# LIBERATING AGENCY AND TRANSFORMING COMPETENCE THROUGH MATHEMATICAL PLAY

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*Many scholars have argued that mathematics classrooms often offer narrow conceptions of mathematical excellence, recognizing only some kinds of thinking and some kinds of people as valuable, and conflating mathematical aptitude with overall intelligence. Play offers the potential to disrupt such classroom mathematical practices, by offering new and broader ways to exercise agency, and, relatedly, more expansive visions of who is seen as mathematically capable. Offering an in-depth analysis of the participation of two students as they engage in mathematics in their Kindergarten class, whole group rug time and small group play centers, we investigate how different activity structures create space for students to exercise agency in ways that demonstrate multiple forms of competence, creating liberating mathematical spaces.*

Keywords: Social Justice, Early Childhood Education, Equity, Inclusion, and Diversity

## Objectives

Many scholars have argued that mathematics classrooms often offer narrow conceptions of mathematical excellence (Ladson-Billings, 1998), recognizing only some kinds of thinking and some kinds of people as valuable (Joseph et al., 2017), and conflating mathematical aptitude with overall intelligence (Martin, 2019). Taken as a whole, such practices serve to establish mathematics classrooms as spaces for white supremacy. This is in part because such classrooms are particularly problematic for students of color who are more likely to have less-qualified teachers (DeMonte & Hanna, 2014), to be removed from school by suspension (Gregory & Roberts, 2017), and to face stereotyped low expectations for their mathematical success (McGee & Martin, 2011; Nasir & Shah, 2011). Play offers the potential to disrupt such classroom mathematical practices, by offering new and broader ways to exercise agency, and, relatedly, more expansive visions of who is seen as mathematically capable. This is in comparison to other more traditional math class activities like whole group instruction which by nature require compliance and thus limit students' agency, choice, and interest, decreasing the multiplicity of ways to demonstrate mathematical competence. Playful mathematics learning allows for students to make choices about their own problem solving, to test and critique their own reasoning and the reasoning of others, and to seek out their own mathematical interests and inquiries.

Consequently, mathematical play has the potential to resist problematic and often racist perpetuations of normalizing homogeneous ways of doing mathematics.

This study seeks to investigate how different types of activity structures create space for students to exercise agency in ways that demonstrate multiple forms of competence, creating liberating mathematical spaces. We offer an in-depth analysis of the participation of two students, Tiana and Quentin, as they engage in mathematics in their Kindergarten class in two contexts, whole group rug time and small group play centers. On the rug, opportunities to exercise agency are relatively limited students can generally either comply or resist. In contrast, small group play centers are designed to allow for mathematical exploration, collaboration,

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decision making, and problem solving, thus providing more diverse ways for students to exercise agency. In the more constrained space of the whole group rug activity, Tiana and Quentin typically exercise agency in different ways (one generally complies, and the other often resists). We follow these two students to the play stations to explore the following question: How do playful mathematical learning spaces serve to create new opportunities for students to explore mathematical ideas and be seen as mathematically competent?

## Theoretical framework

To make sense of student engagement and the potential of mathematical play to disrupt ingrained classroom practices, we leverage the ideas of *agency* and *competence*, which simultaneously highlight both the individual and collective nature of classroom participation (citations). Agency refers simply to action; students can exercise agency in different ways depending on the organization of the classroom space. In some tightly-controlled classrooms, students can exercise agency primarily by rejecting or complying (Golann, 2018). In such spaces, only a very narrow range of behaviors might be endorsed as competent. Alternatively, other classrooms might create multiple ways for students to exercise agency that are seen to be competent; such classrooms often invite students to share their own ideas or debate ideas with others, rather than practicing or reproducing a set of procedures (Boaler & Staples, 2008). Such classroom organization is not solely under the control of teachers; Yoon and Templeton (2019) note that student agency is often limited in schools, particularly when classroom teachers face pressure to meet curricular or assessment demands. Such climates stifle opportunities for students to participate in ways that are meaningful to them, in exchange for the prioritization of adults' demands, such as standards or assessment goals.

Conversely, Yoon and Templeton (2019) point to "flexible spaces" (p. 58), such as free play, as promising contexts in which students can insert their own voices into curricula or classrooms. Although there is variation amongst the definitions of play, most involve characterizations of spontaneity, interest, choice, and pleasure (Brown, 2009; Burghardt, 2010), thus always, by definition, inviting agency. Although often play in classroom settings is seen as separate from learning and a reward for good behavior, play also offers a context for learning (Parks, 2020; Wager & Parks, 2014), and it is this use of play that is our focus here. Many scholars have highlighted how play allows one to think beyond oneself, to test and explore the limits of ideas (Gadamer, 1975; Vygotsky, 1978), thus opening new spaces to exercise agency and explore nuances of mathematical ideas. In such a context, a wider range of behaviors would be seen as competent—not simply engaging a task as directed, but exploring the boundaries of the task, working with others on the task, and inventing modifications of the task. All these actions are central to play, and thus are likely to be recognized as versions of competent behavior. In the analysis that follows, we examine interactions to explore the ways that students can exercise agency, and how their behavior is recognized and marked as more or less competent.

## Methods

This analysis focuses on the contrasting cases (Flyvbjerg, 2006; Stake, 1995) of Quentin and Tiana, two kindergarten students in Ms. Lane's class. We chose these cases for close analysis as we seek to offer in-depth accounts of the interrelation between structure and agency, between individual action and collective recognition. Case study lends itself to such an analysis as we can look in depth across interactions and over time. We have chosen contrasting cases that help

illuminate our question, specifically to explore the ways that different activity structures serve to create new opportunities for students to explore mathematical ideas and be seen as competent.

These cases were identified as part of a larger study of integrating mathematical play into kindergarten classrooms. The study site, a public charter school in the southeastern United States, serves a community of students that is both racially and socio-economically diverse. Approximately half of the student population is White (48%) and half is African American (42%), with 55% of students qualifying for free and reduced lunch. The larger study takes place in partnership with six kindergarten teachers in three classrooms at the school; the data for this paper come from one of the three classrooms. As part of this study, each classroom is observed twice per week, with the observer taking in-depth ethnographic field notes to document student activity, classroom participation structure, teacher questioning, and students' affective experiences. The two focal students for this paper, Quentin and Tiana, were identified early on as contrasting cases through these observations and discussions.

Quentin is a black boy who joined Ms. Lane's class about six weeks into the semester, following some disruption with hiring and classroom reorganization in the entire kindergarten cohort. Quentin had already established a reputation as a child who struggled during whole-group time; in the first classroom he was assigned to he typically was asked to leave the collective mathematical time (that usually took place at the rug) and sit in a chair separate from the group as an effort to calm himself and not distract his peers. He was moved to Ms. Lane's class because of his disruptive behavior. Upon moving to Ms. Lane's classroom, Quentin was able to participate more regularly in whole class instruction, responding well to Ms. Lane's more structured and consistent classroom management style. However, Quentin continued to stand out as a child who needed frequent reminders about where to look and how to control his body.

Tiana is a black girl who was enrolled in Ms. Lane's class from the start of the school year. Tiana is attentive during whole group time, frequently raising her hand, almost always following directions (indeed, anticipating directions), and responding correctly to questions when invited to share. Tiana was often a leader in the class, at times reminding other students about what they should be doing or how they should be behaving during different classroom routines.

After identifying Tiana and Quentin as focal cases, we reviewed video of the students during whole-class mathematical time (almost always on the rug), and at the play centers. Using methods of interaction analysis (Jordan & Henderson, 1995), we examined the videos attending to when and how the students exercised agency, what evidence we had of the ways those behaviors were recognized and interpreted, and ultimately, when the students were seen as being competent either by an adult or peers.

## Findings

We begin with an overview of the typical ways the two students exercised agency during whole-class work time, and the resulting ways their behaviors were typically recognized as more or less competent. It is important to note that there was variation across these videos; at times Quentin was recognized as competent for his contributions and Tiana was reminded of the expectations for behavior. Thus, this overview is a representation of typical behaviors rather than a comprehensive account of every interaction. We then follow with examples of each child's interaction in one mathematical play center, offering those examples as contrasts with the typical engagement that was observed during whole class time.

**Whole-Class Instruction**

Tiana and Quentin typically experienced mathematical activities in this classroom in very different ways, which was most evident in the whole group activities that took place on the rug at the front of the room. In this environment, each child was assigned to sit on their own square.

The class had established a significant number of procedures about how students should position their bodies, talk in groups, distribute and use materials, respond to questions, and transition between activities. In this context, it was readily apparent who was complying with expectations, and who was resisting or rejecting them. Tiana thrived in this environment, consistently remembering and following these rules and procedures and often instructing others to do the same. Overall, Tiana typically exercised agency in ways that were compliant with the expectation, and was seen as mathematically competent for doing so, either though not being called out for correction, or at times being complimented directly for her behavior. In contrast, Quentin struggled in these constrained conditions and was often dismissed from his spot on the carpet to the "safe space," a chair that sits up against a wall away from the carpet, for not following these rules or procedures. While sitting in this safe space he was often redirected to continue to engage in the whole group mathematics activities, but was physically separated from his classmates. Such removal and public corrections served to mark Quentin as not behaving competently in the space. Although this participation structure led to different outcomes for different students—Tiana remains on the carpet while Quentin is removed—it reinforced a system of mathematical competence for all students that favored particular forms of behavior over others. Consequently, agency was ultimately limited for both students.

On the other hand, when Tiana and Quentin transitioned to the small-group play stations, they engaged with open-ended tasks that allowed for multiple forms of participation. Although each play station differed in its goals and materials, all play stations created expectations of and opportunities for exploration and imagination. As a result, Tiana and Quentin had more opportunities to exercise agency and to be seen as mathematically competent. Below, we highlight Tiana's experience from a video from September at the pattern block station, and a video from November of Quentin's time at the "zoo" station—highlighting the ways in which both learners engaged with mathematics and ultimately asserted their agency throughout each episode.

**Tiana's Participation in a Play Space**

Tiana's participation in the play space resembled her participation in the whole class episode, in that she was comfortable reminding others of rules and expectations. However, in this space Tiana was also likely to propose her own ideas and invent new games, thus exercising agency in ways that were not simply about compliance. In this episode, Tiana was playing with three other students at the pattern block station: Will, Nicholas, and Nora. Tiana's playfulness led to the invention of a song and game that all four students contributed to in the same way. For example, when Will stated that he needed a square for his puzzle, Tiana sang, "*Squares, squares, squares! Squares are everywhere!*" Tiana continued this same melody for the entirety of the station, but for different shapes. She even assisted Will, Nicholas, and Nora with shape names so that they could continue to play along. For example, in turn 10 Nicholas asked Tiana to identify a shape.

After Tiana stated that it was a trapezoid, Nicholas returned to the game and said, *"I need a trapsa-toid!"* so that Tiana could sing the song: *"Trapezoids, trapezoids, trapezoids! Trapezoids are everywhere!"* Tiana also took several opportunities to reshape the rules of the game. Forinstance, she eventually declared herself the "singing robot" and that the other students could only take a pattern block from the bin after Tiana finished singing about their shape. Later, Tiana

changed the rules again so that she would now be the one to hand over the shapes after singing. Tiana's final alteration of the rules occurred when she played with her own leadership role by asking Nora to sing so that she could be the one to ask for shapes.

| | | |
|---|---|---|
| 1 | Tiana: | Only if one of y'all need a square or a different type of shape. I'll sing the shape song. |
| 2 | Nicholas: | I made a fish! *Clears fish puzzle and switches to giraffe puzzle.* |
| 3 | Nora: | I need a triangle. *Grabs triangle from bin.* |
| 4 | Tiana: | Triangles, triangles, triangles! Triangles are everywhere! *Sings.* |
| 5 | Will: | I need a hexagon! *Looks at Tiana and smiles. Grabs hexagon from bin.* |
| 6 | Tiana: | Hexagons, hexagons, hexagons! Hexagons are everywhere. *Sings.* |
| 7 | Nora: | *Smiles and laughs.* |
| 8 | Will: | I need a triangle! *Looks at Tiana and smiles. Grabs triangle from bin.* |
| 9 | Tiana: | Triangles, triangles, triangles! Triangles are everywhere! *Sings.* I'm the singing robot. There's a voice box in me. There's a radio in my mouth. |
| 10 | Will: | What is this called? *Grabs trapezoid from bin and looks at Tiana.* |
| 11 | Nora: | Um…hexagon. |
| 12 | Tiana: | Trapezoid. |
| 13 | Will: | I need a trapsa-toid! |
| 14 | Tiana: | Trapezoid, trapezoid, trapezoid! Trapezoids are everywhere! *Sings.* |
| 15 | Will: | I need a diamond! |
| 16 | Tiana: | Diamonds, diamonds, diamonds! Diamonds are everywhere! *Sings and Hands Will a blue diamond.* |
| 17 | Will: | No, not that diamond, the other diamond. *Grabs white diamond from bin.* Oh, here, here, here! |
| 18 | Tiana: | That's not a diamond. Um that's a rhombus. |
| 19 | Nicholas: | Rhombus, rhombus, rhombus! Rhombuses are everywhere! *Sings.* |
| 20 | Tiana: | When I sing it means you can get it. |

Although Tiana was the leader of the game, a role that is typical for her participation in general, her song created a space in which all students could play and engage with mathematics together. This is particularly noteworthy given that pattern block puzzles are not an inherently collaborative task. However, the ill-structured nature of the activity opened opportunities for Tiana to exercise agency in ways that were creative and ultimately extended the original goal of the play station. Furthermore, Tiana's continual shifting of the rules throughout the song and game (turns 1 and 20) were in contrast to her typical compliant nature in mathematical spaces. As Tiana continued to mold and reshape the direction of the game, she played with her own role and position in relation to the other students. Although she was on task and engaging with mathematics throughout the entirety of her time at the play station, her song was a form of personalization and ultimately agency that went against her usual tendency to stick within the bounds of the teacher's exact instructions. What's more, these different ways of exercising agency were collectively recognized as competent, seen in her peers' desire to play the game with her, and in students' requests for shape names (for example, Will's request in turn 10).

**Quentin's Participation in a Play Space**

For Quentin, a shift to a playful center created opportunities for him to exercise agency in several ways, all of which kept him in the activity and supported his mathematical thinking.

While playing at a "zoo" center, which involved placing animals in pens to match their capacity (indicated with a number written on a post-it note in the corner of each pen), Quentin appeared to prioritize mutual play, and made many bids to the student across the table, King, to play together. Although King did not take up these bids, he also did not reject them, and the two boys continued to place animals on the same small square without argument, resulting in a kind of parallel play. This form of collegial, conflict-play was completely in-line with the expectations of the classroom; evidence that Quentin (and King's) play was viewed as competent can be seen when Ms. Lane approached the group, observing them and saying only: "*After you finish, you have to make sure that you count, to make sure that the amount of animals matches this number (tapping on the 13).*" This reminder galvanized King who immediately started to count the animals the boys had placed. At first Quentin simply observed, but after animals began to fall over upon being touched, Quentin made a new bid to play a new game, saying: "*We can knock all of them down, and count up. I'm gonna count while I put 'em on there."* What followed was

a cooperative counting task, which was initiated and sustained by Quentin; although King was happy to count, he did not initially coordinate with Quentin to do so.

| | | |
|---|---|---|
| 1 | Quentin: | So. Onnnneeeee *places an animal.* |
| 2 | King: | *Places an animal* One. |
| 3 | Quentin: | No. That hasta be two. *Touches the animal J just put down*. So onnnn/eeee, *Touching the tiger* |
| 4 | King: | *touching the animal he put down* /onnneeee (*reaches into the bin for another animal)* |
| 5 | Quentin: | *Touches the animal J just put down* Two |
| 6 | King: | No, No. *grabs the tiger Patrick had placed, and then picks up all animals on the board, placing each down as he counts* one, two |
| 7 | Quentin: | *Placing down a third animal* Three |
| 8 | King: | *Puts down the third animal that was in his hand* three |
| 9 | Quentin: | *Puts down another animal* four |
| 10 | King: | *Points at each animal as he counts* one two three four five |
| 11 | Quentin: | *Places a sixth animal* six |
| 12 | King: | *Places a seventh animal* seven |
| 13 | King: | *Both boys put down an animal at the same time* |
| 14 | Quentin: | Eiggggghh/hhtttt |
| 15 | King: | /Eight |
| 16 | King: | Nine *placing an animal* |
| 17 | Quentin: | Ten *placing an animal* |
| 18 | King: | *Both place another animal* |
| 19 | Quentin: | Eleven, *places another animal*, twelve, *places another animal* thirteen. There. |
| 20 | King: | No, that's thirteen. There. We're done! *Tidying the animals. There are 15 animals on the page.* |

In this exchange Quentin was again able to exercise agency in ways that appear to be satisfying to him; he made several bids for cooperative play (turns 1 and 3), and then coordinated

his counting with King, seen in turn 11 when, for the first time, his count came after Quentin's last count (instead of his own last count). Here his competence is widely apparent, as he counts to the number that is indicated on the chart, noting his completion with a satisfied "There." Evidence that King viewed this work as mutual and competent can be seen in turn 20, when he said "We're done." Overall, this play space created opportunities for Quentin to exercise agency in ways that were satisfying to him while also allowing him to be seen as competent both by the teacher and by other students. In so doing, he created new opportunities to engage with mathematical ideas in ways that were personally satisfying and creative, fulfilling his own preference for collective and collaborative play.

## Discussion and Significance

These brief excerpts offer examples of how these mathematical play stations served to offer new and different forms of participation, inviting students to exercise agency differently and creating space for a wider range of behaviors to be seen as competent. Both Tiana and Quentin were seen as competent in the mathematical play spaces, a shift from how they were generally recognized during more traditional math instruction. Looking across instances, it appears that the more permissive norms around the play stations—anticipating that students will find their own path through the task and are allowed to explore and imagine—was an important aspect of this disruption. What's more, the open-ended nature of the tasks themselves that were designed to invite more than one pathway, additionally served to offer more diverse opportunities for students to exercise mathematical agency (Esmonde, 2009).

Broadening notions of competence is important in any mathematics classroom, particularly when such broadened notions create spaces for new and different ways of engaging with mathematics. In addition, such broadened notions influence students' own perceptions of themselves and their mathematical capabilities. In this context, we wonder whether and how play offers a potential inroad to the question posed by Martin: "What can and should refusal of dehumanizing and violent mathematics education look like in principle and practice?" (2019, p. 461). As Joseph et al., (2019), wrote: "A second area for further research is the idea of humanizing Black girls by creating a space for play—a place for them to be happy, gregarious, social, and "goofy" …. Our emerging analysis illuminates that when the Black girls in this study were afforded the opportunity to be both serious and silly, they were more engaged in mathematics learning" (pp. 149). The examples shared here offer an index of Yoon and Templeton's (2019) notion of "flexible spaces" (p. 58), which allow students to insert their own voices into curricula. At times, student voice may seem irrelevant, like Tiana's song; however, these moments of agency can be conceptualized as "acts of resistance" (p. 80) in an adult-regulated world. When classroom structures enable students to assert their agency, adults send the message that students' "dispositions, discourses, and actions" (p. 80) are in fact valuable forms of knowledge.

However, before making the fundamental error of romanticizing children's activity, we note that play is not a panacea—it characterizes a form of human interaction that is fraught with the same perils, biases, and oppression that characterizes all interactions (Bryan, 2020). We know that children's play can be racist, mean, gendered, and exclusionary. Although it has potential to liberate students from the conventions of the discipline, it by no mean transcends the everyday structures that dictate our interactions. Therefore, even as we look for sites of potential disruption, we must stay vigilant at noticing the ways it might fail, lest we contribute to the continued reproduction of mathematics reform that ultimately serves to change nothing for students who are already being oppressed by its structures (Martin, 2019).

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