PROSPECTIVE MATHEMATICS TEACHERS' EXPERIENCES ENGAGING IN AN OPPORTUNITY FOR PRODUCTIVE STRUGGLE

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This study investigated what 12 prospective mathematics teachers (PTs) in a middle school mathematics method course reported during a video-stimulated recall interview about their experiences when they were engaged in a doing math task that yielded an Opportunity for Productive Struggle (OPS). We investigated their reported feelings during the OPS, what mathematics they made sense of as a result of it, and the relationships between their feelings and sense making. We found that PTs' feelings did not predict the nature of their sense making and that regardless of how they felt during the OPS, the majority of them (66.67%) reported that engaging in the OPS resulted in mathematical sense making. Other PTs reported pedagogical sense making. We suggest future research to expand on our findings.

Keywords: Instructional Activities and Practices, Preservice Teacher Education

Productive struggles are opportunities for learners to make sense of mathematics within their *zone of proximal development* (Vygotsky, 1978) and deepen their understanding of mathematical ideas and the relationships among those ideas (e.g., National Council of Teachers of Mathematics [NCTM], 2014; Peterson & Viramontes, 2017). Hence, this concept is crucial for all learners at all learning levels, including prospective mathematics teachers (PTs) who will be charged with enacting the teaching practice *support productive struggle in learning mathematics* (NCTM, 2014) in their future classrooms. Research about productive struggle has identified ways PTs struggle as learners and what might make their struggles productive. For example, researchers (e.g., Zeybek, 2016) who have investigated PTs' productive struggles when engaging them in *high cognitive demand tasks* (Stein et al., 1996) have highlighted the important role of such tasks in supporting PTs to gain a deeper mathematical understanding. Thus, it seems that the use of high cognitive demand tasks might generate opportunities for productive struggle. Past researchers have focused on better understanding PTs' struggles (productive or otherwise) once they have been identified (e.g., Ducloux et al., 2018). Existing studies have provided some information about how to support PTs to engage in productive struggle themselves (e.g., Rahman, 2022) and to support their students' productive struggle (e.g., Anthony, 2021). Based on prior work, we can anticipate which aspects of instruction provide rich opportunities for productive struggle, but little is known about how the various PTs in a class respond to that same opportunity. Better understanding how PTs experience such opportunities—what they report having felt and made sense of—would provide useful information for better supporting productive struggle. Thus, we investigated the research question *What do prospective mathematics teachers report having experienced when reflecting on their engagement in the same opportunity for productive struggle?*

Literature Review

Researchers who study struggle have used the term *productive struggle* in different ways. Some have used it broadly to encompass when learners engage in a task that has an unclear path for them to solve, as long as they work towards the goal of the task without the teacher

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1). University of Nevada, Reno.

decreasing the level of cognitive demand of the task (e.g., Warshauer, 2015; Zeybek, 2016). Others have used the term to reflect a specific research situation, such as "a student persisting in a digital learning task while maintaining a likelihood of future success" (Krumm et al., 2022, p. 514). Kamlue and Van Zoest (2022b) defined *mathematically productive struggle* as the type of struggle that occurs when students "delv[e] more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (NCTM, 2014, p. 48). Across all these uses is the idea that productive struggle supports learning and that other types of struggles do not.

Two themes arise across studies that have focused on productive struggle with PTs. Some researchers have explored how PTs facilitated productive struggle in their classes. For example, Anthony (2021) found that 10 middle grades mathematics and science pre-service teachers who learned about productive struggle in their mathematics methods courses still struggled with creating and sustaining productive struggle with students in their field experience. Similarly, Rahman (2022) investigated the learning opportunities of seven PTs enrolled in a middle school mathematics methods course who were teaching high cognitive demand tasks to their peers. They reported that these PTs had opportunities to learn about productive struggle through the process of selecting tasks and responding to students' struggles.

Other researchers have investigated PTs' productive struggle when engaging with high cognitive demand tasks. For example, Ducloux et al. (2018) directly interviewed 32 prospective elementary, middle, and secondary teachers from three different mathematics content courses for teachers to investigate their struggles after engaging in a non-routine problem-solving task. The participants in the study reported experiencing both negative aspects of struggles (e.g., frustration, too challenging) and positive aspects of struggles (e.g., perseverance, collaborative struggle). Zeybek (2016) investigated 48 middle grades pre-service teachers' struggles in a geometry class and stated that for her participants' struggle to be productive, the participants needed to engage in a high cognitive demand task similar to what Warshauer's (2015) study suggested. Finally, Kamlue and Van Zoest (2022a) investigated 18 PTs' struggles when engaging with a *doing math* task (Stein et al., 1996) and noted that mathematically productive struggle could occur outside the intended learning goal. That is, since *doing math* tasks do not have an obvious solution path, the authors observed the PTs introducing ideas that did not help them solve the problem, but that they came to better understand as a result of trying to use these ideas.

From the above, we notice that high cognitive demand tasks provide learners with opportunities to struggle productively because of the nature of these tasks—they require students to engage with conceptual ideas and often do not have an obvious path to the solution (Stein et al., 1996). Warshauer (2015) also pointed out that if teachers maintain the high level of cognitive demand of a task when noticing students' struggles, those struggles can become productive because the teachers let the students make sense of mathematics with which they are struggling. However, what we as a field do not know is how various PTs in a class respond to the same opportunity for productive struggle. Thus, the purpose of this study was to answer this research question: *What do prospective mathematics teachers report having experienced when reflecting on their engagement in the same opportunity for productive struggle?*

Theoretical Perspectives

Our research is based on a participationist perspective (Vygotsky, 1987). That is, we see learning as taking place through learners' interactions with more knowledgeable others, such as

their teacher and their peers. Drawing on this perspective and the research described above, we defined an Opportunity for Productive Struggle to be a situation where (a) the PTs were engaged in a *doing math* task (Stein et al., 1996) and (b) the teacher had positioned the class to engage in, and was facilitating a discussion about, collaborative sense making of a peer's high-leverage contribution (Leatham et al., in press).

To further unpack PTs experiences, we drew on Goldin's (2000) theoretical framework that described the relationships between affect and heuristics. Hannula et al. (2004) provided an example of how Goldin illuminated interactions between students' feelings and their cognitive processes:

> The feeling of *bewilderment* in approaching a problem in mathematics may simultaneously suggest that certain standard problem interpretations or problem-solving strategies do not work… [a]ffective states may evoke heuristic strategies; thus frustration may evoke a major change in strategy. (p. 110)

This framing suggests the importance of asking PTs about their feelings when engaging in an Opportunity for Productive Struggle.

Methodology

The participants in this study were twelve (of nineteen) PTs in a middle school mathematics methods course who agreed to be interviewed. Five (of seven) PTs were taking it as the first mathematics methods course in a program leading to a secondary mathematics education degree. Seven (of twelve) were taking it as their second mathematics methods course as middle school mathematics majors in an elementary teacher education program. We used video-stimulated recall interviews to ask the PTs about what they had experienced when engaged in an Opportunity for Productive Struggle. The context was a lesson centered on the *doing math* (Stein et al., 1996) Frog Problem (see Figure 1; for more details, see Andrews, 2000, and Dixon and Watkinson, 1998). The PTs engaged in activities around the Frog problem for three ninety-minute sessions, beginning with two teams of PTs physically modeling their peers' suggested ways to achieve the fewest number of moves using chairs set up at the front of the room. They had opportunities to develop their ideas about the mathematics of the Frog Problem and the *mathematical practices* (NCTM, 2014; NGACBP & CCSSO, 2010) they used in solving the problem by participating in whole-class discussions, small-group discussions, and written reflections.

What is the fewest number of moves to switch each group of frogs from one side to the other? *(Allowed moves are jumping over one frog to an empty spot or sliding to an adjacent empty spot.)*

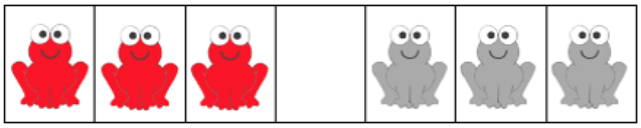

Figure 1: The Frog Problem Prompt and Representation of Two Frog Teams of Size Three

The researcher-identified Opportunity for Productive Struggle discussed in this report occurred at the beginning of the third session and was six minutes long. It was chosen because it fit the two criteria of an OPS identified in the Theoretical Perspective section above: (a) the PTs were engaged in the Frog Problem, a task that met the *doing math* criteria (for more details, see Kamlue & Van Zoest, 2022b) and (b) the teacher had identified an incorrect PT's explanation (See Figure 2) as a high-leverage contribution and positioned the class to engage in mathematical

sense making about it. This OPS provided the opportunity to better understand the difference between exponential and quadratic equations and included multiple PTs' contributions to the whole-class discussion about the PT explanation in Figure 2.

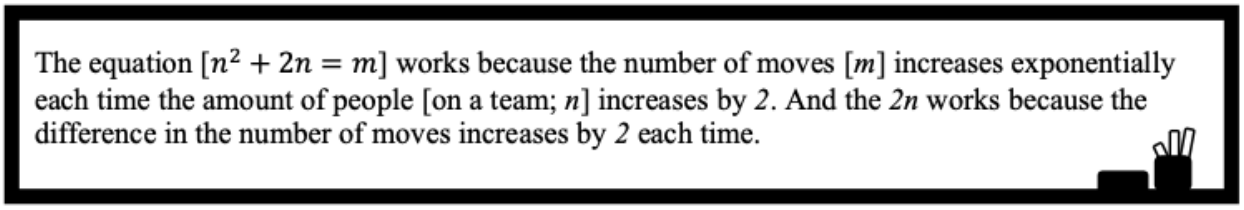

Figure 2: The (Incorrect) Explanation that Initiated the Opportunity for Productive Struggle

The audio-taped and transcribed video-stimulated recall interviews began by showing the PT a video of the selected Opportunity for Productive Struggle (OPS) to help them to recall the experience. This report analyzed the responses to these questions (and follow-up probing):

1. Please describe what you were experiencing in this [OPS].
    a. Do you remember what kind of thoughts you had?
    b. Do you remember how you felt at that time?
2. What mathematics did you make sense of as a result of that [OPS]?

The first and third author independently read through the transcripts and holistically identified a word that captured how each PT described feeling during the Opportunity for Productive Struggle and a phrase that captured what they described making sense of as a result of their experience. They then discussed their coding, developed code names and definitions, and refined them with the help of the second author. The data was then re-coded using the refined codes (see Figure 3 and 4 in Results & Discussion section for the resulting code names and definitions).

Results & Discussion

We report here on the PTs' reflections on what they experienced as they engaged in the interaction that we had identified as an OPS. (See Figure 2 for the explanation that initiated the OPS). We first report on the code names and definitions that we developed in our first level analysis of the data (Figures 3 and 4). We then report on our second level analysis of those codes (Figure 5).

Figure 3 shows code names and definitions that arose from our analysis of the PTs' responses to the first interview question—*Do you remember how you felt at that time?*

Code Name	Definition	Illustrative quote from our data
unsettled	When PTs said "nervous", "confused", or "frustration"	"I think I might have been a little **confused** on where [the classmate's idea] was coming from…"
neutral	When PTs expressed no feeling or said "bored" or "fine"	"Honestly, kind of **bored**…I expected like way more math because [this class] seems to me more like education class…"

settled	When PTs said "acceptable", "connected", "validated, or "good"	"I felt **good** because I was able to connect it back to something I already knew…"

Figure 3: Code Names and Definitions for PTs' Feelings during the Opportunity for Productive Struggle

Figure 4 provides code names and definitions that arose from our analysis of the PTs responses to the second interview question—*What mathematics did you make sense of as a result of that [OPS]?*

Code Name	Definition
Mathematics	An insight or outlook that provides definitions, justifications, questions, explanations of the logical concepts, techniques, tools, or skills identified when applied to a problem.
Teaching	The teacher's use of techniques, tools, or skills in an articulate form intended to accomplish a goal for students.
Other	A characteristic that belongs to a person (Trait) and responses where what the PT made sense of could not be inferred (CNI)

Figure 4: Code Names and Definitions for What PTs Made Sense of from the Opportunity for Productive Struggle

Figure 5 provides the following information for each PT: the PT's reported feelings during the OPS (column 2), their reported sense making as a result of the OPS (column 3), and the type of sense making (column 4). We noticed that the PTs experienced the same OPS differently in two ways: different feelings and different types of sense making. First, they felt differently when engaging in the same task. Six PTs felt unsettled, three PTs felt neutral, and three PTs felt settled. Second, the PTs responded differently when they were asked to answer the same question: *what mathematics did you make sense of as a result of that [OPS]?* Eight PTs made sense of mathematics, three PTs made sense of teaching, and two PTs made sense of other categories than mathematics and teaching—all as a result of engaging in the same OPS.

There were no clear patterns between PTs in the elementary and secondary programs (column 1). There also were no clear patterns related to the feelings the PTs expressed across the types of their sense making; it is noteworthy that each of the sense-making categories had one PT categorized as having a *neutral* feeling. Two PTs did not provide clear evidence of something they had made sense of, and thus their responses were categorized as *other*. PT11 talked about increasing their "confidence" and PT12 simply described what happened during the OPS.

There were three PTs' whose sense making was categorized as *teaching*, one in each feeling category—*settled*, *neutral*, and *unsettled*. Although each of these PTs talked about a different aspect of teaching, they all focused on supporting students. PT8 expressed making sense of the critical role of clarifying questions to support students' justifications, PT9 gained insight into the selection of tasks that support student learning, and P10's response was about supporting students as they struggle to communicate their mathematical thinking. The fact that all three of the PTs in this category thought deeply about a critical aspect of teaching as a result of participating in this OPS suggests that engaging PTs in making sense of their peer's thinking

about a high cognitive demand task in a methods course has both pedagogical as well as mathematical benefits.

PT	Reported feeling	Reported sense making as a result of the OPS	Type of sense making
1	unsettled	"I know when I said increased by two each time I was talking about there [CNI*] and the problem was like a pattern inside a pattern...[the little pattern], was the little pattern in [the big pattern], was by twos and then the big obvious pattern wasn't by twos..."	mathematics
2**		"...I kept thinking this is an exponential because there needs to be an *x* in the exponent…"	
3		"I would just say I have learned about the importance of variables in an equation and like the importance of explaining your variables."	
4		"I would say that everything has structure. You just need to like dig a little deeper and find [the answer]. And then I, the problem will be easier."	
5	neutral	"...look carefully into the equations…"	mathematics
6	settled	"Like breaking apart student work and understanding the different parts"	mathematics
7		"...I was able to understand, um, what, like what the numbers meant in the equation, like what they represented"	
8a***		"I think like justification in mathematics, like an explanation. There needs to be a proper justification of like why it [CNI] works…"	
8b	settled	"...So I think thinking really deeply about like, clarifying questions I could ask if this was like a student of mine, or um, like asking questions that might get them to provide me with a proper justification…"	teaching
9	neutral	"...it's more like just good teaching practices than math itself…[for example:] What makes a good problem? What makes a bad problem?"	teaching
10	unsettled	"...[difficulty explaining equations to peers] is testing my patience a lot, which is good because I know that that will be pushed when I'm a teacher [because my students will also struggle to explain equations]"	teaching
11	unsettled	"...So [the discussion] was very nice; hearing other people's thought processes and ideas and trying to make sense of something that confuses me as well. … when the entire class and the teacher agree[ed] with [me] [it increased my confidence]"	other
12	neutral	"...I would say [the course instructor] put a lot of emphasis on us knowing about exponential functions, and [the instructor] had us do an assignment…"	other

Notes: *CNI means cannot be inferred. **Bold indicates a PT in the secondary education program.
***PT8 reported two explicit pieces of information.

Figure 5: PTs' Reflections on their Engagement with the Opportunity for Productive Struggle

For the eight PTs whose sense making was categorized as *mathematics*, four expressed feelings that were categorized as *unsettled*, three *settled*, and one *neutral*. There were no noticeable patterns related to the feelings the PTs expressed. What is noteworthy was the range of mathematics that the PTs reported making sense of during the OPS. PT1 described "a pattern inside a pattern," reflecting their increasing awareness of nonlinear situations. PT2 discussed the structure of an exponential equation and their realization that the Frog Problem did not fit that structure. PT3 and PT7 reported making sense of representations, such as "variables" and "numbers," respectively. PT4 and PT6 emphasized techniques for solving challenging mathematics problems, and PT8 focused on the importance of justification. Thus, it appears that regardless of how they felt during the interaction, the majority (66.67%) reported that discussing a PT's high-leverage contribution during a *doing math* task resulted in mathematical sense making.

Conclusion

This study discussed what 12 prospective mathematics teachers (PTs) reported having experienced when reflecting on their engagement in an Opportunity for Productive Struggle in their middle school mathematics methods course. The way in which the PTs reported making sense of mathematics suggests that most of them were engaged in *mathematically productive struggle* (Kamlue & Van Zoest, 2022b). Thus our work provides further evidence that *doing math* tasks (Stein et al., 1996) support productive struggle.

Our finding that the PTs' feelings did not predict the nature of their sense making suggests that even when PTs feel unsettled, they can make sense of important mathematical and pedagogical concepts. This finding supports the idea that rushing in to relieve their struggle may undermine the benefits of high cognitive demand tasks (Warshauer, 2015). Our findings also suggest that generating Opportunities for Productive Struggle in mathematics methods courses may support PTs to learn about teaching as well as about mathematics. Rahman (2022) found that PTs learned about productive struggle through selecting tasks. It may also be possible that engaging PTs in worthwhile tasks in a way that models effective pedagogy can lead to *pedagogically productive struggle*—the type of struggle that occurs when delving deeply into understanding the relationship between ideas about teaching and student learning—as well as *mathematically productive struggle* (Kamlue & Van Zoest, 2022b).

This study provided some insight into how different PTs experience the same Opportunity for Productive Struggle. The variety of ways that the PTs in the class experienced this same opportunity illustrates the complicated nature of teaching—teachers need to generate learning opportunities for each of their students knowing that the students will not all respond to that opportunity in the same way. Better understanding the different ways PTs experience rich tasks, such as the Frog Problem, can inform teacher educators' preparation for using these tasks with their PTs.

Extending the research reported here to a larger participant group might reveal additional patterns. Our data was based on PTs' reports of what they made sense of as a result of an Opportunity for Productive Struggle; it would also be useful to have a measure of whether PTs actually gain the better understanding of a mathematical idea that a given Opportunity for Productive Struggle provides.

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