ENHANCING OUR THEORETICAL LENS: SECOND-ORDER MODELS AS ACTS OF EQUITY

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In this theoretical paper, we respond to a call for all Mathematics Education researchers to become equity researchers (Aguirre et al., 2017) by articulating how equity is foundational to making second-order models of students' mathematics. First, based on prior research, we view equity to be about power and respect. We define an act of equity as acting on social boundaries with the intent of changing them in order to address known inequities. Second, we explain why making second-order models is an act of equity, showing how it respects students and can affect power in research settings. Third, we demonstrate how attention to social identity categories and social identities can enhance current second-order models to better support acts of equity.

Keywords: Equity, Inclusion, and Diversity; Learning Theory; Research Methods

In 2017, the Research Committee of the National Council of Teachers of Mathematics called for all Mathematics Education researchers to engage in the intentional, collective, and professional responsibility of becoming equity researchers (Aguirre et al., 2017). To take up this call, we consider two conceptions of equity in relation to doing research in Mathematics Education. In the first, researchers articulate how their research addresses equity-related issues. In the second, researchers articulate how equity is foundational to their research. With the first conception, equity issues may be added on to a research program without considering the principles, assumptions, and aims of the program (Barajas-Lopez & Larnell, 2019). In this paper, we make an account grounded in the second conception of equity.

We are researchers who make second-order models of students' mathematics (Steffe, et al., 1983) that are rooted in radical constructivism (von Glasersfeld, 1995). These models are explanatory accounts of students' learning that can be used to orchestrate future mathematical interactions with other students. They can also be used as a basis for curricular design and work with teachers. We view the making and using of second-order models to be acts of equity in research settings and to be a basis for engaging in acts of equity both in curricular design and work with teachers. However, this position has seldom been articulated (cf. Ellis, 2022).

We propose that researchers who make second-order models should define what they mean by an act of equity (Gutierrez, 2002), explain how these models can support acts of equity in Mathematics Education, and identify how these models could be enhanced to better support acts of equity. We take on these three tasks in this theoretically-focused paper. In addition, we view our paper as one example of how researchers working within a specific research tradition might respond to the Research Committee's (Aguirre et al., 2017) call. Our aim is to engage other researchers—those who use a radical constructivist framework and those who do not—in conversations that make explicit how equity informs their research.

We make two notes before proceeding. First, since we are researchers of learning, when we articulate how building second-order models can support equity, we focus primarily on the outcomes of supporting learning and opportunities for learning. Second, we view the call to become equity researchers as an active, incomplete process of *becoming*.

Defining an Act of Equity

In order to respond to the Research Committee's (Aguirre et al., 2017) call, we organized themes in research on equity issues in the field, highlighting those relevant for learning. These themes include: identifying structural and institutional inequities in the field (e.g., Battey, 2013; Berry, 2015); disrupting deficit narratives in order to focus on asset-based framings of knowing (e.g., Adiredja, 2019; Gholson & Martin, 2014; Maloney & Matthews, 2020; Stinson, 2013); understanding the impact of people's identities on mathematical learning (e.g., Walshaw, 2013; Shah, 2017); supporting students to learn mathematics as a tool for inquiry into issues of social justice (e.g., Gutstein, 2016; Kokka, 2022); and reconsidering power and access by broadening what counts as mathematical knowledge and who generates it (e.g., Gutiérrez, 2018).

Based on this literature, we conceive of equity to be about how power in society affects people (Ball, 2013; Foucault, 1980; Hayward, 1998) combined with respect for people in all of their varied ways of being and identifying in the world (Gutiérrez, 2018; Walshaw, 2013). We conceive of *power* as the "social boundaries that, together, define fields of action for all actors" (Hayward, 1998, p. 12). *Social boundaries* include "laws, rules, norms, institutional arrangements, social identities and exclusions" (Hayward, 1998, p. 2). Our definition of power leads to a particular view of *freedom*: "the capacity to act upon the boundaries that constrain and enable social action, for example, by changing their shape or direction" (Hayward, 1998, p. 12).

We define *respect* as "an act of giving particular attention, including high or special regard" (Merriam-Webster dictionary online). We view respect for people as an important component of what the literature on re-humanizing mathematics education (Gutiérrez, 2018) calls for: respect for students in math classrooms. For example, Gutiérrez has called for students to be regarded as thinkers and as whole people, stating, "not until we seek to stand in the shoes of our students, to understand their conceptions, will we be on the path toward recognizing and embracing their humanity" (p. 2). We concur with this view and seek to show in this paper what we mean by it.

The focus on respect for each and every person means taking an interest in how people identify themselves and how others identify them. A person's *identity* refers to the person's ways of associating themselves with (or outside of) a racial group, gender, etc., as well as with or against mathematics (Cobb et al., 2009; Martin, 2000). Walshaw (2013) characterized identities as changeable, even across contexts, and as partly shaped by the expectations of others and social boundaries. People have partial agency over their identities within these social boundaries (Cobb et al., 2009). Furthermore, one's individual sense of agency may be linked to the extent to which a person experiences freedom as defined above.

With these definitions in mind, we define an *act of equity* as acting on social boundaries with the intent of changing their shape or direction in order to address known inequities. This definition is very close to our definition of freedom, but we view freedom as a capacity or potential, whereas an act of equity is an action. To complete an act of equity requires identifying how the changes produce more equitable outcomes (i.e., intent alone is insufficient). For example, in math classrooms, students are often trained to learn the thinking of their teacher, as we will explain. The first part of an act of equity (i.e., the intent) would be changing that norm so that students' mathematical ways of thinking, and especially the thinking of students at the margins, are a central and sustained subject matter of the class. We would consider these changes to be theoretical until an actor (e.g., a teacher-researcher) establishes markers that the changes produce more equitable outcomes. Because we are focused on mathematical learning, this documentation would focus on how the change in social boundaries positively affected the learning of the students or the opportunities for students' learning.

To be able to engage in acts of equity as a researcher requires, at the very least, knowledge of historical inequities, knowledge of how one has participated in and benefited from the institutions of one's society, and a willingness to question and continue to learn about both. Within the U.S., we consider these institutions to have been infused with racial, gender, linguistic, and other biases, and so we offer brief information about our positionalities (D'Ambrosio et al., 2013). We are former public middle and high school teachers who are white, cisgender, and monolingual. We have all participated in groups working for equity in schools that have been led by both scholars of color and white scholars. All of us have taught racially, socio-economically, linguistically, geographically, gender, and cognitively diverse students, both as former classroom teachers and in our research. All of us studied issues of equity during our academic training, and one of us has a Ph.D. in Urban Education Studies, with an explicit focus on racial equity, as well as a Ph.D. in Mathematics Education. All of us are makers and users of second-order models of students, two of us experienced. All of us consider it our professional responsibility to inquire into and disrupt inequities, to shape our research and teaching to support equity, and to continue to learn. Although we have space for only a few details, we hope this paragraph helps readers see why we are compelled by the issues in this paper.

How Second-Order Model Building is an Act of Equity

In this section we explain how building second-order models of students' mathematics is an act of equity that demonstrates respect for students and that can influence power. We write about ourselves as teacher-researchers in research settings and classrooms.

First- and Second-Order Knowledge

We view rational human knowing as an active process of perceiving and conceiving (von Glasersfeld, 1995). Humans can never get outside of their ways of perceiving and conceiving in order to check whether their ways of perceiving and conceiving are the way things "really" are in the world. Humans can check with sources outside themselves, certainly! But the check a person makes is performed via their ways of perceiving and conceiving. Therefore, the function of knowing is to organize one's experiential world rather than to discover an objective ontological reality. Yet, this position on knowing is not solipsistic, as humans repeatedly encounter constraints through interaction that help them shape their constructions. These constraints include social boundaries, as well as actions or attempted actions upon these boundaries.

Within this framing of knowing, we differentiate between first-order and second-order knowledge. First-order knowledge is the knowledge that a person constructs "to order, comprehend, and control his or her experience" (Steffe, et al., 1983, p. xvi). In contrast, second-order knowledge is the knowledge that a person constructs about another person's knowledge "in order to explain their observations (i.e., their experience) of the subject's [another person's] states and activities (Steffe et al., p. xvi)." A *second-order model* is a constellation of constructs that a teacher-researcher makes to describe and account for another person's mathematical reasoning and learning (Steffe et al., 1983; Ulrich et al., 2014). So, a second-order model is a *scientific* model of someone else's first-order knowledge.

Why is Making a Second-Order Model an Act of Equity?

Making a second-order model is an act of equity because it is an act of respect for the thinking of the students whose mathematical thinking is being modeled. The message of making a second-order model is: "I see your way of thinking, and I am going to hold it out as important and unique." Making second-order models is a time-intensive endeavor, on the order of months and years; it is not something to engage in lightly. In our view, the time-intensive nature of model-making deepens it as an act of respect for the students whose thinking is being modeled. The making of a second-order model is never fully complete due to the complexity of each individual student's thinking. However, teacher-researchers can get to the point where they have a model that holds up across all of their interactions with the student (Steffe & Thompson, 2000).

And yet, if the making of second-order models was only about highlighting the individual thinking of particular students, it would be an honorable pursuit with limited impact. A central reason making second-order models is an act of equity is because making them can contribute to the construction of epistemic subjects (Beth & Piaget, 1966), although we use the term *epistemic students*.

What is an Epistemic Student?

An *epistemic student* is "that which is common to all subjects [students] at the same level of development, whose cognitive structures derive from the most general mechanisms of the co-ordination of actions" (Beth & Piaget, 1966, p. 308). We view an epistemic student as a generalization that teacher-researchers make of their second-order models in the context of using them with future students. That is, creating an epistemic student comes from a researchers' iterative process of making and refining second-order models of student thinking, using these models in future interactions, and engaging with other researchers' second-order models.

For example, in a first-grade classroom with 25 students, a teacher has 25 individual thinkers. Yet, in terms of how the students think about number, second-order model building has shown that there are similarities among students that can help teacher-researchers organize interactions with them (Steffe & Cobb, 1988; Steffe et al., 1983). Teacher-researchers have found that children construct four different number sequences as they construct whole number knowledge. These number sequences are epistemic students, and they are one support in managing the complexities of interacting with a multitude of students; see Hackenberg et al. (2022) for more.

However, epistemic students should be orienting for a teacher-researcher, not deterministic. What this means is that a teacher-researcher uses their knowledge about epistemic students as a guide to support teaching interactions, but they do not use that knowledge to create a list of forbidden topics. In fact, students operating with different number sequences are likely to work on some topics or problems in different ways, but they are not barred from working on certain topics or problems. As Steffe (1992) has explained, he lets the students be the guide regarding what topics to work on, as he adapts to their ways of thinking.

How is Using Epistemic Students an Act of Equity?

Now we turn to how using epistemic students in interaction with particular students is an act of equity. In our view, using epistemic students in interaction is a higher level of respect for particular students than simply making second-order models. It is a higher level of respect because it involves taking action in order to interact supportively with these students in the moment and with students who have similar ways of thinking in the future. A teacher-researcher can use their epistemic students to inform their learning goals, task design, and plans for interaction (e.g., questioning). In so doing, a teacher-researcher is trying to open possibilities for students to engage and learn. A teacher-researcher is also simultaneously building their knowledge of how to interact with students who think similarly in the future. We view using epistemic students in interactions with particular students as "respect for students in action."

In addition, using epistemic students in interactions in research settings and classrooms can allow teacher-researchers and students to act upon social boundaries to change their shape or direction in order to rectify past inequities. For example, a norm that is typical in secondary math classrooms is that students are to learn the first-order knowledge of the teacher (Teuscher et al., 2016). In other words, in typical secondary math classrooms, teachers present mathematical ideas neutrally, as "the mathematics" to be learned. However, the mathematics to be learned in any given classroom is determined by the teacher's first-order knowledge. Most of the time, teachers attempt to convey their first-order knowledge to students. And, students often tacitly agree to this norm and try to learn these meanings (Liljedahl, 2021; Thompson, 2016). And yet, students cannot learn these meanings directly; they can only interpret the teacher's activity with the meanings that they have constructed at that point—with their own first-order knowledge. Nevertheless, many of them try. And thus, the norm of students being tasked with learning the teacher's knowledge is generated and reinforced. This norm becomes one of the central boundaries that shapes the fields of action for teachers and students.

We view this norm as a fundamental inequity because of the heavy burden it places on students and because of the way in which it keeps student thinking hidden. First, trying to learn the first-order knowledge of the teacher means that students are tasked with trying to learn what they often do not have the ways of thinking to learn. Second, student thinking stays hidden. It may surface at times, but when it does, it usually is as a curiosity or anomaly. There is no sense that students' thinking could be a body of knowledge that is different from the teacher's first-order knowledge, and yet is still knowledge, with coherence and depth, and with ways that it could be modified and advanced under supportive pedagogical environments.

A teacher-researcher who is using epistemic students in interaction with particular students can subvert this larger norm of students being tasked with learning the teacher's first-order knowledge. In a research setting, the typical norm may surface by students waiting for the teacher-researcher to tell them what to do, or by not sharing their ideas. A teacher-researcher who is using epistemic students in interaction must learn the mathematical ways of thinking of the particular students. So, this teacher-researcher will have goals of bringing student thinking to the forefront of discussion and using that thinking to support the progress of all students involved. This orientation and these goals, if enacted, can disrupt the typical norm of students being tasked with learning the first-order knowledge of the teacher. This orientation and goals can instead work to establish the norm that the teacher-researcher must build second-order knowledge of the students, and that the students' mathematics should be the subject matter of the research setting—even in whole classrooms. Thus, using epistemic students in interaction with particular students can influence how power affects students and the teacher-researcher in research settings by shifting whose mathematical knowledge is valued and centered.

How Can We Enhance Second-Order Models?

We now turn to our third task, to identify how second-order models could be enhanced to better support acts of equity. To do so, we focus on the inclusion of social identities in the creation of second-order models. In addition, we adopt two terms from van Es et al. (2022), expanse and stretch. *Expanse* is "the breadth and range of what teachers [-researchers] identify as noteworthy" (p. 115) in a research interaction. *Stretch* is the ways in which a teacher-researcher "reaches back historically and forward futuristically" (p. 117) to consider their own and their participants' pasts and futures. We apply these terms to using social identities in the creation of second-order models because researchers making second-order models have typically focused on making accounts of students' mathematical knowing without including how teacher-researchers' and participants' social identities may impact these interactions. To situate our discussion of expanding and stretching current second order models, we first define social identity categories and differentiate them from social identities.

Social Identity Categories and Social Identities

We use the term *social identity categories* to refer to the range of identifiers commonly used to categorize people, including in the reporting of research, such as race, gender, socioeconomic status, etc. (Langer-Osuna & Esmonde, 2017; Martin, et al., 2017). Researchers have argued that social identity categories are deeply ingrained in society in that they are encoded in law (e.g., Feagin, 2010; Lipsitz, 1995) and used to shape institutions like schools (Battey & Leyva, 2016; Ladson-Billings, 2006; Tyack, 1974). Therefore, they have material consequences for all individuals, and these material consequences are differential, depending on how an individual is categorized by others and how the individual categorizes themselves within the broader system (Battey, 2013; Leonardo, 2009). Moreover, individual members of a society have highly differential access to creating or making changes to these categories or to the consequences of the system of categorization (Gimenez, 2014). And, the categories can be used by those who have more access as one way to define the social boundaries of action for those who have less access (Freire, 1993). Thus, the extent to which individuals experience a sense of agency to change the shape or direction of these social boundaries depends on the access they have to mechanisms that can effect change, often requires collective action over time, and is related to the material consequences an individual may experience for working to make such changes.

With these observations in place, and commensurate with radical constructivist epistemology, we position social identity categories as constraints, which function similarly to physical or conceptual constraints (von Glasersfeld, 1995), in that they are the reality that "kicks back" as individuals' construct their *social identities* in interactions with each other. We note that *social identity categories are themselves constructions*, but because they have a legal basis and history, the categories have a different level of durability and social sanctioning than, for example, individuals' construction of their social identities in a mathematical interaction. Moreover, the durability of these categories, and the way that they constrain individuals' interactions, depends on the context in which they are invoked, and on how they have been encoded and used within that context. We further note that the constraints that individuals experience in constructing their social identities are differential across social identity categories. That is, laws that result in differential benefits across different social identity categories (e.g., access to well-funded schools) form the basis for individuals to experience differential constraints in their construction of their social identities within those contexts.

With social identity categories positioned as constraints that individuals experience in their construction of their social identity, we return to an epistemological point that von Glasersfeld (1995) frequently made: An individual's first order knowledge is constructed in relation to, but not determined by, their experience of constraints. That is, we propose that social identity categories form one set of constraints in individuals' constructions of their social identities, but that there are multiple *viable* responses to these constraints. We see this theoretical point as an important way to understand differences in the way members of the same social identity categories construct their social identities: There are multiple viable responses to any set of constraints. We note, too, that the intersection of multiple of these social identity categories are often at play as individuals construct and enact their social identities within a particular context (Collins & Bilge, 2020).

These considerations allow us to differentiate between first- and second-order knowledge within the arena of teacher-researchers' and students' social identities. We have described how social identity categories can form constraints in a person's construction of their first-order knowledge of their social identities. Second-order knowledge of social identities, then, entails a teacher-researcher making second-order models of the ways that their own and their participants' social identities impact interactions, and in particular impact interactions aimed at mathematical learning. We now turn to two examples of stretching and expanding second-order models by attending to the teacher-researchers' and participants' social identities. Due to space limitations, we can only outline them briefly but will elaborate on them more fully in the presentation.

Example 1: Addressing Gender Equity in Interactions

During a recent teaching experiment that the second author conducted with two pre-service secondary mathematics teachers, a graduate student interested in gendered patterns of interaction pointed out that the two participants were engaged in some gender normative patterns of interaction related to their expressions of confidence in their mathematical thinking (Lubienski & Ganley, 2017; Bench, et al., 2015) and how they each attended to the other persons' thinking. Specifically, the female participant was quite attentive to her male partners' mathematical language and reasoning. Although she was a powerful reasoner herself, she also often doubted her mathematical thinking in these interactions. Her male partner, on the other hand, rarely doubted his mathematical thinking and at the same time had difficulty making sense of his female partners' thinking (see Ippolito, et al., 2021). These gender normative patterns of interaction, which were linked to the social identities of the participants and researcher, were one component of the mathematical interactions in the teaching experiment.

As part of the experiment, the teacher-researcher formed the goal of disrupting these gender normative patterns of interaction by framing each participant as possessing a strength—the female participant being a powerful mathematical reasoner who could make strong interpretations of her partner's mathematical thinking, and the male participant having a high level of confidence in his mathematical thinking. He asked each participant to work on the other person's strength relative to themselves—the female participant to work on being more confident in her mathematical thinking, and the male participant to work on understanding his partners' thinking. The pre-service teachers appeared to appreciate the teacher-researcher's suggestion at the time he made it, and they willingly took it up in future teaching episodes.

We see the teacher-researcher and participants' actions as an example of an act of equity in that the researcher and participants acted on a social boundary with the intent of changing its shape or direction in order to address known inequities in patterns of participation related to the gender identities of the participants and researchers. In presenting the example, we will show evidence that the teaching action produced a more equitable outcome regarding the gender normative patterns of interaction. However, that by itself does not tie the act of equity to learning or opportunities for learning. In the presentation, we will identify how making second-order models that attend to the teaching action aimed at disrupting gendered patterns of interaction could be linked to participants' mathematical learning.

Example 2: Designing to Address Equity

A second way to expand second-order models is by designing mathematical work on an issue of social justice into a study in order to make models of both students' mathematical reasoning and students' understanding of an equity-related issue. For example, the third author designed a study in which he asked middle grades students to explore racial bias in jury selection (Gatza, 2021). There were five participants in his study, most of whom were students in his eighth-grade algebra class. Of the five participants, three self-identified as Black/African-American, one self-identified as Hispanic, and one self-identified as multi-racial, Black and White.

Gatza (2021) conducted the design study in an after-school setting, and his analysis focused on two students, one African-American male and one Hispanic male, who participated in 18 teaching episodes. Gatza's study began with an initial interview in which he identified key features of students' multiplicative reasoning coupled with key features of their understanding of race, racial identity, racism, and racial bias. This initial interview was designed so that the researcher could share information about how he thought about his own racial identity, and to allow the students to share their understandings of their racial identity.

The broad intent of the design study was to understand how students' mathematical reasoning impacted their understanding of racial bias, and how their understanding of racial bias impacted their mathematical reasoning. As part of his study, Gatza (2021) also worked with students on differentiating conscious from unconscious bias as one way to support explanations of how an actual outcome could differ significantly from an expected outcome. He elicted initial ideas about conscious and unconscious bias by having students examine a flyer from the American Red Cross about pool safety that showed children phenotypically presenting as white labeled as acting good and children phenotypically presenting as children of color as acting bad. The students initially interpreted the flyer as not involving bias, but simply as happenstance. Through multiple discussions of the meaning of both conscious and unconscious bias that occurred over time, the students were able to interpret what each form of bias would "look like" when they analyzed artifacts like the flyer. They were, then, able to apply these ideas to the jury selection process to reason about factors that could produce differences between actual and expected outcomes. Thus, their development of schemes related to racial bias supported the development of mathematical understandings.

Within the interactions, Gatza's (2021) design study aimed to cultivate respect for the individual participants by basing the teaching episodes on the students' evolving mathematical and racial bias schemes. Moreover, the design was tied to a social and historical inequity, racial bias. Researchers have found that this particular inequity impacts many facets of Black and Latinx students' lives, including their experiences in schools (Gutiérrez, 2018; Kozol, 2005; Ladson-Billings, 2006; Martin, 2009), and that Black and Latinx students who have strong understandings of race, racism, and racial bias are more likely to thrive in schools (Carter, 2008; Oyserman, Brickman, & Rhodes, 2007). Thus, the teacher-researcher explicitly designed contexts to open opportunities for students to develop capacity to act upon the social boundaries that constrain and enable fields of action by changing their shape or direction. Moreover, throughout the study, there was indication that the participants willingly engaged, working to develop their ideas about race, racism, and racial bias. In the presentation, we give details about the relationship between the students' mathematical schemes and their schemes of racial bias.

Conclusion

Our basic stance in writing this paper is that radical constructivism, as a theory of knowing, has to be paired with first- and second-order models in many domains in order to develop the theory and its usefulness. For example, radical constructivism has already been augmented by a model of communication (Thompson, 1999, 2013), as well as second-order models of specific mathematical domains like students' construction of whole numbers (Steffe & Cobb, 1988; Steffe et al., 1983). We consider this paper to be a step in contributing to a model of equity. We invite discussion to produce refinements of constructs and continued enhancements of second-order models and their uses. The goals are to advance equity and disrupt inequities.

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