MUTABILITY OF STEM MAJORS' ABSTRACTED QUANTITATIVE STRUCTURES

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Recently, abstracted quantitative structures (AQS), a construct from quantitative reasoning, has been offered as a means to conceptualize and study mathematization during mathematical modeling. Extending this theoretical work, we provide empirical evidence that an intervention targeting participants' AQS can assist in aligning modelers' models with normatively correct models. We report on a pre/post intervention study designed to elicit alignment between symbolic forms and AQS and alignment between AQS and modeling scenarios. We used the Sørenson-Dice coefficient and cluster analysis to identify shifts in student associations of symbolic forms with modeling scenarios.

Keywords: modeling, mathematical representations, undergraduate education

Developing students' capacity to apply their mathematical knowledge to real-world scenarios is a central goal of mathematics education, especially for undergraduate STEM majors. Transfer of mathematics knowledge to a non-mathematical domain is difficult for students to do (Lesh & Zawojewski, 2007; Wake, 2014) and is challenging for researchers to study (Carraher & Schliemann, 2002; Evans, 1999; Lobato, 2006). Within research on the teaching and learning of mathematical modeling, the idea of transfer is captured by the idea of *mathematizing* or recognizing mathematical structure within a real-world scenario (Maaß, 2006; Zbiek & Conner, 2006). Mathematization has been difficult to study because it largely occurs as a modeler's mental actions, and despite this fact, has been under-theorized (Cevikbas et al., 2021). Recently, the field has made considerable efforts towards finding theoretically-grounded ways to operationalize *mathematization* such that it can be studied; specifically, scholars have made progress in explaining modeler's mental actions from a cognitive constructivist perspective by examining modeling from a quantitative reasoning lens (Czocher & Hardison, 2019; Czocher et al., 2022; Kularajan, 2023; Niss, 2010). Such approaches are necessarily based in the students' own interpretations of real-world scenarios and representations of them. Some hypotheses have emerged. One is that transfer of mathematical knowledge between scenarios can be traced by attending to individuals' *abstracted quantitative structures* (AQS), which are networks of quantitative operations an individual has interiorized to such an extent that it is independent of figurative material (Moore et al., 2022). Another is that operationalizing mathematization through the theoretical machinery of quantitative reasoning provides leverage for designing interventions and supports that may improve students' overall modeling skills. This paper adds to the conversation by addressing both hypotheses.

If the approach of operationalizing modeling in terms of quantitative reasoning is viable, then the field needs empirical evidence that abstracted quantitative structures are mutable during modeling and that real-world scenario can be assimilated into schema associated with modelers' AQSs. In this study, we operationalize AQS in terms of Sherin's (2001) symbolic forms and report on a pre/post intervention conducted with undergraduate STEM majors. We answer the research questions: *Are participants' ways of reasoning with abstracted quantitative structures stable, when subjected to a learning environment focused on modeling with those structures?* and *Is there an impact on participants' associations between symbolic forms and scenarios*?

Theoretical Perspectives on Modeling & Empirical Background on Mathematical Concepts

We adopt the cognitive perspective on mathematical modeling which is suitable for studying modelers' processes of rendering a real-world problem as a mathematical problem to solve (Kaiser, 2017). The cognitive perspective articulates the phases a modeler may pass through to specify a mathematical problem. These phases include anticipating mathematical structures that may be useful in representing and solving the problem, carrying out a solution process, and interpreting and validating the solution in terms of real-world constraints. *Mathematizing*, one phase of mathematical modeling, refers specifically to introducing conventional representational systems (e.g., equations, graphs, and tables) to represent mathematical "properties and parameters that correspond to the situational conditions and assumptions that have been specified" (Zbiek & Conner, 2006, p. 99). Others have pointed out that mathematizing entails "anticipating mathematical representations and mathematical questions that, from previous experience, have been successful when put to similar use" (Stillman & Brown, 2014, p. 766). That is, modelers need an idea of what to try out as a model that might be adequate. While the idea of *anticipation* and *implemented anticipation* (Niss, 2010) has gained traction in modeling research for their descriptive power, they lack both explanatory and predictive power with regards to how to aid students during mathematization. We view the idea of implemented anticipation through the lens of quantitative reasoning to explain *what* is anticipated and *how* anticipated structures are formed.

Sherin (2001) elaborated on the construct of *symbolic forms* that contemplates modelers' associations between algebraic *templates* and *conceptual schema*. The template is a format (e.g., $\_ \times \_ = \_$) that can express a mathematical idea or relationship (e.g., rate of change is proportional to amount present). Symbolic forms help explain how and why a modeler might choose to use $\times$ instead of $+$ when constructing an equation to represent a scenario. Theoretical work on quantities, quantification, and quantitative reasoning helps explain how individuals imbue the templates, variables, and conceptual schema with situationally relevant meanings. Quantitative reasoning means conceiving quantities and relationships among quantities; those relationships may be arithmetic (numerically evaluated) or quantitative (mental operations) (Thompson, 1990). Thompson gave an example using two individuals' heights to demonstrate that an *additive comparison* (mental operation) can be evaluated using *subtraction* (arithmetic operation). However, a comparison of the difference in two individuals' heights to the difference in another two individuals' heights (the difference between A and B is N times more than the difference between C and D) does not require evaluation of the differences in order to conceive the quantitative meaning. An abstracted quantitative structure (AQS) is a network of quantitative operations that an individual has interiorized and can operate *as if* it is independent of a figurative material (Moore et al., 2019). The core idea is that when a modeler has constructed an AQS, it is available to the individual when the scenario that engendered its construction is no longer present. Moore et al. (2019) clearly demonstrated evidence of the construction of AQSs and evidence of assimilation of new-to-the-student scenarios to previously constructed AQSs. Moore et al. (2019) also hypothesized that AQSs play an important role in transfer due to cognitive reorganization of previous experiences. Here, by transfer, we mean recognizing the applicability of symbolic forms for the purpose of mathematizing a given scenario, that is, *implemented anticipation* (Niss, 2010).

Mathematization of a real-world scenario is notoriously difficult for students across grade bands and content areas. Even students in advanced mathematics find it challenging because they face more complex scenarios where *rate of change* takes on dual roles as both dependent and independent variables. In such scenarios, students struggle with the idea that time is an implicit variable (Keene, 2007) and often interchange a differential equation with its solution (Donovan, 2002). Scaffolding for these students includes placing emphasis on the system being modeled and making explicit connections between the equation and the system it represents (Baker, 2009; Czocher, 2017; Myers et al., 2008; Pennell et al., 2009). For these reasons, we designed a study to generate empirical evidence as to whether participants' ways of reasoning with AQSs are stable throughout a learning environment focused on modeling with those structures. With reference to previous work on differential equations, our study focused on the templates of symbolic forms in Figure 3 and the learning environment emphasized these forms as representing conceptual schema like *rate of change is constant (A), rate of change is proportional to quantity present (B), and net rate of change (C)*. Throughout, we will refer to the answer choices in the Matching Tasks as "templates," intending to correspond to a family of *symbolic forms* and we will use "AQS" to refer to the *abstracted quantitative structures* we observed participants create, use, and re-use during the open modeling tasks comprising the learning environment. The templates operationalize the AQS as generic formats.

Methods

We conducted a pre/post intervention study within a design research project, which we evaluated using a paired samples t-test and a follow-up study using cluster analysis techniques. The intervention was a learning environment – task sequence and scaffolding – designed to build participants' mathematizing competencies. The learning environment comprised 10 hour-long task-based interviews with each of 23 undergraduate STEM majors recruited from courses listing differential equations as a prerequisite. The 10 sessions were organized so that the first and last sessions – our focus in this report – featured prompts intended to (a) document the criteria participants used to classify scenarios and (b) document participants' association of templates with the given scenarios. The middle 8 sessions consistently emphasized recognizing symbolic forms and associating them with quantitative structures participants recognized in the real-world scenarios presented in the tasks.

We describe the intervention's tasks using Yeo (2007) classification framework. The tasks were mathematical modeling problems with well-defined goals (develop a model for the scenario) but ill-defined answers (multiple valid models). Earlier tasks in the sequence provided built-in guidance and were relatively simple in that they called for fewer quantities and quantitative operations. Later tasks were not guided and were also more complex, being open to constructing many quantities and manipulating them with quantitative operations and relationships. In this paper, we focus on the first and final sessions, which featured pre/post items intended to document the participants' associations between scenarios and quantitative structures. In Sessions 1 and 10 we gave the Matching Task to document stability of reasoning with the target templates.

The Matching Task prompt (Figure 3) asked participants to match scenarios to the templates (a) – (e) based on what *they* saw as relevant within the problem scenario. For example, Item 9 is an abbreviated prompt based on a canonical salty tank problem from differential equations. It was an open problem during the modeling sessions and as a pre/post item was reduced to indicating the template perceived as appropriate to the participant. Participants were asked for explanation of their answers, information which we use to enrich our analysis.

Consider the following mathematical expressions:

a) $\frac{dQ}{dt} = k$ b) $\frac{dQ}{dt} = k \cdot Q(t)$ c) $I = a \pm b$ d) $I = k \cdot a \cdot b$ e) $\frac{dQ}{dt} = kP(t)Q(t)$

where $Q(t)$ and $P(t)$ are quantities, $Q'(t)$ is the rate $Q(t)$ changes with respect to time, k is a constant, and $a, b,$ and I are quantities that possibly depend on time (or not) or may be composed of other quantities (or not). Which of the mathematical expressions above (**either individually or as a composition**) can be used to model the real-world scenarios below?

Item 4: Consider a natural habitat where bobcats are natural predators of rabbits. Bobcats are very good hunters, but they aren't perfect. Therefore, not all of the bobcat/rabbit encounters result in a rabbit's death. Model *the number* of rabbit deaths due to only predation by bobcats.

Item 9: Consider a tank where water and a salty solution enters the tank while the well-mixed liquid inside the tank exits the tank. Model *the rate* at which the amount of salt in the tank changes with respect to time.

Figure 3: Abbreviated statement of Matching Task

A total of $N = 23$ undergraduate STEM majors participated in the design research project whose result was the modeling intervention reported in this paper. The participants were generally high preforming, reporting high grades in mathematics ($M = 3.4, SD = 0.5$) and a high level of confidence with relevant mathematical concepts ($M = 364, SD = 69$ out of 500 points on a confidence scale). We describe the sample composition by gender and major, but do not analyze the data according to these subgroups because of the small resulting sample sizes. Approximately 22% of participants identified as female, 74% as male, and 4% as non-binary. Approximately 30% of participants were pursuing a degree related to computer science, 35% in electrical engineering, 17% in civil engineering, 13% in physics, and 4.3% in mechanical engineering. Because the overall project used design methodology to develop the intervention, the Matching Task was revised after Implementation 1 ($N = 6$) to provide response E. We exclude $N = 9$ participants' responses from the t-test and pre/post response comparisons because the pre/post response options were not the same for participants in the earliest implementations ($N = 7$), and some participants did not complete the post-test ($N = 2$). However, we did include all participants' responses in the pre-test cluster analysis ($N = 23$) and all except the latter in the post-test analysis ($N = 21$) by using the response similarity metric (described below).

To quantify similarity of participants' ways of reasoning with the templates, we established a metric for response pattern proximity, as follows. Participants were encouraged to consider the templates as composites. For example, the normatively correct response for Item 9 was keyed as B&C, where the template B reflects the fact that the rate at which salt leaves the tank is proportional to the current amount of salt in the tank, and C reflects additive comparison of the inflow and outflow (net rate). Selecting C was regarded as closer to the keyed answer than selecting A and scoring reflected this. We chose the Sørenson-Dice coefficient (SDC) as a similarity metric to calculate scores because it both emphasizes similarities in response patterns and ignores template options which were neither included in the student answer nor the keyed answer. The SDC provides a 0 to 1 score for each item which can be interpreted as a percent overlap of the participant and keyed answers and is computed as $SDC = 2|P \cap K|/(|P| + |K|)$ for student answers P and keyed answers K (Sørenson, 1948). We calculated SDC per item and then summed to generate a participant pre- and post-test score between 0 and 9. We conducted a paired samples t-test on these scores, which met all assumptions.

We then explored shifts in associations per-participant and per-item using hierarchical cluster analysis techniques available in SPSS. Cluster analysis is a "data-driven approach" that allows clumping together participants who respond similarly, instead of using *a priori* groups. The goal is to organize heterogeneous samples into smaller groups that are maximally similar within-group and maximally dissimilar across groups (Woods et al., 2020). We estimated similarity by applying the SDC metric on participant responses to the Matching Task items to determine similarity between sets of participant responses. Finally, we teased apart the influence of item- and individual-level response patterns to articulate an account of shifts in participants' associations between symbolic forms (represented by the response choices) and scenarios.

Results

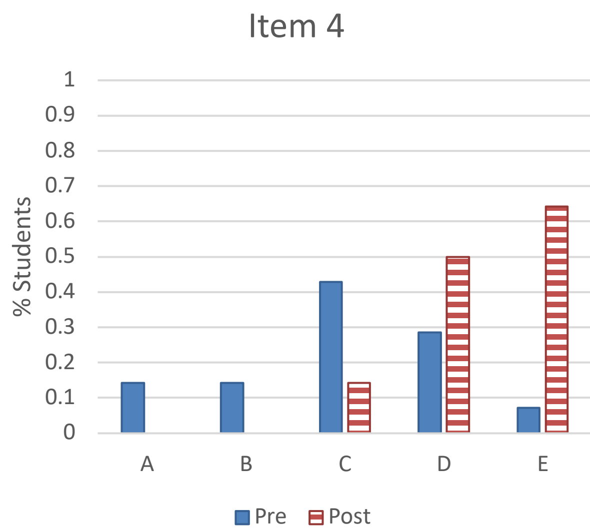
A paired t-test of the scores of the pre-tests ($M = 4.37,\ SD = 1.54$) and post-test ($M = 6.42,\ SD = 1.29$) suggest that the intervention resulted in additional normatively-correct associations between the templates and problems ($t(13) = 4.8, p < 0.001$), with a paired samples effect size of $d = 1.25$. In the aggregate, participants' associations between scenarios and templates shifted. Calculating the SDC between students' pre and post answers to each item (n = 126 pairs of pre/post responses from 9 items and 14 participants), we found that 41% of the post-test responses had no similarity with the corresponding pre-test answer, with an average pre/post-test similarity of 0.50. The cluster analysis revealed that similarity of participants' responses was lower on the pre-test than on the post-test (Table 1). Clustering performed on pre-test response sets yielded low average similarity. The two primary pre-test clusters included 11 of the 23 participants with average internal similarities of 0.63 ± 0.08 and 0.66 ± 0.08. The post-test revealed two clear primary clusters containing 8 and 7 participants respectively, and the 6 remaining participants were clear outliers from either cluster. Table 1 provides the similarity details of the post-test clusters, as well as average performance of students within these clusters.

Table 1: Post-Test Cluster Statistics

		Internal Similarity		Matching Task Performance		
Group	n	Mean	Range	Pre[2]	Post	Improvement[2]
Cluster 1	8	0.76 ± 0.07	(0.59, 0.93)	5.27 ± 1.31	7.43 ± 0.68	2.16 ± 1.65
Cluster 2	7	0.76 ± 0.08	(0.55,0.91)	3.87 ± 1.33	6.10 ± 0.64	2.25 ± 1.58
Outliers	6	0.40 ± 0.10	(0.19,0.56)	3.15 ± 0.52	4.31 ± 0.35	1.17 ± 0.17

- The cluster analyses revealed overall shifts towards the normatively correct response patterns and also indicated non-conforming item- and participant-level response patterns. For example, pre/post response patterns on Item 4 and Item 9 are shown in

[2] These columns only include scores on the final form of the pre/post test, resulting in $n_1 = 6, n_2 = 6, n_{out} = 2$.



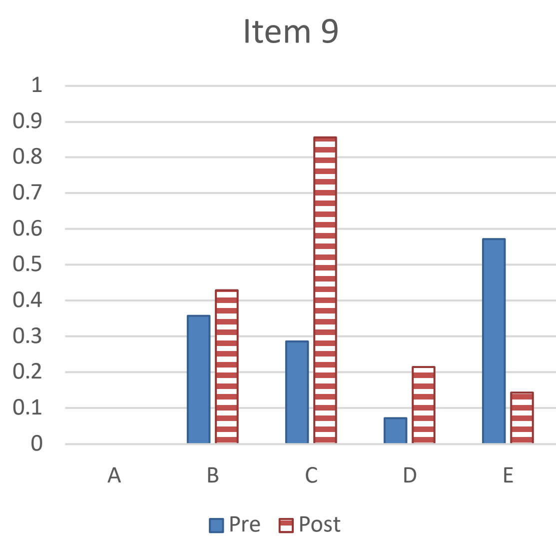


Figure 4. For the $n = 14$ participants who took the same version of the pre/post tests, the graphs illustrate a general shift towards the normatively correct responses for each item. On Item 9, just shy of 60% of participants included E in their responses, while only 29% included C, and 14% selected both. Participants who provided a justification for selecting only E mentioned it looked similar to what they recalled from their differential equations class and that the multiplicative factors represented inflow and outflow. On the post-test, 86% associated template C with the scenario, and only 14% kept E, indicating that many participants learned to associate the conceptual schema for the superposition of flows with template C as necessary to model the salt tank problem.

On some items, participant response patterns did not shift towards the keyed answers. In item 4, the response frequency for E increased though D was the keyed answer. Item 4 describes a predator-prey scenario. The item requests a model for the *number* of rabbit deaths due to predation, explicitly prompting the participant for an amount rather than a rate. The keyed answer D reflects a proportion of the possible interactions between the bobcat population and the rabbit population. On the pre-test, the most selected template was C (43%). Participant justifications indicated they were conceiving the predator-prey interactions as being one-to-one, implying that the number of rabbits killed by bobcats was equivalent to the number of bobcats. On the post-test, the percentage of participants selecting C dropped to 14% while the proportion who chose E rose from 7% to 64% and the percentage who chose D rose from 29% to 50%. One of the open modelling tasks during the sessions featured a predator-prey scenario in which participants leveraged a symbolic form which fits into template E (as in the Lotka-Volterra equations). In comparison, Item 5 presented another predator-prey scenario and requested a model for the species interactive dynamics. The keyed answer was B&C&E to reflect two equations for the two species. On this item, participant responses did shift towards the keyed answer, with B rising from 21% to 50%, C from 29% to 57%, and E from 36% to 86%. No participants selected A in pre or post, and D decreased from 36% to 29%. Thus, the response patterns suggest that many participants learned to associate the template with the predator-prey scenario, but that the finer conceptions involved in transferring an AQS that call for distinction between amounts and rates-of-change of amounts were obscured.

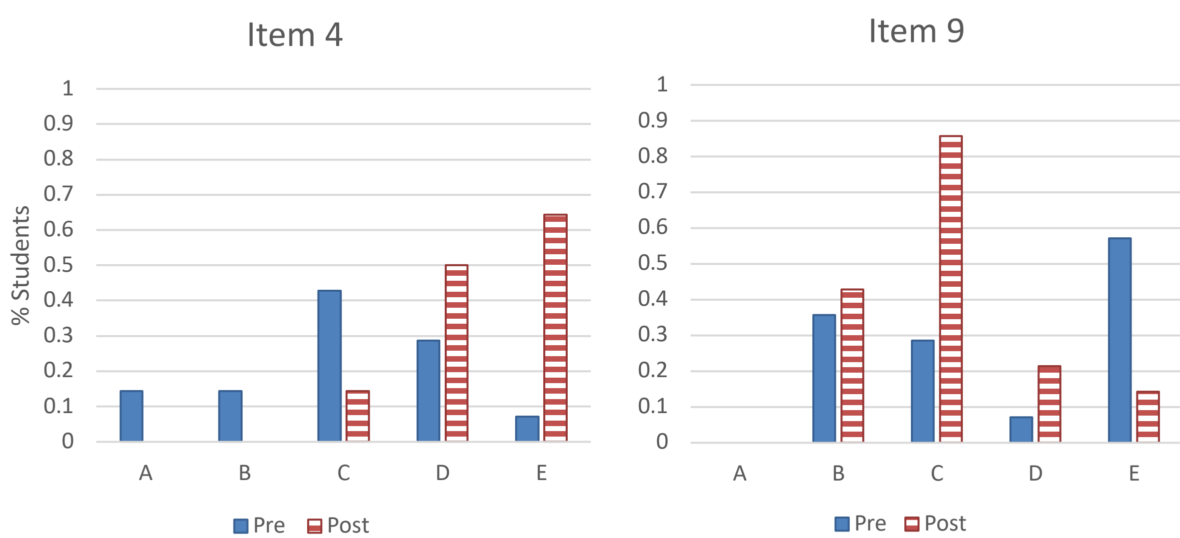


Figure 4 Relative Frequency of Multi-Select Responses to Items 4 and 9

To better understand shifts in patterns of reasoning in relation to the item contexts, we closely examined Yixli, Tien, Niali, and Khriss. These participants were chosen because they exhibited four archetypal cases arising from a 2 × 2 configuration: associations were (not) shifted × associations were (not) normatively correct. The existence of the four archetypal cases demonstrates that shifting associations does not imply a shift to a normative association. For each participant, we describe how their ways of reasoning with AQS may have changed (or not) from pre to post. Yixli revised his responses to only Item 4 and 5 from pre to post; his post score was in the lower third of the sample (5.5/9). Explaining his change from C to E on Item 4, he stated, "because at this point it's, like, beaten into my skull from all the problems that we did" [in the sessions]. Yixli successfully developed a system of differential equations for the predator-prey scenario during the open modeling sessions. Thus, his statement supports the inference that he learned to associate predator-prey scenarios with template E but still inconsistently associated an AQS for rate of change in amount with a prompt requesting an amount, a common conflation when learning to model with differential equations (Rasmussen & King, 2000; Rowland & Jovanoski, 2004). More generally, Yixli demonstrated high levels of recognition of key features of the scenarios but did not associate normatively correct AQS with those features. For example, in items 2 and 4, he selected non-normative responses because he furnished a rate of change equation instead of an amount equation. In item 6, he selects one keyed response accounting for only part of the information presented in the prompt. Finally, on item 8, Yixli associated the scenario with exponential growth (B) instead of linear growth (A) based on the real-world setting rather than its properties. Considering Yixli's response patterns across items explains why the majority of his responses – which were non-normative – did not shift from pre- to post. There were few non-normative associations for the intervention to address and Yixli's tendency to focus on the real-world setting rather than its features was unperturbed by the intervention.

Tien changed responses to 4 items, selecting additional templates on the post while maintaining the templates he selected on the pre. He scored in the top third of the sample on post (7.3/9). On his pre-test, Tien selected at least one keyed response on all except two items; on items 4 and 9 he selected options C and E, respectively. He changed his response on item 4 to C&D&E, explaining that, for him, D reflected the number of rabbit deaths due to bobcats, while C and E reflected other quantities in the scenario. He similarly revised his responses to items 5 and 6 to select C&D&E, which overlapped with the keyed response B&C&E. Items 5 and 6 treated population dynamics, indicating that Tien viewed the C&D&E composition of templates as important for modeling population dynamics. Finally, Tien revised his response to item 9 from C to C&E, overlapping with the keyed response B&C. However, Tien explained that C reflected the sum of salt content and solution content, yielding the total amount of substance in the tank, not the additive comparison of inflow to outflow. We interpret Tien's response patterns as indicating that he began the intervention with mostly normative associations between scenarios and productive symbolic forms and also gained additional association with quantitative structures that would aid him in modeling population dynamics. Additionally, his associations of forms with the salty tank scenario were correct for his conceptions, but non-normative.

Unlike the previous two, Khriss revised his answers on all but 3 items but only slightly improved his score by a single point from 3.7 to 4.7, ending up scoring in the bottom tenth of the sample. On the pre-test, Khriss exclusively utilized the templates B, C, and E. On problem 2 he wrote out a normative equation but selected a non-normative template. Similarly, on problems 3 and 8 he selected the template E to model an exponential growth problem, demonstrating that he had a normatively correct answer on 3 and a common non-normative answer on 8, but simply selected a non-normative template to describe those models. This was continued in the post-test on which Khriss was able to identify normative equations which modelled every problem, but then associated the scenarios with non-normative templates. The primary distinction between Khriss's pre and post tests was that the equations Khriss wrote for the post-test responses were more detailed and normatively-correct than the equations Khriss wrote while taking the pre-test. This indicates that, while Khriss's associations of templates with scenarios did not generally become more normative, the symbolic forms used by Khriss did.

Finally, Niali revised his responses to nearly all items, improving from 2 to 7.2, and scored in the top third of the sample on the post-test (7.2/9). During the sessions, Niali indicated a robust association between template A and scenarios featuring linear growth and between template B and scenarios featuring exponential growth. Thus, we infer that his low pre-test scores were not due to a lack of transferrable AQS's adequate to distinguishing those scenarios mathematically. Instead, he over-selected E for exponential growth scenarios rather than B. By the post-test, Niali associated E with scenarios featuring a rate of change dependent upon interaction between two quantities. Like Yixli, he responded to the real-world setting of item 8 rather than the scenario-specific features in the item.

Conclusions

The paired samples t-test indicates that undergraduate STEM majors' ways of reasoning with abstracted quantitative structures can be modified through modeling with those structures. The cluster analyses also revealed overall shifts towards the normatively correct response patterns and also indicated non-conforming item- and participant-level response patterns. We observed that for some participants, the context was a stronger indicator of response selection than scenario-specific features. In previous literature, this phenomenon has been referred to as focusing on "surface features" vs. "deep structure" (Schoenfeld & Herrmann, 1982) However, we argue that the conclusion is not so simple. Our participants proffered "deep structural" explanations based on their structural conceptions of quantities and relationships among quantities even when seemingly focused on contextual features. That is, many selecting non-normative responses demonstrated evidence of structural ways of reasoning rather than superficial reasoning.

One limitation of our approach is that template-matching is not perfectly predictive of associations between scenarios and symbolic forms, as Khriss's response patterns revealed. Teasing this apart might require developing a more nuanced similarity metric that accounts for the idea that template B is actually a sub-template of E, both of which can be expressed as instances of C (though many participants did not evidence awareness of these insights). However, overall, the Sorensen-Dice coefficient enabled a complex metric that modeled the response pattern data well. It supported a robust comparison of *similarities* in participants' ways of reasoning with sets of responses, providing an additional tool for evaluating complex reasoning patterns. Thus, we are optimistic about the approach to measuring the assimilation of scenarios to AQS' in this way because it opens possibilities for future work on AQS.

Our objective in the present study was to examine the stability of participants' ways of reasoning with symbolic forms when engaging with modeling tasks designed to help them assimilate scenarios to those forms. Recently, Kularajan (2023) argued that a promising approach to studying (and subsequently improving) students' capacity for mathematizing is to examine and respond to their emergent quantitative reasoning about the scenario. Our contribution is providing empirical support to this conjecture. We conclude that participants showed evidence of *changing* which scenarios are assimilated to a given AQS by matching it with a template as a consequence of engaging with modeling scenarios that reinforced the use of symbolic forms matching those AQSs. The next steps in this line of research are articulating the contours of learning environments that may be fruitful for students with differing ways of reasoning.

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