DESIGNING INSTRUCTIONAL RESOURCES TO SUPPORT TEACHING

EL DISEÑO DE RECURSOS EDUCATIVOS PARA APOYAR A LA ENSEÑANZA

José Luis Cortina
Universidad Pedagógica Nacional, México
jcortina@upn.mx

Jana Višňovská
The University of Queensland, Australia
j.visnovska@uq.edu.au

We discuss the importance of bringing teaching to the forefront of instructional design. We do so by describing the process of developing an instructional sequence for early number, using design research. The instructional sequence was developed with the specific aim of supporting teaching, conceived as a complex and demanding job, not reducible to predictable routines. The sequence has caught the interest of an unexpected number of teachers in Mexico. We have followed up with some of them and have documented that the resource has benefited their practice significantly. In our account of the design process, we highlight what—from a theoretical point of view—we have come to regard as three guiding ideas that are central to designing for supporting teaching: (1) designing for a resource to be viable in teachers' classrooms, (2) designing for a resource to be regarded by teachers as relevant to their practice and (3) designing so that a teacher who has just taken an interest in a resource might fruitfully engage with it in her practice.

Keywords: Number Concepts and Operations, Instructional Activities and Practices, Teacher Knowledge.

In the development of resources to support mathematics instruction in schools, instructional designers must necessarily take a position regarding both learning and teaching. Whether implicitly or explicitly, multiple considerations come into play not only about how students are to learn mathematics, but also about the roles that teachers are to play in instruction. A strong inclination has been to prioritize in the designs issues related to student learning. This, of course, is an easily justifiable position since the improvement of students' mathematics learning is central to our aims as mathematics educators. However, this inclination carries the risk of overlooking teaching and even misconstruing it, jeopardizing the suitability of instructional design products to help improve mathematics education.

Commonly, resources are designed with the expectation of them being capable of adequately supporting students' learning, if administered appropriately. Many are developed through research and are based on rather complex and robust conceptualizations of mathematics learning. In contrast, not much attention is typically given in the design process to how teaching is conceptualized. Often, it ends up being framed as a practice in which compliance and adherence are considered to be essential. At the least, it is commonly expected that, for the use of the resource to render positive results, teachers would have to adhere to the indications of the developers. Hence, in instructional design, teaching is frequently conceptualized as a practice of a predominantly administrative and organizational nature, where teachers are seen to be operating within what Biesta (2007) refers to as "a causal model of professional action" (p. 7).

The framing of teaching as a mainly administrative and organizational occupation stands in sharp contrast with how research in the field has come to understand it. It has been shown that teachers necessarily shape how instructional activities play out in classrooms (Brown, 2009; Gueudet & Trouche, 2012; Pepin, 2018), and decide how student learning ultimately gets to be supported (Biesta, 2007; Dewey, 1997; Lampert, 2001). It has been also shown that good mathematics teaching is demanding, uncertain, and not reducible to predictable routines (Ball & Cohen, 1999; Lampert, 2001; Schifter, 1995). This is due largely to the relational and adaptive nature of teaching. Teaching mathematics well "necessarily requires that teachers teach in response to what students do as they engage in solving mathematical tasks"(Jackson et al., 2013, p. 647).

The aforementioned research makes it reasonable to question the value of instructional resources that are designed under the assumption that teaching is a predominantly administrative and organizational occupation. The use of resources that are based on an erroneous conceptualization of teaching might not lead to the improvement of mathematics instruction. But would it even be possible to design resources in which it is assumed that teachers are the ultimate producers of instruction? If so, could the design of such resources contribute to the substantial improvement of the teaching and learning of mathematics in classrooms?

For several years, our work as researchers and instructional designers has focused on exploring what the development of resources for supporting mathematics teaching entails. These are resources to be used by teachers to support their students' learning, developed with the understanding that good mathematics teaching requires functioning effectively in uncertain and indeterminate situations, where it is necessary to constantly make autonomous judgments, based on a pertinent rationale (see Hoyle, 2008).

In the paper, we first explain the research approach we have taken in developing the instructional sequence, which is based on the methodological principles of design research. Next, we give an account of the process of developing of the instructional sequence, which involved three research cycles, each of which led us to develop theoretical insights into the design of instructional resources to support teaching, and to modify the instructional sequence. At the end of paper, we discuss how, of the three guiding ideas, we came to regard the second as the leading one: designing for a resource to be regarded by teachers as relevant.

Research Approach

Our research has been conducted following the general guidelines of design research (e.g., Bakker, 2018; Cobb et al., 2003), where we have pursed both, the crafting of an educational innovation and the development of theory. It has consisted, so far, of three research cycles. Each cycle has been conducted with the purpose of both improving an instructional sequence on early number and of developing and refining theoretical ideas. In the following three sections of this paper, we explain what each of these cycles has entailed.

First Research Cycle, a Dual Classroom Design Study

The first research cycle in the development of the instructional sequence consisted of conducting a *dual* classroom design study (Gravemeijer & van Eerde, 2009), in a third-grade preschool classroom (equivalent to kindergarten in the USA). The design study was conducted in collaboration with Jesica, a preschool teacher who at the time was enrolled in a Master program and was being supervised by the first author. The decision to conduct the design study was prompted by Jesica's concerns about the lack of success she had had in teaching her preschool students to solve additive word problems, required by the Mexican curriculum.

The first two authors were familiar with the resource known as the *Patterns and Partitioning* instructional sequence (P&P; Cobb, Boufi, et al., 1997; Cobb, Gravemeijer, et al., 1997; McClain & Cobb, 1999), which had been demonstrated to be viable in supporting students to develop sophisticated early number ideas. These include developing notions that allow them to solve simple additive problems by reasoning about composing or decomposing quantities, instead of by counting by ones (Steffe, 1992; Steffe et al., 1988). Although the P&P was developed as part of the initial phase of a classroom design study conducted in a first-grade classroom, its designers considered that it could be a valuable instructional resource in kindergarten (McClain & Cobb, 1999).

The P&P was designed to support the collective development of early number ideas by providing opportunities for students to reason about patterns and partitions of collections of up to ten items, leveraging whole-class discussions. For example, in one of the initial classroom activities, students are supported to develop familiarity with pairs of numbers that add up to five, by discussing the different ways in which five monkeys could be in two trees .

The dual classroom design study was conducted with two chief goals. The first entailed investigating the practicality of using the P&P in a classroom like Jesica's. This goal included inquiring about the adaptations that would be necessary to make to the instructional sequence to increase its possibilities of becoming a viable resource in this type of classroom.

The second goal was to inquire about Jesica's educational practice in relation to the use of the P&P. Among other things, we planned to investigate how she would make sense of the instructional sequence, whether she would regard it as a useful resource and, if so, how, and why. In addition, we wanted to find out if the use of the sequence would lead to positive changes in her practice. If so, what would those changes be, and which elements of the instructional sequence would have favored them.

In preparing for the classroom design study, we assessed the students' elementary understandings of the number, that were prerequisites for children to be able to productively engage with the instructional tasks of the P&P sequence. It was found that the vast majority of Jesica's students had not yet developed those elementary understanding. Some children in Jesica's classroom were only successful with the word number sequence up to three and could correctly identify the names of only one or two single digit numerals (Cortina & Peña, 2018; Peña, 2018).

The results of the assessment presented the research team with two problems directly related to developing instructional resources for supporting teaching. The first one concerned the viability of the resource. The activities proposed at the starting point of the P&P sequence did not connect to what students in a classroom like Jesica's already understood about numbers and could do. Clearly, the P&P sequence could not readily be used in a classroom like Jesica's, if expected to be a resource that would help a teacher in supporting the development of numerical notions in their students.

In addition, the resource offered no guidance as to how it would be viable to start to work with children in such a classroom, so that they could be supported to eventually become readily capable of participating in the instructional activities of the P&P sequence. This meant that the P&P sequence not only lacked prompt viability in Jesica's classroom but also relevance for a teacher like her. A teacher in Jesica's position could reasonably—and correctly—consider the P&P sequence to be unsuited to her teaching practice, given the educational profile of the students with whom she worked.

The results of the assessment led us to significantly modify the instructional sequence, in an attempt to make it viable in Jesica's classroom and relevant for her. The starting point was changed and a whole new phase was included at the start, aimed at supporting children's development of essential number notions. The version of the P&P Instructional Sequence that was used in the classroom design study, with the modification we included, is presented in Table 1.

In an intensive out-of-classroom collaboration with the first author, Jesica conducted the classroom design study in her preschool (Peña, 2018; Peña et al., 2018). The first author and Jesica met after each lesson to analyze classroom events and co-design the following classroom activities. During these interactions, the first author both supplied elaborations of the rationale that justified the instructional sequence and kept a record of what clarifications and elaborations were needed.

Table 1. The First Version of the Modified P&P Instructional Sequence

Phase	Overarching teaching goal	Specific learning goals
1	*Support the development of the essential number understandings up to five*	*Master the number word sequence* *Enumerate with one-to-one correspondence* *Use fingers to represent numbers* *Identify the names of written numerals*
2	Support students' reasoning about patterns and partitioning with numbers up to five	Reason about (and subitise) spatial patterns Reason about (and subitise) finger patterns Reason about number partitions in the 10-frame Subitise and reason about spatial patterns in the 10-frame Reason about how to solve arithmetic problems by composing or decomposing quantities, instead of by counting by ones
3	Support the development of the essential number understandings up to ten	Master the number word sequence Enumerate with one-to-one correspondence Use fingers to represent numbers Identify the names of written numerals
4	Support students' reasoning about patterns and partitioning with numbers up to ten	Reason about (and subitise) finger patterns Subitise and reason about spatial patterns in the 10-frame Reason about number partitions in the rekenrek Reason about how to solve arithmetic problems by composing or decomposing quantities, instead of by counting by ones

In an intensive out-of-classroom collaboration with the first author, Jesica conducted the classroom design study in her preschool (Peña, 2018; Peña et al., 2018). The first author and Jesica met after each lesson to analyze classroom events and co-design the following classroom activities. During these interactions, the first author both supplied elaborations of the rationale that justified the instructional sequence, and kept a record of what clarifications and elaborations were needed.

The classroom design study consisted of 21 instructional sessions that were taught over a 5-month period. In Phase 1, the teacher supported collective engagement in repeated counting with words and symbols through activities such as singing number songs, playing number-word games and board games. In Phase 2, for example, students reasoned about number partitions up to five in the 10-frame with a narrative involving a watermelon stall with two decks (see Figure 1, left). They were asked to advise a teacher's friend on arranging a number of watermelons on her market stall. While students proposed different arrangements, the teacher kept a record of these on the board, specifying how many watermelons would be in the top and bottom decks (see Figure 1, right).

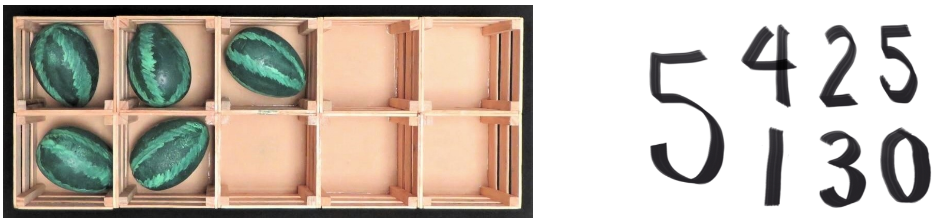

Figure 1. A watermelon stall and the capture of the record the teacher kept.

Phase 3 included similar activities to Phase 1, but with number words, collections, and written numerals up to 10. Phase 4 culminated in students reasoning about composing and decomposing quantities when solving problems about passengers getting on and off a Tour Bus.

> The bus left a park with 4 tourists, made a stop at a museum, and arrived at the destination with 10 tourists onboard.

When the teacher first asked the whole class to explain what happened at the museum, it was considered obvious that more tourists had boarded the bus. Lupe explained: "Six got on because six are missing for ten". Hernan, when asked whether he understood Lupe's response, said: "Yes! Six are missing because there were four, and six are ten". Both Lupe and Hernan were amongst the children who showed the least understanding of early number at the beginning of the classroom design study. Their responses to the problem illustrate how the great majority of the students not only came to solve rather advanced additive problems correctly, but how they did so by reasoning composing or decomposing quantities, not by counting by ones.

Our analysis indicated that the modified instructional sequence was viable for the targeted preschool classrooms (Peña, 2018; Peña et al., 2018). At the completion of the classroom design study, the team members had a strong practice-embedded understanding of how the designed resource could be used to support the reasoning about patterns and partitions up to 10, in educational settings like the one in which Jesica taught.

Second Research Cycle, Developing an Online Resource

The second research cycle was the result of the two authors remaining in communication with Jesica and maintaining a collaboration. After concluding her master's studies, Jesica started to teach in a new school. There, she was questioned by her concerned principal and colleagues about why she did not follow the recommendations of the Ministry of Education regarding how to teach mathematics in preschool. She was using whole class activities rather than small group work, and was focusing on small numbers instead of "maintaining the challenge" by teaching tasks with larger numbers. Instead of responding to pressure by reverting to institutionally legitimate forms of teaching, Jesica defended her teaching decisions by referring to her students' initial assessments, the results of the classroom design study in which she had participated, and the research literature.

Her teaching soon became of interest to her principal and supervisor when they realized how Jesica's students became much more eager to participate in mathematics than was typical in the classrooms of the school zone. Jesica was asked to give short workshops during the staff meetings, both at the school and the school-cluster levels. Several of her colleagues started to approach her for advice. We understood that Jesica alone could not provide support to her colleagues comparable to that she had received during her Master study. Hence, the aim of the second research cycle was to develop resources that could help Jesica in her efforts to support her colleagues in improving their early number instruction.

A website was created (www.sentidonumerico.com) with online resources intended to aid Jesica in leading the workshops and helping the teachers in their planning and decision making. We understood that if the website was to become a resource capable of *participating with teachers* on their selection of goals and tasks that were suitable for their students, our first job was to facilitate its plentiful, independent, and confident use by teachers. We assured her that the website was designed well for Jessica's colleagues most accessible, and often the only, computer: a smartphone. It included descriptions and short videos of classroom activities, and printable tools that teachers could adapt and use in their classrooms (e.g., game boards). In addition to the immediate physical accessibility of the tasks and tools, our aim was to make the conceptual resources accessible. Each activity was thus labelled to identify a specific learning goal and a phase of the sequence (see Table 1).

Based on access data and teachers' questions, we noticed that Jesica's colleagues were much more interested in the classroom activities aimed at supporting essential number understandings (Table 1, Phases 1, 3), than in the rest of the online resources. They readily recognized the importance of their students developing basic counting skills, despite this not been a priority explicitly stated in the Mexican curriculum. In contrast, they did not recognize the relevance of the learning goals within P&P phases of the sequence and found it difficult to justify the considerable investment of time and effort that pursuing these goals required.

The contribution of P&P activities to supporting early number learning was the key innovation of the resource. Like any substantial teaching innovation, this presented teachers with what research on implementation refers to as an ecological disruption (Koichu et al., 2021). Unsurprisingly in hindsight, the teachers initially responded to this by non-participation with the innovative parts of the resource.

The research team construed this situation as a design problem that entailed both the relevance of the resource and its clarity, for the teachers. The resolution required finding ways of communicating effectively that the innovation addressed problems that the teachers already considered relevant to their teaching, and how it did so.

It is worth clarifying here that, over several decades, "problem solving" has been a key goal in the Mexican mathematics curricula (Secretaría de Educación Pública, 2011), including at the preschool level. Not surprisingly, the concern for making students problem solvers has entered the prevailing teacher culture. We knew that there was much frustration amongst Jesica's colleagues, because only a few of their students ever became proficient in solving word problems. In addition, it was apparent to us that if the teachers were to focus solely on supporting the development of essential number understandings, their students would not come to solve the Tour Bus problems with flexibility when larger numbers were involved, as the only number patterns at their disposal would be those of sequential order (i.e., counting up and perhaps down by ones, cf. Graven, 2016).

Our instructional design challenge thus became to find ways of adjusting the instructional sequence so that the teachers could more easily recognize the close relation between the concerns they already had regarding their students' difficulties with problem solving and the pursue of the P&P objectives. We made three modifications to the instructional sequence with the expectation that it would help the teachers to recognize that (a) by supporting their students to reach the key learning goals of the instructional sequence, they would be providing them with valuable means for problem solving, and (b) while the essential number understandings were necessary for becoming a proficient problem solver, they were not sufficient.

The first modification involved renaming "number understandings" to "number skills", to align with the language in which teachers made connections to their practice. The second modification involved renaming the teaching goals as addressing "basic" vs. "advanced" number skills (see new Phases 2 and 3 in Table 2). This removed the non-transparent "patterns and partitioning" language and presented the advanced number skills as a continuation of the basic skills, aimed at enhancing students' problem solving beyond the activity of counting. The third modification involved downgrading the Phase 3 (Table 2) into a transition stage to further support teachers in recognizing the advanced number skills goals as the key ones they needed to support.

Table 2. The Second Modification of the P&P Instructional Sequence

Phase	Overarching teaching goal	Specific learning goals
1	Support the development of the essential number understandings up to five	Master the number word sequence Enumerate with one-to-one correspondence Use fingers to represent numbers Identify the names of written numerals
2	Support students' reasoning about patterns and partitioning with numbers up to five	Reason about (and subitise) spatial patterns Reason about (and subitise) finger patterns Reason about number partitions in the 10-frame Subitise and reason about spatial patterns in the 10-frame Reason about how to solve arithmetic problems by composing or decomposing quantities, instead of by counting by ones
3	Support students' reasoning about patterns and partitioning with numbers up to ten	Reason about (and subitise) finger patterns Subitise and reason about spatial patterns in the 10-frame Reason about number partitions in the rekenrek Reason about how to solve arithmetic problems by composing or decomposing quantities, instead of by counting by ones

Formal and informal data indicated to us that the modifications encouraged considerable increase in teachers' engagement with both the advanced number skills activities and the rationale of the instructional sequence. However, our aim here is to illustrate the type of criteria that were followed when trying to design a resource to support teaching, when teaching is understood to be demanding, complex and independent, and teachers are considered to be the ultimate producers of instruction. Our design failure, which manifested as teachers' lack of use of the key parts of the resource, shows where the designers' work was unfinished. As we have already illustrated, the resource was not ready, for teachers who had not participated directly in its development, to be easily recognized by them as worthwhile of fully engaging with it.

Third Research Cycle, Adjusting the Instructional Sequence and the Online Resources

In facilitating teacher workshops with Jesica, we became aware that there was one issue underlying the design of the sequence that was far from being clear to the teachers, namely, that it was critical for children to experience fun, joy, belonging and success as they got involved in the instructional activities, particularly at the beginning of the instructional sequence (Phase 1, see Table 2). During the classroom design study, we had attempted to ensure that the children had these kinds of experiences, regardless of how competent they were.

The modifications that we made to the instructional sequence during the second research cycle, seemed to make it easier for teachers to recognize the importance of supporting their students' development of relatively complex number skills with numbers up to ten. However, they seemed to think that the pursue of such a goal would require a kind of instruction in which they would have to be instructing and correcting the children, constantly. The instructional activities of Phase 1 (see Table 2) were based on stories and games, and when Jesica used them, she focused on conveying to her students that they were good at what she was asking them to do. In the case of Jesica's colleagues, it seemed that engaging students in mathematical activities expecting the children to enjoy them, and without correcting their mistakes, presented a significant pedagogical innovation.

From a learning perspective, our consideration about trying to ensure that children experienced fun and joy, when engaging in mathematics instruction, was justified in the literature (Boaler, 2019; Parks, 2020). In addition, we viewed teaching reliant on correction of errors as a case in point of what Rancière (1991) refers to as stultification in schooling: a mechanism through which students are subtly but persistently positioned as incompetent and lacking. At this point in the development of the instructional sequence, we became aware that supporting students to experience fun, joy, belonging and success was also of critical importance for teaching. As we explain next, it was directly related to the possibility of the instructional sequence being viable in the teachers' classrooms and, also, relevant to them.

Students' willingness to engage in mathematical activities makes teaching more manageable for the teacher, and the teacher more likely to participate with the resource. Conversely, if students come to view mathematical activities as tiresome, and themselves as incompetent and lacking, they will probably become reluctant to participate. This could easily create classroom management complications and lead a teacher to consider an instructional resource as unfitting for her class.

Once again, we construed the situation as being an instructional design problem, one that concerned the clarity of the instructional sequence and also, as we already explained, its viability and its relevance. We realized that the sequence of goals (Table 2) that described the instructional sequence addressed only mathematical content, keeping the forms of classroom engagement hidden. We then decided to include a new phase at the start of the in the instructional sequence (Phase 0; see Table 3), where the main goal is to support children's willingness to engage in early number activities, and enjoyment of doing so. We developed activities and teaching routines aimed at supporting teachers in coming to value and pursue this initial teaching goal based on noticing their students' enthusiasm.

Encouraging Results

At the current stage of our research project, we feel that there is still much that we need to understand in relation to designing resources that support teaching. However, there have been promising developments that suggest that we are on a proper path. Some of them are related to the interest that Jesica's colleagues have shown in the instructional sequence, which we have already mentioned. Others are related to later developments that have taken place on a larger scale.

Table 3. The Final Modification of the P&P Instructional Sequence

Phase	Overarching teaching goal	Specific learning goals
0	Support the development of an interest in and a taste for counting and numbers	Become interested and show joy when engaging in activities that involve counting or working with numbers

The last version of the instructional sequence has caught the interest of an unexpected number of teachers. In 2020, after Mexican schools closed for COVID-19, we started to collaborate with several teaching organizations and offered intensive online workshops, organized in 2-hour increments over three consecutive days. Although we do not know how the attending teachers adapted the designed resource in their teaching, they valued the experience positively, to a surprising degree. One of the workshops was attended by 850 teachers who were present during all three days, even though no external incentives were provided to participate. The Facebook community that we created to keep in touch with the teachers has reached 7000 members.

In the summer of 2022, we offered a 27-hour online teacher professional development course, to be delivered in eleven Saturday sessions, throughout the 2022-2023 school year. The course was advertised in the Facebook community. We designed the course contemplating that we would have between ten and fifteen participants. However, the number of applications exceeded one hundred. We decided to select thirty participants, giving priority to the teachers who worked in public school and, amongst them, those who worked in schools located in rural areas, where professional development opportunities are scarce.

At the time of writing this paper, nine of the eleven sessions have been held. There are twelve teachers who have participated in all the sessions. Overall, the course has helped us identify issues worth looking at closely, to improve the way in which the instructional sequence is formulated, and also the suitability of the resources we provide teachers online. It has also helped us to identify clearly that the instructional sequence can become a valuable resource for teachers who work in significantly different kinds of schools, one that helps them make judgments about how their groups are progressing, provides them with a pertinent rationale for deciding how to continue the educational work, and offers them clear suggestions about the activities, manipulatives and other means that they can employ in pursing specific learning goals.

Discussion

Our work as researchers and instructional designers has focused on exploring what the development of resources for supporting mathematics teaching entails, when good mathematics teaching is understood as being independent, intellectually demanding, and complex. The resources we seek to develop would be for teachers to use, with their full consent, and they would be regarded by teachers as useful and beneficial to achieve their educational endeavor.

Our research has led us to propose three theoretically oriented ideas to guide the design, refinement, and improvement of these types of resources: (1) designing for a resource to be viable in teachers' classrooms, (2) designing for a resource to be regarded by teachers as relevant to their practice and (3) designing so that a teacher who has just taken an interest in a resource might fruitfully engage with it in her practice. In retrospect, we recognize the second as being the leading idea. In fact, the importance of the other two can be understood in terms of how they can make a resource come to be regarded by teachers as relevant.

Concerning the first guiding idea, teachers would not be likely to consider a resource as relevant if when trying to engage with it, they find that it is incompatible with their students' current mathematical abilities, or with their real possibilities of progress. Teachers will also not likely consider a resource as relevant if they find that the proposed means of support are not well suited to achieve the specified learning objectives. In other words, a resource that is not viable in a teacher's classroom, is unlikely to be regarded as relevant, at least in the long run.

In developing resources to support teaching, it is thus important that they be crafted through a careful and rigorous instructional design process, such as that involved in conducting a classroom design study. The reader will recall that the P&P sequence was developed as part of a classroom design study, and the adaptations we made to it during the first research cycle were the result of another study of the same type.

But the viability of an instructional resource might be necessary condition for it to be relevant for teachers, but not sufficient. A resource might not be regarded by teachers as worthwhile of expending time and effort to engage with it and learn how to use it, if they do not see it as useful in pursuing goals they already consider important—regardless of how rigorous the research process of developing it was, how innovative it is, and that it has been used successfully in other classrooms. Hence, as we did during the second research cycle, the development of a resource to support teaching may require extra design efforts, tailoring it so that teachers can clearly recognize it as worthwhile. To be sure, this redesign process should not harm the viability of the resource, or else its relevance would be compromised.

The importance of the third guiding idea can also be seen in terms of relevance to the teacher. From the perspective of developing resources to support teaching, the proposed starting point of a resource, crafted in the form of an instructional sequence, must not only be viable in terms of students' current capacities to participate in the proposed activities. The activities must also be viable in terms of teachers' current capacities to engage with them productively. If a teacher can not readily engage in the use of an instructional sequence in a way that is productive and meaningful to her, it is likely that she will not regard it as worthwhile.

Conclusion

Instructional design has long been seen as a good way to seek improvements in mathematics education. For the most part, the focus has been placed on students' learning. Resources have been developed with the intention of making it possible to recreate in classrooms what research has found to be important in learning mathematics, both in general and of specific fields and ideas. Teachers, for their part, have been commonly seen as those responsible for implementing the innovations. They have been expected to have faith in them and to be keen to assume the expenditures of time, effort and learning that incorporating the innovations into their teaching will involve.

In this paper we have introduce a different approach to instructional design, one that entails considering that the successful support of mathematical learning requires good teaching, and that this kind of teaching is achieved by teacher who act with conviction, will, judgment, and autonomy. This generates new challenges for instructional design. As we have explained here, these challenges include ensuring that the designed resources can be recognized by teachers as relevant to their teaching, that they are viable in teachers' classrooms, and that they allow teachers to engage fruitfully with the resources, from the start.

References

Bakker, A. (2018). Design research in education: A practical guide for early career researchers Routledge. https://doi.org/10.4324/9780203701010

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Towards a practice-based theory of professional education. In G. Sykes & L. Darling-Hammond (Eds.), Teaching as the learning profession: Handbook of policy and practice (pp. 3-32). Jossey-Bass.

Biesta, G. J. J. (2007). Why "what works" won't work: Evidence-based practice and the democratic deficit in educational research. Educational Theory, 57(1), 1-22. https://doi.org/10.1111/j.1741-5446.2006.00241.x

Boaler, J. (2019). Developing mathematical mindsets: the need to interact with numbers flexibly and conceptually. American Educator, 42(4), 8-33.

Brown, M. W. (2009). The teacher–tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), Mathematics teachers at work (pp. 17-36). Routledge. https://doi.org/10.4324/9780203884645

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. Journal for Research in Mathematics Education, 28, 258-277. https://doi.org/10.2307/749781

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in education research. Educational Researcher, 32(1), 9-13. https://doi.org/10.3102/0013189X032001009

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification in one first-grade classroom. In D. Kirshner & J. A. Whitson (Eds.), Situated cognition: Social, semiotic, and psychological perspectives (pp. 151-232). Lawrence Erlbaum.

Cortina, J. L., & Peña, J. (2018). Nociones numéricas de alumnos mexicanos de tercero de preescolar (Numerical notions of Mexican third-year preschool students). Educación Matemática, 30(3), 103-123. https://doi.org/10.24844/EM3003.05

Dewey, J. (1997). Experience and education. Free Press.

Gravemeijer, K., & van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. The Elementary School Journal, 109(5), 510-524. https://doi.org/10.1086/596999

Graven, M. (2016). When systemic interventions get in the way of localized mathematics reform For the Learning of Mathematics, 36(1), 8-13.

Gueudet, G., & Trouche, L. (2012). Teachers' work with resources: Documentational geneses and professional geneses. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), From text to 'lived' resources: Mathematics curriculum materials and teacher development. Springer. https://doi.org/10.1007/978-94-007-1966-8

Hoyle, E. (2008). Changing conceptions of teaching as a profession: Personal reflections. In D. Johnson & R. Maclean (Eds.), Teaching: Professionalization, Development and Leadership (pp. 285–304). Springer Dordrecht. https://doi.org/10.1007/978-1-4020-8186-6_19

Jackson, K., Garrison, A., Wilson, J., Gibbons, L., & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. Journal for Research in Mathematics Education, 44(4), 646-682 https://doi.org/10.5951/jresematheduc.44.4.0646

Koichu, B., Sánchez Aguilar, M., & Misfeldt, M. (2021). Implementation-related research in mathematics education: The search for identity. ZDM Mathematics Education, 53, 975–989. https://doi.org/10.1007/s11858-021-01302-w
Lampert, M. (2001). Teaching problems and the problems of teaching. Yale University Press.
McClain, K., & Cobb, P. (1999). Supporting students' ways of reasoning about patterns and partitions. In J. V. Copley (Ed.), Mathematics in the early years. National Council of Teachers of Mathematics.
Parks, A. N. (2020). Creating joy in PK–grade 2 mathematics classrooms. The Mathematics Teacher, 113(1), 61–64. https://doi.org/10.5951/mtlt.2019.0250
Peña, J. (2018). El sentido numérico en preescolar (Number sense in preeschool) [Unpublished Masters Thesis]. Universidad Pedagógica Nacional]. Mexico City.
Peña, J., Cortina, J. L., & Visnovska, J. (2018). What happened at Frida's museum? Teaching Children Mathematics, 25(3), 175-179. https://doi.org/10.5951/teacchilmath.25.3.0174
Pepin, B. (2018). Enhancing teacher learning with curriculum resources. In L. Fan, L. Trouche, C. Qi, S. Rezat, & J. Visnovska (Eds.), Research on mathematics textbooks and teachers' resources: advances and issues (pp. 359-374). Springer. https://doi.org/10.1007/978-3-319-73253-4
Rancière, J. (1991). The ignorant schoolmaster. Stanford University.
Schifter, D. (1995). Teachers' changing conceptions of the nature of mathematics: Enactment in the classroom. In B. S. Nelson (Ed.), Inquiry and the development of teaching: Issues in the transformation of mathematics teaching. Education Development Center.
Secretaría de Educación Pública. (2011). Programa de estudio 2011. Guía para la educadora. Educación básica. Preescolar (Program of Studies 2011. Teacher guide. Preschool). Author.
Steffe, L. P. (1992). Learning stages in the construction of the number sequence. In J. Bideaud, C. Meljac, & J.-P. Fischer (Eds.), Pathways to number: Children's developing numerical abilities (pp. 83-88). Lawrence Erlbaum. https://doi.org/10.4324/9780203772492
Steffe, L. P., Cobb, P., & von Glasersfeld, E. (1988). Construction of arithmetic meanings and strategies. Springer. https://doi.org/10.1007/978-1-4612-3844-7