors for students' MSB are Reciprocity (Factor 1), Dynamism (Factor 2), Groundedness to an External Referent/Mathematics Utility (Factor 3), and Subjectivity/Mathematics Identity (Factor 4). The results of the EFA lend support to internal structure validity evidence supporting the conclusion that the scores from this instrument are a valid assessment of the sub-scales MSB (based on four of the five transdisciplinary themes), thereby justifying the retention of the theoretical names. Factor values bolded in Table 2 were chosen for the final EFA analyses of internal consistency.

Table 2: Exploratory Factor Analysis- Factor Loadings Pattern Matrix

Variables	Factor 1	Factor 2	Factor 3	Factor 4
S1 C01A 9th grader sees himself/herself as a math person		0.208	0.162	**0.837**
S1 C01B Others see 9th grader as a math person		0.153	0.135	**0.790**
S1 C11A 9th grader's fall 2009 math teacher values/listens to students' ideas	**0.757**	0.283		
S1 C11B 9th grader's fall 2009 math teacher treats students with respect	**0.869**	0.138	0.110	
S1 C11C 9th grader's fall 2009 math teacher treats every student fairly	**0.857**	0.161		
S1 C11D 9th grader's fall 2009 math teacher thinks all student can be successful	**0.720**	0.159	0.164	
S1 C11E 9th grader's fall 2009 math teacher thinks mistakes OK if students learn	**0.620**	0.175		
S1 C11F 9th grader's fall 2009 math teacher treats some kids better than others	**0.520**	0.170		
S1 C07A 9th grader thinks fall 2009 math course is useful for everyday life	0.111	0.214	**0.648**	
S1 C07B 9th grader thinks fall 2009 math course will be useful for college	0.134		**0.690**	0.117
S1 C07C 9th grader thinks fall 2009 math course is useful for future career		0.129	**0.803**	0.135
S1 C06A 9th grader is enjoying fall 2009 math course very much	0.266	**0.621**	0.246	0.339
S1 C06B 9th grader thinks fall 2009 math course is a waste of time	-0.212	**-0.466**	-0.321	-0.152
S1 C06C 9th grader thinks fall 2009 math course is boring	-0.232	**-0.629**	-0.203	-0.178
S1 C11G 9th grader's fall 2009 math teacher makes math interesting	0.508	**0.616**	0.102	
S1 C11I 9th grader's fall 2009 math teacher makes math easy to understand	0.510	**0.545**		0.105
S1 C11F 9th grader's fall 2009 math teacher treats some kids better than others	0.395	0.113		
SS Loadings	4.129	2.035	1.882	1.570
Proportion Variance	0.243	0.120	0.111	0.092
Cumulative Variance	0.243	0.363	0.473	0.566
The degrees of freedom for the model was 74 and the fit was 0.5888				

Subscales were measured for internal consistency via Cronbach's alpha (α) of reliability. Table 3 provides the results of the final variable components retained for each factor with the corresponding Cronbach's α indices. All Cronbach's α measures in Table 9 were greater than the recommended .70 for acceptable internal consistency reliability (UCLA Statistical Consulting Group, 2022). These analyses were conducted in R.

Table 3: Mathematical Sense of Belonging (MSB) Subscale Reliability

Transdisciplinary Theme	Component Variables	Cronbach's Alpha Coefficient
Subjectivity/Mathematics Identity	C01A, C01B	$\alpha = .838$
Groundedness to an External Referent/Mathematics Utility	C07A, C07B, C07C	$\alpha = .779$
Reciprocity	C11A, C11B, C11C, C11D, C11E, C11F	$\alpha = .878$
Dynamism	C06A, C11G, C11I	$\alpha = .809$

Lastly, the binary items were theoretically mapped onto the fifth transdisciplinary theme of self-determination. This mapping was based on parallel syntax and semantics of the survey items and the intersecting, transdisciplinary self-determination theme's description (Mahar et al., 2012). As binary items need a different correlation matrix to generate, it was not yet possible to apply the complex design in R nor SPSS with the available packages. Therefore, these survey items were assigned a weighted, composite index.

Discussion

This study used exploratory factor analysis and long-standing belongingness and mathematics educational theoretical research to establish a set of sub-scales for creating an empirically constructed mathematical sense of belonging (MSB) construct. The data from the HSLS:09 were used to analyze mathematics affective survey variables and establish external validity (using base and replicate weights) and internal consistency and reliability (alpha scores greater than .70). The large-sized ($N = 13{,}534$) and nationally representative sample contributed to the success of this research as the results are generalizable. Results showed four of five *transdisciplinary* themes of belonging (namely Subjectivity, Groundedness to an External Referent, Reciprocity, and Dynamism) empirically unveiled statistically reliable subscales of a new mathematical sense of belonging (MSB) construct within a complex sample (Table 3).

Although the EFA was successful in this study, there are limitations. First, the participants were high school students, therefore the generalizability is limited to secondary students. Further studies could be conducted on primary/elementary students to establish an MSB for younger students. Finally, one of the five transdisciplinary themes of belonging, namely self-determination, could only be theoretically mapped to binary items from the HSLS:09 and were not a part of the EFA. The authors attempted to reach out to other R community experts, but none were able to apply a complex survey design to the EFA in order to include the binary items. Hopefully, appropriate R-packages and corresponding codes will be available in the near future. Further detailed statistical results may be requested of the corresponding author.

Implications

The results of this study provide a foundation for identifying subscales for an empirically established, discipline specific MSB. Mathematics teacher educators are encouraged to implement and instruct preservice teachers on the importance of each of transdisciplinary themes of belongingness and the themes' relationships to existing literature on supporting students' positive MSB (Table 1). Practitioners could use this research to justify the need for professional development which focuses on a holistic approach to mathematics instruction.

References

Aguirre, J. M., Mayfield-Ingram, K., & Martin, D. B. (2013). *The Impact of Identity in K-8 Mathematics: Rethinking Equity-based Practices*. Reston, VA: National Council of Teachers of Mathematics.

Alhadabi, A., & Li, J. (2020). Trajectories of academic achievement in high schools: Growth mixture model. *Journal of Educational Issues, 6*(1), 140-165.

Anderson, R. K., Boaler, J., & Dieckmann, J. A. (2018). Achieving Elusive Teacher Change through Challenging Myths about Learning: A Blended Approach. *Education Sciences, 8*(3), 98.

Arends, F., & Visser, M. (2019). The contribution of South African teachers to students' sense of belonging and mathematics achievement: Students' perspective from the 2015 Trends in International Mathematics and Science study. *South African Journal of Childhood Education, 9*(1), 1-11.

Ball, D., & Moses, R. (2009, October 1). *Equity and mathematics: an interview with Deborah Ball and Bob Moses*, 54-59. (J. Richardson, Interviewer) Phi Delta Kappa.

Barbieri, C., & Booth, J. L. (2016). Support for struggling students in algebra: Contributions of incorrect worked examples. *Learning and Individual Differences, 48*, 36-44.

Barbieri, C., & Miller-Cotto, D. (2021). The importance of adolescents' sense of belonging to mathematics for algebra learning. *Learning and Individual Differences, 87*, 101993.
Baumeister, R. F., & Leary, M. R. (1995). The need to belong: a desire for interpersonal attachments as a fundamental human motivation. *Psychological bulletin, 117*(3), 497.
Bonitto, C. M. (2020). Comparing male and female student self-efficacy, identification, and achievement in Mathematics: A cross-lagged panel analysis of causal effects. *[Doctoral dissertation]*. University of Kansas.
Briggs, C. (2014). Mathematics: Self-efficacy, identity, and achievement among African American males from the high school longitudinal study. *Publication No. 3583106*. ProQuest Dissertations and Theses Global.
Brown, T. A. (2006). *Confirmatory factor analysis for applied research.* Guilford Press.
Chakrabartty, S. N. (2017). Composite index: methods and properties. *Journal of Applied Quantitative Methods, 12*(2), 25-33.
DeRoyston, M. M., Lee, C., Nasir, N. S., & Pea, R. (2020, October 26). *Rethinking schools, rethinking learning*. Retrieved from Kappanonline.org: https://kappanonline.org/rethinking-schools-rethinking-learning-mckinney-de-royston-lee-nasir-pea/
Finn, J. D. (1989). Withdrawing from school. *Review of educational research, 59*(2), 117-142.
Gay, G. (2000). *Culturally responsive teaching: Theory, research, and practice.* New York, NY: Teachers College Press.
Good, C., Rattan, A., & Dweck, C. S. (2012). Why do women opt out? Sense of belonging and women's representation in mathematics. Journal of personality and social psychology. *102*(4), 700-717.
Graesser, A., & D'Mello, S. K. (2011). Theoretical perspectives on affect and deep learning. In R. A. Calvo, & S. K. D'Mello (Eds.), *New Perspectives on Affect and Learning Technologies* (pp. 11-21). New York, New York: Springer.
Hahs-Vaughn, D. L. (2016). Applied multivariate statistical concepts. Chicago: Routledge.
Hannula, M., Leder, G. C., Morselli, F., Vollstedt, M., & Zhang, Qiaoping, Z. (2019). Affect and mathematics education: Fresh perspectives on motivation, engagement, and identity. Springer.
Henson, R. K., & Roberts, J. K. (2006). Use of exploratory factor analysis in published research. *Educational and Psychological Measurement*, *66*(3), 393–416. https://doi.org/10.1177/0013164405282485
Holian, L., & Kelly, E. (2020). *STEM Occupational Intentions Stability and Change through High School. Stats in Brief.* National Center for Education Statistics.
Ingels, S. J., Pratt, D. J., Herget, D. R., Burns, L. J., Dever, J. A., Ottem, R., ... & Leinwand, S. (2011). High School Longitudinal Study of 2009 (HSLS: 09): Base-Year Data File Documentation. NCES 2011-328. *National Center for Education Statistics*.
Isler, J. C., Berryman, N. V., Harriot, A., Vilfranc, C. L., Carrington, L. J., & Lee, D. N. (2021). Defining the Flow—Using an Intersectional Scientific Methodology to Construct a Vanguard STEM Hyperspace. *Genealogy, 5*(1), 8.
Jackson, C., & Delaney, A. (2017). Mindsets and practices shifting to an equity centered paradigm. In A. Fernandes, S. Crespo, & M. Civil (Eds.), Access and Equity Promoting High-Quality Mathematics in Grades 6–8 (pp. 143-155). Reston: National Council of Teachers of Mathematics.
Joseph, N. M., Tyler, A. L., Howard, N. R., Akridge, S. L., & Kelsi, R. R. (2020). The role of socialization in shaping Black girls' mathematics identity: An analysis of the high school longitudinal study 2009. *Teachers College Record*.
Kebreab, L., Bush, S. B., & Jackson, C. (2021). Leveraging student identities to develop pedagogical fluency. Mathematics Teacher: Learning and Teaching PK-12, 114(12), 948-955.
Kilpatrick, J., Swafford, J., & Bradford, F. (2001). *Adding it up: Helping children learn mathematics (Vol. 2101).* Washington, DC: National Academy Press.
Korpershoek, H., Canrinus, E. T., Fokkens-Bruinsma, M., & de Boer, H. (2020). The relationships between school belonging and students' motivational, social-emotional, behavioural, and academic outcomes in secondary education: A meta-analytic review. *Research Papers in Education, 35*(6), 641-680.
Kurban, E. R., & Cabrera, A. F. (2020). Building Readiness and Intention Towards STEM Fields of Study: Using HSLS:09 and SEM to Examine This Complex Process Among High School Students. *The Journal of Higher Education, 91*(4), 620-650.
Ladson-Billings, G. (1995). But that's just good teaching! The case for culturally relevant pedagogy. *Theory Into Practice, 34*(3), 159-165.
Lerner, R. M., Lerner, J. V., Almerigi, J. B., Theokas, C., Phelps, E., Gestsdottir, S., & ... & Smith, L. M. (2005). Positive youth development, participation in community youth development programs, and community

contributions of fifth-grade adolescents: Findings from the first wave of the 4-H study of positive youth development. *The Journal of Early Adolescence, 25*(1), 17-71.
Leonard, J., & Martin, D. B. (2013). *The brilliance of Black children in mathematics.* IAP.
Lund, E. M. (2020). Post-school employment and education outcomes of young adults reporting deafblindness on the High School Longitudinal Study of 2009. *Journal of the American Deafness & Rehabilitation Association (JADARA), 53*(3), 11-22.
Mahar, A., Cobigo, V., & Stuart, H. (2012). Conceptualizing belonging. *Disability and rehabilitation, 35*(12), 1026-1032.
Maslow, A. H. (1943). A theory of human motivation. *Psychology Review, 50*(4), 370-396.
Muhammad, G. (2022, May 18). *The Blueprint of Our Lives.* Retrieved from HILL Pedagogies: https://hillpedagogies.com/resources/
Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Civil rights from Mississippi to the algebra project.* Boston: Beacon Press.
NCTM/NCSM. (2019). *Building, S. T. E. M. Education on a Sound Mathematical Foundation.* Retrieved from https://www.nctm.org/Standards-and-Positions/Position-Statements/Building-STEM-Education-on-a-Sound-Mathematical-Foundation
National Council of Teachers of Mathematics. (2014). *Principles to actions: ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics.
National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations.* Reston, VA: National Council of Teachers of Mathematics.
National Council of Teachers of Mathematics. (2020). *Catalyzing change in middle school mathematics: initiating critical conversations.* Reston, VA: National Council of Teachers of Mathematics.
National Governors Association. (2010). *Common core state standards*. Retrieved from http://www.corestandards.org/Math/
OECD. (2017). Retrieved 10 2019, from PISA 2015 results (volume III): Students' well-being: https://doi.org/10.1787/9789264273856-en
Schoenfeld, A. H. (2015). What counts, when? Reflection on beliefs, affect, attitude, orientations, habits of mind, grain size, time scale, context, theory, and method. In *From beliefs to dynamic affect systems in mathematics education* (pp. 395-404). Switzerland: Springer International Publishing.
Schunk, D. H. (2012). *Learning theories and educational perspective sixth edition.* New York, NY: Pearson.
Sharpe, S. T., & Marsh, D. D. (2021). A systematic review of factors associated with high schoolers' algebra achievement according to HSLS: 09 results. *Educational Studies in Mathematics*, 1-24.
Tate, W. F. (2013). Race, retrenchment, and the reform of school mathematics. In E. Gutstein, & B. Peterson (Eds.), *Rethinking mathematics: Teaching social justice by the numbers, second edition* (pp. 42-51). Milwaukee, WI.
Thomas, C. A., & Berry III, R. Q. (2019). A qualitative meta-synthesis of culturally relevant pedagogy & culturally responsive teaching: Unpacking mathematics teaching practices. *Journal of Mathematics Education at Teachers College, 10*(1), 21-30.
UCLA Statistical Consulting Group. (2022). *OARC Stats. Retrieved May 9, 2022, from*. Retrieved from UCLA Advanced Research Computing: Statistical Methods and Data Analytics: https://stats.oarc.ucla.edu/spss/faq/what-does-cronbachs-alpha-mean/
U.S. Department of Education. (2017). *College Applications of 2009 High School Freshman: Differences by Race/Ethnicity.* Data Point, IES-NCES.
U.S. Department of Education. (2022). U.S. Department of Education Launches New Initiative to Enhance STEM Education for All Students: Teacher development and partnerships with external organizations are key elements. (2022, December 7). *Home-U.S. Department of Education*. Retrieved February 22, 2023, from https://www.ed.gov/news/press-releases/us-department-education-launches-new-initiative-enhance-stem-education-all-students
Wang, Q., & Pomerantz, E. M. (2009). The motivational landscape of early adolescence in the United States and China: A longitudinal investigation. *Child Development, 80*(4), 1272-1287.