INVESTIGATING STUDENT'S UNDERSTANDING OF THE AREA AND PERIMETER OF RECTANGLES THROUGH PROBLEM-SOLVING

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Numerous studies (Anwar et al., 2016; Carpenter et al., 1980; Huang & Witz, 2013; Outhred & Mitchelmore, 1996; Machaba, 2016; Van de Walle, Karp & Bay-Williams, 2014; Winarti et al., 2012) have shown that students need help understanding the concept of area and perimeter. This study aimed to investigate what students know about finding the area and perimeter of rectangles through problem-solving. The participants for the study were five 4th Graders from a public school located in a mid-size suburb setting outside the Chicago metropolitan area. Participants were asked to solve five problem-solving tasks, after which an interview was conducted for three out of the five participants. Results showed that students did not understand the concept of the area and perimeter of rectangles. Implications for the study, as well as future research, were also discussed.

Keywords: geometry, relational and instrumental understanding, area, perimeter, problem-solving

Introduction/Statement of the Problem

Recommendations regarding geometry and measurement as essential content standards in kindergarten through 12th-grade mathematics are clearly stated in the National Council of Teachers of Mathematics documents (NCTM, 1989, 2000, 2010). It is believed that having geometric knowledge, relationships, insight, and the ability to measure helps children see that mathematics is practical in everyday life and that it also helps them develop many mathematical concepts and skills (NCTM, 1989, 2000, 2010). Research also shows that children learn about the properties of shapes and sharpen their intuitions and awareness of spatial concepts as they explore patterns and relationships with models, blocks, geoboards, and graph paper (King, Rojo, Bryant, 2022; Winarti et al., 2012). Area and perimeter skills have been highlighted as critical foundations of algebra and as a prerequisite for more advanced mathematics courses (National Mathematics Advisory Panel [NMAP], 2008). The learning of geometry and measurement encourages and develops students' problem-solving and mathematical thinking skills, as well as the ability to apply mathematical knowledge (Putra, Panjaitan, Putri, Wulandari, & Hermita, 2021).

Research in the field of mathematics education, locally and internationally, often reveals a poor understanding of the concepts of area and perimeter (Anwar, Yuwono, As'ari, & Dwi, 2016; Carpenter, Corbitt, Kepner Jr, Lindquist, & Reys; Huang & Witz, 2013; Machaba, 2016; Outhred & Mitchelmore, 2000; Winarti, Amin, Lukito, & Gallen, 2012). Results showed that most learners do not realize that it is possible to have many rectangles with the same area but different perimeters and vice versa (Outhred & Mitchelmore, 1996; Machaba, 2016; Van de Walle, Karp & Bay-Williams, 2014). Similarly, some learners thought that rectangles with the same area should possess the exact dimensions and have the same perimeter, and vice versa (Machaba, 2016). Another way that students demonstrated a lack of proper understanding is by giving procedural definitions (formula definitions) for the area and perimeter of rectangles (Huang & Witz, 2013; Machaba, 2016).

Unfortunately, a gap in the research is that the concept of area and perimeter is an underdeveloped skill area for many school-age children (King et al., 2022). Therefore, this study aimed to investigate 4th graders conceptual understanding of and their ability to solve problems related to the area and perimeter of rectangles by applying mathematical knowledge gained on area and perimeter to new experience. It is believed that this study will further emphasize the need to develop children's conceptual understanding of the area and perimeter of shapes (regular and irregular) and promote problem-solving to measure the current state of students in terms of their understanding of the area and perimeter of rectangles. To achieve the stated purpose of this study, I proposed the following research questions:

- What are students' conceptual understanding of the area and perimeter of rectangles?
- How did students' prior knowledge and /or conceptual understanding assisted them in solving tasks about the relationship between area and perimeter of rectangles?
- What knowledge if any were students' able to construct about the area and perimeter of rectangles?

Background of the Study

Despite their importance in mathematics teaching and learning, studies have revealed a poor understanding of the concepts of area and perimeter (Anwar et al., 2016; Carpenter et al., 1980; Huang & Witz, 2013; Outhred & Mitchelmore, 1996; Machaba, 2016; Van de Walle, Karp & Bay-Williams, 2014; Winarti et al., 2012). According to (Winarti et al., 2012), some of the reasons for this continual source of confusion might be because (1) both area and perimeter involve measurement and (2) students are taught formulae for both concepts at about the same time, therefore getting formulae confused. Suggestions on how to address the issue were provided by some researchers (Carpenter et al., 1980; King et al., 2022; Putra et al., 2021; Winarti et al., 2012). For example, strategies that can be used to demystify area and /or perimeter include using manipulatives (Outhred & Mitchelmore, 2000; Winarti et al., 2012), focusing on the variable, using visual-chunking representations, and contextualizing instruction (King et al., 2022). Designing and implementing GeoGebra learning activities on the area and perimeter of rectangles have also been found to be helpful for primary school students (Putra et al., 2021). By using GeoGebra, students can increase their involvement in understanding the concepts of the area and perimeter of rectangles and visualize rectangular shapes of different sizes (Putra et al., 2021).

Problem-solving has also been found to aid students' understanding of the concept of as well as the development of relational understanding (Skemp, 1976) between the area and perimeter of rectangles (Carpenter et al., 1980; King et al., 2022). An effective way to achieve this is to contextualize instruction by incorporating real-world scenarios and experiences that would allow connection to student's prior knowledge (King et al., 2022; Machaba, 2016). Through problem-solving, students can apply knowledge and skills acquired during mathematics instruction to solve new tasks (Carpenter et al., 1980). Also, since students learn mathematics to be able to reason mathematically and not just to memorize mathematical formulas or facts, problem-solving has been found to help develop understanding and explain the processes used to arrive at solutions, rather than remembering and applying a set of procedures (Klerlein & Hervey, 2019; Skemp, 1976). Through problem-solving, students develop a relational understanding of mathematical concepts (Skemp, 1976), become more engaged (Bada, 2015), and appreciate the relevance and usefulness of mathematics (Klerlein & Hervey, 2019).

Theoretical Framework

The theoretical framework for this study is based on Piaget's constructivist theory which focuses on children's ability to assimilate new learning into their existing schema. Constructivism is a theory about how people learn (Machaba, 2016). It states that people construct their own understanding of the world through their experiences with the world (Bada, 2015; Hein, 1991; Suhendi & Purwarno, 2018). When we encounter a new experience, we must reconcile it (assimilate) with our previous ideas and experiences (Bada, 2015; Machaba, 2016; Andang, 2018). The ability to reconcile a new experience (e.g., novice mathematical task) with existing schema is a way that students construct knowledge (Skemp, 1976). This view of learning sharply contrasts with one in which learning is the passive transmission of information from one individual to another, a view in which reception, not construction is key. For students to be able to construct knowledge, they must actively participate in the teaching and learning process (Bada, 2015; Hein, 1991). Some of the advantages of constructivist theory is that it produces students who are independent, able to think, able to express ideas and to solve problems (Bada, 2015; Machaba, 2016).

The process of assimilating or reconciling the unfamiliar with what one already knows in mathematics education is a way of identifying and establishing relationship(s) between mathematical topics or concepts (Machaba, 2016; Skemp, 1976). Identifying and establishing relationship(s) between mathematical topics or concepts creates a type of understanding known as relational understanding (Skemp, 1976). Skemp (1976), simply defined relational understanding as "knowing both what to do and why" (p. 2). Skemp expected that students in the mathematics class should not only know how to solve a problem (by using formulas or rules), but they should also be able to state or justify why the method(s) works. For example, Skemp mentioned that it is not sufficient for students to memorize the formula for calculating the area of a triangle = 1/2 base * height but should also be able to relate this with finding the area of a rectangle (p. 9). In contrast to relational understanding, Skemp (1976), defined instrumental understanding as "the application of mathematical rules to find answers to mathematical tasks without reasons" (p. 2). For example, many learners know that the formula to calculate the area of a rectangle is length multiplied by breadth, but they do not know why this is so. He argues that learners should develop a relational understanding of area and perimeter.

For relational understanding to take place, students also need to have conceptual knowledge of mathematical topics or concepts (Skemp, 1976). This is because lack of conceptual understanding results into having misconceptions (Machaba, 2016). Studies have shown that the inability of learners to have a relational understanding of area and perimeter of figures is due to a misunderstanding of the concepts of area and perimeter (Outhred & Mitchelmore, 1996; Van de Walle et al. 2014). To have a conceptual understanding of area of a rectangle means to be able to find the amount of space inside the edges of the figure. Whereas having a conceptual understanding of the perimeter of a rectangle is to be able to find the measurement of the length of the edges of the figure (Machaba, 2016). To be able to have these conceptual understandings, students also need to be taught how to use tiles or unit cubes to cover up surfaces of rectangles as a way of finding their perimeters (Winarti et al., 2012) and /or areas (Outhred & Mitchelmore, 2000; Winarti et al., 2012). When students have good conceptual understanding of perimeter and area of rectangles, it becomes easy for them to see and understand why rectangles with same perimeter does not need to have same area measurement and vice versa. They also come to understand that rectangles with the same area have dimensions that are factors of the fixed area (Machaba, 2016).

Methodology

The research methodology used in this study was qualitative. It was organized around a problem-solving task administered to five 4th-grade students (two boys and three girls) between the age of 9-10 years as well as a clinical interview carried out with only three of the students (two boys and one girl). The study took place at a public school located in a mid-size suburb setting outside the Chicago metropolitan area. The student population consists of 48% female students and 52% male students. The school is racially diversified, with 75% White and 24.4% Minority Enrollment: 9.7% Hispanic/Latino, 8.5% two or more races, 3.8% Asian or Asian/Pacific Islander, and 2.3% Black or African American.

There are two parts to this study. In the first part, students were asked to answer four problem-solving tasks (Table 1) to test their understanding of and ability to solve problems related to the area and perimeter of rectangles. The students spent approximately 45 minutes on the tasks. The second part of the study was an interview session. Three students each participated in an interview that lasted for 15 minutes. The interview session (Table 2) enabled me to find out if the students had a conceptual understanding of the area and perimeter of rectangles, how each student's prior knowledge and /or conceptual understanding assisted to problem solve, and whether they were able to construct new knowledge. The interview session was audio recorded. The instruments used for data collection were problem-solving tasks and interview questions (see Table 1 for excerpts of interview questions).

Table 1: Problem-Solving Tasks and Excerpt of Interview Questions

The Perimeter and Area Puzzle	Excerpt of Interview Questions
Draw as many rectangles as you can that have a perimeter of 24 units. Find the area of each rectangle you drew. What do you notice about the rectangles?	Can you please read this question and explain to me in your own words what the problem is asking you to do?
Draw as many rectangles as you can that have an area of 24 sq. units. Find the perimeter of each rectangle you drew. What do you notice about the rectangles?	What does it mean to find the perimeter of a rectangle?
Find all the rectangles that have an area of 36 sq. units. Fill in the table with the length, width, and perimeter of each rectangle. What patterns did you notice?	What does it mean to find the area of a rectangle?
From your solutions to questions 1, 2, and 3 above, can you tell what happens to the rectangles when the area stays the same? Can you tell what happens to the rectangle when the perimeter stays the same?	Can you explain what you did and why you did it that way?
	Does this rectangle have a perimeter of 24

units?

What can you notice about the rectangles?

Data Analysis and Coding Scheme

Two sets of data were analyzed. The first set of data was focused on the children's written solutions to the four problems. A student is said to have a conceptual understanding if s/he can come up with different rectangles (that has same perimeter but different area and vice versa) with the help of manipulative or by drawing rectangles with surfaces covered with squares. For the second research question, I identified students' prior knowledge: compose two-dimensional shapes, partition a rectangle into rows and column of same-size squares and count to find the total number of them, understand concepts of area and relate area to multiplication and to addition, and recognize perimeter as an attribute of plane figures (Common Core State Standards for Mathematics, 2010) used during problem-solving. By using their prior knowledge, students should be able to realize that: (1) we can have many rectangles with same perimeter but different areas and vice versa, (2) rectangles with the same area have dimensions that are factors of the fixed area (3) when the difference between the dimensions of a rectangle with a fixed area is the smallest, you will have the smallest perimeter (same is applicable when the difference between the dimensions of a rectangle with a fixed area is the largest), (4) given a fixed perimeter, the rectangle with the largest area will be the one with the dimensions that are closet together (a square), (5) given a fixed perimeter, the rectangle with the smallest area will be the one with the dimensions farthest apart. For the third research question, students can construct knowledge if they are able to establish the above relationships about the area and perimeter rectangles.

During the interview session, a student was said to have conceptual understanding if s/he didn't provide formula definitions of area and perimeter of rectangles (Machaba, 2016). The definitions or description could also be given in relation to when tiles or unit squares are used as coverings for the surface of rectangles. Two out of the questions asked during the interview are: (1) Can you explain what you did and why you did it that way? (2) What do you notice about the rectangles drawn? These two questions were asked to know if the students were able to form relationships between area and perimeter of rectangles (as stated above) using their prior knowledge (CCSSM, 2010). The interviews with the three students were transcribed. The reason for selecting these students was due to their responses to the tasks.

Results

In this section, I will briefly discuss students' conceptual understanding of one of the problem-solving tasks (see Figure 1 & Table 1), type of prior knowledge used, whether students were able to establish relationship between area and perimeter using their prior knowledge, and their responses to part of the interview questions (see Table 2).

Coded Student's Work

It is important to mention that the students were already working on finding the area of rectangles using tiles/unit squares, completing multiplication facts, and solving multiplication equations before carrying out this study. As they began to solve the tasks, I provided them with manipulative. While student 1 and student 2 demonstrated a conceptual understanding of area of rectangles by drawing unit squares to cover the surfaces (Outhred & Mitchelmore, 2000), the question instead asked them to first draw as many rectangles as possible with perimeter 24 units, and then find the area of each rectangle. Based on this, it is unclear if the students had a conceptual understanding of perimeter of rectangles. The solution provided by student 3 showed that she doesn't have a conceptual understanding of area and perimeter of rectangles (Outhred & Mitchelmore, 1996). Overall, I think all three students attended to finding the area of the rectangles because that was the topic they were learning at that time.

All three students work demonstrated the use of some prior knowledge of area and perimeter of rectangles. For instance, the students were able to compose two-dimensional shapes, partition a rectangle into rows and column of same-size squares (except for student 3) and count to find the total number of them, understand concepts of area and relate area to multiplication. But none of the students were able to recognize perimeter as an attribute of plane figures (Machaba, 2016). It was expected that the students would be able to relate area and perimeter of rectangles using their prior knowledge. Unfortunately, they were unable to notice any of such relationships. During the interview session on question 2 (not shown), only student 2 was able to notice that when the area is fixed, the rectangles have different perimeters. One of the reasons for this may be because the students lacked conceptual understanding of area and perimeter of rectangles.

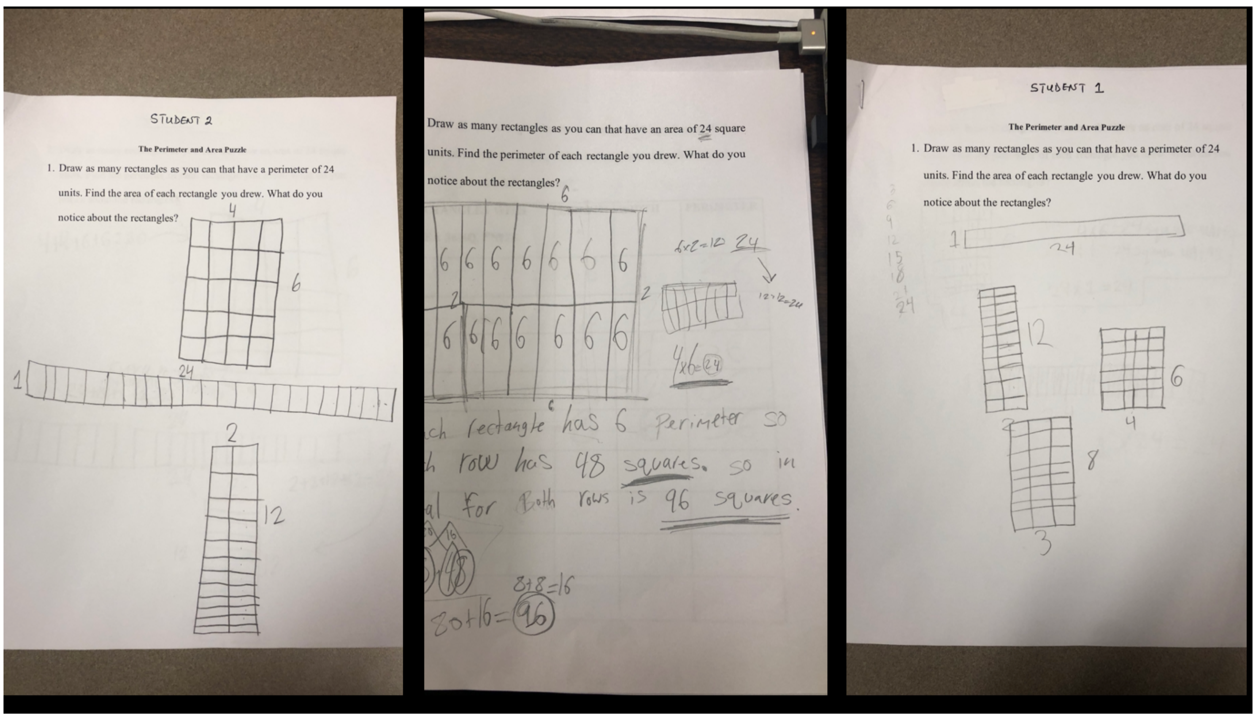

Figure 1: Sample of Student's Work

Student's Responses to Segment One of the Interview Questions. The following session explains students' understanding of the concepts of the area and perimeter of rectangles (see Table 2 below). I purposely asked each of the student the first interview question to know if they have a good understanding of the questions. The ability to interpret and make sense of mathematical tasks is one of the skills required during problem-solving. (Carpenter et al., 1980; King et al., 2022). Contrary to some of the student's work (Figure 1 above), the interview session showed that only students' 1 and 2 had a conceptual understanding of what a perimeter of rectangle is. Student 3 however explained perimeter in terms of finding area. She said that "…No, the product, like times this and this". When asked about what it means to find the area a rectangle, students' 1 and 2 answered by giving a procedural/formula definition (Machaba, 2016). Student 1 however gave a conceptual definition of area of a rectangle. He stated that "stuff on the inside…and you count on if it was a big 12 on that side and 12 on that side. That would be 24-foot area…cos it's the inside, not outside". It seems that the interview session helped one or two of the students to realize and correct some of the mistakes made in their work. Also, when student 1 was asked if the rectangle he drew in question 2 (not shown) had a perimeter of 24 units, he was able to provide a correct explanation that showed he had a conceptual understanding. He mentioned that "you would count 1234 to 24 for that then add then 4646". What this tells us is that student 1 counted the units squares and added up all the sides of the rectangles.

Table 2: Student's Responses to Segment One of Interview Questions

Interview Questions	Student's Responses	Conceptions
I: Explain to me in your own words what the problem is asking	S1: it's asking you to draw a rectangle with an outside of 24. S2: …I think it's like, like, like the whole rectangle equals 34. No, 24, and just like perimeter of 24. S3: I should have done 12 times 12, but I did 4 times 2	
I: What does it mean to find the perimeter of a rectangle?	S1: it's asking you to draw a rectangle with an outside of 24. S2: 4 + 4 + 6 + 6. Like, add the top and bottom twice and then add them to each other. S3: perimeter is like the side kind of. No, the product, like times this and this	Conceptual understanding Conceptual understanding Lacked conceptual understanding.
I: What does it mean to find the area of a rectangle?	S1: stuff on the inside…and you count on if it was a big 12 on that side and 12 on that side. That would be 24-foot area…cos it's the inside, not outside. I: so, you mean for area you have to add? S1: no, you can use multiplication. S2: you have to times the top and the width. So, like the length and the width S3: the area I know for sure is	Conceptual understanding Procedural understanding Procedural understanding

	length times width, but I don't know about the perimeter	
I: Does this rectangle have a perimeter of 24 units?	S1: you would count 1234 to 24 for that then add then 4646	Conceptual understanding

Discussions & Summary

Like previous research, the findings of this study showed that students lacked a conceptual understanding of the concept of the area and perimeter of rectangles which also prevented them from establishing relationships between area and perimeter of rectangles (Anwar et al., 2016; Carpenter et al., 2013; Machaba, 2016; Outhred & Mitchelmore, 1996; Winarti et al., 2012). The students' work also showed that they didn't attempt the part of the task which required that they write down what they noticed among the rectangles they created. For example, student's 3 work was totally wrong and was unrelated to the task. While she was been interviewed, she struggled to interpret the questions and demonstrated a poor understanding of area and perimeter of rectangles (Machaba, 2016; Outhred & Mitchelmore, 1996). When asked to interpret the questions, she stated that "I should have done 12 times 12, but I did 4 times 2". Student 3 thought 12 times 12 will gives her a perimeter of 24 units.

However, during the interview session, after reading and interpreting the questions, only student 1 demonstrated conceptual understanding of area and perimeter. Student 2 was able to conceptualize perimeter but not the area as a measure of surface or a region of the rectangle. He described the area using the formula definition (Huang & Witz, 2013; Machaba, 2016). Generally, I would say that having a one-to-one discussion was helpful for the students (Cobb & Yackel 1996)

Implications for Research

During the interview session, students had the opportunity to review their work and correct some of their mistakes. This shows the importance of one-to-one interviews, creating a psychological construct between the student and researcher (Cobb & Yackel, 1996). Since the study of geometry requires thinking and doing (NCTM, 2000), it is essential that both researchers and teachers engage students in problem-solving to develop their mathematical reasoning ability and for them to be able to apply mathematical knowledge.

Although the students graded work indicated that they switched the definition for the area of a rectangle to mean the perimeter of a rectangle, this does not necessarily mean that they do not understand the concepts. During the interview, it was discovered that either the students did not understand the questions, or they interpreted the questions based on the current topic (finding the area of rectangles using arrays and equations for tens) they were learning. It is suggested that teachers or researchers should revise their tasks and activities so that they are interpreted by students as intended.

One limitation of the study is that some of the questions were not correctly worded, so students were unsure what to do or how to begin solving the problems. Future research should include more population of students (preferably lower grades), a pre-test, and a teaching session (depending on students' performance on the pre-test).

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