AGREEING ON OBJECTIVES OF GEOMETRY FOR TEACHERS' COURSES: FEEDBACK FROM INSTRUCTORS ON AN INITIAL LIST[13]

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We report on an effort to vet a list of 10 student learning objectives (SLOs) for geometry courses taken by prospective geometry teachers. Members of a faculty online learning community, including mathematicians and mathematics educators who teach college geometry courses taken by prospective secondary teachers developed this list in an effort to reach a consensus that might satisfy various stakeholders. To provide feedback on the final list of 10 SLOs, we constructed and collected responses to a survey in which 121 college geometry instructors ranked a set of potential SLOs, including the 10 proposed SLOs as well as 11 distractors. The 10 SLOs were, for the most part, among the highest ranked by the sample.

Keywords: Geometry and Spatial Reasoning, Research Methods

Secondary mathematics teachers' mathematical preparation is a critical issue influencing the quality of mathematics instruction. Mathematicians are essential stakeholders within higher education because these courses are often offered within mathematics departments; nevertheless, they are not the only stakeholders. Additional stakeholders include the teacher education programs that require students to take these courses and mathematics education scholars, who often operate at the nexus of disciplinary knowledge (i.e., mathematics) and teacher preparation. Numerous practical considerations guide the ongoing joint investment of mathematics departments and teacher education programs in such courses. Mathematics departments frequently face difficulties in being able to provide courses required for teaching accreditation. Related difficulties—particularly vis-à-vis college geometry courses (see Grover & Connor, 2000)—include finding faculty members who are willing to teach those courses and shaping the curriculum of those courses to attend to both the sensibility and knowledge generated by the discipline of mathematics and the need to develop capacity to handle the mathematical content of high schools. Still, mathematics departments are inherently invested in the articulation of secondary mathematics with undergraduate mathematics curricula, and so they continue to actively navigate dilemmas related to the varied constituencies of mathematics courses taken by prospective teachers. In prior work, we identified in our interviews with university geometry instructors describing their positions as having to manage five different tensions (Herbst et al., 2023). One of them, the *content* tension is subtended by two distinct perspectives that could guide the design of mathematics coursework taken by preservice mathematics teachers. The first emphasizes that those preparing to be high school mathematics teachers should study the same

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mathematics curriculum as mathematics majors, which can provide a comprehensive understanding of the field and its development then can be applied to the school curriculum (Matthews & Seaman, 2007). The other perspective emphasizes that high school mathematics teachers should focus on the mathematical knowledge needed for teaching, which includes understanding the high school mathematics curriculum (Ebby, 2000). The tension between determining course content purely based on disciplinary considerations and doing so attending to the diverse sources of the school curriculum is an ongoing challenge that mathematics departments must navigate in their efforts to provide courses necessary for teaching accreditation (Herbst et al., 2023; Brown et al., in press).

We consider the Geometry for Teachers (GeT) course to be a useful case for examining challenges to and opportunities for assessing and developing consensus across the multiple stakeholders involved with undergraduate mathematics curriculum. GeT courses are often required for teaching certification and are typically taught within mathematics departments at universities. Instructors are sometimes prepared as mathematicians and other times as mathematics educators. However, GeT courses typically have fewer students than other service courses in mathematics departments, such as calculus or linear algebra, making it difficult for mathematics departments to create local communities to support the course. This often means that decisions about the content of GeT courses are left up to the individual faculty members teaching them. While diversity in GeT curriculum is not in and of itself a problem, it is directly implicated in a dual set of related challenges: (a) secondary mathematics teachers have access to widely variable geometry coursework but are expected to teach a relatively cogent high school geometry curriculum and (b) assessing college geometry students' learning outcomes in some systematic way is complicated by the wide variability of curricula. So, in spite of some reasonable variations in curriculum and instruction, GeT courses present a compelling case for developing and sustaining some convergence around essential student learning outcomes.

To address the issue of increasing capacity for high school geometry instruction in the absence of resident GeT communities in mathematics departments, an inter-institutional online professional learning community (OPLC) of GeT instructors (GeT: A Pencil; see getapencil.org) was formed and has been working together for the last past 5 years (An et al., 2023). The authors of this paper have been involved in efforts to convene and support the community in several ways, including facilitating interactions among participants and collecting data that informs their efforts. One issue that surfaced early in discussions within the OPLC was a lack of a clear shared understanding of what should be in the GeT course. A subgroup of 11 members of the community (SLO-WG, hereafter), including mathematicians and mathematics educators, have come to a consensus on a set of 10 Student Learning Objectives that they consider essential for any prospective secondary mathematics teacher to attain. For this list of objectives to have impact on the large systemic challenges named above, it was important to share it with the larger community of GeT instructors (including those outside of GeT: A Pencil) and obtain their feedback. As the group contemplated how they might share their work with and gauge reactions from this broader community, we considered how we might support their efforts to measure and systematically analyze notions of convergence. Thus, we designed and distributed a survey to a larger group of GeT instructors to provide feedback on the work of the SLO-WG: Would the community of GeT instructors at large endorse these 10 SLOs as those most important to aim for in a GeT course?

In this paper, we begin by providing a brief background of the SLO-WG and their approach for creating the list of SLOs for the GeT course. Next, we provide a perspective on the practical challenges involved in the work of soliciting feedback on a consensus on a set of student learning objectives for a course, from the view of survey methodology design. We illustrate the use of Balanced Incomplete Block Design to elicit understanding at scale in a quick multiple-choice format. We conclude the paper with a discussion on the implications of this work with the teaching of GeT courses and how this framework could be used to examine sets of learning goals produced by other transdisciplinary communities interested in coming to a consensus on essential learning outcomes.

Background on SLO-WG

Members of the SLO-WG decided to use a winnowing strategy to create a list of essential SLOs, with each group member contributing a set of student learning objectives they thought were essential to one master list (An et al., in press). Discussions and reflections on learning objectives drafted as part of earlier work guided their work. In developing learning objectives, instructors drew on or were informed by the following sources: (1) instructors' previous course syllabi and materials, (2) secondary geometry standards documents (e.g., CCSS-M; NGA; 2010; NCTM, 2000), (3) college geometry curricular guidelines and recommendations (e.g., CBMS, 2012; Venema et al., 2015), and (4) descriptive research on undergraduate geometry courses (e.g., Grover & Connor, 2000). Following the initial drafting, the SLO-WG culled the master list based on common themes. These themes became the focal point of subsequent meetings in which the group worked toward the development of common statements that all participants agreed were essential learning objectives. Ongoing discussions have been influenced by additional reflection and discussion on the interpretations of standards and guidance documentation.

Table 1: Brief Statements of the 10 SLOs for the GeT Course Produced by SLO-WG

SLO	Description	SLO	Description
1	Derive and explain geometric arguments and proofs.	2	Evaluate geometric arguments and approaches to solving problems.
3	Understand the ideas underlying current secondary geometry content standards.	4	Understand the relationships between axioms, theorems, and different geometric models in which they hold.
5	Understand the role of definitions in mathematical discourse.	6	Effectively use technologies to explore geometry and geometric relationships.
7	Demonstrate knowledge of Euclidean geometry, including its history.	8	Be able to carry out and justify basic Euclidean constructions.
9	Compare Euclidean geometry to other geometries such as hyperbolic or spherical.	10	Use transformations to explore definitions and theorems about congruence, similarity, and symmetry.

In Spring 2020, SLO-WG begun writing elaborations of the SLOs in a community newsletter and these have been collected in getapencil.org. These elaborations are seen as part of a living document and feedback is expected from all stakeholders that might result in continuous improvement of the list of SLOs as well as their elaborations. A companion edited book is also in development (Brown et al., in preparation) that will help extend the community of those interested in shaping an emerging consensus. To inform this emerging consensus, the SLO-WG posed the question: “What does the larger community of instructors who teach the geometry course for secondary teachers think about the list of SLOs we have developed?” While the SLO-WG collected feedback on the SLOs in many modalities (e.g., dissemination in writing, delivering seminar talks, and hosting getapencil.org), we took upon ourselves to construct and disseminate a survey to elicit feedback from a larger group of instructors of GeT courses.

When we began the challenge of gathering feedback from the greater community of educators on the proposed SLOs, we realized that developing a method for framing the task in a way that would elicit responses for a valid inference was critical. In the rest of this section, we describe some practical problems that constrain our ability to solicit and analyze open-ended feedback from individuals and undertake a more synthetic study of consensus and convergence.

Practical Challenges

When we took on the task of asking the community of instructors at large for their views on the proposed SLOs, we realized that an important element was figuring out a strategy to pose the question that would arguably elicit answers for a valid inference. Some practical problems that we could anticipate were the following:

1. If we asked people whether they would endorse the 10 SLOs as the student learning objectives for all GeT courses, we could expect a wide range of negative answers–from those who would disown all 10 to those that would disown 1 of the 10. This strategy seemed biased against our goals.
2. If we asked people to list the 10 learning objectives they would support, then coded the responses and compared those with our list of 10, we would not be asking an endorsement question but seeking to reproduce the accomplishment of our working group but without the benefit of their discussion. This strategy seemed one that would not do what we wanted to do, regardless of possible outcomes (e.g., some objectives might find confirmation in their popularity).
3. A combination of the first and second strategy, whereby we’d give all 10 objectives and asked for participants to add what they saw missing seemed promising but also seemed that in could contain the same problems as 1 and 2 and perhaps others depending on what was asked of people.
    a. If people were asked to choose 10 after listing what they wanted, we might be prompting bias toward their individual choices. While the SLOs that remained highly chosen (because kept in most lists) might be endorsed by the collective, the aggregate of written-in objectives might not be so easy to make sense of just yet.
    b. If people were asked to cross from the 10 those they did not want and add however many they wanted, providing a list of possibly more than 10 choices, for us to later choose the 10 with the most choices, this might likely bias the task toward the initial 10.
4. We considered giving people the list of 10 and an equal number of distractors whose statements we drafted to read similar to statements of the 10 and considered having people rank order all SLOs and distractors to then tally the ranks and choose the 10 highest ranked. This strategy seemed unbiased but cognitively very demanding as the number of pairwise comparisons that would be entailed by any one accomplished ranking would be $20*19/2 = 190$. That is, it would be hard to guarantee that after reading 20 statements and putting them in order of preference, the respondent would be able to defend the entailed preference order of any two potential objectives, considering how many pairs would need to be sorted. However, the idea of having a ranking task that did not make so high a cognitive demand on the participants seemed compelling.

These considerations took us to look for a survey design and statistical procedure that we could use to satisfy the following requirements:

1. To have a set of potential SLOs roughly twice as large as the 10 drafted by the WG, that is one in which it would be just as likely for a sampled set of 10 objectives to include an SLO or a distractor. Call the size of this set *N*.
2. To figure out how many objectives *N* should have, and how many sets of elements $n < N$ (call this number of sets *k*) we should have so that
    a. Any one participant would be given one set of n elements to rank, sampled from the *N*
    b. The number of comparisons entailed by each individual ranking would be small enough not to cause high cognitive demands on raters
    c. Any one pair of elements from the *N* would have equal chance of appear within all the *k* sets
3. If such a model existed, we would need to calculate how many people to ask to rank one set of n elements so that each of the sets had an equal chance to be used in the survey
4. We needed to figure out how to tally all the completed ranks so that an aggregate rank could be determined based on the aggregate of the ranks.
5. Further, if we wanted to figure out which 10 objectives out of the *N* ranked would be endorsed by the community, we would need to identify a statistical model and a sample size that could be used to evaluate the ranked list of size *N* to detect significant differences (e.g., between the objectives 10th and 11th ranked SLOs in the aggregate).

Table 2: Distractor Statements of the 11 dSLOs Included in the Questionnaire

dSLO	Description
11	Use geometric properties to find the measures of angles or sides
12	Distinguish between necessary and sufficient conditions in a mathematical claim
13	Understand the ideas underlying advanced Euclidean and absolute geometry
14	Make connections between geometry and other mathematical subjects such as linear algebra, mathematical modeling, and group theory.
15	Understand the importance and role of diagrams in geometric communication.
16	Effectively use digital proof tools to practice writing geometric proofs with feedback.
17	Understand the role played by practices like building structures, navigation, stargazing, and art in the historical development of geometry.
18	Be able to demonstrate that the three classical problems of geometry are not solvable with straightedge and compass
19	Prove theorems about circles and tangent lines.
20	Investigate advanced properties of projective geometry
21	Have experience with the mathematical modeling cycle in geometry problems.

Methods

A Balanced Incomplete Block Design (BIBD) is a survey research strategy that ensures that all survey items are asked the same number of times while minimizing response biases (Alvo & Cabilio, 1991). Participants in a BIBD are randomly assigned to groups or blocks, and each group is shown a subset of the survey items. Each survey question is divided into a defined number of blocks, guaranteeing that all items are asked the same number of times while also allowing for fewer things to be included in each survey. This reduces respondent burden and survey fatigue while simultaneously guaranteeing that key survey items are not overlooked. To employ a BIBD in a questionnaire, we must determine the survey items that will be included and the number of blocks that will be used. A balanced incomplete block design is one in which all pairs of treatments occur together in the same block at the same frequency; this number is indicated by λ. A total of *nb* judges rank *t* items *k* at a time based on *n* replications of a BIBD with *b* blocks. The BIBD must meet the following requirements: 1) $bk = tr$ and 2) $\lambda = r \times \frac{k-1}{t-1}$. In our case, since we have 10 SLOs, in order to satisfy the conditions of this BIBD, we needed to come up with 11 'distractor' SLOs (dSLOs, hereinafter) for a total of *t* = 21 objects to rank. The remainder of the conditions are as follows: 1) $b = 30, k = 7, t = 21, r = 10 \Rightarrow (30)(7) = (21)(10)$; 2) $\lambda = 3;\ r = 10, k = 7, t = 21 \Rightarrow 3 = 10 \times \frac{(7-1)}{(21-1)} \Rightarrow 3 = 10 \times \frac{3}{10}$.

Table 3: Balanced Block Design for our Distribution of the Questionnaire

block	1	2	3	4	5	6	7
1	2	5	10	11	17	19	20
2	3	6	11	12	18	20	21
3	4	7	12	13	19	21	15
4	5	1	13	14	20	15	16
5	6	2	14	8	21	16	17
6	7	3	8	9	15	17	18
7	1	4	9	10	16	18	19
8	3	4	8	13	17	19	20
9	4	5	9	14	18	20	21
10	5	6	10	8	19	21	15
11	6	7	11	9	20	15	16
12	7	1	12	10	21	16	17
13	1	2	13	11	15	17	18
14	2	3	14	12	16	18	19
15	1	6	9	12	17	19	20
16	2	7	10	13	18	20	21
17	3	1	11	14	19	21	15
18	4	2	12	8	20	15	16
19	5	3	13	9	21	16	17
20	6	4	14	10	15	17	18
21	7	5	8	11	16	18	19
22	1	2	4	8	9	11	21
23	2	3	5	9	10	12	15
24	3	4	6	10	11	13	16
25	4	5	7	11	12	14	17
26	5	6	1	12	13	8	18
27	6	7	2	13	14	9	19
28	7	1	3	14	8	10	20
29	1	2	3	4	5	6	7
30	8	9	10	11	12	13	14

In our questionnaire, we have *t* = 21 treatments (i.e., number of all SLO and dSLO statements), *k* = 7 statements per block (i.e., number of statements each participant gets to rank), *r* = 10 times each SLO statement appears across the block design, and *b*=30 blocks. Additionally,

$k = 3$, how many times an item pair goes together in the block. The block design is in Table 3 below. Each block represents a set of statements that participants in the block had to rate against each other, answering the question: "Please rank the following statements of student learning outcomes of a geometry course in order of preference, where 1 is the highest (top) and 7 is the lowest (bottom) by dragging and dropping them into place.” In the next section, we describe the findings from our study with 121 GeT instructors of the course nationwide, collected in 2022.

Table 4: Descriptive Statistics of the Findings

(d)SLO	Average	Rank	# ranked 1/7	# ranked 7/7	Std Dev.
1	2.1	1	19	0	1.29
2	2.5	3	21	3	1.93
3	3.1	6	7	1	1.66
4	2.4	2	17	1	1.69
5	2.7	4	10	0	1.40
6	3.5	9	5	2	1.67
7	2.8	5	12	1	1.72
8	4.3	13	1	2	1.64
9	3.5	8	4	1	1.52
10	3.6	10	6	1	1.69
11	4.6	15	0	7	1.65
12	4.3	14	1	3	1.63
13	3.3	7	7	6	2.03
14	5.0	17	1	5	1.41
15	3.7	11	3	2	1.80
16	6.3	21	0	23	1.05
17	5.3	18	1	12	1.58
18	6.2	20	0	22	0.92
19	4.2	12	3	2	1.58
20	6.1	19	0	18	1.07
21	4.7	16	3	9	1.95

Findings

Overall, the group seemed to find consensus with the SLO-WG when comparing the brief statements of their 10 SLOs with the statements of the comparable distractors provided. In Table 4, some descriptive statistics about the average score given to each SLO (where lower score means higher priority to be in a course), where the SLO ranks with respect to the others, how often the SLO was rated the most or least important of the 7 provided to the participant, as well as a measure of variance for the item to show how varied the instructors were with respect to how they rated that SLO in comparison to others and amongst themselves.

Of the ten SLOs proposed, the 121 participants who took the survey on average ranked nine of the ten amongst the highest ranked statements. Additionally, all the SLOs that were ranked the most important of the seven 10 or more times came from the original list, and all the SLOs that were ranked the least important of the seven 10 or more times came from the list of distractors. This helps demonstrate some key aspects of the construct and content validity of these SLOs.

Discussion and Limitations

In this report, we provided a methodological framework for studying student learning outcomes at scale created by online learning communities. In designs where information needs to be gleaned from large amounts of researchers and practitioners without a tremendous number of resources or time, questionnaires such as these can provide a lever to gain a pulse on the field, and send a signal that this work of developing student learning objectives is being worked on. We continue to bolster the tremendous efforts of the members of this OPLC over the past few years in developing a set of student learning outcomes that can impact the teaching and learning of high school geometry. We hope this can be one of several ways to triangulate on a set of established student learning outcomes for a geometry course for teachers.

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