ALGEBRA INTERVENTIONS AT THE ELEMENTARY AND SECONDARY LEVELS: SEARCH FOR A DEFINITION

Sheree T Sharpe
University of New Hampshire
Sheree.sharpe@unh.edu

Matthew Mauntel
University of New Hampshire
Matthew.mauntel@unh.edu

This paper is a part of a larger research study that we are conducting to develop a framework consisting of the data-driven best practices around teaching and learning algebra in K-12 classrooms. The purpose of this paper is to develop a usable definition for algebra interventions for the second stage of screening within the larger study, which will be done by reviewing seminal or systematic reviews in the algebra field across grades K-12. Combining the algebra definitions with the type of interventions produces three questions to ask during the second stage of screening to decide if a study is an algebra intervention.

Keywords: Algebra, Algebraic Thinking, Types of Interventions

It is well known that algebra is essential and a gatekeeper to higher level mathematics and to the pursuit of a career in the Science, Technology, Engineering, and Mathematics (STEM) fields (Hughes et al., 2014; National Mathematics Advisory Panel, 2008). Despite this, many middle and high school students struggle with understanding and achieving in school algebra (Bednarz, 2001; Kieran, 2007). Sharpe (2019) found that difficulties occur when students transition from arithmetic to school algebra. Hence, researchers have argued that students need to gain algebraic thinking in the elementary grades (Carraher, Schliemann, & Schwartz, 2008). This paper is a part of a larger research study that we are conducting to develop a framework consisting of the data-driven best practices around teaching and learning algebra in the K-12 classrooms. The purpose of this paper is to describe what we mean by algebra interventions in grades K-12 classrooms to aid in identifying relevant research.

Perspective

Meta-research is the quantitative (meta-analysis) and the qualitative (meta-synthesis) analysis of studies (both published and unpublished). The advantage of this type of research is the cumulative power of analyzing many samples on a specific topic, the result of which has much greater explanatory power than one study in isolation (Cooper, 2017). In the overarching study, the authors are in the process of conducting a meta-research study. To conduct this larger research the authors will need to search databases and gray literature, screen studies, extract data from the studies, and analyze the data via meta-analysis and qualitative meta-synthesis. The screening stage consists of three levels of screening: (1) Title/Abstract screening, (2) Full-text screening, and (3) Methods screening. During the title and abstract screening stage of over 11,000 studies, conflicts between raters arose regarding whether or not the studies included an intervention involving algebraic thinking, especially at the elementary level. A screening protocol was developed with seven questions, where the screener needs to answer *yes*, *no*, or *more info is needed*. If all questions received a yes or more info is needed, then the abstract moved from level 1 to level 2 of screening, however, if any of the questions received a no, then the abstract moved to the rejected folder. The two screening questions that the screeners had conflicts with while trying to answer "yes/no/more info" are:

- Does the study involve algebra, an algebra-related topic (equation, function, variable), or algebraic reasoning (patterns, relations, quantitative relationships, change in various contexts)?
- Does the study involve at least one of the following teaching approaches:
    - A strategy or intervention for learning about algebra, functions, or equations?
    - A program or instructional technique for learning about algebra, functions, or equations?
    - A game, technological tool, or curricular materials for learning about algebra, functions, or equations?

Hence, the aim of this paper is to develop a usable definition for algebra interventions for the second stage of screening, which will be done by reviewing seminal or systematic reviews in the algebra field across grades K-12.

Modes of Inquiry

Our process for developing screening questions began by first identifying key studies about early algebra and algebraic thinking. A search of ERIC was conducted using the term "algebraic thinking." The top search results were inspected, snowballed, and reversed snowballed (Sayers, 2007) to identify key articles that were referenced in the field. Five papers were included from this search based upon their being referenced by multiple publications in algebraic thinking and presenting a definition of algebraic thinking (Carraher and Schliemann, 2007; Kaput, 2008; Blanton et al., 2018; Chimoni et al., 2018; and Kieran, 2021). Kieran (2022) was not cited extensively but was included because several of her algebra papers were referenced frequently and this work was the latest iteration. Also included were the NCTM standards (2000) for algebra since they are the basis for most US mathematics teaching standards.

Next, we investigated systematic reviews in algebra to develop a list of potential interventions. A search of ERIC on EBSCOhost, APA PsycInfo, and Education Source was conducted using the terms: "algebra interventions AND systematic reviews." This initial search produced three reviews all related to learning disabilities and algebra instructions (Bone et al., 2021; Hughes et al., 2014; Watt et al., 2016). A search of Hughes et al., (2014) produces an additional two reviews (Haas, 2005; Rakes et al., 2010) on algebra instructions. And a search of Bone et al., (2021) produces one additional review (Hwang et al., 2019). This resulted in a total of six reviews to investigate the type of algebra interventions.

Results

Algebra Definition

The goal of this section is to operationalize a definition of algebra to be utilized in the meta-research. This means producing a set of yes/no questions that screeners can utilize to determine if a study pertains to early algebra. This was accomplished by looking at surveys of literature, systematic reviews, and organizational reports (such as NCTM) that attempt to define algebra and early algebra. The goal of this section is to synthesize the working definitions together to produce a list of characteristics of an algebra study across grades K to 12. The six publications that we review in this section include: the NCTM standards for algebra (2000), Carraher and Schliemann (2007), Kaput (2008), Blanton et al. (2018), Chimoni et al. (2018), and Kieran (2021). After reviewing these works we looked across the definitions to formulate a set of questions that could be used on a screening protocol to detect if a study involves algebraic thinking.

In 2000, NCTM issued standards relating to mathematics across the K-12 curriculum including algebra standards for grades K-2 and 3-5. These standards while enacted differently across the two groups highlighted the importance of four key standards (p.296): “Understand patterns, relations, and functions; Represent and analyze mathematical situations and structures using algebraic symbols; Use mathematical models to represent and understand quantitative relationships; and Analyze change in various contexts.” Examples of the standards include students using a variety of notations and objects including pictures, lists, and equations; students engaging in learning about the properties of numbers and the rules governing them such as the commutative property; and students modeling a variety of real-life scenarios such as growth of plants. As students enter grades 6-8, the same standards remain, but their subjects differ as more emphasis is placed on functions, graphs, equations, tables, algebraic expressions, and linear equations. Finally, as students transition to grades 9-12 greater emphasis is placed on a wide variety of functions including exponential, logarithmic, and period functions and more complex work with functions including composing, combining, and inverting. Students are recommended to study global behavior of graphs including finding the intercepts, asymptotes, and zeros of a graph. Students are encouraged to symbolize the relationships between different expressions.

Carraher and Schliemann (2007) build on the idea of early algebra by posing five questions that will help define early algebra. The core themes behind these questions are the relationship between arithmetic and algebra, the role of procedural mathematics in algebra, the role of modeling in algebra, when formal mathematics notation should be introduced, and the role of non-conventional notations such as student illustrations and informal student language. They highlight the idea that “algebra is latent in the existing early mathematics curriculum,” (p.25) through introducing ideas that could relate to functions, properties of number systems, properties of the number line, and algebraic equations. Carraher and Schliemann make it clear that there are opportunities for learning algebra throughout elementary school and there are activities such as modeling, representing, and generalizing that extract the deeper algebraic structure from lessons.

Kaput (2008) defines algebra as consisting of two core aspects that are embodied by three main strands which view algebra “as a cultural artifact expressed mainly as conventional symbol systems and as a certain kind of human activity” (p.10). The first core aspect involves generalizing and symbolizing patterns or regularities that are noticed. The second core aspect takes these symbolized generalizations and reasons with them to elaborate on structure and this is where the traditional rules of algebra are established. The first core aspect embodies much of early algebra and the second core aspect often applies later in one’s algebraic career, but this is not guaranteed. The three strands are much like topics that are studied in algebra. The first strand concerns computations, arithmetic, and quantitative reasoning. The second strand encompasses relations, joint variations, and functions. The final strand concerns modeling situations both inside and outside of mathematics.

Blanton et al. (2018) builds on the framework of Kaput et al. (2008) to define four key aspects/activities involved in early algebraic thinking. These activities include condensing information through generalizing, representing with pictures and symbols, justifying by developing arguments to explain their reasoning, and expanding their built knowledge to other situations by reasoning with generalizations. In their proposed curriculum they discuss some of the contexts in which students will be using these activities. These include the relational understanding of the equal sign, looking at the fundamental properties of arithmetic, and modeling with linear equations. In Blanton's work we continue to see the importance of generalizing as well as several other activities including representing, justifying, and relating. The topics include looking at equations, equality, arithmetic, and relations.

Chimoni et al. (2018) looked at algebra for grades 4-7 and developed four different dimensions of algebraic thinking based partially on the work of Kaput. The first dimension was the strands from Kaput (2008). The second dimension involves key concepts/topics which are essential to algebra, and they identify as "equal signs, equality, equations, properties of numbers, properties of operations, variables, unknown quantities, symbols, co-variation, and correspondence" (p. 60). The third dimension consists of processes which expand upon activities from earlier works and include "noticing, conjecturing, representing, generalizing, justifying, and validating" (p. 60). The final dimension involves types of reasoning and includes both inductive and deductive reasoning. Through Chimoni et al.'s (2018) we see common topics that arise and the importance of certain forms of activity such as generalization.

Finally, Kieran (2022) conducted a historical analysis of the development of early algebra and identified three dimensions: analytical, structural, and functional. The analytical dimension looked at how students came to understand unknowns and issues relating to equations such as equivalence. The structural dimension referred to "expressing structure and properties within numbers, operations, and expressions" (p.1144). This involved comparing and relating numbers while representing and symbolizing them with expressions. The functional-thinking dimension involved generalizing especially when it was extracted from covariational and relational reasoning. While generalizing was only referenced by name in the last dimension, Kieran admits that generalizing is "the scarlet thread that runs through all three dimensions" (p.1134).

Looking across these different definitions of algebra there are several things that we noticed. First there is an importance given to certain human activities or processes. These are most often represented by -ing words. The most important example which is prominent across almost every definition is the act of generalizing. Other activities include modeling, representing, and symbolizing, justifying, relating, validating, expressing, solving, and graphing. These human activities extract the structure from observations such as generalizing or relating and then apply that structure to other problems through activities such as solving, justifying, and validating. In this way, we feel the activities communicate the essence of Kaput's (2008) core aspects, Chimoni et al's processes, and the thread that runs through Kieran's (2022) three dimensions.

While human activities are important, it is also important to collect the strands that they explore while engaging in algebraic thinking. These are the topics of their activity and include quantities, generalized arithmetic, co-variation, equations, equality, properties of numbers systems, operations, variables, change, patterns, graphs, models, expressions, or functions. Some of these topics include reifications (Sfard, 1991) of the mentioned activities. The topics allow us to distinguish between other mathematical subjects such as geometry where generalization, validating, and justifying are also important activities. The activities allow us to distinguish between early algebra and middle or high school algebra by looking at the topics of the activity. If we look at the NCTM standards (2000) for example, the standards are the same for grades K-12, but the topics are different. As students progress through early algebra, they begin by looking at patterns and formulating generalizations of these patterns by extending them and drawing early graphs and understanding equivalence. In middle school they are expected to model linear equations and solve equations. Finally, in high school they solve multiple different types of equations, graph inequalities and model a variety of situations using algebraic concepts they have developed along the way.

The goal of this section was to develop a set of questions to distinguish algebra from other topics of mathematics to distinguish early algebra from topics like arithmetic. First, we want to ensure that if a study claims to represent algebra, pre-algebra, algebraic thinking, or early algebra then it is included. If the study does not describe itself as algebraic then we will include it if it includes both an algebraic activity and topic or rather if both of the following questions are yes:

- Does the study involve generalizing, modeling, representing and symbolizing, justifying, relating, validating, noticing, conjecturing, reasoning, expressing, solving, or graphing?
- Does the study involve any of the following topics: quantities, generalized arithmetic, co-variation, equations, equality, properties of numbers systems, operations, variables, change, patterns, graphs, models, expressions, or functions?

Types of Interventions

The goal of an algebra intervention is to intervene in the regular practices of the classroom as a way to improve the teaching or learning of algebra. The goal of this section is to explore the different types of algebra interventions used in a classroom for inclusion in our meta-research. This was accomplished by synthesizing six studies (i.e., meta-analyses, systematic reviews, reviews, and synthesis) related to algebra intervention as a means to capture the different types of algebra interventions. The six studies that were synthesized are Haas (2005); Rakes et al. (2010); Hughes et al. (2014); Watt et al. (2016); Hwang et al. (2019); and Bone et al. (2021).

Haas (2005) conducted a meta-analysis of studies published between 1980 and 2002 at the secondary school level (grades 7 to 12) wherein the authors conducted an experimental investigation with algebra instruction as the focus. Haas' (2015) meta-analysis resulted in the inclusion of 35 independent experimental studies which he used to classify (by effect sizes) six teaching method categories correlated to positive effects on students' achievement in the secondary-algebra classroom. The following are the six teaching method categories as described by Haas (2015): "Cooperative learning, Communication and study skills, Technology-aided instruction, Problem-based learning, Manipulatives, models, and multiple representations, and Direct Instruction" (p. 27-28).

Rakes et al. (2010) conducted a systematic review and meta-analysis of studies published between 1968 to 2008 on algebra instructional improvement strategies that used random experiments and quasi-experimental designs. Rakes et al. (2010) meta-analysis resulted in the inclusion of 82 relevant studies from mostly the secondary level with one study at grade 3. Five categories of improvement strategies emerged: "Technology-based curricula, Non-Technology curricula, Instructional strategies, Manipulatives, and Technology tools" (p.382).

Hughes et al. (2014) conducted a meta-analysis of Algebra Interventions aimed at improving algebra performance of students with disabilities and struggling learners published between 1983 and 2013. Hughes et al. (2014) included studies that were either experimental or quasi-experimental designs which resulted in 12 relevant articles both at the elementary and secondary levels. The type of interventions described by the authors included: Cognitive/Model-based Interventions, Co-teaching (general ed teacher and special ed teacher), CRA - Concrete-Representational-Abstract Instructional Sequence, Graphic Organizers, Single-sex Instruction, and Technology.

Watt et al. (2016) conducted a review and meta-analysis of studies published between 1980 and 2014 for teaching algebra to students with learning disabilities using either experimental, quasi-experimental, or single-subject designs. The review included the results of 15 studies (10 experimental and 5 single-subject designs) at both the elementary and secondary level. Effect sizes were computed for the experimental studies. The interventions were: Concrete-Representational-Abstract Instructional Sequence, Cognitive Strategy instruction, Enhanced Anchored Instruction, Tutoring, and Graphic Organizers.

Hwang et al. (2019) conducted a review of algebra interventions published after 1990 about the cognitive processes involved in problem solving and their instructional strategies. These algebra interventions focused on secondary students (grades 6 to 12) with learning disabilities. The review included the results of 11 effective algebra interventions. Four conceptualizations of the cognitive processes involved in problem solving were identified: (a) sequential concrete–semi-concrete–abstract, (b) sequential virtual-abstract, (c) integrated concrete–semi-concrete–abstract, and (d) abstract only. Hwang et al. (2019) also discussed several instructional strategies incorporated in the interventions: advance organizer, modeling, guided practice, independent practice, post-test, feedback/rewards.

Bone et al. (2021) conducted an evidence-based synthesis on algebra instruction and interventions for secondary students (grades 6 to 12) with learning disabilities published between 1999 and 2019 by critically analyzing the practices for teaching appropriate algebraic concepts. The synthesis included 18 studies across 17 articles, where only 14 met the Council of Exceptional Children standards of high quality. Twelve of the 18 studies used single-case design and the other 6 used group design methodology. Across the 18 studies, 8 instructional categories emerged: (a) concrete-representational-abstract (CRA) framework (b) schema-based instruction, (c) enhanced anchor instruction (EAI), (d) manipulatives, (e) peer-assisted learning strategies, (f) virtual-abstract (VA) framework (g) graphic organizers and diagrams; and, (h) explicit inquiry routine. However, Bone et al. (2021) found that none of the instructional approaches met the criteria for evidence-based, and only 5 were found to be potentially evidence-based (i.e., CRA, manipulatives, EAI, schema based instruction, and peer assisted learning strategy).

Looking across these definitions of interventions, we found six general intervention types for algebra across the grade levels: Instructional, learning, curricula, technology, tutoring, and manipulatives. Although these categories form a general base from which to discuss effective algebra interventions, one type of intervention not covered in the aforementioned reviews was teacher development. Sharpe and Schliemann (2017) provide a good example of the effectiveness of this as an intervention. In their study, Sharpe and Schliemann analyzed the impact of a teacher development program based on a functional approach to algebra on their grade 7 students' understanding of equations. The teacher development intervention was a three semester-long online graduate level courses on algebra, functions, multiple representation, modeling, applications, and student thinking. The teachers were not given a class-level intervention or told what to implement in their classrooms. Sharpe and Schliemann's (2017) found that the students of the teachers who participated in the teacher development intervention showed significantly greater improvement than the students in the control teachers' classrooms.

We have included teacher development as a potential algebra intervention measure to be considered further in this context. The results of our synthesis of the six studies produced the following categories of interventions:

- Instructional Strategies – this consists of different instructional methods such as cooperative learning, focus on communication and study skills, explicit inquiry routine, mastery learning, cognitive strategies, schema-based instruction, enhanced anchored instruction, concrete-representational-abstract instructional sequence, co-teaching, single-sex instruction, and direct instruction (Bone et al., 2021, Haas, 2005; Hughes et al., 2014; Hwang et al., 2019; Rakes et al., 2010; Watt et al., 2016). Enhanced anchored instruction is an inquiry-based learning where students solve problems or complete tasks using video anchors of real life situations (Bone et al., 2021, Watt et al., 2016). Concrete-Representational-Abstract Instructional Sequence is an instructional sequences which moves students through the concrete (manipulatives), representational (pictures) to the abstract stage of symbols (Bone et al., 2021, Haas, 2005; Hughes et al., 2014; Hwang et al., 2019; Watt et al., 2016).

- Learning Strategies – this includes problem-based learning and model-based learning which is learning to solve real-world or word problems using problem solving strategies (Bone et al., 2021; Haas, 2005; Hughes et al., 2014).
- Curricula – this consisted of reformed-based curricula and researcher-developed curricula (Rakes et al., 2010).
- Technology-aided Instruction/Curricula/Tools – this includes using computers, calculators, software, graphic organizers, or virtual representation as a basis for curricula, instruction, or a tool in the classroom (Bone et al., 2021; Haas, 2005; Hughes et al., 2014; Hwang et al., 2019; Rakes et al., 2010; Watt et al., 2016).
- Tutoring - this can consist of one-on-one tutoring with a peer, tutoring from preservice teachers, special education teachers, or paraprofessionals, or incorporating peer-assisted learning strategies (Bone et al., 2021, Watt et al., 2016).
- Manipulatives – this is when concrete objects are used to help students understand a concept (Bone et al., 2021, Rakes et al., 2010).
- Teacher Development - this is when the intervention is geared towards the teachers with no explicit instructions on how to implement a specific strategy or curricula with the students (Sharpe & Schliemann, 2017).

The goal of this section was to explore the different types of algebra interventions that may be expected when screening studies for inclusion. If a study claims to intervene in the regular practices of the classroom as a way to improve the teaching and/or learning of algebra via instructional strategies, learning strategies, curricula, technology, tutoring, manipulatives, or teacher development, then we will include it into the pool for review in the larger study.

Discussion and/or Conclusions

Recall that this paper is a part of a larger study that we are conducting to develop a framework consisting of data-driven best practices around teaching and learning algebra in the K-12 classroom, by conducting both a meta-analysis and a qualitative meta-synthesis. The aim of this paper is to develop a usable definition for algebra interventions for the second stage of screening within the context of the larger study. Combining the algebra definitions with the type of interventions produces three questions to ask during the second stage of screening to decide if it's an algebra intervention (Figure 1).

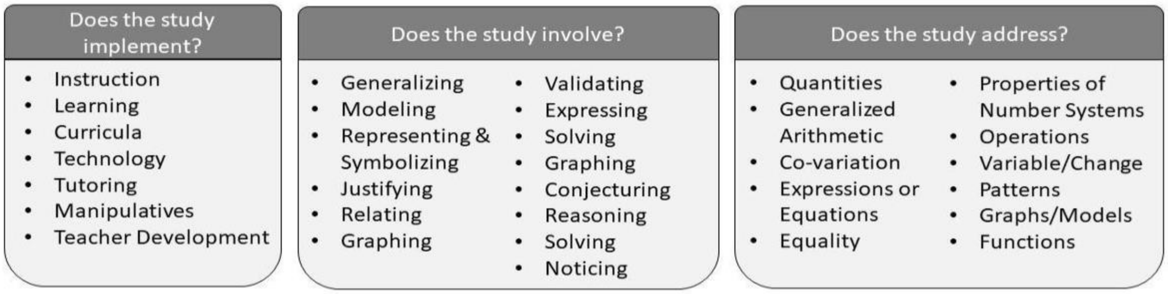

Figure 1: What is an algebra intervention?

Next, we will provide an example of an algebraic thinking intervention in elementary school, as it relates to our model in the figure above. Blanton et al. (2019) conducted an intervention study in three school districts across 46 schools with 23 schools randomly assigned to treatment and the other 23 randomly assigned to the control condition. The authors describe the intervention as follows:

> "The intervention consisted of 18 one-hour lessons at each of Grades 3 to 5, with lessons taught throughout the school year (approximately September through March). As noted earlier, lessons were designed to engage students in the algebraic thinking practices of generalizing, representing, justifying, and reasoning with mathematical structure and relationships within the Big Ideas of generalized arithmetic; equivalence, expressions, equations, and inequalities; and functional thinking" (Blanton et al., 2019, p. 1942).

As underlined in the above paragraph, the type of intervention is instructional, the type of activities are generalizing, representing, justifying, and reasoning, and the type of topics are generalized arithmetic; equivalence, expressions, equations, and inequalities; and functional thinking.

An additional example elucidates how the information in the figure above can be applied to studies involving algebra interventions in secondary school. Thompson and Senk (2001) examine the performance of 8 pairs of algebra classes from four schools (Atlanta, Chicago, Mississippi, and Philadelphia), where one of the pair uses the University of Chicago Mathematics Project (UCSMP) Advanced Algebra Curriculum and the other their regular textbook. A total of 150 students were in the UCSMP classes and 156 in the comparison classes, all students were either in 10th or 11th grade. Topics across the intervention and comparison schools included "work with linear, quadratic, higher degree polynomial, exponential, logarithmic, and trigonometric expressions, equations, functions, … powers and roots, sequences and series, systems of equations and inequalities, matrices, and conic sections" (p. 64). The UCSMP textbook "emphasizes four dimensions of understanding: skills, properties, uses, and representations."

As underlined in the above paragraph, the type of intervention is curricula, the type of activity includes representing, and the type of topics include functions and equations. What is an algebra intervention? It is a study of a practice that intervenes in the regular practices of the classroom as a way to improve the teaching and/or learning of algebra, and if the study claims to address algebra, pre-algebra, algebraic thinking, or early algebra then it's an algebra intervention. Our analysis produced a protocol for our screeners to decide if a publication included an algebra intervention. The following three new screening questions will replace the initial two screening questions for the full-text screening each being answered with a dichotomous yes vs no:

- Does the study implement any of the following: instructional strategies, learning strategies, curricula, technology, tutoring, manipulatives, or teacher development?
- Does the study involve generalizing, modeling, representing and symbolizing, justifying, relating, validating, noticing, conjecturing, reasoning, expressing, solving, or graphing?
- Does the study involve any of the following topics: quantities, generalized arithmetic, co-variation, equations, equality, properties of numbers systems, operations, variables, change, patterns, graphs, models, expressions, or functions?

Acknowledgement

This material is supported by the National Science Foundation DRK-12 Grant #2142659.

References

Bednarz, N. (2001). A problem-solving approach to algebra: Accounting for the reasoning and notations developed by students. In H. Chick, K. Stacey, J. Vincent, & J. Vincent (Eds.), The future of the teaching and learning of algebra: Proceedings of the 12th ICMI Study Conference (Vol. 1, pp. 69-78). Melbourne, Australia: The University of Melbourne.

Blanton, M., Brizuela, B. M., Stephens, A., Knuth, E., Isler, I., Gardiner, A. M., ... & Stylianou, D. (2018). Implementing a framework for early algebra. Teaching and learning algebraic thinking with 5-to 12-year-olds: The global evolution of an emerging field of research and practice, 27-49.

Bone, E., Bouck, E., & Witmer, S. (2021). Evidence-Based Systematic of Literature on Algebra Instruction and Interventions for Students with Learning Disabilities. Learning Disabilities: A Contemporary Journal, 19(1), 1-22.

Carraher, D. W., Martinez, M. V., & Schliemann, A. D. (2008). Early algebra and mathematical generalization. ZDM, 40, 3-22.

Chimoni, M., Pitta-Pantazi, D., & Christou, C. (2018). Examining early algebraic thinking: insights from empirical data. Educational Studies in Mathematics, 98, 57-76.

Cooper, H. (2017). Research synthesis and meta-analysis: A step-by-step approach (5th ed.). Thousand Oaks, CA: SAGE Publications, Inc.

Haas, M. (2005). Teaching methods for secondary algebra: A meta-analysis of findings. NASSP Bulletin , 89 (642), 24-46.

Hughes, E. M., Witzel, B. S., Riccomini, P.J., Fries, K. M., & Kanyongo, G. Y. (2014). A meta-analysis of Algebra interventions for learners with disabilities and struggling learners. The Journal of the International Association of Special Education, 15(1), 36 – 47.

Hwang, J.., Riccomini, P.J., & Morano, S. (2019). Examination of Cognitive Processes in Effective Algebra Problem-Solving Interventions for Secondary Students with learning Disabilities. Learning Disabilities: A Contemporary Journal, 17(2), 205-220.

Kaput, J.J., Carraher, D.W., & Blanton, M.L. (Eds.). (2008). Algebra in the Early Grades (1st ed.). Routledge.

Kieran, C. (2007). Learning and teaching Algebra at the middle school through college levels: Building meaning for symbols and their manipulation. In F. Lester (Ed.), Second handbook of research on mathematics teaching and learning (Vol. II, pp. 707–762). Charlotte, NC: Information Age Publishing.

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: background, overarching dimensions, and new directions. ZDM–Mathematics Education, 54(6), 1131-1150.

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. National Council of Teachers of Mathematics.

National Mathematics Advisory Panel. (2008). Foundations for success: the final report of the National Mathematics Advisory Panel. U.S. Department of Education.

Rakes, C.R., Valentine, J.C., McGatha, M.B., & Ronau, R.N. (2010). Methods of Instructional Improvement in Algebra: A Systematic Review and Meta-Analysis. Review of Educational Research, 80(3), 372-400.

Sharpe, S.T. (2019). An algebraic translation task solved by grade 7-9 students. Mathematical Thinking and Learning, https://doi.org/10.1080/10986065.2019.1564970.

Sayers A. (2007). Tips and tricks in performing a systematic review. The British journal of general practice : the journal of the Royal College of General Practitioners, 57(538), 425.

Watt, S. J., Watkins, M, & Abbitt, J. (2016). Teaching Algebra to Students with Learning Disabilities: Where Have We Come and Where Should We Go? Journal of Learning Disabilities, 49(4), 437-447.