COORDINATED MULTI-HAND INSCRIPTIONS WITH COLLABORATIVE IMMERSIVE SPATIAL DIAGRAMS

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Immersive spatial diagrams (three-dimensional diagrams rendered with immersive stereoscopic displays) offer learners the opportunity to extend practices of diagramming in school geometry into a human-scale, spatial context. Learners can use two or more hands to inscribe simultaneously with digital spatial painting tools, a spatial analog of the input affordances of multi-touch tablets. In this study, we describe how learners coordinate their use of multi-hand spatial painting tools to inscribe diagrams. In particular, with human-scale spatial diagrams, learners' embodied inscriptions can center them within the diagram (e.g., with one's torso as an axis) and realize mathematical relationships (e.g., an arm sweeping a circle as a radius). Our analysis suggests that the design of spatial diagramming environments should consider the opportunities for embodied connections afforded by large-scale, collaborative, and multi-handed interactions.

Keywords: Technology, Geometry and Spatial Reasoning, Embodiment.

In school geometry, diagrams are often inscribed on plane surfaces with styluses or mediated by interactions with a mouse pointer. Diagrams can be extended out of the plane with physical and digitally rendered media. While both the extrusion of plastic with 3D pens (Ng & Ferrara, 2020; Ng & Sinclair, 2018) and immersive spatial diagrams (Kaufmann & Schmalstieg, 2006; Dimmel & Bock, 2019; Cangas et al., 2019) extend diagrams out of the plane, these tools often restrict collaborative authorship of a diagram to modes of turn-taking or roles of inscriber and observer. Like their plane stylus counterparts, the 3D pen stylus often momentarily obstructs access to the inscription it produces - limiting simultaneous contributions. Immersive spatial diagrams (three-dimensional diagrams rendered on immersive stereoscopic displays) can provide opportunities for learners to collaborate with immersed and observer roles (Price et al., 2020; Bock & Dimmel, 2021a; 2021b), and as co-immersed learners (Kaufmann & Schmalstieg, 2006; Rodríguez et al., 2021; Walkington et al., 2022).

Collaboration with Immersive Spatial Diagrams

In immersive spatial diagrams that are digitally rendered with virtual reality, unequal access to the virtual environment with immersed and observer roles limits both the ability of the immersed participant to see the observers' gestures and the ability of the observer to take turns inscribing and manipulating the diagram (Bock & Dimmel, 2021a; 2021b). When virtual-reality (VR) head-mounted displays are used for rendering, the learners' views of their physical surroundings are obstructed and replaced with their virtual environment. Price et al. (2020) use this asymmetric access to the diagram to make salient features of a two-dimensional coordinate grid to support the immersed learners' embodied navigation of the spatial diagram, with analysis of the learners' use of verbal communication to coordinate their inscriptions.

Immersive spatial diagrams can also offer multiple learners' simultaneous immersion in a shared virtual environment. When rendered with virtual-reality (VR) head-mounted displays, these environments may depend on avatars to mimic gestures and body movement in non-verbal discourse (Walkington et al., 2021; see also, Rodríguez et al., 2021). When rendered with augmented-reality (AR) or mixed reality (MR) head-mounted displays, participants can have shared access to the virtual environment that is visually overlaid on their physical surroundings. With simultaneous access to the virtual and physical environments, learners can directly observe gestures and body movements without an avatar proxy (Flatland XR Channel, 2021).

Coordinated Inscriptions

Across emerging virtual environments for immersive spatial diagrams, we are interested in the potential of learners' coordinated diagramming. Spatial diagrams might be manipulated by grasping and dragging vertices (Bock & Dimmel, 2021a; 2021b), through the use of virtual tools to embed relationships (*e.g.*, a spatial compass (Dimmel & Bock, 2019; Dimmel et al., 2020), an orthogonal tool (Rodríguez et al., 2021)), gestures that vary parameters of the diagram (Bock & Dimmel, 2021a; 2021b; Flatland XR Channel, 2021) or by digitally painting inscriptions in space (Google, 2016; Mozilla, 2017; Icosa Foundation, 2021; Rendever, 2022). Learners' coordination of manipulations of the diagram might include turn-taking, but the virtual spatial canvas can also allow learners to coordinate simultaneous manipulation of the diagram. For example, two learners might grasp the vertices of a triangle and drag them in opposing directions. In this study, we asked: how do learners' coordination of spatial inscription embody geometric relationships in their diagrams?

Theoretical Framework

We organized our investigation of learners' co-ordination of around virtual elements of a diagram (*i.e.,* potentials) that are realized through learners' embodied interactions. Sinclair et al. (2013) describe how relationships between potentials of a diagram and the actual (*i.e.,* realized potentials) are remade through learners' multi-modal interactions with a diagram. For example, a radius may be embedded as a potential of a diagram of a circle, but the diagram is transformed when a learner realizes the radius in the diagram (*e.g.,* with a gesture) even though the radius may still be invisible in the inscription of the diagram. In this study, we describe how learners' use of coordinated, embodied inscriptions bring potentials of the diagram into the actual.

Methodology

We conducted a set of semi-structured interviews where pairs of students at a public research university worked together in an immersive spatial diagramming environment. In these semi-structured interviews, both participants and one interviewer were immersed in the virtual environment with virtual-reality head-mounted displays (*e.g.,* HTC Vive Pro, Oculus Quest). We prompted participants to work together and asked, "how many ways can you make a [geometric figure]?" Prompts included making plane and solid geometric shapes. Our intention in asking participants "how many ways" was to generate diversity in the methods that participants used to make inscriptions. The immersed interviewer was instructed to encourage participants to discuss how they knew that their inscription would be the figure they intended. The immersed interviewer encouraged participants to continue their discussion with each other with prompts including "why would that make a [geometric figure]" and "how do you know".

We captured and composited video recordings of (1) the virtual view of each participant, (2) the virtual view of the immersed interviewer, (3) a physical view of the participants in the laboratory classroom. From these records, we identified 103 episodes across interviews with three pairs of participants where one or both of the participants attempted to make a diagram of a mathematical figure. These episodes were then coded in an iterative process for the tools used in the virtual environment and the features that the participants coordinated while inscribing (*e.g.,* fixing one's torso as an axis, and rotating around that axis), starting with an a priori analysis of the virtual environment. In this analysis, we were particularly interested in how participants' coordination of their bodies' motions while inscribing might make salient mathematical properties and relationships in their diagram.

A Priori Analysis of the A-Painter Environment

Spatial painting environments that were developed for artistic use can offer learners a spatial analog of a paper and stylus environment for diagramming. Knispel and Bullock (2017) describe how users' collaboration in a spatial painting environment can emphasize non-verbal, embodied aspects of discourse. We chose to investigate a spatial painting environment as a context where a multi-modal discourse including spatial inscriptions might produce mathematical diagrams where the body's role in constructing the diagram might make salient mathematical relationships within the diagram.

Spatial painting environments can separate each learners' brush into their own layer, so that their inscriptions visually co-exist but are composed of strokes from separate brushes (Rendever, 2022). In a fork of Mozilla's (2017) A-Painter spatial painting environment, collaborative spatial inscriptions were not separated into distinct layers (Lee, 2018; Bock, 2022). Instead, a bug in the software connected the brushes with line segments between all hands drawing in the same moment in time. As multiple hands move through space, these segments inscribe approximations of the surfaces that the motion of the segments would sweep out. We embraced this bug in the spatial diagramming environment and investigated learners' exploration of the bug to make mathematical inscriptions.

For example, a horizontal segment between two hands raised vertically together could inscribe an approximate covering of the surface of a trapezoid (Figure 1). In Figure 1, two participants extrude a line segment into a trapezoid after discussing how one needs to move their hands faster than the other. Before making this inscription, the pair of participants rehearsed their movements without inscribing. We see avatars of each of the participants, seen through their (left, center) and the immersed interviewer's (right) view, where one of each participants' hand forms the end point of the segment.

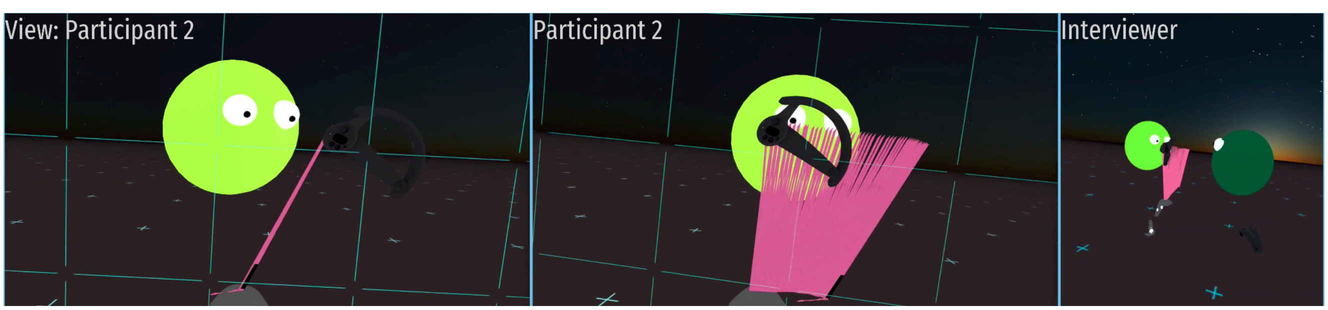

Figure 1: Two Participants Sweep Out A Trapezoid

Before conducting semi-structured interviews, we documented our understanding of the affordances of the diagramming environment. We focused on how multiple, simultaneous embodied interactions might shape learners' interactions with a spatial diagram. Mozilla's (2017) A-Painter environment allowed learners to inscribe curves by pulling a trigger on their control and moving their hand through space. The networked A-Painter environment (Lee, 2018; Bock, 2022) added support for teleportation, simple avatars with a spherical head and VR controllers for hands, and the surface-sweeping multi-hand brush bug.

We describe surface-sweeping as a tracing of a one-dimensional curve through space while filling an area with color. This sweeping action has similarities to research on area-coverings with paint-rollers in two-dimensional contexts (Altindis & Raja, 2021), however, our analysis focuses on a spatial generalization as a resource to support learners' mathematical discourse without specific attention to measurement.

Embracing the multi-hand brush bug, we hypothesized that learners would: (1) trace curves through space with one hand, (2) trace surfaces through space with two hands, and (3) inscribe line segments by using the brush to connect the location of two hands without moving. With these modes of inscription, we expected learners to be able to make (a) polygons as networks of line segments, (b) quadrilaterals as extrusions of line segments, and (c) circles, cones, cylinders, and disks as revolutions of swept points or segments around an axis. We expected that learners might inscribe diagrams at a variety of scales, including diagrams larger than themselves, both external to their bodies and – particularly when rotating around an axis – around themselves. Finally, we expected that learners might use their shoulder as a pivot, their torso as an axis, and their extended arms as a radius to realize relationships in the diagram through their embodied inscription.

Results

Three pairs of participants completed semi-structured interviews with the A-Painter spatial diagramming environment. For the first pair to complete the semi-structured interview, the multi-hand brush bug presented was expected to connect the two hands of one participant, but not connect between hands of different participants. During the interview, the participants experienced the multi-hand brush inscribing only between one's own hands; simultaneously, the immersed interviewer's environment drew connections between participants. Before the second and third pairs' interviews, the multi-hand brush was patched to always connect drawings across participants.

Every pair of participants traced curves through space with one hand, traced surfaces through space with two hands, inscribed line segments by using the brush to connect the location of two hands without moving. Every pair of participants made (1) polygons and polyhedrons as networks of line segments, (2) quadrilaterals as extrusions of line segments, and (3) circles, cones, cylinders, and disks as revolutions of swept points or segments around an axis.

Unexpectedly, each pair of participants also continued to trace curves or surfaces while they teleported, which extended their drawings between distant locations (Figure 2). In Figure 2, a participant teleports while drawing [left], producing a long line segment [right]. The blue arc selects the participant's teleportation destination. We see this from the participants' first person virtual perspective; in the right panel, they have turned around to see their inscription.

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2). University of Nevada, Reno.

Figure 2: Drawing While Teleporting

The second and third pair of participants traced surfaces extending from one to another at both short (~1 meter, Figure 3) and long (2-50 meters, Figure 4) distances. In Figure 3, two participants are sweeping out a pair of cones. In the left panel, we see the participants in the laboratory classroom aligning their perspectives in the virtual environment. We see the participants looking at each other in the left participant's view [center left] and the right participant's view [center right]. In the right panel, we see the immersed interviewers view of the participants' avatars. While their perspectives are not aligned in the physical space, they are in the virtual environment. In Figure 4, we see one participants' perspective of the virtual environment, facing another participant's avatar. The participants have traced out a cylinder.

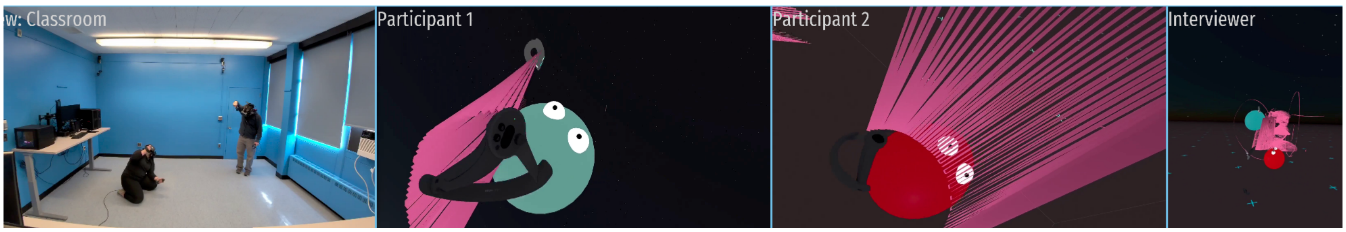

Figure 3: Two Participants Sweep Out a Cylinder at a Short Distance

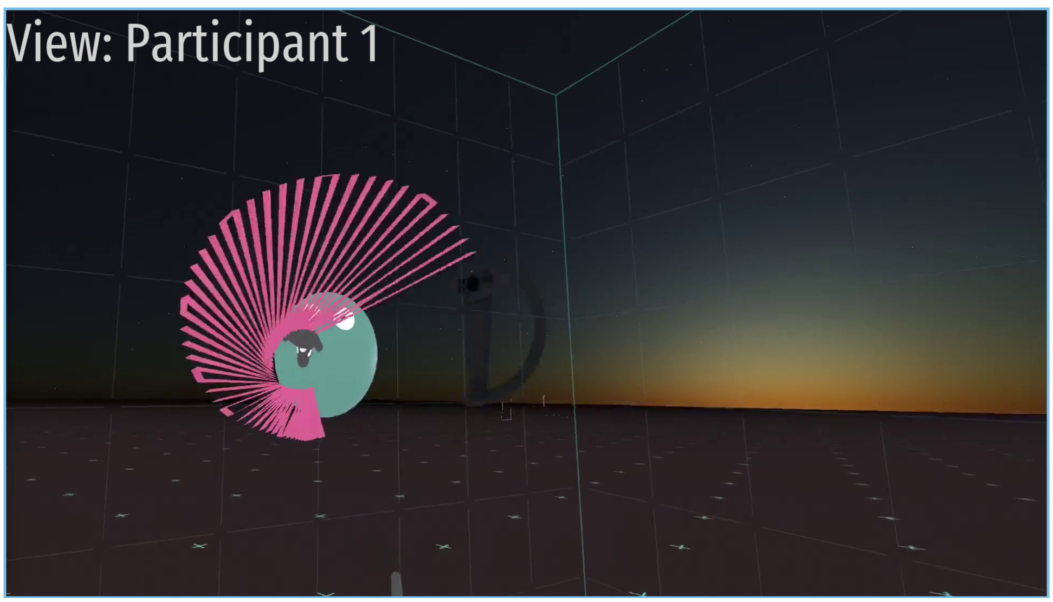

Figure 4: Two Participants Sweep Out a Cylinder at a Moderate Distance

All elements of our a priori analysis appeared in participants' use of the spatial painting environment; they also attended to a number of other features to coordinate their embodied inscription, both while inscribing alone and with a partner. Participants attended to (1) maintaining parallel relationships during their movement, (2) co-varying rates of the movement of two or more hands, (3) reflective symmetries in movement, (4) both shoulders and elbows as pivot points, (5) arms and forearms as radii, (6) arbitrary axes imagined in space, horizontally and vertically aligned perspectives as axes, self and others' bodies as axes, and (7) rotations fixed around an axis or pivot point.

Each of these embodied features of the diagram brings forth a potential of the diagram; for example, by extending an arm as a radius, fixing ones' torso as an axis, and rotating around that axis while tracing a curve the diagram of a circle that is produced (Figure 5) has realized embodied potentials of the radius and axis or rotation that are invisible if the diagram is reduced to its rendering. In Figure 5, a participant extends their forearm as a radius [left], turns around their torso while using their controller to draw in the virtual environment [center], completing the circle [right].

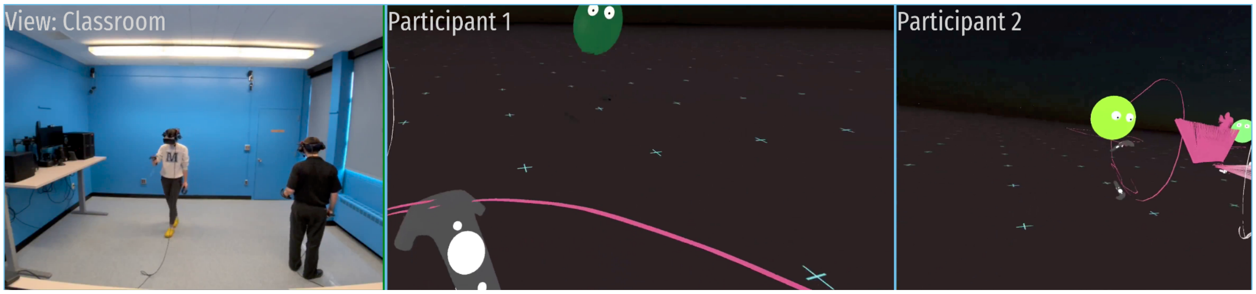

Figure 5: A Participant Traces a Circle

Table 1 describes select examples of the relationships between these coordinated features and the realized potentials of the diagram. For example, in Figure 6, the participant anchors one hand above their head and stretches out a forearm [left], turns to sweep out the segment [center] and stands inside the completed cone [right]. We see the participant coordinating a rotation, with their torso as an axis, while holding one hand above their head (on the axis) and another forearm extended as a radius. Here, the participant embodies the rotational symmetry of the diagram, a circle formed by the revolution of a radius as the base of the cone, and the necessary intersection of the axis and the segment that forms the cone.

In Figure 7, two participants hold their hands to form a rectangle [left], sweep the rectangle around an imagined axis [center-left, center-right], inscribing a thick washer [right]. In this example, the participants' collaboratively coordinate their motion around an axis to realize a washer as a revolution of a rectangle.

Table 1: Coordination of Movement during Inscription of Select Diagrams

Subject	Coordinated Features	Realized Potentials
Parallelogram	co-planar motion, moving at the same rate	length of one-dimensional cross-sections is fixed
Trapezoid (Figure 1)	co-planar motion, moving at related rates	length of one-dimensional cross-sections grows
Circle (Figure 5)	rotation, fixed around self as axis, extended arm as radius	rotational symmetry, radius
Circle	rotation, fixed around shoulder as pivot, extended arm as radius	radius, center
Cylinder (Figure 4)	rotation, fixed around external vertical/horizontal axis, of vertical/horizontal segment	rotational symmetry with axis parallel to revolved segment
Cylinder (Figure 3)	rotation, fixed around axis formed by aligned perspective (vertical/horizontal), each drawing	rotational and reflective symmetries, circles as base of cylinder

Subject	Coordinated Features	Realized Potentials
	circle, opposite direction of movement, matching rate of movement	
Cone (Figure 6)	rotation, fixed around self as axis, one hand above head, one arm extended as radius	rotational symmetry, circles as base of cone, segment intersecting axis at endpoint
Washer (Figure 7)	rotation, fixed around one participant's torso as axis, four hands composing rectangle along radius & height	rotational symmetry, washer as revolved rectangle, radius as direction, height

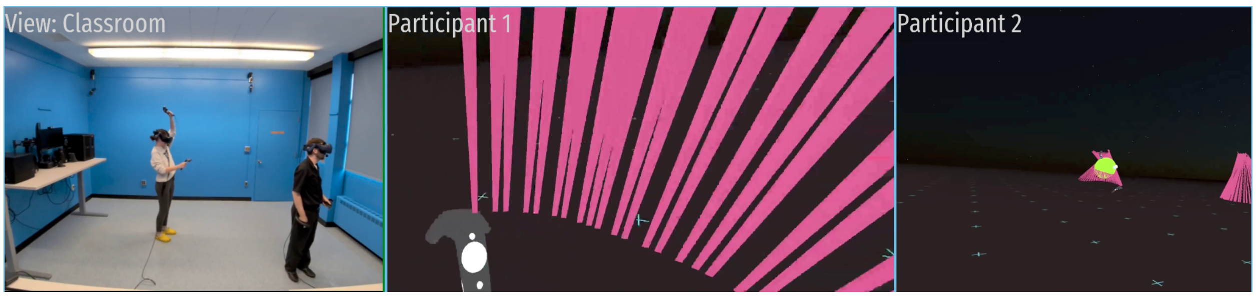

Figure 6: A Participant Anchors a Segment Above Their Head While Rotating

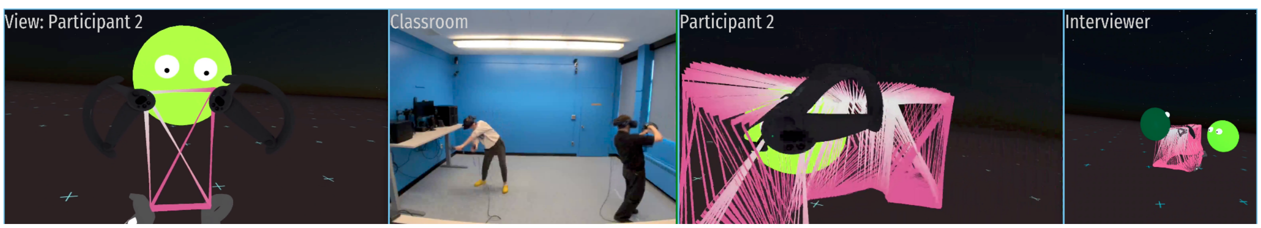

Figure 7: Two Participants Extrude a Rectangle around an Axis

Discussion & Implications

In two-dimensional, digitally rendered dynamic geometry diagrams, the multiple, simultaneous inputs of multi-touch tablets supported new potentials for both one and multiple learners' interactions with the diagram, that can center embodied and spatial relationships in the diagram (Jackiw, 2013; Chorney & Sinclair, 2018). These multi-touch inputs can be found in multi-hands transformations of dynamic diagrams in immersive spatial diagram environments (*e.g.,* Bock & Dimmel, 2021a; Dimmel et al., 2020; Rodríguez et al., 2021); we find learners capitalizing on the potential of these affordances with A-Painters multi-hand brush.

Attention to Covarying Rates of Movement

In diagrams of parallelograms, trapezoids, cylinders, and cones, participants attended to the covering motion or change in position of their hands. Pairs of participants coordinated moving their hands “at the same rate” to construct parallelograms and cylinders, at related rates to construct trapezoids, and “the same in opposite directions” to construct cones. This embodiment of covariation drew one pair of participants to focus their investigation on how the paths of the endpoints of a segment that is revolved around an axis can produce a variety of figures, including a pair of cones, a cylinder, and a variety of self-intersecting surfaces.

This presents exciting opportunities to investigate how multiple inputs can reshape how learners draw mathematical diagrams. As an inscription tool, the relationships between two participants' embodiments of circles are realized in a visual trace. Looking at the potential of multi-hand tools in spatial diagram environments, we take inspiration from York et al.'s (2022) Graph Tracer, where students embody varying quantities as one moves a stylus and the other moves the canvas of inscription, and the relationship between their embodiments is found in the graph that is produced. Further work is needed to understand how the affordance of multi-hand inputs can be used in the design of immersive spatial diagram environments, including considering redesigning tools implemented as analogs of mouse pointer input systems.

Scale and Ego-centric Diagrams

Finally, participants' use of embodied, spatial inscription tools often produced diagrams that surrounded the learner (*e.g.,* Figure 6) or extended between learners (*e.g.,* Figure 3), instead of small, external diagrams that are common in school geometry. Immersive spatial diagrams offer the affordance of scale, which can support learners' taking new perspectives in investigations of geometry (Benally et al., 2022). In future iterations of A-Painter, we plan to investigate how learners use the scale of immersive spatial diagrams when digital painting tools are available to markup dynamically rendered representations of mathematical figures. For example, spatial painting tools could be combined with other dynamic spatial renderings to be used to markup spatial realizations of surfaces of revolution in calculus contexts.

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References

Atlindis, N., Raja, W. (2021). Enacted Task Characteristics: Setting An Infrastructure For Students' Quantitative Reasoning. Proceedings of the 43rd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Philadelphia, PA.

Benally, J., Palatnik, A., Ryokai, K., & Abrahamson, D. (2022). Charting our embodied territories: Learning geometry as negotiating perspectival complementarities. For the Learning of Mathematics.

Bock, C. G. (2022). Immersive-math/naf-a-painter (1e02bad0b16fb5890cd22f2a60f22f142d6183b5) [JavaScript]. The University of Maine. https://gitlab.com/immersive-math/naf-a-painter

Bock, C. G., & Dimmel, J. K. (2021a). Dynamic Spatial Diagrams and Solid Geometric Figures. Psychology of Mathematics Education North America.

Bock, C. G., & Dimmel, J. K. (2021b). Digital Representations without Physical Analogues: A Study of Body-Based Interactions with an Apparently Unbounded Spatial Diagram. Digital Experiences in Mathematics Education, 7(2), 193–221. https://doi.org/10.1007/s40751-020-00082-4

Cangas, D., Morga, G., & Rodríguez, J. L. (2019). Geometry teaching experience in virtual reality with NeoTrie VR. Psychology, Society, & Education, 11(3), 355–366.

Chorney, S., & Sinclair, N. (2018). Fingers-on geometry: The emergence of symmetry in a primary school classroom with multi-touch dynamic geometry. Using Mobile Technologies in the Teaching and Learning of Mathematics, 213–230.

Dimmel, J., & Bock, C. (2019). Dynamic Mathematical Figures with Immersive Spatial Displays: The Case of Handwaver. In G. Aldon & J. Trgalova (Eds.), Technology in Mathematics Teaching: Selected Papers of the 13th ICTMT Conference (pp. 99–122). Springer International Publishing. https://doi.org/10.1007/978-3-030-19741-4_5

Dimmel, J., Pandiscio, E., & Bock, C. (2020). The Geometry of Movement: Encounters with Spatial Inscriptions for Making and Exploring Mathematical Figures. Digital Experiences in Mathematics Education. https://doi.org/10.1007/s40751-020-00078-0

Flatland XR Channel (Director). (2021, December 13). AR Collaboration. https://www.youtube.com/watch?v=HFdOBiHzZes
Google. (2016). Tilt Brush. Alphabet, Inc. https://www.tiltbrush.com/
Icosa Foundation. (2021). Open Brush. https://openbrush.app/
Jackiw, N. (2013). Touch and multi-touch in dynamic geometry: Sketchpad explorer and "digital" mathematics. Proceedings of the 11th International Conference on Technology in Mathematics Teaching, 149–155.
Kaufmann, H., & Schmalstieg, D. (2006). Designing Immersive Virtual Reality for Geometry Education. IEEE Virtual Reality Conference (VR 2006), 51–58. https://doi.org/10.1109/VR.2006.48
Knispel, J., & Bullock, F. (2017). Collaborative VR painting in web browsers. SIGGRAPH Asia 2017 VR Showcase, 1–2. https://doi.org/10.1145/3139468.3148451
Lee, H. (2018). Haydenjameslee/networked-a-painter (a58aec460128d5902aaedac3ef68799c2fdf397b) [JavaScript]. https://github.com/haydenjameslee/networked-a-painter
Mozilla. (2017). Aframevr/a-painter [JavaScript]. A-Frame. https://github.com/aframevr/a-painter (Original work published 2016)
Ng, O.-L., & Ferrara, F. (2020). Towards a Materialist Vision of 'Learning as Making': The Case of 3D Printing Pens in School Mathematics. International Journal of Science and Mathematics Education, 18(5), 925–944. https://doi.org/10.1007/s10763-019-10000-9
Ng, O.-L., & Sinclair, N. (2018). Drawing in Space: Doing Mathematics with 3D Pens. In Uses of Technology in Primary and Secondary Mathematics Education (pp. 301–313). Springer.
Price, S., Yiannoutsou, N., & Vezzoli, Y. (2020). Making the Body Tangible: Elementary Geometry Learning through VR. Digital Experiences in Mathematics Education. https://doi.org/10.1007/s40751-020-00071-7
Rendever. (2022). MultiBrush on Oculus Quest. https://www.oculus.com/experiences/quest/3438333449611263/
Sinclair, N., Freitas, E., & Ferrara, F. (2013). Virtual encounters: The murky and furtive world of mathematical inventiveness. ZDM, 45(2), 239–252. https://doi.org/10.1007/s11858-012-0465-3
Rodríguez, J. L., Romero, I., & Codina, A. (2021). The Influence of NeoTrie VR's Immersive Virtual Reality on the Teaching and Learning of Geometry. Mathematics, 9(19), Article 19. https://doi.org/10.3390/math9192411
Walkington, C., Nathan, M. J., Hunnicutt, J., Washington, J., & Holcomb-Webb, K. (2022). Learning geometry through collaborative, embodied explorations with augmented reality holograms. Proceedings of the 16th International Conference of the Learning Sciences–ICLS, 1992–1993.
Walkington, C., Wang, M., Nathan, M. J., Kageyama, K., Edwards, L. D., & Ferrari, G. (2020). Collaborative gestures among secondary students conjointly proving geometric conjectures. Paper Accepted for Presentation at the 14th International Congress on Mathematical Education, Shanghai, China.(Modified Format Due to COVID-19), Shanghai.
York, T., Greenstein, S., & Akoum, D. (2022). Embodying covariation through collaborative instrumentation. In A. F. Lischka, E. B. Dyer, R. S. Jones, J. N. Lovett, J. Strayer, & S. Drown (Eds.), Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Middle Tennessee State University.