ERROR PATTERNS OF AT-RISK STUDENTS IN SOLVING ADDITIVE WORD PROBLEMS BEFORE AND AFTER THE INTERVENTION

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Students with learning disabilities/difficulties in mathematics often apply ineffective procedures to solve word problems. Given that current mathematics curriculum standards emphasize conceptual understanding in problem solving as well as higher-order thinking and reasoning, the purpose of this study was to evaluate the impact of a model-based problem-solving (MBPS) intervention program on elementary students' word problem-solving performance through analyzing the error patterns. Results indicate that after the MBPS intervention, participants significantly improved their problem-solving performance and made less errors on solving problems across a range of problem situations. Implications of the study will be discussed in the context of National Council of Teachers of Mathematics' calling for teaching big ideas to help students develop a deep understanding of mathematics knowledge.

Keywords: Mathematical Representations, Number Concepts and Operations, Problem Solving, Special Education

Introduction

According to the Nation's Report Card (National Assessment of Educational Progress [NAEP], 2022), mathematics scores of all students declined when compared to 2020. Particularly, lower performing students exhibited greater achievement decline than their average or high performing peers based on 2022 long-term trend mathematics assessments for age 9 students. Currently, majority (84%) of American 4th graders with disabilities performed below the proficiency level. By 8th grade, 93% of the students with disabilities performed below the Proficiency level (NAEP, 2022). In fact, students with learning disabilities or difficulties in mathematics (LDM) lag well behind their peers from very early on in their educational trajectory ; they often continue to fall further behind as they transition from elementary to secondary schools (Carcoba Falomir, 2019).These findings present a pressing issue for all teachers and educators, because legal mandates (e.g., Every Students Succeeds Act, 2015) and current standards (e.g., Common Core State Standards Initiative [CCSSI], 2012; National Council of Teacher of Mathematics [NCTM], 2000) require that ***all*** students, *including students with learning difficulties*, in the US be taught to high academic standards that will prepare them to succeed in college and careers. As mathematical problem solving is an important part of school mathematics (Verschaffel et al., 2020), it is imperative that all students achieve proficiency in word problem solving.

Students with LDM often treat word problems mechanically and apply ineffective procedures such as searching for keywords to identify the operation. That is, they focus on whether to add, subtract, multiply or divide rather than whether or how the problem makes sense. When encountering a word problem, they often just find the numbers in the problem and apply an operation without comprehending the problem and understanding the mathematical relations in the word problem (Xin, 2008, 2007). On the other hand, NCTM (2000) and Common Core State Standards (CCSSI, 2012) both emphasize mathematical thinking and reasoning. Specifically, NCTM is calling for teaching “big ideas,” which is defined as “mathematical statements of overarching concepts that are central to a mathematical topic and link numerous smaller mathematical ideas into a coherent whole” (Caldwell, et al., 2011, p. 9). Mathematical big ideas draw students’ attention to fundamental concepts, link small fact/ideas together, and connect previously learned ideas to new concepts. As such, teaching big ideas can help students develop a deep understanding of mathematics knowledge (Caldwell et al., 2011).

Conceptual Framework: Perspectives in Math Word Problem Solving

According to existing literature in mathematics education (de Corte & Verschaffel, 1987; Carpenter, & Moser, 1984), the semantic structure of the word problem significantly influenced young children problem solving strategies. Children apply a range of addition and subtraction strategies even before the formal education in the elementary school. Therefore, it is suggested that early math education pay attention to selection or construction of elementary arithmetic word problems. That is, it is preferred that the problems present in the textbook would reinforce the use of children’s pre-existing knowledge or make that knowledge useful and necessary. As such there are significant amount of research in psychology of math education and recently in the field of special education promoting the instruction or intervention that focuses on the semantic structure of the word problems (Verschaffel et al., 2020).

On the other hand, scholars in mathematics education support the emphasis on mathematical relations identified in word problem solving (Davydov, 1982; Savard & Polotskaia; 2017, Xin et al., 2008). To promote students conceptual understanding of mathematical problem solving, specifically designed tasks were presented to students for them to represent a range of variously constructed word problems in a cohesive mathematical model equation promoting students’ construction of the mathematical relationships (Savard & Polotskaia, 2017; Xin et al., 2008). These researchers argue that mathematical relations play a crucial role in mathematical learning; however, too often students were taught to rely on keywords or semantic feature of the word problem story in determining the operation for the calculation of the answer (Savard & Polotskaia, 2017).

Effective Instructional Features in Teaching Word Problem Solving

To guide instructional practice for students with LDM, Institute of Educational Science (IES)’s latest Practice Guide suggests the use of *systematic instruction* through sequencing, using worked-out examples, providing visual and verbal supports, and teaching of precise mathematical language (Fuchs et al., 2021). As the outcome of a collaborative work from math education and special education we have developed, with the support from the National Science Foundation (NSF, Xin et al., 2015)[i], a web-based computer tutor that emphasizes *conceptual model-based problem solving* (MBPS) (Xin, 2012). The MBPS program integrates research-based practices that are consistent with the latest IES practice guide (Fuchs et al., 2021), including concrete (e.g., virtual manipulatives), representational (e.g., bar models), and abstract (mathematical model equations) instructional sequences, visual and linguistic support, and teaching of precise mathematical language.

Specifically, the MBPS program pay particular attention to making the reasoning behind mathematics explicit to students through nurturing fundamental mathematical ideas (e.g., the conception of number as a composite unit) that would lead to the additive reasoning (e.g., part and part makes up the whole or P+P=W). As part of the MBPS approach, linguistic and visual support were integrated parts of the MBPS program. For instance, *Word Problem Story Grammar* prompting questions (Xin et al., 2008, 2012) were used as a series of linguistic scaffolds to facilitate students' representation of word problems in mathematical model equations (e.g., part + part = whole) for accurate problem solving. In addition, visual were used to support student understanding of the part-part-whole (PPW) mathematical model. For instance, "name tags" generating from specific problem situations were used to denote each of the elements in the PPW diagram equation to facilitate the representation process.

Empirical studies have shown the effectiveness of MBPS in improving students' word problem solving performance (Witzel et al., 2022; Xin et al., 2011, 2017, 2023). To understand the impact of the MBPS on students' conceptual understanding of additive word problem solving, this study analyzed participating students' error pattern when solving addition and subtraction word problems before and after the MBPS intervention. Analysis of error patterns assists identifying areas of instructional needs (Kingsdorf & Krawec, 2014). Specifically, we focused on the participating students' success or error pattern as well as strategy use in solving additive word problems.

Method

Participants and Setting

This study was conducted within the larger context of the NSF-funded project[i] (Xin et al., 2015-2020). Participants included in this study were nine third graders with LDM from one elementary school in the mid-western United States. See Table 1 for demographic information of the participants.

Table 1: Participant Demographics

Variable/Name	S1	S2	S3	S4	S5	S6	S7	S8	S9
Gender	Female	Female	Male	Male	Male	Female	Male	Male	Female
Ethnicity	Hispanic	Black	Hispanic	Hispanic	White	White	White	Black	Black
Age(year-month)	8-11	9-5	8-8	8-8	8-6	8-3	8-9	9-1	8-1
Socioeconomic status	Low	Low	Low	Low	Low	Low	Low	low	Low
Years in special education	0	0	1 (LD)	0	3(LD)	0	0	3(LD)	0
RtI support	Tier 2	Tier 2	Tier 3	Tier 2	Tier 3	Tier 2	Tier 2	Tier 3	Tier 2
% in general Class	100%	100%	>80	100%	>80	100%	100%	>80	100%
OtisLennon/Full-scale	No test	76	77	61	90	79	72	82	No test
Verbal		74	79	73	92	81	73	77	
Performance		79	77	50	89	79	75	87	

Note. LD= learning disabilities; RtI = Response-to-Intervention; Tier = RtI Tiers

Measures

To measure participating students' performance before and after the intervention, we used a researcher-developed 14-item WPS criterion test (Xin et al., 2020). It involves eight part-part-whole problems (including *combine*, *change/join-in*, and *change/separate* story situations) with either the part or the whole as the unknown, and six additive compare problems (including "more than…" or "less than…" story situations) with either the *compared quantity*, *referent quantity*, or the *difference* as the unknown. The WPS test was designed in alignment with the NCTM and Common Core standards (CCSSI, 2012), which emphasize varying construction of word problems for assessing conceptual understanding of mathematics problem solving. Cronbach's Alpha of the criterion test was .86 and the test–retest reliability was .93 (Xin et al., 2020). Table 1 presents sample word problems included in the WPS Test.

Table 1: Sample Word Problem Situations in the Test

Combine	
"Whole" unknown CMB-W	Mr. Samir had 61 flashcards for his students. Mrs. Jones had 27 flashcards. How many flashcards do they have altogether?
"Part" unknown CMB-P	Together, Jamie and Daniella have 92 books. Jamie says that he has 57 books. How many books does Daniella have?
Change-join in	
"Whole" unknown ending amount CJ-WE	Leo has 76 math problems for homework. His Dad gives him 22 more problems to solve. How many math problems in total does Leo need to solve?
"Part" unknown change amount CJ-PC	Sam had 8 candy bars. Then Lucas gave him some more candy bars. Now he has 15 candy bars. How many candy bars did Lucas give Sam?
"Part" unknown beginning amount CJ-PB	Selina had several comic books. Then Andy gave her 40 more comic books. Now, Selina has 67 comic books. How many comic books did Selina have in the beginning?
Change-separate	
"Whole" unknown beginning amount CS-WB	Alex had many dolls. Then she gave away 12 of her dolls to her sister. Now Alex has 26 dolls. How many dolls did Alex have in the beginning?
"Part" unknown ending amount CS-PE	Davis had 62 toy army men. Then, one day he lost 29 of them. How many toy army men does Davis have now?
"Part" unknown change amount CS-PC	Ariel had 41 worms in a bucket for her fishing trip. She used many of them on the first day of her trip. The second day she had only 24 worms left. How many worms did Ariel use on the first day?
Compare-more	
Compared quantity unknown CM-C	Denzel has 28 toy cars. Gabrielle has 15 more toy cars than Denzel. How many toy cars does Gabrielle have?
Referent quantity unknown CM-R	Tiffany collects bouncy balls. As of today, she has 42 of them. Tiffany has 20 more balls than Elise. How many balls does Elise have?
Difference unknown CM-D	Logan has 52 rocks in his rock collection. Emanuel has 12 rocks in his collection. How many more rocks does Logan have than Emanuel?
Compare-less	
Referent quantity unknown CLS-R	Ellen ran 62 miles in one month. Ellen ran 29 fewer miles than her friend named Cooper. How many miles did Cooper run?
Compared quantity unknown CLS-C	Kelsie said she had 82 apples. If Lee had 32 fewer apples than Kelsie, how many apples did Lee have?
Difference unknown CLS-D	If Laura has 41 candy bars and another student named Paula has 70 candy bars, how many fewer candy bars does Laura have than Paula?

As for scoring, one point was given if a correct answer was given to a problem. In the case that the answer to the problem was incorrect, however, the algorithm or model equation was correctly set up, half point was awarded.

MBPS Intervention

MBPS is web-based interactive tutoring program. Sessions were monitored by supervisors. The participants worked with the MBPS computer tutor "one-on-one" during the afterschool program, Monday through Thursday, for a total of 18 sessions (ranged from 15 to 23 across different individuals) with each session lasting for about 25 minutes. The session supervisor helped each of the participants log onto the MBPS computer tutor program in the beginning of each of the sessions. Then the student followed the direction of the computer tutor and engaged in the activities in Modules A through C. **Module A** engaged students in a series of activities involving the use of virtual manipulatives such as unifix cubes, to nurture fundamental mathematical ideas that are crucial for the development of additive reasoning and problem solving. It focuses on students' conception of "number as the composite unit" (e.g., any number that is larger than 1 can be decomposed into a combination of two numbers, for instance, 4 is made of 3 and 1, or 2 and 2, or 1 and 3) and the development of multi-digit numbers as quantities of tens and ones. The aim of Module A is to challenge children's counting acts to provoke changes in their mental operations, which will bring about the development of the composite unit. **Module B** engaged students in representing and solving various *combine* and *change* problem types (see Table 2 for problem types) using one cohesive mathematical model equation (part and part makes up the whole, or P + P = W). **Module C** engaged students in representing and solving a range of *additive compare* problems using the same model equation, however, the denotations of each of the elements in the PPW diagram equation were adapted to the problem situations accordingly. Name tags wee used to help students anchor "who has more?" which would be the "bigger" quantity or the "whole," and "who has less?" which would be the "smaller" quantity or the "part." After solving the comparison problems, students were given opportunities to represent and solve mixed additive word problems to further strengthen students' construction of the mathematical model, P + P = W, for generalized problem solving. See Figure 1 for Sample screenshots of Module A (left column) and Module C (right column).

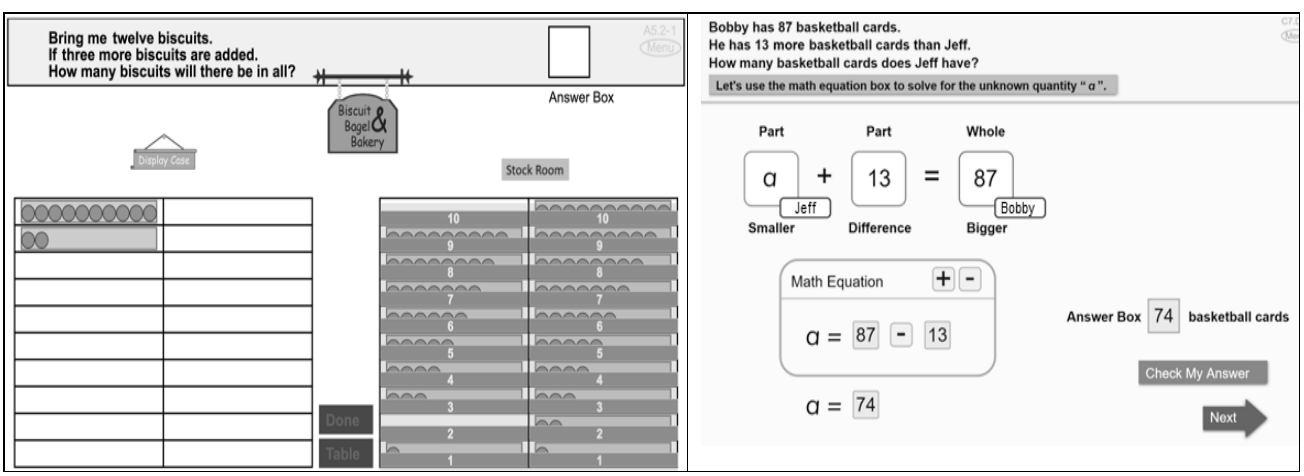

Figure 1: Screenshots of Modules A & C of the MBPS tutor (©Xin, Kastberg, & Chen,

2015-2020)

Results and Discussion

The purpose of this study was to explore the impact of web-based MBPS computer tutor on students' error patterns on solving a range of additive word problems. Overall, students significantly improved their performance from an average of 35.7% correct on pretest to an average of 71.4% correct on posttest. As for error patterns, results indicate that during the pretest, most of the participating students were just grabbing the numbers in the problem and adding them all together regardless of how the word problem was constructed and/or the mathematical relations described in the problem. After the MBPS intervention, students made their attempt to represent information, based on their comprehension of the problem, in the PPW diagram equation and then solve for the problem. Figure 2 presents percentage of students who correctly solved each of the 14 problems (the problem type is noted at the bottom of the bars, please refer to Table 1 for coding of problem types) before and after the intervention.

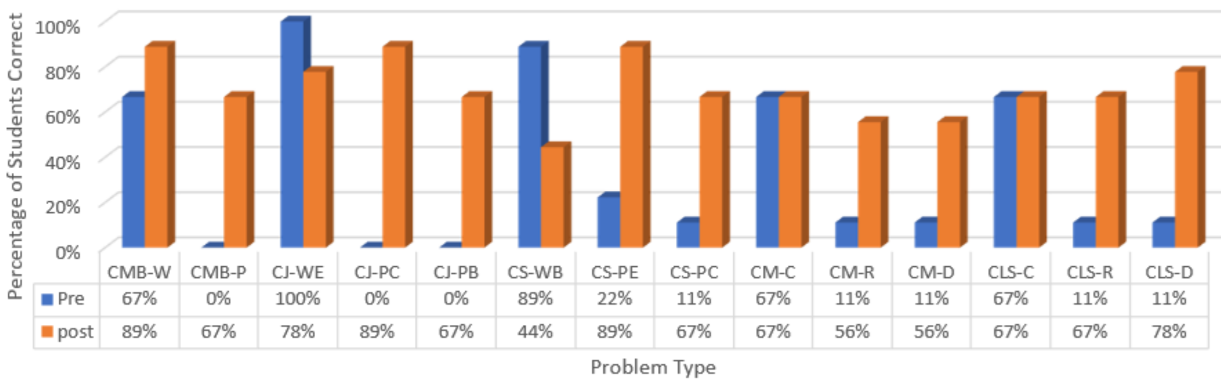

Figure 2*: Percentage of Students solved each of the problem types before (blue/dark color) and after (orange/light color) the MBPS Intervention*

As shown in Figure 2, after working with MBPS tutor, participants made less errors on solving problems across a range of problem situations (over 71% of all problem situations) except for the following two problem situations (a) *change-join*, ending total (whole) as the unknown (CJ-WE, e.g., Leo has 76 math problems for homework. His dad gives him 22 more problems to solve. How many math problems in total does Leo need to solve?"), (b) Change-separate, "whole"/beginning amount unknown (CS-WB, e.g., Alex had many dolls. Then she gave away 12 of her dolls to her sister. Now Alex has 26 dolls. How many dolls did Alex have in the beginning?). Both problem types require adding the two given numbers to get the total (or "whole"). Upon careful examining students' work in pretests, it was discovered that, during the pretest, most of the students simply took the two numbers given in the problem and added them together to get the answer. This senseless strategy would win them the luck in solving problems with above two problem situations. In addition, problems such as *change-join* with ending total (or "whole") as the unknown (CJ-WE) are the easiest problem situations as students could either relying on the *keyword* ("more" or "total" signifies an operation of addition) to solve the problem, or blindly using addition to solve all problems would win them the luck. For the rest of the problem types, particularly those with missing part or missing addend problems (e.g., CMB-P, CJ-PC), problems with the beginning amount as the unknown (CJ-PB), comparison problems with the referent quantity as the unknown (e.g., CM-R, CLS-R, the so-called “inconsistent language” problems, Lewis & Mayer, 1987; e.g., Ellen ran 62 miles in one month. Ellen ran 29 fewer miles than her friend named Cooper. How many miles did Cooper run?), it seems that the MBPS strategy benefited the students in solving these problems as shown in Fig.2 (significantly improved percentage of students who solved the problem correctly). However, for comparison problems with “consistent language” (e.g., Denzel has 28 toy cars. Gabrielle has 15 more toy cars than Denzel. How many toy cars does Gabrielle have?), students’ performance stayed the same after the intervention. It should be noted that to solve *compare* problems with “consistent language,” the “keyword’ strategy would get them the correct answer, although it might involve no mathematical reasoning or understanding of the mathematical relations depicted in the problem. Figure 3 presents sample student work before (left column) and after the MBPS intervention (right column). As shown in Figure 3 (left column), during the pretest, the students simply added two numbers together for the answer regardless of the problem situations. After the intervention (right column), students used “name tags” to represent the information in the PPW diagram equation and then solve the problem.

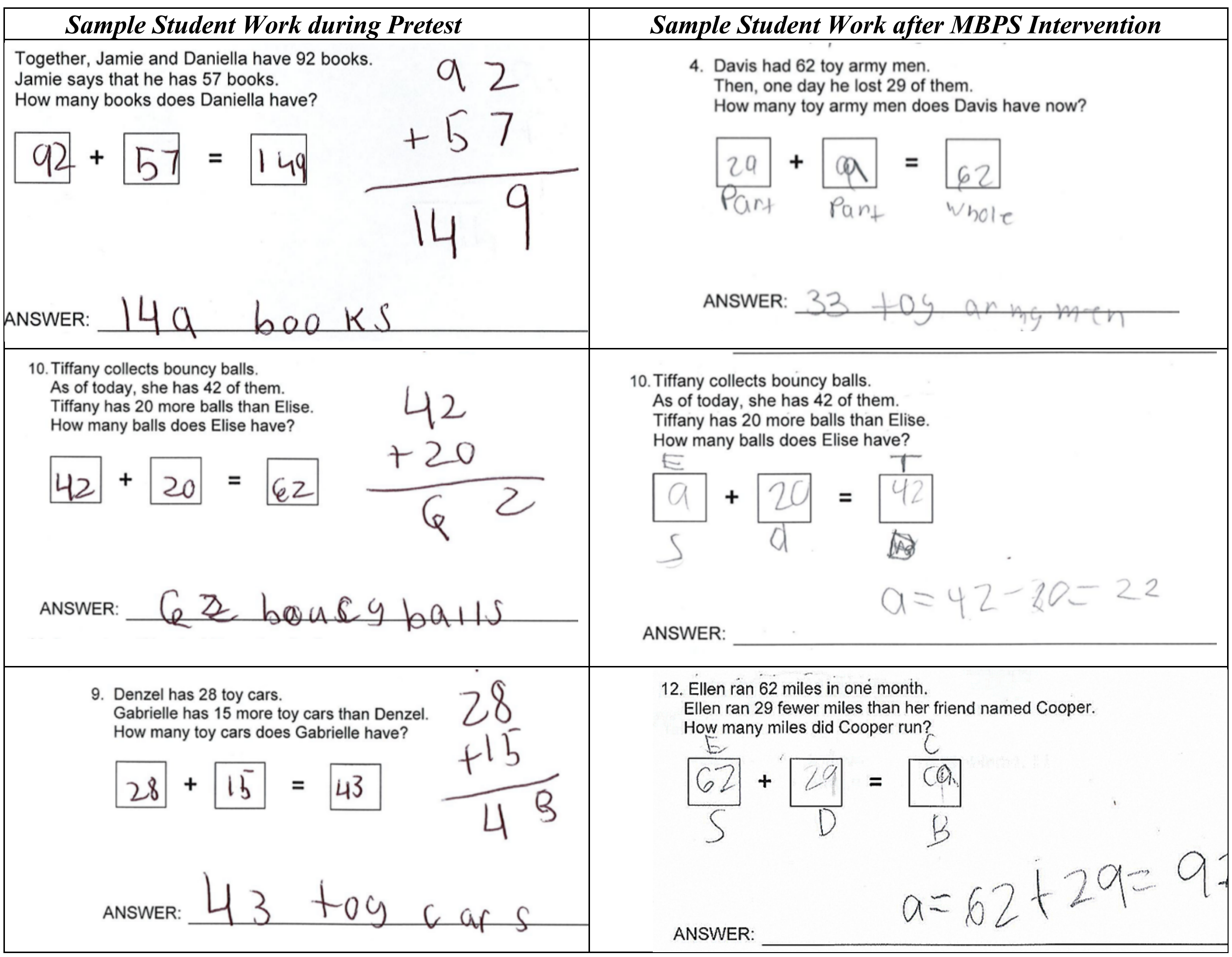

Figure 2: Sample Student Work Before and After the MBPS Intervention

Implications for Practice

The MBPS intended to teach “big ideas” (e.g., “part and part makes up the whole”) in additive word problem solving to promote generalized problem-solving skills. The analysis of success or error pattern shows that participating students improved on solving most of the problems after the MBPS intervention. On the other hand, it should be noted that when teaching students a new strategy, perhaps it won’t be like switching a light bulb—"turning on” the new strategy and “turning off” the old strategy. Often there might be a delay of the use of the newly learned strategy or a mix-up in the use of newly learned strategy and the old strategy (Zhang et al, 2013). As teachers /educators make their effort to promote students’ conceptual understanding of word problem solving and make connections between mathematical ideas, it is important to connect the new concepts to students’ existing knowledge. Students should be provided with abundant learning opportunities, through teachers’ strategically designed learning tasks, for them to experience the advantages and/or the power of the new strategy, for instance, the MBPS which is applicable to solve a range of additive word problems, and therefore “undo” the existing “robust” however “ill” conceived strategies such as the keyword strategy.

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