WHAT DOES IT MEAN FOR GEOMETRY TEACHERS TO IMPROVE A LESSON? A MULTIMODAL ANALYSIS

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A central goal of lesson-centered professional development programs (PD) for mathematics teachers is to learn by constructing an artifact, for example, by designing and improving a lesson plan together. That leads to the questions, what does it mean, for mathematics teachers, to improve a lesson? And how can improvements be accounted for in the analysis of the resulting artifacts, especially when these are multimodal? This paper lays the groundwork for answering such questions by drawing on empirical data from a lesson-centered PD program for secondary geometry teachers. We show how semiotic choices were made to convey that the teacher would need to support students when geometry instruction moves from a construction task to a proof, by (1) addressing students' confusion; and (2) creating a shared language to discuss diagrams. We relate these findings to teachers' professional growth and the conference theme.

Keywords: Geometry and Spatial Reasoning; Problem-Based Learning; Professional Development; Research Methods.

Objectives of the Study

As part of the emphasis on practice in professional development programs (PD) (Cohen & Ball, 1999) for mathematics teachers (MTs), lesson-centered PD – that focus on collaborative work around one lesson (e.g., Morris & Hiebert, 2011) – have risen in popularity. A key activity in these programs is the iterative process of revising artifacts such as mathematical tasks, storyboarded lessons, vignettes, or lesson plans (e.g., in Lesson Study programs; Lewis et al., 2009). Such activities are aimed at the professional growth of the MTs through their involvement in the construction and revision of artifacts (building on the *constructionism* approach proposed by Papert & Harel, 1991). While numerous studies have sought to account for MTs' professional growth when they design lessons together (see Huang & Shimizu, 2016), it is less common to find studies that analyze the revised artifacts themselves. Thus, some questions remain unexplored, in the context of a lesson-centered PD: What does it mean, for MTs, to improve a lesson? How can improvements be accounted for in the analysis of the resulting artifacts, especially when these are multimodal? And how can changes in artifacts be linked with MTs' professional growth, if at all? These lines of inquiry align with recent calls for theorizing teachers' multimodal professional learning (Lefstein et al., 2020). By multimodal we allude to communication that relies on two or more of the communication modalities including written, oral, visual, kinesthetic, and gestural. With the proliferation of lesson-centered PDs, and particularly such that include storyboarding or vignette-making, there is a need for a framework that: 1) captures and compares multimodal semiotic choices that MTs make while creating and revising instructional artifacts (analyzing written and visual modes), and 2) relates these choices to MTs' oral arguments to provide a multimodal characterization of teachers' professional

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learning (analyzing, in addition, oral and perhaps gestural expressions). To lay the ground for developing such a framework, we focus on the first component by comparing two artifacts from a lesson-centered PD in which secondary MTs created versions of a geometry lesson, represented through storyboards of cartoon characters. We ask:

How do changes made over time to the storyboard of a geometry lesson evince the kinds of improvements that MTs were attending to in the context of a lesson-centered PD?

In this study, storyboarded lessons serve as visual lesson plans that MTs created and constantly revised over two years. In this sense, our use of the word *improvement* is based on the assumption that all revisions made by MTs were intentional. We do not view these revisions as objective improvements, but rather as expressions of what the MTs considered improvement.

Theoretical Framing

This section describes how we theorize revisions in storyboards, and how we relate such revisions to MTs' opportunities for professional growth. Our focus on improvements in storyboarded lessons leads us to take a closer look at visual as well as written semiotic choices. By semiotic we mean the "intricate web of signs signifying signifiers" (Radford et al., 2008, p. vii). That is, we are looking for the meanings that are made through expressive choices in the two modalities (visual and written). To decode visual and linguistic meanings, we draw on aspects of systemic functional linguistics (SFL) (Halliday & Matthiessen, 2004; Matthiessen et al., 2010). While SFL originally offered a lens to explore meaning in language, various adaptations to other semiotic media (e.g., in children's picture books by Painter et al., 2013; or in film by O'Halloran, 2004) have allowed Herbst et al. (in press) to extend it to the interpretation of storyboarded lessons.

In SFL, a system can be thought of as a set of options that are available to construe particular meanings. Herbst et al. (in press) discuss several systems that use multimodal resources to construe meanings in the design of storyboarded lessons: Orientation, Visibility, Temporality, and Individuality. The system of *orientation* relates to the choices designers of storyboards make to tell viewers where to focus their attention. Similar to how cameras are used in filmmaking, this system includes the choices of which points of view will be used, and when the storyboard will zoom-in and out (see Figures 2b and 2c for different point of views). Another resource to orient viewers is the use of captions (see Figure 1a) that provide textual information which is not represented in speech bubbles, such as "five minutes later". A second system is *visibility*, which supports the construal of meanings by making classroom experience more or less visible to the viewer, to convey "what is important for the viewer to see" (ibid, p. 11). For example, the choice to inscribe a diagram on the board versus invoking it as present without actually providing it (e.g., by using the legend "diagram here"), or to transcribe a question that was posed by the teacher but only invoke how students responded (e.g., by having student captions "blah blah"). *Temporality* supports the development of a sense of timeliness and duration in the representation of a lesson (e.g., by ordering frames in a certain way, ordering speech bubbles within a frame, or using the gutter between frames to move time forward). A fourth system is *individuality*, which alludes to choosing how characters or artifacts will be made distinct from each other (e.g., by choosing how characters look like). To further interpret some of the linguistic choices we draw on Christie's differentiation (2002) between *instructional* and *regulative* registers in classroom discourse. The instructional register refers to the language, symbols, and visual representations involved in representing the mathematical content of the lesson (e.g., "Reflect point A over line 1", Figure 1b), while the regulative register refers to the language, symbols, and visuals the

teacher use to tell students what they are requested to do (e.g., "Read the instructions", Figure 1a). The systems noted above (orientation, visibility, temporality, individuality) characterize yet a third register, which projects the regulative and instructional registers when a lesson is represented for an audience (as opposed to enacted by its participants for instruction's sake), and that Herbst et al. (in press) name the *transactive* register.

As noted above, our second goal is to relate improvements in artifacts to MTs' professional growth. To do that, we draw on Clarke and Hollingsworth's (2002) interconnected model for professional growth, which consists of four domains of the teacher's work that interact with each other: the external domain (e.g., adoption of a new textbook), the personal domain (e.g., knowledge, beliefs, dispositions), the domain of practice (e.g., the actual teaching of a lesson), and the domain of consequences (e.g., students' interest in the subject). Our prior research showed how the work MTs do when improving storyboards is a form of "*virtual professional experimentation*" (Milewski et al., 2018, p. 111) that takes place in the domain of practice, but has consequences in other domains, especially in the internal domain (see Herbst et al., 2020). In other words, we showed how differences that can be traced during virtual professional experimentation are related to processes of professional growth. In this paper, we draw on the assumption that it is worthwhile to better understand the differences that can be accounted for in the domain of practice (e.g., improving the storyboarded lessons here), hypothesizing that observable changes in the domain of practice suggest that professional learning occurs.

Methods

Context and Data Collection

Between 2015 and 2017, a two-year practice-based PD was implemented to engage secondary MTs in designing and improving storyboarded representations of lessons that use reform-oriented instructional tasks. The goal they pursued was to demonstrate for other teachers how these tasks could be used in classrooms and the finished storyboards were meant to become part of a library of practice. Sixteen MTs from six schools in a Midwestern state were divided into four groups (two for geometry and two for algebra). In Year 1, each group was assigned four novel tasks and developed four storyboarded lessons (one for each task). In Year 2, representatives from the groups taught the lessons in their classrooms and later shared their experiences in the PD. These experiences were used to inform further revisions and improvements of the storyboarded lessons. The blended PD included yearly face-to-face meetings, monthly video-conference meetings, and weekly online forums. For this study, we focus on one geometry lesson, developed by six geometry teachers who had between 12 and 26 years of teaching experience. The main task of the lesson aims at students' understanding of composite reflections over two intersecting lines. Assuming that students had previously engaged with construction tasks that required transformations of objects in the coordinate plane, and in addition practiced proving, the lesson goal was to connect plane transformations and geometric proofs. The first part of the task (hereafter, construction task) gives a line and a point outside the line. Students are asked to draw a point on the line, connect the two points, and reflect the resulting segment across the line. This construction leads them to find out that the angle measures (between the segment and the line) are preserved and congruent (see the diagram on the board in Figure 1a). The following task (hereafter, proof task) reads: "The composite of two reflections over two intersecting lines produces a rotation centered at the intersection of the lines that is double the rotation of the first line to the second line." The attached diagram shows two intersecting lines and a point next to one of the lines (see the diagram on the board in Figure

1b). In a previous analysis of the interactions that occurred in this PD (Milewski et al., 2019) we found that MTs had trouble framing this task (because it is neither a proof nor a construction) when implementing the lesson in their classrooms (in the beginning of Year 2). Our goal for this analysis was to explore how coping with this challenge was apparent, if at all, in the improvement of the lesson over time. The data collected includes video records of group meetings and classroom implementations, forum logs and all artifacts, including versions of the lessons storyboard. The sources used for the current data analysis are two versions of the lesson storyboard from the end of Year 1 and Year 2 (comprising 12 and 20 frames, respectively).

Data Analysis

Our multimodal analysis looks at the use of written and visual modalities (Christie, 2002; Herbst et al., in press) drawing on SFL (Halliday & Matthiessen, 2004), and also informed by principles of visual data analysis (Kress & Van Leeuwen, 2020). Similar to the steps in the content analysis of written texts, visual data analysis comprises image description, segmentation, coding, memo writing, and iterative interpretation of these. In more detail, we have conducted the following steps: (1) storyboards are made of a sequence of frames. To find improvements, we compared the frames and sought for places where MTs made changes to frames or added frames. At this stage we found that no changes were made in the first nine frames of the storyboard, and so we focused on the 10th frame onward. We described what is happening in each frame. (2) Based on the descriptions we wrote; we identified which frames in one storyboard were comparable to the frames in the other. We deemed the frames comparable when the action taking place in each frame was similar (e.g., the teacher launches the task), and defined the pairs of comparable frames as our unit of analysis – how "same" periods in the lesson were represented in the different storyboards. For example, we found that a period that was represented with one frame in Year 1 was represented with three frames in Year 2. We also identified new periods in Year 2 that had no equivalences. This process of segmentation included creating tables that displayed the frames side by side and detailed the differences found between them. (3) We identified the semiotic changes in the comparable sets of frames, in relation to the semiotic registers. By analyzing the semiotic choices using the theoretical framing presented above, we found themes of improvements in the lesson (similar to the identification of themes in textual content analysis), on which we elaborate below.

Findings

The participants' semiotic choices elaborate the transition between a task in which students were asked to make a *construction* and a subsequent task in which they were asked to do a *proof*. As mentioned above, the lesson started with nine frames that narrate how the teacher launches the construction task and how the class makes sense of it. These frames were identical in Year 1 and Year 2 (Y1 and Y2, hereafter). The lesson refinement starts when the teacher launches the proof task, and is illustrated by the following improvements: (1) representing the teacher's address of students' confusion, and (2) representing the teacher's displaying of a student-generated diagram

Improvement #1: Representing the Teacher's Address of Students' Confusion

The first improvement is illustrated by three periods in the lessons that were revised between Y1 and Y2 (Figures 1, 2 and 3). In the following, we analyze each period by comparing and contrasting frames. The first change is the launch of the proof task (Figure 1): in Y1 the teacher asks the students to read the task instructions (Figure 1a), while in Y2 the teacher reads them (Figure 1b). This change is apparent in several semiotic choices: In terms of orientation, the additions to the caption in Y2 orient the viewers to notice that the teacher is the one who reads

the instructions. Another difference is in the use of registers in the teacher's speech bubble. In Y1 (Figure 1a) the teacher uses only the regulative register (asks students to to "read", "tell", and "show"), whereas in Y2 (Figure 1b) the teacher uses also the instructional register when reading the instructions ("Reflect point A over line 1 to A'"), that is, the viewers can see what mathematical actions the students are asked to do. While the storyboarded lesson in Y1 glosses over the details of the task, the Y2 lesson draws the viewer's attention to the details of the task, to make sense of the students' confusion that follows. Furthermore, there is a change in the system of visibility with the different display of a diagram in each frame. In Y1 (Figure 1a) the diagram on the board is a remnant of the previous phase of the lesson that appeared on the previous frames as well; in Y2, the teacher projects a new diagram which coheres with the proof task described in the teacher's speech bubble, illustrating a multimodal representation of the teacher's use of the instructional register. This diagram was handed to students as part of the task sheet.

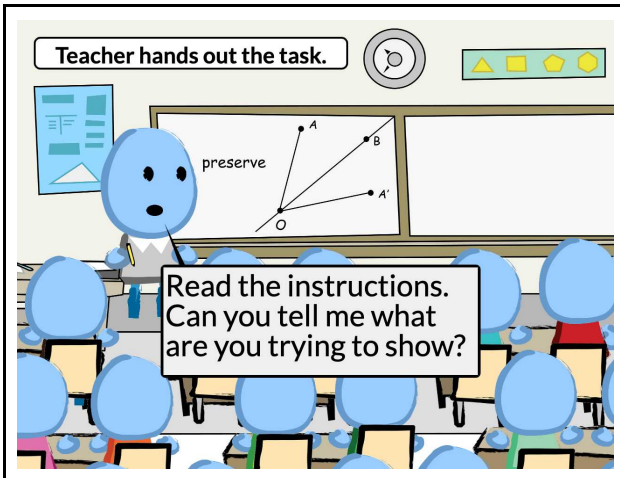

Figure 1a. Y1: Frame 10

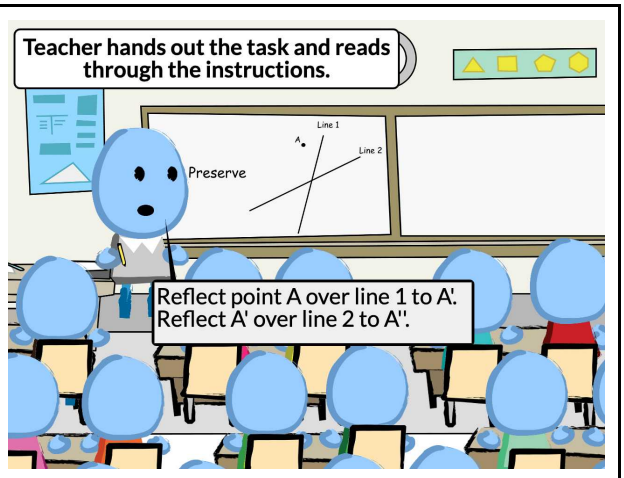

Figure 1b. Y2: Frame 10

Figure 1. Launch of the task.

The second comparison (Figure 2) elaborates what happens in the lesson once the task is read (either by the teacher or the students). In Year 2, the teacher asks the students to solve, and the viewers are told that the students are confused. MTs used several semiotic resources to draw attention to this confusion, changing or adding frames (including a different point of view), facial expressions, caption, and text. While in both Figures 2a and 2b the orientation is the student's point of view, focusing on the teacher – frame 11 in Y2 (Figure 2b) is zoomed in, the teacher is looking at the students in silence, which creates a dramatic pause in the lesson. This is followed by a frame (Figure 2c) where the orientation shifts to one similar to the teacher's point of view facing the students, but that includes the teacher as one being observed. For the first time in this storyboard, several students have different facial expressions (a choice to represent students' individuality). These facial expressions are explained by the caption, "some groups are confused." The students' confusion is presumably induced by the shifting from construction to proof without explicitly discussing it. The changed representation of time (temporality) – the elaboration of the period after the task was issued accomplished by adding an additional frame – lends even more emphasis to this challenging shift. In terms of visibility and temporality, some actions that could possibly happen in the lesson are not narrated: In Figure 2a, viewers do not see

if the students reacted to the teacher's question in the previous frame (Figure 1a). In Figure 2b, viewers do not see who added points A' and A'' to the diagram, yet they may guess that it was the teacher as they are in control of the board. The lack of dialogue adds more focus to other components of the storyboard (e.g., facial expressions, caption).

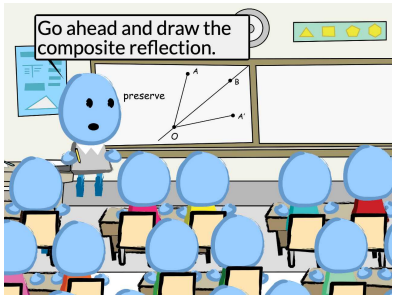

Figure 2a. Y1: Frame 11

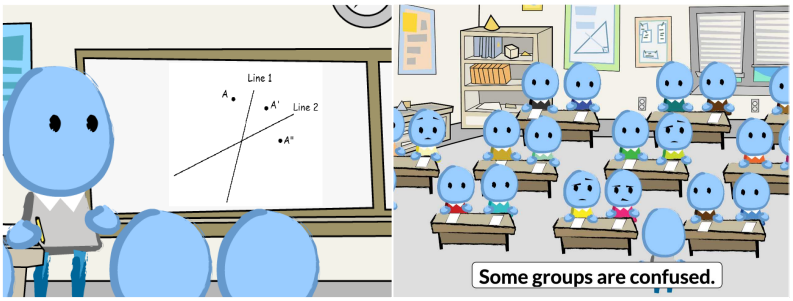

Figure 2b and 2c. Y2: Frames 11 and 12

Figure 2. The period in the lesson after the task was read.

The third comparison (Figure 3) shows the teacher walking around the room as the students engage in the proof task. A salient improvement made in this moment is the addition of dialogue and the camera's focus and zoom-in on the students and the teacher's faces (Figures 3b, 3c), as opposed to focusing only on the teacher in Figure 3a. Individuality is construed not only by using different facial expressions and color of the shirts, but also by distinguishing the students' interactions with the teacher: Yellow answers the teacher's question hesitantly, alluding to the angle as "something," while Pink seems more convinced and talks about angles. These individual characteristics display the heterogeneity of the class, since the students are not represented as a monolithic whole. In addition, some linguistic choices were made to convey the interpersonal teacher-students relationships: In Figure 3b, the teacher asks "what are *we* trying to show?", suggesting that making sense of the task is a joint effort. In Figure 3c, the caption says that the teacher "addresses confusion", while the teacher is asking a question – which means that asking questions is the teacher's method of addressing confusion (instead of, for example, answering).

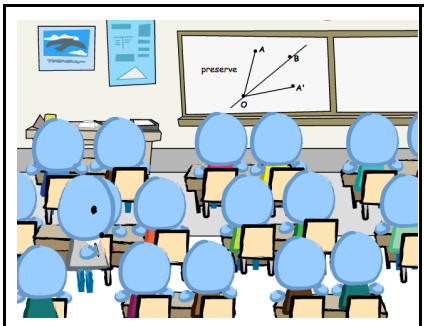

Figure 3a. Y1: Frame 12

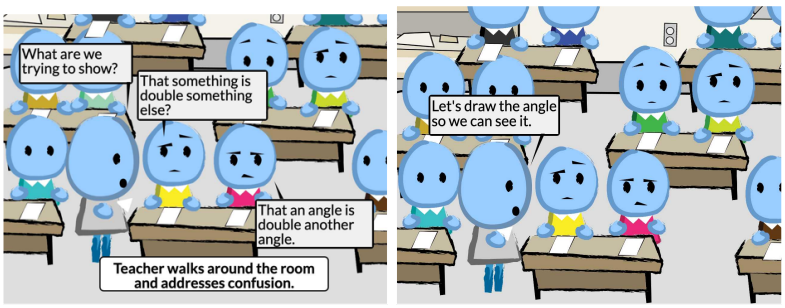

Figure 3b and 3c. Y2: Frames 13 and 14

Figure 3. Teacher walks around the classroom.

Improvement #2: Representing the Teacher's Displaying of a Student-Generated Diagram

The second improvement is the added representation of a discussion around student-generated diagrams in the storyboarded lesson in Y2 (see Figure 4). While this part of the lesson was not represented in Y1, it was represented using six frames in Y2. In the following, we analyze the first three frames of this period. The teacher is inviting a group to share their work, which the teacher deliberately selected when walking around the room (Figure 4a). Then, the teacher labels the diagram, aiming at creating a shared language (Figures 4b, 4c). Comparing these frames to the Y1 storyboard, which lacked representation of how the task is handled, suggests that over Y2 the MTs decided that the intricacies of the discussion around student work should be spelled out through storyboarding. Since each student drew their own diagram, the lesson improvement is manifested in detailing the processes in which the teacher turns this diversity of expressions into a coherent discussion. In what follows, we detail how semiotic choices led us to conclude this.

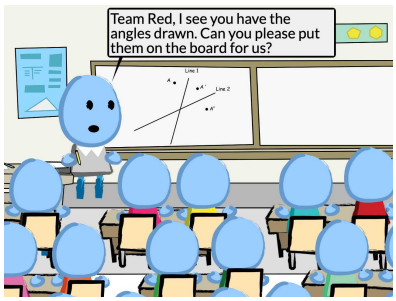

Figure 4a. Y2: Frame 15

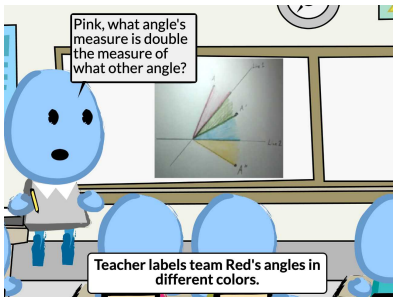

Figure 4b. Y2: Frame 16

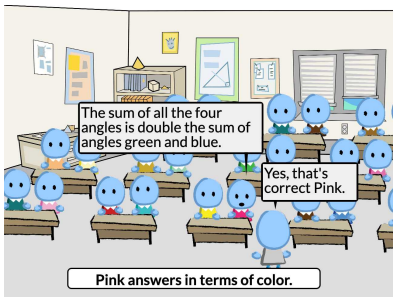

Figure 4c. Y2: Frame 17

Figure 4. Teacher labels and presents a student-generated diagram.

The change in orientation between Frame 14 (Figure 3c) and Frame 15 (Figure 4a) indicates the transition from work time to classroom discussion. In Frame 15, The speech bubble indicates the teacher's specific request for Team Red to share their work, rather than inviting any student to do so. The teacher words ("for us") aims at building interpersonal connections. The teacher is also attending to the specific mathematics that the class should focus upon: "the angles drawn". which is another evidence for the teacher's language becoming more subject-specific. In Frame 16 (Figure 4b) we see changes in orientation by zooming in on the teacher and the diagram and adding a caption. The caption provides a sense of timeliness, explaining that the diagram was already put on the board and that the teacher was the one who colored it. This draws viewers' attention to the actions that the teacher is making to support the discussion. In terms of visibility, the choice to include the diagram on the board means that it is important for the viewers' understanding of the course of the lesson. The teacher's speech bubble shows that they ask Pink directly to use the colors in the diagram to answer a question they were previously asked, in Frame 13 (Figure 3b). The repetition of the same question, yet with more precise fashion, draws viewers' attention to the importance of precision in the process of proof. In the next frame, the orientation changes, which lets viewers see that the students' facial expressions are all the same, possibly suggesting they are no longer confused. The caption "Pink answers in terms of color," together with Pink's speech bubble, illustrate how coloring the diagram was a key decision for overcoming the confusion: By using the colored diagram, Pink is able to say what is to be proven. The teacher reassures this by saying "Yes, that's correct, Pink." Pink's emerging ability to use precise language suggests that the teacher's moves in Frames 10-17 were productive.

Discussion

This study set out to examine what MTs perceive as improvements in a representation of a geometry lesson. By comparing two versions of a storyboarded lesson, we identified some of the semiotic changes that MTs made over time which allow us to articulate what was improved in the lesson. We found that the improved representation of this lesson, as was reflected in the changes in the artifact, included not only a detailed depiction of how the teacher launches a task, but also of the students' reaction to the task and how the teacher addresses their confusion or other complexities that the task brings up. We identified two themes which overall suggest that the MTs were interested in representing the shift from a construction situation to a proof in a greater depth and resolution. In particular, the MTs improved the lesson in ways that drew viewers' attention to the students' confusion and the teacher's moves in response to the confusion (first improvement), and to the teacher's way of creating a shared language about the diagram during the whole-class discussion (second improvement). These foci were reflected in the following semiotic choices: addition of frames, inclusion of different points of view (camera positions), zooming in on the teacher or students, addition of captions that oriented viewers to notice particular details, addition of students' facial expressions and speech bubbles, and more.

Furthermore, a noticeable change is that the students' perspective on the task is represented in the Y2 version. The lesson revision through storyboarding revealed an aspect of improvement that is often subtle in spoken/written discourse: The added frames showed that an enhanced noticing of students' perspectives (including their puzzlement and quandaries) is an essential component of the teacher's expertise. This finding, related to the conference theme regarding MTs' attention to student engagement, reminds us that a novel task coupled with the teacher's good will are not sufficient to ensure productive student engagement in a problem-based lesson. The Y2 lesson suggests that the anticipation of student confusion is crucial for deciding on the moves and scaffolds that the teacher uses. Another feature of the suggested improvements is the increased specification of the mathematical content in the teacher's talk, illustrated by the increased use of the instructional register. We argue that using a more subject-specific language when storyboarding suggests an increased engagement with the mathematical details of the lesson. Thus, it points to how, during the PD, the domain of practice shapes and is shaped by the participants' internal domain, since their tacit knowledge about launching tasks and address students' confusion was shared with others and represented in the artifact. We argue that this kind of work with MTs enables them to attend more to "the detailed and intricate nature of practice" (Ball & Forzani, 2009, p. 507), thus, it contributes to their professional growth. This illustrates how professional learning environments can help MTs address student engagement.

Theoretically, this paper contributes an analytic description of semiotic choices, drawing on methods from SFL and its extensions. The question we raised – how to account for changes in artifacts and relate them to professional growth? – is yet to be fully answered. We aim for this work to serve as a foundation for future analyses of storyboarded lessons and other artifacts like lesson plans. This would be a fruitful area for further work that uses multimodal data and multiple teacher groups and lessons to conceptualize MTs' professional growth. The main limitation of this study is the focus on one lesson. Although the corpus includes additional data that could be analyzed similarly, we chose to focus on one lesson to ensure that we provide enough contextual details and are able to present the analyzed frames. Despite this limitation, this study offers insights into how MTs operationalize the goal set to them of improving a lesson.

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