GUIDED REINVENTION AS TEACHING PRACTICES THAT AIM TO PROMOTE STUDENT LEARNING TO DEFINE

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Our study explores teaching practices that aim to promote students' learning to define an analytical object. A Calculus II instructor conducted a teaching experiment (TE) in which 11 students reinvented a formal definition of a limit over five class periods with the instructor's guidance. During the TE, the instructor's teaching practices were based on what principles of definitions from the literature inform about defining discourse. Our analysis of the instructor's teaching practices and students' follow-up work revealed several teaching practices that seem to promote development of students' narratives towards a formal definition of an analytical object: Providing testing methods to check if students' narratives can be considered as a completion of the defining task, asking students to place components of their definitions on graphs with related quantities, and asking them to reflect their illustration of the definition in their written definition.

Keywords: Calculus, Classroom Discourse, Undergraduate Education

Recently, there has been growing interest in students' learning at times when the rules of mathematical discourse change and the teaching practices promoting such learning. The Commognition approach (Sfard, 2008) defines such learning as meta-level learning because it involves students learning about new rules of discourses as well as expanding and changing properties of mathematical objects that the students are already familiar with (Valenta & Enge, 2022). The instructor's role in meta-level learning is particularly important because those new discursive rules are historically established and thus difficult for learners to discover on their own or appreciate their value in contrast to the ones that are familiar to them (Nachlieli & Elbaum-Cohen, 2021; Valenta & Enge, 2022). There have been recent studies about the teaching practices that aim to promote meta-level learning and calls for more studies (Cooper & Lavie, 2021; Nachlieli & Elbaum-Cohen, 2021; Schüler-Meyer, 2020; Valenta & Enge, 2022).

In this study, we consider students' learning to define as meta-level learning because this learning involves students learning about the rules of defining discourse, which impacts both their defining procedure and the end product, i.e., their definitions as well as expanding their narratives about mathematical objects. Studies about students' engagement with definitions have examined their difficulties with formal definitions and how they come to agreement on how to use mathematical words according to their formal definitions (e.g., Tabach & Nachlieli, 2015) and how students develop their own definition based on iterative refinements of their intuitive thinking about a mathematical object (Oehrtman et al., 2014; Swinyard, 2011; Schuler-Meyer, 2018 & 2020; Swinyard & Larsen, 2012). This study aims to expand our understanding of students learning to define by examining teaching practices that aim to promote students' development of a definition of an analytic object in a guided reinvention setting with the research question:

What teaching practices can promote students' reinvention of a formal definition of an analytic object?

Theoretical Background

We adopted commognition as our theoretical framework by conceptualizing learning to define as meta-level learning and building on existing studies about teaching practices that promote meta-level learning. We adopted guided reinvention as an instructional approach that could promote meta-level learning. Moreover, we built on existing guided reinvention studies that reported how students developed their narratives about mathematical objects and expand those results by investigating the teaching practices that could promote such development.

Learning and Teaching in Commognition

Commognition is a theoretical framework that views mathematics as a type of discourse characterized by its distinctive use of words and visuals, narratives about mathematics objects, and routines that are task-procedure pairs. A task is a setting where a learner considers "herself bound to act," and a procedure is the prescription of the actions "that fits both the present performance and those on which it was modeled" from her past experience (Lavie et.al., 2019, pp. 160-161; Sfard, 2008). Commognition defines learning as change in one's discourse: at the object-level which involves new words, routines, narratives about familiar objects and at the meta-level which involves changes in meta-rules, which impact routines. The role of experts is important in meta-level learning because such learning is often not self-motivated by students due to the fact that those rules are historically developed and the value of adopting them is not clear to non-experts (Nachlieli & Tabach, 2012; Valenta & Enge, 2022). Commognition defines teaching as "communicational activity the motive of which is to bring the learners' discourse closer to a canonical discourse" (Tabach & Nachlieli, 2016, p. 299) and teaching practice also as task-procedure pairs, i.e., "the task as seen by the performing teacher together with the procedure she executed to perform that task" (Nachlieli & Elbaum-Cohen, 2021, p. 3).

Teaching practices that promote meta-level learning became the subject for a growing field of research (e.g., Cooper & Lavie, 2021; Nachlieli & Elbaum-Cohen, 2021; Schüler-Meyer, 2020; Valenta & Enge, 2022). These studies have revealed teaching practices that help students transition from their old discourse that is no longer aligned with meta-rules of the target discourse towards the new discourse by making "boundaries between previous and new discourses" (Nachlieli & Elbaum-Cohen, 2021, p. 11) clear, by providing students tasks with which they could use features of their old discourse (e.g., routines) in emerging ways that are "appropriate in the eyes of the teacher" (Cooper & Lavie, 2021, p. 3; Schluer-Meyer, 2020), and helping them interpret the task towards a more formal discourse (Valenta and Enge, 2022).

Defining as Meta-level Learning & Guided Reinvention as Teaching Practice Promoting It

The meta-discursive nature of defining and its importance as a mathematical practice have been established (Martín-Molina, 2018; Martín-Molina, 2020; Ouvirier-Buffet, 2011; Zandieh & Rasmussen, 2010). Studies have examined the characteristics of experts' or students' defining processes (Martín-Molina, 2018; Martín-Molina, 2020; Ouvirier-Buffet, 2011; Zandieh & Rasmussen, 2010). There have been recent studies considering defining as meta-level learning, which involves learning about rules about definitions and constructing a definition according to them (Martín-Molina, 2020; Schüler-Meyer, 2018). Those meta-rules are informed by properties of definitions such as that they should be "unambiguous and precise", "operable, so that it can be decided whether an object belongs to a category or not", "complete and sufficient" (Schüler-

Meyer, 2020, p. 238), and “invariant under a change of representation” (Zaslavsky & Shir, 2005, p. 320). Activities aiming to produce a definition that can be endorsed in formal mathematical discourse involves adoption of metarules that those properties inform (Schüler-Meyer, 2020).

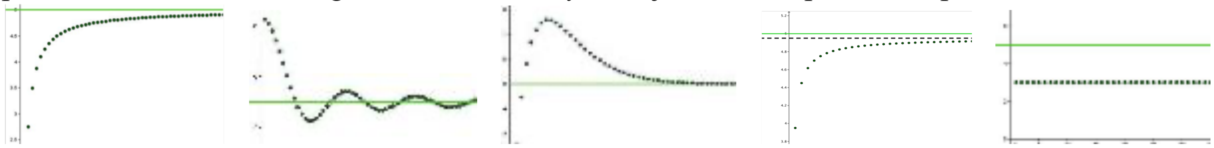
We adopted a guided reinvention instructional approach in which students reinvent a formal definition of an analytical object: the limit of a sequence. In this approach, students learn about mathematical objects through “a process by which students formalize their informal understandings and intuitions” through their own experiences and activities with carefully designed instructor’ guidance (Gravemeijer et al. 2000, p. 237). Guided reinvention studies have investigated the development of students’ narratives towards a formal definition from the constructivist view by analyzing the data with pairs of students (Oehrtman et al., 2014; Swinyard, 2011; Swinyard & Larsen, 2012). Recent guided reinvention studies addressed the meta-discursive nature of defining through commognition (Schuler-Meyer, 2018 & 2020). Schuler-Meyer (2020) guided secondary students towards formal mathematical discourse by using a task where students could use their familiar secondary mathematics routines of “categorizing, describing, symbolizing, and calculating” in a specific task situation (i.e., epsilon-strip activity) and attempt to transform them into routines of formally defining limits.

Our study also views learning to define as meta-level learning and aims at examining teaching practices that can promote it. Like Valenta and Enge (2022) who documented teaching practices aiming at helping students in a proving context transition from providing an empirical argument using examples to providing a deductive argument “to show the validity of the concluding narrative” (p. 5), we examined the teaching practices that promote students learning to define (a limit in our study) by transitioning from their intuitive and descriptive narrative toward a formal narrative about quantities and relations between quantities. We are particularly interested in how the metarules of defining, which are informed by the principles of definitions from the literature, could guide such teaching practices and their potential impact on students’ work. In comparison to Schuler-Meyer (2020) where students’ defining activities built on their completion of a specific task, which involves the components and deductive order of a formal definition of limit, we explored more general aspects of students’ defining process and the teaching practices that seemed to help students progress toward a formal definition. The results of this study will also contribute to the field by providing a mechanism behind students’ learning trajectory that was revealed in existing guided reinvention studies by providing teaching practices that might have helped such learning.

Research Design

We collected data in a Calculus II class at a public U.S. university where an instructor (the 2nd author, the TR- teacher researcher) conducted the guided reinvention TE in which 11 students generated definitions of limit for five class periods (50-75 minutes). Students confirmed that they had not seen a formal definition of limit before. In the beginning of the TE, students were asked to generate a broad range of example and non-example graphs of sequences converging to 5 (Figure 1) and discuss their convergent or non-convergent behaviors. Then, students worked in small groups (TE group – GTE, G1 and G2) to define a limit of a sequence. We videorecorded the activity of one group of 4 students (GTE with participants – P1, P2, P3, and P4) and whole class interactions with the TR, transcribed the recordings using Transana, and collected and digitized copies of students’ written work.

For analysis, we catalogued the GTE’s individual and group definitions, their interactions with the instructor and other groups, problems the GTE identified with their definitions, solutions that they suggested, and problematic issues with their definition (issues that experts may recognize but the GTE had not noticed). We analyzed teaching practices that aimed to promote students' learning to define an analytic object as task-procedure pairs. We first



Example A Example B Example G Non-example 5 Non-example 7

Figure 1. Selected example and non-example graphs used in TE (solid horizontal line: 5)

examined the sequences in data where the TR intervened in students' work by looking at the TR's actions, writing the general procedure based on the actions and trying to identify the task that the TR wanted to accomplish. We, then identified meta-level or object-level rules that seemed to inform each teaching practice. We also documented what seemed to be an impact of the teaching practice on students' subsequent work. It should be noted that in the beginning of TE, the TR laid the groundwork for students to adopt some of the metarules, such as that a definition should "include all examples and all nonexamples," "not be synonymous," and "be concise and precise" by discussing them with students. These can be considered as a teaching practice prompting students' learning to define. However, we decided to focus on the teaching practices in the results that are responsive to what students created as definition to document a potential impact on their work. During TE, the TR used the metarules in his teaching practice to help students shift their narratives about the limit towards a formal definition.

Results

This section will report the teaching practices that aim to promote students' reinvention of a formal definition. For each teaching practice, we present the status of students' work at the time it was observed, describe the teaching practice, and explain what seems to be its impact on students' work. Due to limited space, we only present teaching practices (TP) that seemed to lead to substantial changes in students' subsequent definitions.

TP1: Communicating to students that their definition should not exclude examples or include non-examples.

Consistent with the results of other studies, the GTE's beginning definitions were descriptive of salient examples including "terms …ultimately approach 5 as *n* increases," to which the TR asked if their definition captures an example. Table 1 summarizes this teaching practice.

> TR: I could see a student coming here who hadn't seen your definition before, and saying something to the extent of, well it appears to me that these, since you described it [Graph B] as terms go away from 5, and then they come back and then they go away and then they come back, uh that the terms in B, the values of them, the a sub n values... don't always approach 5. [6 second pause] (Day 1, 45 minutes)

Table 1: TP1: Communicating to students that definitions should not exclude any examples.

Task	Procedure	Empirical Observation
Leading students to realize that their definition does not include all	TR uses a hypothetical student outside the GTE, pointing out that their	"A student…something to the extent of … terms go away from 5, and then they come back…don't

examples, thus violating Metarule 1.	definition excludes an example.	always approach 5."

This teaching practice provided a method for students to check if their definition can be considered as a completion of the defining task in terms of meeting the metarule that a definition should include all examples and exclude all non-examples. This teaching practice seemed to have an impact on the GTE's work, as in follow-up conversations, they used this method to test their revised definition that included "the distance between the terms and 5 decreasing," ultimately rejecting it because, "That's not true either cause the distance is increasing at the last point" (P1).

TP 2: Communicating to students that words in definition should be quantified through placing them on graphs

In the GTE's initial definitions, they used "ultimately" with "terms…approach" and justified it with inclusion of graph G. The TR then asked what and where "ultimately" is on the graph:

TR: In G, what's this ultimately?
P2: Because it starts out not approaching 5, like you don't think it is.
TR: Yeah. Can you get specific, what count one, two, three, four, five, you know, where- what's this ultimately? In G, what's this ultimately?
P3: It approaches from a one and a two, but then from a three through four, five, six, it's going away from 5.
P1: It'd be six on.
TR: Six on. Is that what you think ultimately was getting at?...I'm looking for you kinda illustrating this definition [points to a GTE definition] on here [waves hand over graphs B, G & 5 on their board] (Day 2, 35 minutes).

In this excerpt, the TR explicitly chose one of the GTE student's responses to "where/what ultimately is" (P1) and asked them to illustrate the word on the graph. Table 2 summarizes this teaching practice.

Table 2: TP2: Communicating to students that words in definition should be quantified through placing them on graphs

Task	Procedure	Empirical Observation
Making student aware of quantities related to words that are used in their definition	The TR asked students to place the words on graphs with specific numbers that are associated with it.	"What's this ultimately?...Can you get specific, what count one, two, three, four, five, you know, where- what's this ultimately?" ([3-5 & 6])

In contrast to TP1 which provided students a method to test if they completed the task of defining, TP2 provides a procedure through which they can quantify not-yet quantified words in the definition, thus progress towards a formal definition. Note that TP2 is based on the following metarule of defining discourse: components in written definition should have places on graphs. This metarule is our operationalization of a principle that a definition should be "invariant under a change of representation" (Zaslavsky & Shir, 2005, p. 320). This invariance principle was applied when a form of students' narratives changed from written to graphical. In subsequent conversations, the GTE placed ultimately on graph G (Figure 2), and added “a specific value for *n*” in their next definition:

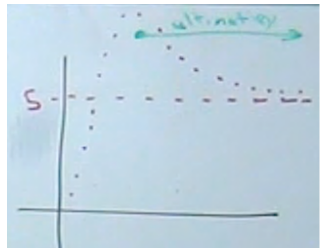


Figure 2. GTE’s illustration of “ultimately” (Day 2, 41 minutes)

> **GTE Definition 6: There is a specific value for n, such that a_n for all n's after that is so close to 5 that it basically equals 5. (like 0.999=1) (Day 2, 43 minutes).**

Although there is no direct evidence that the phrase “a specific value for n…for all n’s after that” is transformed from “ultimately”, the phrase seems closely related to them identifying “ultimately” with “six on” and drawing it as an arrow starting at a specific value of *n* and on, which seemed to be an impact of TP2.

TP 3: Asking students to illustrate not-yet-defined elements on graphs and then to capture the illustration to explain the elements

On Day 4 of the TE, the GTE was provided the opportunity to read other groups’ definitions, and decided to adopt some of their notations in their own definition:

> **GTE Definition 16: If $|a_n - 5| \leq \varepsilon$ for any value of n, $n_c < n$ with error→0, then a_n converges to 5 (n_c=some n) (Day 4, 8 minutes).**

In this definition, n_c and ε are not explained but the GTE did not seem to notice this problem even after illustrating the definition on graphs multiple times. To address this issue, the TR asked them to illustrate the definition on graphs explicitly including those components, then to check if their definition captured their illustrations, and then to revise it if not.

TR: So where exactly is n sub c? …

P1: I think it would be this one [points to the 14th point]. Well, that's the last *n* that all the n's after that [same point], the terms for these *n*'s would be within the error bound.

TR: So then how would you label n sub c?

P3: Well this would be 14 [labels n_c on the n axis] I think. That'd be the n sub c.

…

TR: Do you feel like your definition captures everything you’ve illustrated because you keep saying there is this n sub c [points to n_c in Graph B], and then you said all the terms [looking at P1]

…

P1: All the terms after it aren’t within-

TR: All the terms after it- so do you think this definition [waving hand over their definition] captures that idea that-

P1: We should probably add that though, all the terms after it?

In this excerpt, the TR asked the GTE to illustrate an undefined term and describe the illustration, and then ask them to reflect the descriptions in their definition, which Table 3 summarizes.

Table 3. TP3: Asking student to illustrate a not-yet-defined component on graphs and then asking them if their definition captures how they illustrated the component.

Task	Procedure	Empirical Observation
Leading students to realize an undefined component in their definition and have them define it.	Asking students to illustrate the definition including the not-yet-defined component on graphs and asking them if their definition captures how they illustrated the component.	"Where exactly is n sub c?" [After GTE answered] "Do you think this definition captures that idea?"

We note that TP3 is also based on a metarule of defining discourse: a written definition should capture how it is illustrated on example and non-example graphs. This metarule is also our operationalization of the principle that a definition should be "invariant under a change of representation" (Zaslavsky & Shir, 2005, p. 320), previously mentioned in TP2. This time, it was applied when the form of students' narratives changed from graphical to written. This teaching practice seemed to have an impact on the GTE's definition, as after this conversation, they revised their definition consistent with how they illustrated n_c:

GTE Definition 17: If $|a_n - 5| \leq \varepsilon$ for an n_c being an n that all terms after n_c are within the given error bound then $\lim_{n\to\infty} a_n = 5$ (Day 4, 29 minutes).

Using the same practice, by directing the GTE's attention to the component that they constantly placed first while illustrating their definition on graphs, which was "$\boldsymbol{\varepsilon}$"**,** the TR asked GTE to reflect the illustration of $\boldsymbol{\varepsilon}$ in their definition. This teaching practice also seemed to have an impact; students eventually revised their definition by starting with "within a given error bound".

Discussion and Conclusion

The goal of this study was to identify teaching practices that aim to promote students' learning of defining. We analyzed five days of a guided reinvention TE in which students reinvented a formal definition of limit, identified such teaching practices (TP), and here we reported on three of the practices that seemed to lead to substantial changes in students' subsequent definitions:

- TP1: Communicating to students that definitions should not exclude any examples.
- TP2: Communicating to students that words in definition should be quantified through placing them on graphs.
- TP 3: Asking students to illustrate not-yet-defined elements on graphs and then to capture the illustration to explain the elements.

These teaching practices show how metarules of defining could be used to help students transition from their informal descriptive narrative about a mathematical object towards formal ways of taking about it, i.e., by defining the object using quantities and their relationships. First, TP1, which provided students with a method to test if their definition could be considered as completion of the task of defining, is based on the metarule that a definition should include all examples and exclude all nonexamples. The instructor used TP1 to point out that students' initial definition violated this metarule, so it could not be considered as completion of the defining task. Students adopted this testing method to test their subsequent definitions, and rejected them when they violated this rule. In comparison, TP2 and TP3 provided students with guidance on how to proceed to create a definition that meets the metarule. TP2 and TP3 were based on our operationalization of another principle of definition, namely that a definition should be "invariant under a change of representation" (Zaslavsky & Shir, 2005, p. 320), which enables and reflects flexibly moving between a written definition of an analytic object (e.g., limits) and its graphical mediation on a plane. Specifically, TP2 seemed to impact students' subsequent work where they started to quantify not-yet quantified words in their definition, thus moving towards a formal definition of limit, which consists of quantities and their relations (e.g., Changes in the GTE's definitions: "ultimately"→"six on"→an arrow on the graph in Figure 1→"a specific value for n, such that a_n for all n's after that"→"n_c… all terms after n_c"). TP3 also seemed to impact their subsequent work, such as when they explained previously undefined terms (i.e., n_c and ε) in their definition by reflecting how they placed those terms on graphs while illustrating their written definition. We also view TP2 and TP3 as using components of students' existing discourse (not-yet quantified or explained terms in their current definition) according to routines governed by the metarules of the targeting discourse (i.e., formal discourse of defining), by having students treat those terms as if they were already part of a formal discourse (e.g., asking them to place not-yet quantified terms like "ultimately" on a graph with numbers and place not-yet explained terms "n_c and ε" based on how they think about them as if they were explained in the definition). Such uses of components of existing discourse in the context of emerging discourse "in a manner that is appropriate in the eyes of the teacher" (Cooper & Lavie, 2021, p. 3) could be considered another way of helping students transition to more formal discourse governed by new metarules. Activities that are interdiscursive in nature should be further investigated in terms of their role and impact on development of students' defining narratives in general and their definitions of the limit in particular. We suggested that those teaching practices explain a mechanism behind students' reinvention of a formal definition of limit. In particular, we suggested that changes of students' narratives about the limit seemed to be impacted by those practices based on metarules of defining.

Our study contributes to the ongoing discussion about teaching practices that promote students' metalevel learning (Martín-Molina, 2020 & Schüler-Meyer, 2020) by providing empirical evidence of such practices in the context of defining, which is evidence that existing studies have called for (Nachlieli and Elbaum-Cohen, 2021). It also contributes to the existing guided reinvention research, which has documented learning trajectories through which students develop a formal definition of limit, by providing a potential mechanism through documenting teaching practices that seem to promote such development. We also note that such practices, which are based on metarules of defining, provided an alternative way to use and communicate those rules with students instead of by explicitly telling students about the rules and asking them to adjust their definition accordingly, which Schüler-Meyer (2020) found “very demanding for the teacher and students” (p. 245).

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