FRAMING INSTRUCTIONAL TASKS FOR INTERACTION WITH CONTENT: INTRODUCING DERIVATIVES GRAPHICALLY WITH INQUIRY

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In this study, I present how eight U.S. college calculus instructors with different patterns of inquiry practices used instructional situations to frame instructional tasks for introducing derivatives graphically to students. During four interviews, the instructors proposed up to eight tasks for introducing derivatives physically, graphically, verbally, and symbolically (Zandieh, 2000). The findings focus on the two tasks proposed by each instructor that centered the graphical representation of the derivative: derivative at a point as the slope of a tangent line, and derivative as a function presented by its graph. While no two instructors proposed the same tasks with the same instructional situations for teaching these concepts, they relied on graphing, conjecturing, and calculating situations to frame their tasks.

Keywords: Undergraduate Education, Calculus, Instructional Activities and Practices, Mathematical Representations

This study contributes to our understanding of teaching of calculus and implementing inquiry in undergraduate mathematics classrooms by investigating the ways calculus instructors frame instructional tasks in relation to the content to teach a fundamental calculus concept— derivatives—with inquiry. There is still very little basic research on the instruction of calculus and even less on those that promote 'active learning' (Larsen et al., 2017). The existing research has mainly attended to the learning of calculus ideas and the content and cognitive orientation of non-instructional tasks (e.g., exams and textbooks; Tallman et al., 2016; White & Mesa, 2014).

This scarcity of knowledge about what happens in calculus classrooms is discouraging, given that the course has been notorious for being a gateway for many students to follow a STEM pipeline or graduate from college (Blair et al., 2018; Bressoud, 2012). While lecture still dominates calculus lessons (Laursen, 2019), implementing more inquiry has been promoted to make calculus more accessible. However, research on inquiry teaching in undergraduate mathematics, which has mainly focused on pedagogical variables rather than the content, does not always suggest positive student outcomes (e.g., Johnson et al., 2020). Alternatively, I attend to the content instructors and students interact with in these classrooms to operationalize inquiry teaching in relation to the content. Using inquiry as the context to study calculus tasks is valuable, as it allows uncovering a wide range of calculus-specific instructional tasks, given that inquiry in undergraduate mathematics education is often characterized by students' discovery of mathematical ideas through engagement with sequences of mathematical tasks (Laursen & Rasmussen, 2019). Here, I specifically focus on the tasks that instructors proposed for teaching the graphical representations of derivatives: slope of tangent line, as derivative at a point; and the derivative function, as a graph. More precisely, I address the following research question: How do college Calculus I instructors, who teach with various inquiry approaches, frame instructional tasks to introduce the graphical representations of the derivative?

Literature Review

The ample research on the learning of derivatives graphically suggests its importance for students' understanding of derivatives (e.g., García-García & Dolores-Flores, 2021; Ubuz, 2007).

Borji et al. (2018) and Hähkiöniemi (2006) confirmed the importance of the graphical representations of derivatives by carrying out teaching interventions designed with the APOS (Action, Process, Object, and Schema) theory that emphasized visual representations. In Borji et al.'s study, students who used graphing software performed better on derivatives tests later. Hähkiöniemi found that students could recognize derivatives visually early on, concluding that visual representations are a good fit for introducing and developing understanding of derivatives. However, the scarce research on teachers' actual teaching of derivatives graphically shows that teachers do not approach them similarly. Kendal and Stacy (2001) compared two high school calculus teachers' use of graphical, symbolic, and numerical representations with a computing and graphing tool. The teachers used the graphing aspect of the tool differently based on their beliefs about the importance of various representations and whether their students were capable of learning different representations simultaneously. Park (2015) studied how three college calculus instructors defined derivative at a point and as a function, showing that: 1) although they used secant and tangent lines to graphically illustrate derivative at point, they did not make the connections to symbolic representations explicit; and 2) they often showed derivative as a function with zero, positive, or negative values of the derivative "rather than numerical values that change over an interval" (p. 248). Delos and Thomas (2003) studied four teachers' teaching of derivatives, finding that while there was not a common way of teaching derivatives among the teachers, all four used the approach of moving secants toward a tangent. The study presented in this paper dives deeper into instructors' use of graphical representations for teaching derivatives by revealing the different ways they frame their instructional tasks to teach slope of tangent line as derivative at a point and the graph of derivative function.

Theoretical Framework

Goffman defines framing as the explicit or implicit answer to the question "What is it that's going on here?" that individuals must answer in a social situation to proceed "with the affairs at hand" (1974/1986, p. 8). In education research and the learning sciences, framing has been used in various ways to: theorize students' understanding of themselves and the intellectual activities they engage with in the classroom (Greeno, 2009), investigating opportunities to learn within learning environments (Hand et al., 2012), characterizing teachers' shaping of mathematics and science lessons and activities (Engle, 2006; Herbst et al., 2020), and bridging learning in mathematics and sciences between contexts (Chapman, 2022; Hammer et al., 2005). For the purposes of this study, I use Herbst and colleagues' (2020) notion of framing a mathematical problem, originally introduced by Bateson (1954/2003) and later developed by Goffman, to identify how interactions within a specific moment in time during calculus instruction are interpreted and governed. To unpack college calculus instructors' framing of instructional tasks for teaching derivatives with inquiry, I answer Goffman's question of 'what is going on here' with two types of framing: *Framing for Interaction with Content* and *Framing for Social Interaction* (also known as activity structure; Doyle, 1984). Due to space, I only focus on the former here. To frame a task for interaction

with the content, I use instructional situations (or situations as I use them interchangeably) within the notion of didactical contract, a contract with an often-implicit set of expectations that regulates the relationship between the teacher and students regarding the work of teaching and learning mathematics (Brousseau, 1984).
Instructional situations as "*distinct types of problems* [emphasis added] used in a course of studies" are the customary ways by which the teacher's and students' actions and interactions are *framed* into appropriate units of work regarding the knowledge at stake (Herbst et al., 2020, p.
5); given an instructional situation, students know what kind of problem they are presented with and what kind of mathematical work and interactions they should prototype (Herbst & Chazan, 2012). By focusing on these subject-specific framings in tasks, we can see what learning opportunities are offered to students during instruction (Herbst et al., 2018).

Methods

The data for this study comes from a larger project that seeks to understand how eight U.S. calculus instructors frame their instructional tasks for teaching derivatives with inquiry. The eight instructors were purposefully selected using cluster analysis from a pool of 48 calculus instructors to represent different patterns of inquiry-oriented practices (Shultz, 2020). The participants were tenured and had more than four semesters experience teaching Calculus I with inquiry. Five participants used he/him pronouns (Justin, Adrian, Barry, Matthew, Gopher; pseudonyms); one used she/her (Monica); one used they/them (Max); and one used all pronouns (Alex). I conducted four semi-structured 1–2-hour long interviews with each instructor, as they proposed up to eight tasks, organized by Zandieh's (2000) framework (each interview was dedicated to one representation of the derivative; Gerami, 2023). Zandieh's framework organizes students' conceptions of derivatives by representation (graphical, verbal, physical, symbolic) and process-object layers (ratio, limit, function). The process-object layers are hierarchical, as each layer is found by taking the process of that layer over the previous layer as an object. For example, the limit layer is found by the *process of* finding the limit of the ratio as an *object*. The limit layer corresponds to when the denominator approaches zero; the function layer is presented as an array of numbers or set of ordered pairs of differences. The instructors were asked to propose a task (from their teaching materials or to create new ones they could use in the future) that would help transition students' conceptions from one layer to the next (the target layer) within a representation. Although not prompted, they could also reach out to other representations as needed. Here, I focus on the two tasks that the instructors proposed during Interview 2 for the graphical representation: from the ratio to the limit layer via Prompt 3 ("Assume that you have already taught about slope of secants and want students to learn about slope of tangents. Propose a task involving the graph of a function, where students have to figure out the slopes of some tangent lines"), and from the limit to the function layer via Prompt 4 ("Assume that you have already taught about slope of tangents and want students to learn about the graph of the derivative function. Propose a task involving the graph of a function, where students have to visualize the graph of the derivative function.").

To analyze framing for interaction with content, I used inductive/deductive hybrid thematic analysis, which entails using pre-ordinate themes "through the application of an explicit theoretical framework developed through engagement with the literature" (the deductive element) to generate themes from the data (the inductive element; Proudfoot, 2022).

Because instructional situations have not yet been identified in the context of calculus instruction, I used instructional situations identified in other content areas (mainly geometry and algebra) to find eight generic types of problems that students could do in any mathematics classroom: *graphing*; *calculation*; *exploration & conjecturing*; *doing proofs*; *generating a new definition or installing a new concept*; *installing a new theorem, property, or formula*; *solving equations with known methods*; and *solving word problems*. I used these to identify the emerging instructional situations (i.e., the mathematical work that students are expected to do) in each task using the written descriptions of the tasks and triangulated the findings by listening to the interviews and reading the transcripts to find information about the situations that instructors mentioned but did not include in the task description.

Findings

The eight instructors proposed a variety of tasks for the two prompts (Prompt 3: ratio → limit and Prompt 4: limit → function), all of which consisting of three generic types of problems— graphing, calculating, and conjecturing. Although some instructors used similar calculus-specific instructional situations within the three generic problem types, nearly no two instructors proposed the same tasks consisting of all the same instructional situations. Although I cannot show all 16 tasks due to space, I overview the similarities and differences among the instructional situations found within each. I also differentiate between explicit and embedded situations: a situation is explicit if students know what type of problem they are working on after reading a task, whereas a situation is embedded within an explicit (larger) situation if while working on the explicit problem, students find out that they must solve another smaller problem in order to solve the explicit situation.

Tasks Proposed for Prompt 3: Ratio → Limit

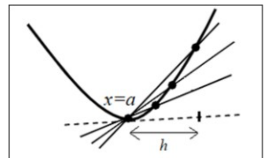
To frame the tasks proposed for prompt 3, the instructors relied on a variety of graphing, calculating, and conjecturing situations involving the limit layer (the target layer), with all but two instructors (Alex and Justin) also explicitly using graphing and calculating situations at the ratio layer as precursors to the situations with the target layer (Figure 1). The tasks proposed consisted of 2-4 explicit situations, with only Justin using two to frame his task.

Situations involving the target layer (limit)

❖ **Graphing**
- Graphing tangent line at a point-*Alex, Barry, Matthew*

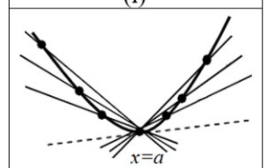
❖ **Conjecturing**
- Conjecturing how a smooth function's graph looks like if we zoom in enough (iii)-*Gopher*
- Conjecturing slope of a tangent line at a point from a pattern of slopes of secants
  - o *h*-units distanced to the right, with *h* decreasing-*Adrian*, *Justin*
  - o to the left and right side of a fixed point (ii)-*Monica, Max*

❖ **Calculating:**
- Calculating slope of tangent line at a point
  - o by calculating the limit of slope of secants *h*-units distanced to the right side of a fixed point with *h* approaching zero (i)-*Gopher, Justin*



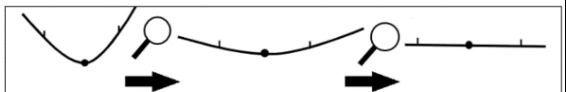
(i)

- Estimating slope of tangent line at a point
  - o by estimating slope of the line on plain background-*Alex*
  - o {by estimating slope of the line on plain background} (finding equation of a line)-*Alex*
  - o by calculating slope of secants *h*-units distanced to the right side of a fixed point for very small *h*-*Alex*



(ii)

  - o from pattern of slope of secant lines to the left and right side of a fixed point-*Matthew*
  - o by zooming in (iii)-*Barry*


(iii)

Situations not involving the target layer (ratio)

❖ **Graphing**
- Graphing secant lines
  - o to the left and right side of a fixed point (ii)-*Matthew, Monica, Max*
  - o *h*-units distanced to the right side of a fixed point with *h* decreasing (i)-*Barry, Adrian*

❖ **Calculating:**
- Calculating slope of secant lines between two points-*Adrian, Max*
  - o to the left and right side of a fixed point (ii)-*Monica, Matthew, Gopher*
  - o *h*-units distanced to the right, with *h* decreasing- *Gopher*
  - o {to the left and right side of a fixed point (ii)} (within finding equation of a line)-*Monica*

Note$_1$. Brackets, [xxx], denote embedded calculus-specific instructional situations. Braces, {xxx}, denote an embedded calculus-specific instructional situations within non-calculus-specific situations, which is noted in parenthesis outside the braces.

Note$_2$. The images are adapted from Haghjoo et al. (2023).

Figure 1. Calculus-Specific Instructional Situations within Tasks Proposed for Prompt 3

Matthew, Monica, Max, Barry and Adrian started their tasks at the ratio layer (not the target layer) by asking students to graph secant lines given two points, one of which being the point they wanted students to eventually find the slope of the tangent line at (*Graphing secant lines or segments*). Matthew, Monica, and Max asked students to use points on the function on both sides of the fixed point, while Barry and Adrian defined the second points h-horizontal-units to the right, with *h* decreasing (image (ii) and (i) in Figure 1 respectively). Monica, Max, Matthew, and Adrian also asked their students to find the slopes of the secant lines either explicitly (*Calculating slope of secant lines between two points*) or as an embedded situation (finding equation of a secant line). From these instructors, Barry and Matthew went on to ask students to draw a tangent line at the point (*Graphing tangent line at a point*), thus involving the target layer (limit) in a graphing situation. Alex asked students to draw tangent lines to the graph of a continuous function, but they did so without mentioning secants first. Given that tangent lines as derivative at a point are the target layer of the task, it is interesting that only Alex, Barry, and Matthew explicitly asked students to draw tangent lines. Although Gopher and Justin used graphs as a representation, they did not frame the tasks with any graphing situations, meaning that they did not expect students to do the work of graphing to complete their tasks.

Although only three instructors used a graphing situation at the limit layer (target), all the instructors relied on conjecturing and/or calculating situations to engage students with slope of tangent. A conjecturing situation involving the limit layer, *how a smooth function's graph looks like if we zoom in enough*, was used by Gopher at the beginning of his ask after providing students with a quadratic graph in a graphing application: "What do you observe about how the graph appears as you view it more and more closely [at $xx = 0.8$]?". However, Gopher's situations later in the task did not require students to use their conjectures to find the slope of tangent at a point, as he asked them to *calculate slope of secant lines.* The other conjecturing situation—*conjecturing slope of a tangent line at a point from a pattern of slopes of secants*— was used by Adrian, Monica, Justin, and Max. While Adrian and Monica had their students graph and calculate slopes of secant lines before this situation, Justin and Max had their students use technology to collect slopes of secants (after finding the slope of one secant line, Max's students would use Excel to collect the slope of the remaining secants; Justin's students would use an interactive graph with one fixed point [Figure 3a] to collect the slopes of secant lines by moving the second point closer to the first). Adrian, Monica, and Max ended their tasks with this situation, while Justin and Gopher continued their tasks with one more situation: *Calculating slope of tangent line at a point by calculating the limit of slope of secants h-units distanced to the right side of the point with h approaching zero*. Given that Justin and Gopher expect students to find an exact value using limits, I named this situation calculating; I use "estimating" to define the other calculating situations in which instructors did not expect exact values (estimating is still under the umbrella of calculating because students use known formulas to estimate).

Alex, Matthew, and Barry used the *estimating slope of tangent line at a point* situation to have their students find slope of tangents. After having students draw tangent lines at various points to a continuous graph on plain background, Alex asked them to find the slope of the tangent lines. Given that the graph is not on grid paper with labeled axes, students must estimate the slopes. Alex finished the task by asking students: "If the graph of a function weren't readily available, how would we go about finding the slope of a line tangent to its graph?". Because Alex expected students to write the equation of average rate of change and come to the idea that

Δxx or *h* must be "very very small," I captured this situation as *estimating slope of secants h-units distanced to the right side of a fixed point for very small h*. Matthew had a very similar approach as those who used *conjecturing slope of a tangent line at a point from a pattern of* *slopes of secants*, because students started the task by drawing secants and finding their slopes. However, the task directly told students that they are approximating slope of a tangent line and that they should find an overestimate and an underestimate; thus, given the wording of the task, students do not need to make a conjecture about the slope of the tangent line (e.g., whether it exists, whether it is smaller/larger than slope of nearby secants). Lastly, Barry also asked his students to *estimate the slope of tangent line at a point by zooming in* the graph and calculating the slope once the function looks linear.

Tasks Proposed for Prompt 4: Limit → Function

To frame the tasks proposed for prompt 4, all instructors except Justin, used a variety of graphing situations involving the function layer (the target layer), with Adrian, Gopher and Max also explicitly using calculating situations at the limit layer as precursors to the graphing situations (Figure 2). Justin was the only instructor who used conjecturing, and only conjecturing, to frame his task. The tasks consisted of 1-2 explicit situations, with only Adrian using three to frame his task.

Within the graphing situations, the most common situation was *Graphing ff' given the graph of ff*. Although the tasks looked similar on the surface, there is more nuance to this instructional situation when looking at the work that the instructors expect of their students. Adrian, Barry, Max, and Monica provided graphs of f that were either on a grid or plain background, meaning that students should either *estimate* or *calculate* slopes of tangent lines at various points before plotting the slopes as points on the graph of ff' and drawing a curve that fits the points. This means that these situations include embedded calculating situations (*Estimating slopes of tangents* and *calculating exact slopes of tangents*) within them because the instructors did not explicitly allude to these situations in the tasks' descriptions. On the other hand, Gopher used a similar situation (*Graphing ff' by plotting slope of tangents at various points and drawing a curve that fits best through the points*) but pre-framed the situation differently. He first explicitly asked students to calculate slope of tangent lines at multiple given points using the limit definition of the derivative (*Calculating the slope of tangent at an arbitrary point a using the limit definition*) as a precursor to the graphing situation without involving the target layer.

Gopher then instructed students how to draw ff' given their findings from the previous step: "Plot the values of the slope at each value of tt. Find an equation that fits your data."

Situations involving the target layer (function)

- **Graphing**
  - Graphing ff' given the graph of ff
    - [by estimating slopes of tangents] and plotting slope of tangents at various points and drawing a curve that fits best through the points-*Barry, Max & Monica*
    - [by calculating exact slopes of tangents] and plotting slope of tangents at various points and drawing a curve that fits best through the points-*Adrian & Monica*
  - Graphing ff' by plotting slope of tangents at various points and drawing a curve that fits best through the points-*Gopher*
  - Graphing ff given the graph of ff' [by estimating yy-values of ff' as slopes of tangents of ff] and drawing a curve with estimated slopes for their tangents-*Barry*
  - Graphing ff given information about the function's critical points and plotting the points and drawing a curve that fits best through the points -*Alex*
  - Graphing ff given the formula of ff' by [finding the function's critical points] and plotting the points and drawing a curve that fits best through the points -*Alex*
- **Conjecturing**
  - Conjecturing the relationship between two graphs (of ff and ff')-*Justin*

Situations not involving the target layer (limit)

- **Calculating:**
  - Calculating slope of tangent line at a point given graph of ff-*Adrian*
  - Calculating $ff'(aa +/-)$ for given number aa using the limit definition-*Adrian*
  - Calculating the slope of tangent at an arbitrary point using the limit definition-*Gopher*

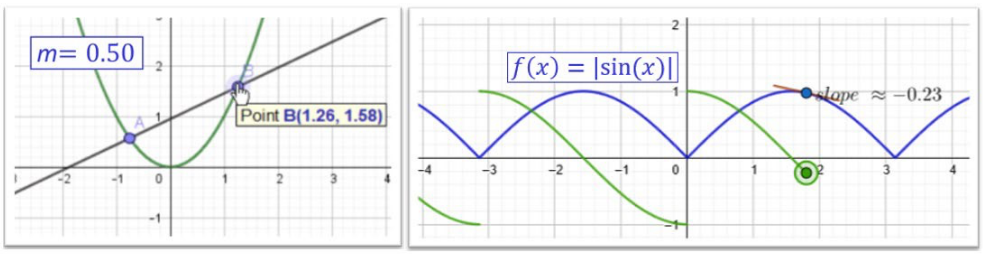
Note. Brackets, [xxx], denote embedded instructional situations.

Figure 2. Calculus-Specific Instructional Situations within Tasks Proposed for Prompt 4

Adrian, like Gopher, also used calculating situations at the limit layer as precursors to the graphing situation at the function (target) layer. He started the task with the formula and graph of $ff(xx) = |xx - 2|$, followed by two calculating situations: *Calculating slope of tangent line at a point given graph of* ff (not 2) and *Calculating* $ff'(2+)$ *and* $ff'(2-)$ *for given number* aa *using the limit definition*. He finished the task by asking students to define ff' symbolically via graphing it. After realizing that the derivative does not exist at $xx = 2$ (because $ff'(2+) \neq ff'(2-)$ by the definition of differentiability at a point), students would use the previous calculations to draw ff' before and after $xx = 2$. Therefore, it seems that given his choice of function (absolute value with two straight lines meeting at a non-differentiable corner), he used the opportunity of graphing ff' to have students practice using the limit definitions of derivative and encounter a situation where ff is not a smooth curve. Monica was the only other instructor that used a non-smooth function in his task of graphing ff' given ff, but did not provide any scaffolds like Adrian.

After asking students to graph ff' given the graph of ff, Barry asked his students to do the graphing in the reverse direction in the second part of his task (*Graphing* ff *given the graph of* ff'*)* using a similar approach of estimating slopes in his first part of the task. Alex, however, started his task by asking students to draw the graph of ff' given information about the original function's critical points, without first asking them to draw ff' given ff's graph. They then continued the task by drawing ff using the formula of ff', wanting students to find the critical points themselves. When asked about why they have students go from ff' to ff, rather than ff to ff', they said: "I don't think if you know the graph of a function, graphing the derivative [is] as a meaningful, like, I don't see that as a question that many students are asking … we know something about the derivative and you don't know something about the function, that's a meaningful epistemological position that they're often in." Alex then explained that they would only do an example later from ff to ff' if a student asked about it.

Justin used a conjecturing situation in his task by utilizing a graphing application (Figure 3b). Students were told to open the application, type in various functions of their choice (the blue box in Figure 3b) and play with the application to find out what the application does. As students moved the blue point on the blue graph of $ff(xx)$ from left to right, another function without any label appeared in green (ff'). Moreover, a small red line segment tangential to the function's graph moved with the blue point with its approximated slope next to it. Although the task clearly involves graphical representations of ff and ff', I captured the situation as conjecturing, instead of graphing, because the work students do in this task is not graphing, but observing the two graphs and making a statement about how, they think, the graphs are related.



(a) (b)

Figure 3. The graphing applications designed by Justin for Prompt 3 (a) and 4 (b)

Discussion and Conclusion

In this paper, I describe the ways eight college Calculus I instructors across the U.S., who teach with inquiry, frame their instructional tasks to introduce the graphical representations of the derivative at a point (Prompt 3) and as a function (Prompt 4). The study goes beyond the prior research on identifying the representations of derivatives that students engage with and investigates the *types* of calculus-specific mathematical problems (i.e., instructional situations) they work on. The instructors mainly relied on three generic types of problems: graphing, calculating, and conjecturing. While the instructors used a variety of calculus-specific situations in their tasks for both prompts, they used more unique situations to introduce the slope of tangent line as derivative at a point, than derivative as a function. The findings add to those of Delos and Thomas's (2003); even within the graphical representation, teachers have other ways of introducing derivative in addition to secants approaching a tangent. Although the tasks were designed for teaching with inquiry, most situations asked students to apply what they know (calculating, graphing). No instructor used *installing* situations (defining a new idea); they instead seemed to rely on conjecturing in the tasks and installed the new ideas via lecture themselves. Moreover, while all tasks were composed of multiple situations, the teachers made most situations explicit. Thus, inquiry can be operationalized as the number of situations instructors summon in their tasks and how explicit they make them; a task would be more inquiry-oriented if it has more situations and/or those situations are more implicit than explicit.

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