BEYOND THE STATISTICS: JOY IN MATHEMATICS

Nicole M. Wessman-Enzinger
George Fox University
nenzinger@georgefox.edu

Laura Bofferding
Purdue University
lbofferd@purdue.edu

In this paper, we share about specific tenets of joy in mathematics and conditions for supporting joy. A pair of students, Julia and Javier, young Latinx mathematicians in Grades 3 and 4 respectively, participated in a study that involved addition and subtraction with negative integers within game play. Drawing on descriptions of joy and supporting joy, we describe specific instances of joy from Julia and Javier and the conditions that supported their joy. Julia and Javier's joy with mathematics above grade level provides a story of empowerment that counters a narrative that they are in an afterschool program where students are not meeting grade level expectations. Further, this qualitative analysis provides explicit instances of joy in mathematics and the conditions supported in a structured mathematical game, which is useful to both teachers and researchers.

Keywords: Affect, Emotion, Beliefs, and Attitudes; Equity, Inclusion, and Diversity; Number Conceptions and Operations

For young children, especially within early childhood education, there is strong support for play-based instruction (Parks, 2015) and advocating for creating joyful mathematics (Parks, 2020). Yet, the importance of play and joy are not exclusive to young children or only social encounters. Students of all ages can engage in mathematics in playful ways (Featherstone, 2000; Wessman-Enzinger, 2018). Parks (2020) advocates that strategies such as supporting play, choice, exploration, social interaction, engagement with materials, relaxing on time, and supporting caring relationships can facilitate joy in mathematics. These strategies for supporting joy in a PreK-2 classroom offered by Parks seem to naturally extend to all mathematical classrooms at all grade levels. Joyful mathematics offers a way of engaging in mathematics that counters the challenges that many students face with mathematics. For example, mathematics instruction focused on conceptual understanding and depth of thinking as opposed to correct solutions and speed moves toward joyfulness (Parks, 2020). Yet, including all of the strategies described above within instruction may prove daunting to teachers (and researcher-teachers) who feel the pressure of getting through particular material.

Joy is also a space of empowerment (Lama & Tutu, 2016). In thinking about joy, we acknowledge that mathematics instruction has historically left some students feeling joyless and excluded (e.g., Battey & Leyva, 2016; Martin, 2019; Joseph et al., 2019). Latinx students are a group traditionally underrepresented in STEM disciplines (Bensimon et al., 2019; Rodriguez & Blaney, 2021). Therefore, by highlighting Latinx students' joy, while they do mathematics far above grade level expectations, we tell a counter narrative that is asset-based rather than deficit-based. Jones and Gomez Marchant (2021) highlight the importance of de-centering Anglos and centering Latinx stories in asset-based ways. Further, this study broadens the work on joy in mathematics to older students and provides a case that illustrates how to include the strategies for incorporating joy, even in what could be interpreted as a structured learning opportunity.

Conceptual Framework: Defining Joy in Mathematics

Joy is often equated with play. Play, according to Huizinga (1955), is voluntary, creates a separate world, creates order, involves tension, and has rules (Featherstone, 2000). Further

definitions even distinguish between mathematical play and play with mathematics (Ginsburg, 2006). For example, mathematical play might involve playing with counting for the sake of counting; whereas play with mathematics involves using mathematics that is taught as part of play. Play (whether mathematical or not) may be joyful, but situations that are not voluntary (or outside the scope of play) might also be joyful. But what is joy in mathematics? Parks (2020) describes joy as going beyond fun. Parks also describes joy as wonder, excitement, pride, and pleasure for practitioners. Other frameworks (see Cottrell, 2016), outside mathematics, describe joy as having temporality (i.e., it is brief and temporary), intentionality (i.e., it is spontaneous and voluntary), awareness (i.e., it involves sudden sense of insight), connection with others, and physical attributes (e.g., clapping, laughing). Parks (2020) suggests a few strategies for supporting joy, such as providing space for play, allowing for choice, social interaction, and caring, and relaxing time constraints. Exploring joy in mathematics has primarily been discussed by Parks with early education students. Understanding experiences and what joy is in mathematics for older students will be helpful for supporting joy in mathematics at all levels.

Based on definitions of play (e.g., Burghardt, 2011; Ginsburg, 2006), most elementary school math might not be considered play because the activities are generally not voluntary and are structured. In this research report, we investigated attributes of joy that emerged when doing mathematics in a structured activity during an afterschool program. In particular, we investigated the following research questions: In what ways does joy emerge (if any) in a structured mathematics game about negative integers? How can we characterize joy in mathematics? What conditions support joy?

Methods

Context and Data Collection

One of the researchers worked with students in an afterschool program at Redwood Elementary in the PNW. At Redwood Elementary, 91% of the students are Hispanic and 15% of the students are at grade level expectations for mathematics according to the state. Students in grades 3 and 4 worked for 20 minutes a day in their afterschool program on mathematics. The researcher, conducting a study on children's formal and invented notation about negative integers, worked with the students individually or in pairs using mathematical games (See Figure 1 for research design). The purpose of the study began in supporting thinking and learning about negative integers. It is within that context that the researchers began thinking about joyful experiences in mathematics, inspired by the students during the structured game play.

We recruited 10 participants from an afterschool program at an elementary school in the Pacific Northwest for students in grades 3 and 4. We considered students from grades 3 and 4 as ideal participants because they have not yet been formally introduced to the concept of negative integers and the representation for negative integers on a number line. On the other hand, Grade 3 and 4 students have already been introduced to and are familiar with the addition and subtraction of *positive* integers (whole numbers), which means that they would be able to leverage and extend their prior knowledge to negative numbers.

Integer Pretest	Game 1: Draw or Discard (Wessman-Enzinger & Bofferding, 2014)	Game 2: Race to a Place	Integer Posttest
Individual Interviews *n=10*	Group 1: Mathematicians' Representations (*n*=5) Group 2: Students' Representations (*n*=5)		Individual Interviews *n=10*

Figure 1: Research Design

In this paper, we use case study methodology (Merriam, 1998) and highlight one of the pairs from group 1 (the mathematicians' group). The students in this group played structured mathematics games that provided negative integers with traditional notation, while the other group used invented notation for negative integers. We have intentionally selected this group for two reasons. First, after this particular session, the first researcher felt "joy" and immediately reached out to the second researcher to discuss the moments of "joy." Because of this, it seemed like an apt set of data to first start to qualitatively explore. Second, we know that mathematical play, which should be inherently joyful, is not necessarily present in the context of a game with rules, provided in a top-down way. This made us wonder how this particular context could be joyful, and it felt like that describing spaces of joy in this particular context would be most beneficial to both researchers and teachers.

Structured Game Play

The students in the session that we selected for this study played the Draw or Discard game (Bofferding & Wessman-Enzinger, 2015; Wessman-Enzinger & Bofferding, 2014) and used traditional notation for negatives, which we called mathematicians' notation. The objective of this game is to have the most points at the end of the game using the cards in Figure 2.

Figure 2: Cards in Draw and Discard

Each player starts with two cards and the players use a pile of cards in the center of the playing space. Players must begin by deciding how many rounds of turns they want to play. On each player's turn, the player must turn over the top card in the center deck so that everyone can see it. Then the player must decide to either draw or discard a card. If the player chooses to draw the card from the center pile, they will add it to their hand. If the player does not want to draw the face up card, they must discard a card from their hand. The next player repeats the same steps, and play continues until the previously determined number of rounds have been completed. A player can have as many or as few cards in their hand as they wish, and the sum of integers in a player's hand represents the total points.

Data Analysis

We began qualitatively analyzing how students engaged with negative numbers and joy by first reading the transcript separately and taking notes. After individual reflections on the video and transcript, we met together and first conducted a form of qualitative analysis called thematic analysis (Braun & Clarke, 2020). After noting themes, we saw ways that these themes aligned with moments of joy we saw in Parks (2020) and also delineated by Cottrell (2016).

Results

We structure and discuss the themes on how the students experienced joy chronologically. We specifically chose this to highlight how the students' experienced joy throughout the game, which looked different towards the end of the game compared to the beginning of the game.

Beginning: Explaining the Rules of the Game

The researcher first shared: "When we play our game, it will be you two on one team against me." Javier and Julia laughed and smiled about this. Both made silly faces and said, "Ok, ok, ok…" with excitement about playing against the researcher. The researcher then explained the

rules of the game. First, the researcher engaged the students in discussion around what the words "draw" and "discard" mean, modeling with the cards. Then, she modeled the rules of the game, employing pedagogical strategies for supporting joy, such as care and social interaction, by asking the students, "Have you ever made a game?" She continued by saying, "We can show each other what our cards are, because it's a strategy game."

Table 1: Tenets of Joy, Joyful Experiences, and Conditions of Joy for Beginning

Tenets of Joy	Joyful Experiences of Students	Conditions for Supporting Joy
Excitement	Joking	Students playing collaboratively *against* the teacher
Physical Attributes	Smiling, laughing, joking	Sharing the cards that were used with each other

Learning: Modeling the Game

The students began by drawing their first two cards: -4 and -3. The transcript excerpt below is Javier and Julia's first interaction with negative integers and these initial cards.

Javier:	(Makes a joke.) It's a draw. It's a draw.
Researcher:	Why are you saying it's a draw?
Javier:	Because it's seven.

In this excerpt we see Javier and Julia laughing together when they draw -3 and -4, perhaps because they are seeing negative integers for the first time. Javier makes a joke to draw new cards again. There is spontaneity in Javier's joke and their first attempt at considering -3 + -4.

Researcher:	You think it's seven?
Julia:	Wait… Wait…
Researcher:	What do you think it is?
Julia:	Three minus… Well, it would be zero.
Javier:	One minus.
Julia:	No it would be one.
Javier:	One minus.
Julia:	(Nods yes.) One minus.

Javier and Julia's initial idea for -3 + -4 is "one minus." The researcher did not comment, instead saying, "How about you help me with my cards (shows cards 0 and -7)? What's my score? I've got 0 and what do you think this is called?" Julia started saying, "Seven minus (referring to -7), and the researcher suggested they could call it negative seven. Then both Julia and Javier said her score would be "Zero!" Instead of correcting them, the researcher created space for student thinking by saying, "I'm going to make an analogy for you. If I have three plus zero what is it?"

Julia:	Three plus zero… three.
Researcher:	Ok. If I have five plus zero, what is it?
Javier:	Five.

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2). University of Nevada, Reno.

Julia:	Oh. So that would be negative seven.	

The researcher used an analogy with the students, comparing "0 + 3" and "0 + 5" to her cards, "0 + -7," helping them make sense of her hand before returning to their hand. She asked students to think about what negative three and negative four points are.

Javier:	One minus.
Julia:	(Nods.) One minus.
Researcher:	Ok. Let's make an analogy here together. If I had three points and I got four points, how many would I have?
Julia/Javier:	Seven.
Researcher:	So if I had negative three points and negative four points, how much would I have?
Javier:	Seven.
Julia:	Negative seven.
Javier:	(makes a joke) We draw…
Researcher:	Can you explain?
Javier:	It's like, it's like, it's just like plus.
Researcher:	So you have -4 points and -3 points, so how many points do you have?
Julia:	Oh, I get it. It's like you added.
Javier:	But minus it.

Through the help of another analogy and her partner explaining, "I'd just like plus," Julia stated "oh I get it" with increased volume, a big smile, and a sense of new insight.

After the students determined the sum of their initial cards, they encountered the first move of the game where they needed to decide if they were going to draw the -5 card that was flipped over or discard one of their cards. Although discarding the -4 would be the strategic move towards getting more positive points, Julia and Javier decided to add -5 (i.e., draw the card) to their sum. The researcher did not comment and supported the students in this choice. Julia and Javier worked collaboratively and spoke in Spanish about their new total. They verbally stated that the sum of -4 + -3 + -5 was "doce" (or +12) and wrote -12 on their recording sheet. The researcher affirmed their language and solution, by nodding at -12 and also calling it "doce."

Table 2: Tenets of Joy, Joyful Experiences, and Conditions of Joy for Modeling the Game

Tenets of Joy	Joyful Experiences of Students	Conditions for Supporting Joy
Physical Attributes	The students laughed at the negative integers and made jokes about re-drawing new cards.	Giving students unfamiliar cards with negative integers on them.
Intentionality and Connection	The students collaborated on what -3 + -4 equals. They both adjusted their thinking.	The researcher supported brief exploration in thinking before moving onto the next card draw.

Awareness	The students made the connection that -3 + -4 = -7.	The researcher modeled making analogies.
Connection	The students made choices, such as what card they wanted to draw or discard and used language to use.	The researcher supported student choices ("one negative" for -1; Spanish "doce" for -12).

Playing: Finding Flow in the Game

Next, with the cards -3, -4, and -5, Javier and Julia flipped +6 in the center pile. They needed to decide if they would draw the +6 or discard one of their cards. As the students made a choice about what to do next they smiled and whispered to each other (truly playing "against" the researcher now). They decided to discard the -3, which would leave them with -4 and -5. The students then needed to decide how many points they had left, the sum of -4 and -5. They worked collaboratively and negotiated together that their sum, or total points, was -9.

On their next move, they flipped over a +3 in the center deck with a hand of -4 and -5. Javier spontaneously started clapping, saying, "Take it, take it, take it." Although Javier and Julia both now agreed that they should take the positive card, +3, they had different ideas about what 3 + -4 + -5 are. Javier thought 3 + -4 + -5 = -6 and Julia thought that 3 + -4 + -5 = -12. The transcript below highlights Javier's argument within the game. Figure 3 highlights their physical smiles and laughing as they argued about the sum of 3 + -4 + -5.

Figure 3: Javier and Julia discuss different sums for 3 + -4 + -5

Javier:	Minus six. Minus six.
Researcher:	Instead of saying minus six, try explaining it to her.
Javier:	These two bro (pointing at the negatives).
Researcher:	Ok. How about this…
Javier:	These two… these two … (pointing at cards) these two bro.
Researcher:	Ok, what two?
Javier:	These two. If you minus these two (points at -5 and -4) to three then that's six minus. You are plussing. You are trying to plus to get higher.

Through discussion, the researcher helped the students remember that -3 + -4 = -7. Then, she asked about -3 + 4.

Javier:	If you minus these two it's one.
Researcher:	Yes, four minus three is one. But, you are saying one minus? Can you explain that?
Javier:	Because this is lower than the four (points at the 3).
Researcher:	What's lower than the four?

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2). University of Nevada, Reno.

Javier: The three.

Eventually Julia stated, "Let's put down negative six. … I'm convinced."

Next, Javier and Julia, flipped over a 0 in the center pile with a hand of -5, -4, and 3. They collectively agreed to discard their -5 and debated the sum of 3 and -4. Julia thought the sum of 3 and -4 was 1 because 4 – 3 = 1 Javier argued the sum of "one minus." And Julia changed her mind to "one minus." Then, Javier and Julia decided to draw a card, +3, and add it to their hand. They worked together to determine the sum of 3 + 0 + -4 + 3. As they decided this, they exhibited excitement. Notably, Julia started obtaining the correct solutions for the sums of negatives now.

Researcher: (Spreads cards out on the table for the students to look at.) Let's see here, we
                        have three, zero, three, and negative four.
Julia: So this would be six (combines the two 3 and 3 cards together).
Javier: (moves cards 3 and -4 together)
Julia: (takes the cards back) Six minus four is two. Two points.
Javier: Yeah, two. Yeah, let's catch up! Let's go!

The students and the researcher continued taking turns. The students, without questions or support from the researcher, decided which cards to discard or draw. The students had emotional reactions to the researcher's choices. When the researcher made a move to draw or discard a card, they laughed, and Javier would cover his face with the hood of his sweatshirt.

Table 3: Tenets of Joy, Joyful Experiences, and Conditions of Joy for Playing

Tenets of Joy	Joyful Experiences of Students	Conditions for Supporting Joy
Connection	They each stated their ideas and questioned each other (e.g., debated the sums of 3 + -4 + -5 and then 3 + -4).	The researcher supported the students' choices in drawing or discarding, even if it veered from the most strategic move.
Awareness and Pride	The students began to solve problems on their own and showed enthusiasm about it.	The researcher encouraged them to look at their recording sheet and talk to each other.
Excitement	"Take it, take it, take it." "Yeah, let's catch up! Let's go!	The students played a game against the researcher.
Wonder	The students each had moments of curiosity about the sums of their cards.	The researcher encouraged them to share their thinking.

Ending: Concluding the Game

As the game concluded, the researcher won the game. Javier put his shirt over his head as the researcher confirmed, "I did win." Both students laughed and asked to play another round. The researcher said, "We do have ten more minutes," which Javier took as a good sign, saying, "Yeah, yeah, yeah!" The researcher confirmed, "We can see who wins after 10 minutes." The researcher

and the students began their second game, with the students playing against the researcher. On the researcher's turn, she had a -8 in her draw pile with -4 and 5 in her hand.

Researcher:	I've got a negative four and five. So negative four and five is one. And, I drew a negative eight. I'm going to…
Javier:	Take it! (Keeps laughing hard, this is a joke.)
Researcher:	I'm definitely not taking the negative eight.
Julia and Javier:	(laughing)

The students made a math joke by telling the researcher to make a move that would not help her win. Adding -8 to the hand would make the total point score as low as possible. Julia and Javier continued teasing the researcher, based on their integer understandings, as they concluded their time playing the game. Javier and Julia continued teasing the researcher the rest of the game. Eventually, the researcher said: "So whatever you are saying is what I should not do… "

Table 4: Tenets of Joy, Joyful Experiences, and Conditions of Joy for Ending

Tenets of Joy	Joyful Experiences of Students	Conditions for Supporting Joy
Excitement	The students wanted to play another round of the game.	The researcher let them play again even with the brief time.
Awareness	The students made math jokes about integer values (indicating cleverness).	The students collaboratively played against the researcher.
Physical Attributes	The students cheered for a second game and laughed at their math joke.	The researcher asked for their opinion on what moves to make.

Implications

Julia and Javier are young Latinx mathematicians in grades 3 and 4, respectively. They not only engaged with negative integer operations in intellectual and mathematical ways, but they also experienced joy. The most significant part of these results is highlighting how two young, Latinx mathematicians not only worked with negative integers (a topic far above grade level), but they also experienced joy in working with negative integers. Recommendations include operations with negative integers in middle school; yet, Julia and Javier cleverly did this mathematics as young mathematicians in Grades 3 and 4. Similarly, Julia and Javier, demonstrated many instances of joy within in mathematics. The context of this school matters: statistics paint these children in an afterschool program where only 15% of the students are at grade level. Yet, we meet Julia and Javier, joyfully engaging in advanced mathematics.

This joy is immersed in a structured study and a structured mathematical game, which may seem like a space where there is little opportunity for joy. Yet, the examples provided here included many different experiences with joy (e.g., wonder, excitement, connection, cleverness). The results here point to an inaugural avenue for thinking about how we can support joy in different pedagogical and research environments, such as structured mathematical game play. As we consider future implications, we think about the tenets and conditions of joy, such as collective

and collaborative. For example, the students played against the researcher, someone positioned in power. Julia and Javier shared their opinions on the game play and expressed themselves mathematically in both English and Spanish. We wonder about ways to modify games to be collaborative and about the potential of collaborative games. Relaxing, allowing choices, and encouraging the students are aspects to supporting joy that centers on caring.

References

Battey, D., & Leyva, L. A. (2016). A framework for understanding whiteness in mathematics education. *Journal of Mathematics Education, 9*(2), 49–80.

Bensimon, E. M., Dowd, A. C., Stanton-Salazaar, & Dávila, B. A. (2019). The role of institutional agents in providing institutional support to Latinx students in STEM. *The Review of Higher Education, 42*(4), 1689–1721.

Bofferding, L., & Wessman, N. M. (2015). Solutions to the integers: Draw or discard game. *Teaching Children Mathematics, 21*(8), 460–463.

Braun, V., & Clarke, V. (2020). Thematic analysis. In H. Cooper (Ed.), *APA Handbook of Research Methods in Psychology* (Vol. 2., pp. 57–71). American Physiological Association.

Burghardt, G. M. (2011). Defining and recognizing play. In A. Pelligrini (Ed.), *The Oxford handbook of the development of play* (pp. 9–18). New York, NY: Oxford University Press.

Cottrell, L. (2016). Joy and happiness: A simultaneous and evolutionary concept analysis. *Journal of Advanced Nursing,*1506 – 1517. doi: 10.1111/jan.12980.

Featherstone, H. (2000). "-Pat + Pat = 0": Intellectual play in elementary mathematics. *For the Learning of Mathematics, 20*(2), 14–23.

Ginsburg, H. P. (2006). Mathematical play and playful mathematics: A guide for early education. In D. Singer, R. Golinkoff, & K. Hirsh-Pasek (Eds.), *Play= Learning: How play motivates and enhances children's cognitive and social-emotional growth* (pp. 145–165). New York, NY: Oxford University Press.

Huizinga, J. (1955). *Homo ludens: A study of the play-element in cult.* Boston, MA: Beacon Press.

Jones, S. R., & Gomez Marchant, C. N. (2021). Narratives of Raza in mathematics education research literature. In D. Olanoff, K. Johnson, & S. Spitzer (Eds.), *Proceedings of the forty-third annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 134–142). Philadelphia, PA.

Joseph, N., Frank, T. J., & Taqiyyah, Y. E. (2021). A call for a critical-historical framework in addressing the mathematical experiences of black teachers and students. *Journal for Research in Mathematics Education, 52*(4), 476–490.

Lama, D., & Tutu, D. (2016). *The book of joy.* Penguin Random House.

Martin, D. B. (2019). Equity, inclusion, and antiblackness in mathematics education. *Race Ethnicity and Education*, *22*(4), 459–478. https://doi.org/10.1080/13613324.2019.1592833

Merriam, S. B. (1998). Qualitative research and case study applications in education. San Francisco, CA: Jossey-Bass.

Parks, A. N. (2020). Creating joy in PK-Grade 2 mathematics classrooms. *Mathematics Teacher: Learning & Teaching PK-12*, 61–64.

Parks, A. N. (2015). Exploring mathematics through play in the early childhood classroom. Teachers' College Press.

Rodriquez, S., & Blaney, J. (2021). "We're the unicorns of STEM": Understanding how academic and social experiences influence sense of belonging for Latina undergraduate students. *Journal of Diversity in Higher Education, 14*(3), 441–455.

Wessman-Enzinger, N. M., & Bofferding, L. (2014). *Integers: Draw or discard! game. Teaching Children Mathematics, 20*(8), 476–480.