

AN ARGUMENT FOR ENGAGEMENT AS A FUNDAMENTAL CONSTRUCT FOR UNDERSTANDING MATHEMATICS LEARNING

UN ARGUMENTO A FAVOR DEL COMPROMISO COMO CONSTRUCTO FUNDAMENTAL PARA COMPRENDER EL APRENDIZAJE DE LAS MATEMÁTICAS

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An extended (and probably unnecessary) parallel is drawn between engagement in mathematics and engagement in musical performance. Key facets of engagement are described and a model of how mathematics engagement plays out in task-level activities is discussed in light of new findings related to its social and emotional facets. Implications for instructional practices that flow from this research are presented. The article concludes with suggestions for future research that incorporates understandings of identity and emotional object as promising directions.

Keywords: Engagement, motivation, emotion, affect, identity

My old mentor, Tom Romberg, he of *Curriculum and Evaluation Standards for School Mathematics* fame, once reflected to me that mathematics is like music. “You don’t become a musician by playing scales. You become a musician by playing music.” In my other life, I am a musician, and I love creating music, writing, and composing songs that try to say something about life and its meaning, and playing these songs to entertain and uplift people. When Teruni asked me to perform a couple of my songs prior to this address, I thought about the parallels between my relationship with music and my relationship to mathematics, and crafted this essay using some of these parallels as examples that might resonate to those of us interested in mathematical engagement—the sense of connection people have with mathematics, and their interaction with it and others as they experience it.

I recently finished a marathon 37-hour studio session with my band. During those 37 hours, our team of 6 band members, one sound engineer, backup singers and “support team” (family) worked collaboratively to develop the structure of sixteen songs, overlaying different instrumental and vocal parts, and troubleshooting problem areas of tempo, pitch, orchestration, and voicing—all areas that together, make a song potentially compelling to the listener. At the end of those 37 hours, we had rough cuts of each of our songs that served as sort of “existence proofs” that our creative ideas were correct. But even at the end of the session, the songs are not polished, the final beauty yet unrevealed.

The process of developing this body of work echoes the general research on engagement in mathematics. Members of our community of practice displayed various facets of engagement: Cognitive, Behavioral, Affective/Emotional, and Social, at different intensities at different times in the process of solving the many problems we encountered when creating something that for us was new and pleasing. One of my musical mentors, Igor Glenn, who played with Glen Campbell, Bobby Weir, The New Christy Minstrels, and John Denver told me, “Jim, being a musician is spending your life collecting licks and then putting them together in new ways.” Much like music, when learners are “engaged” in mathematics, they employ deep processing, instantiating, or developing procedures for combining smaller units of thinking into routines that, when

compiled, constitute a problem solution (Cognitive Engagement). In music we can liken this to composing a song such that the finished product has intellectual and aesthetic rigor.

In mathematics, learners must also perform tasks, using appropriate tools for thinking and communication (Behavioral Engagement). In music, one must be able to play the notes, the licks and riffs that make up the pieces of the composition. Notice that the procedures enacted—the notes, licks and riffs are not the music. They are elements of understanding that are combined to create the music. The tools musicians use—instruments, click-tracks, microphones, and effects—determine how a group of musicians can enact the composition, affording and constraining the possible ways in which a composition can be played.

In mathematics, learners use emotional feedback to gauge task efficacy and to monitor engagement while learning (Affective/Emotional Engagement). Musically, the “feel”—the sense of emotion, whether it is happiness, sadness, frustration, or anger—one experiences while playing is transferred to the finished piece. Sometimes these emotions get in the way of quality performance—inhibiting the singer, for example, from adequately forming the right tone, as their throat closes with sadness. Sometimes emotions related to poor performance engender a negative feedback cycle that prevents the musician from enacting skills that they had previously been able to perform. But sometimes, emotion empowers a player to try something new that elevates the playing of herself and her bandmates to a new level of understanding of not just the song, but of musicianship itself.

Lastly, in mathematics, learners play off their peers, reacting to and modifying behavior to support each other and to get value added from the collaboration (Social Engagement). Sometimes this type of engagement is productive, leading to improved learning, and sometimes it is inhibitive, leading to a breakdown of learning and its associated cognitive and social relationships. In recent work, my colleague Mandy Jansen and our team listened to students reflect on their relationship with their peers as they learned mathematics together. Their responses were laden with references to practices that impacted their own engagement. Such practices included social loafing, disruptive behaviors, disrespecting others, including the teacher, and domineering conversation, as examples of social interactions that inhibited individual engagement. But they also referred to practices such as helping one another, providing encouragement, and explanation as reasons why they remained engaged (Riske, et al., 2021). Likewise in music, members of a team must listen to each other, playing with their eyes and ears, such that each person’s contribution is unique, but coordinated with those of their bandmates. Respect, attention, support, and encouragement are all critical for one’s musical development.

Much of the dynamics of engagement witnessed by teachers is due to the complexity of social acts in service to mathematical discourse. I have used the metaphor of improvisational jazz in previous articles, making the case that members of a community of practice play much the same roles as musicians in a band, each contributing some element to the larger discourse that corresponds to their expertise, confidence in their expertise, and their personal interpretation of the mathematics as well as the roles others take on. Curriculum serves as a basic composition by which the discourse has structure, and where, given the sociomathematical norms established in the classroom, individuals can “riff” off each other, providing creative input to the discourse, such that what ultimately results is a performance unlike any other. Yet, it still maintains the structure of the composition—the curriculum—to ensure coherence and aesthetic wholeness. Also like many bands, the “conductor”—the teacher—listens to the composition as it is being played, correcting mistakes, interpreting, and orchestrating the performance to display rigor and aesthetic quality.

When each of these facets of engagement are entered into in productive ways, students create new mathematical knowledge, feel interested and efficacious, value mathematical thinking as useful and important, and ultimately tend to pursue mathematically intensive work more readily in the future (Middleton, Jansen, & Goldin, 2017). Just like music, a performance becomes an episode that may be called up from memory later and replayed in the head or used as the basis for future performances. With the use of recording devices such as whiteboards, flip charts, graphing utilities and collaborative software, these memories don't have to stay in one's head. They can be made public—replayed—such that others may appreciate the thinking, borrow ideas, and make them their own, critique them and add new contributions (Lamberg & Middleton, 2009).

Now, there is only so far that analogies can take us before they become tiresome. But I hope that those of you who enjoy music and especially those who have played or sang with others resonate (see what I did there?) with this description.

When it comes to research on mathematics engagement, new advances in our understanding are being developed every day. I have boiled these findings down into four fuzzy take-aways that reflect the larger body of research on engagement, but that highlight new and important avenues that may benefit scholars who do not focus on engagement, per se, but who must take it into account when researching learning, teaching, and curriculum. These take-aways are:

Mathematics engagement is embodied—When we engage in mathematical thinking, our motor cortex, endocrine system, attention and arousal centers of the brain, cortical processing, sensory and effector systems are activated—always. This, of which my colleague and friend, Mitch Resnick will speak at this conference, is some of the most exciting work in our field, indeed in the entire field of cognition. For instance, we have evidence that, when we observe others doing a task, the same centers in our brain light up as if we ourselves were doing the task. When we observe others emote, we likewise tend to interpret the experience in ways that evince the same emotion as the observed. When we rotate objects mentally, our heads tilt and our hands wave, mimicking the transformation. Emotional mirroring of this sort is often likened to contagion, wherein one person may begin to laugh triumphantly, and others follow suit (Iacoboni, 2009; Mafessoni & Lachmann, 2019).

Mathematics Engagement is emotional—Emotion is implicated into every facet of engagement. It is not separate from cognition, behavior, or social functioning (Meyer & Turner, 2006). Without taking emotion into account, we cannot understand the reasons students persist in the face of difficulty, nor when they engage in practices that may be maladaptive from a mathematical perspective. It is also becoming more evident that mathematics itself—its doing and the resulting products of that doing—is emotional. Our feelings color the directions we take, the problem-solving processes and tools we employ, and whether potentially productive ways of thinking are followed up upon (Goldin, 2007). Lastly emotion directly impacts the depth of cognitive processing one can employ when engaged in challenging mathematics. Students who interpret their experience as interesting and enjoyable tend to persist in the face of failure versus students who are frustrated and anxious (Tulis & Fulmer, 2006).

Mathematics Engagement is multifaceted—when I say that engagement is multifaceted, I do not mean in a trivial way—that there are many variables that contribute to engagement. What I mean is that, at any time in a learner's experience, they engage in *many* things: Mathematics problem solving, negotiating social relationships, general mood relating to life events, specific state characteristics, and physiological dynamics that each color their behavior in the moment of learning. This multidimensional situatedness implies that not all students will be optimally

engaged at any one moment in time. While this is obvious to all of us who have taught before, it has not always been taken as a design imperative for curriculum and instruction. In the past, this fact has been overlooked in an attempt to create teacher-proof, general materials that turn out to have little meaning for learners (Thompson, 2013). Recent attempts to address this fact have led to true innovations including contextualized curriculum, positioning and other affirming teaching practices, and adaptive curriculum technologies (e.g., Gresalfi & Hand, 2019). Each of these attempts has been successful, such that curriculum, at least, and some instruction is quite different now than it was 40 years ago.

But it begs the question, “Is it possible, or even desirable for all learners to be optimally engaged in all tasks at all times?” My own answer to this question is tied to my belief (and extensive evidence) that each student has different gifts, different skills, preferences, and proclivities they have developed because of their past experiences in and out of school. To address each facet of engagement, cognitive, behavioral, social, and emotional at optimal levels, we must take the long-term approach, while designing tasks for the short term. What I mean by this is that social engagement is the critical variable that will determine the optimization. For any task, if we optimize cognitive engagement, there must be multiple levels of challenge and multiple means by which students are able to enter the task. Similarly with behavioral engagement, some students will be more adept at different times with different tools for different purposes. Emotional engagement will be dependent upon the varied histories of students and their past relationship with mathematics, their peers, and schooling in general. No single task will optimize all these facets for all students all the time.

Social engagement is the lever by which, even though at the task level engagement is not always optimal, over time, the support of the group, if orchestrated with care and inclusion as key principles can bolster lagging engagement, reward exceptional engagement, and offload sub-tasks to different members of the group to account for each learner’s personal preferences and proclivities. Every task may not be optimal, but over time, all students may have the opportunity and the disposition to engage productively.

Mathematics Engagement is the result of feedback—Following up on this, one of the key revelations in the past 20 years in the fields of affect, motivation, engagement, and identity, is the notion that what happens in the short term impacts the long term. This may seem to be a kind of no-brainer—of course what we learn in the moment impacts our longer-term understandings. That is why we teach, to create moments that provide the opportunity for people to learn and grow mathematically. But in the field of educational psychology the focus on general processes—traits to be more precise—has overshadowed the importance of state processes to the extent that the unique nature of mathematics concepts, pedagogical practices, and social environment have been largely ignored, and their respective and collective impact, unexplored (Middleton, Jansen, & Goldin, 2017). But work in situational interest (Renninger & Hidi, 2019), achievement-related emotions (Forsblom, et al., 2022), task-based efficacy (Midgely et al., 1989), and self-regulated learning (Bell & Pape, 2014) show how dramatically what happens in the moment of learning becomes consolidated into longer-term trait-like habits and preferences.

A Model of Task-Level Engagement

This work is being combined with work on social facets of engagement to reveal a rather complicated, but still relatively simple system of influences where feedback from the moment reinforces the long-term attitude of students, which in turn, impacts the student’s initial model of the learning situation, thereby partly determining their patterns of engagement (Middleton et al., 2023).

Together, these newer empirical results can provide some direction for the design of mathematics curriculum and teaching practices. The syllogism I am drawing here is complex. The web of implications from object to predicate flows to areas yet unknown, but here are my understandings of the field of engagement-related research currently, and their implications for the improvement of the mathematical well-being of students (See Figure 1).

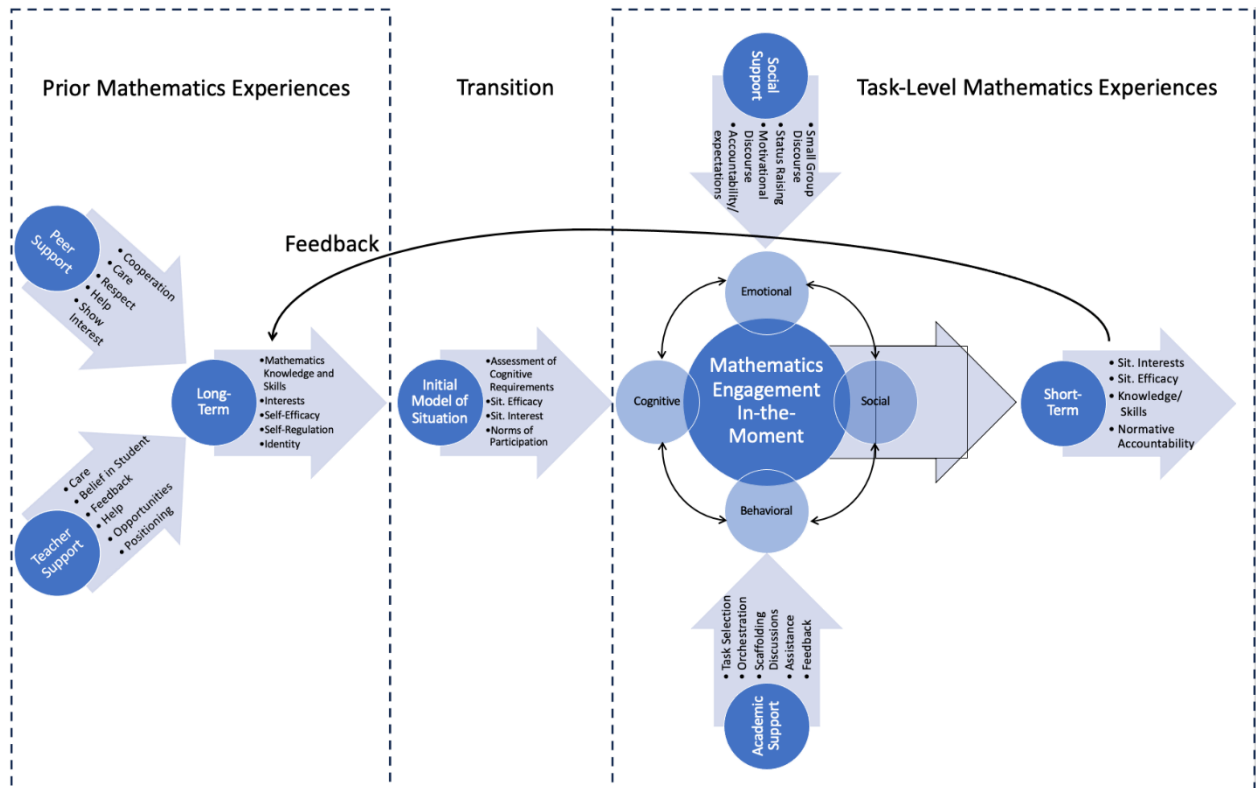


Figure 1. Major interacting facets impacting mathematics engagement.

When we think of prior mathematics experiences, the residue of these experiences makes up a person’s memories. In those memories, students recall support they have received by their peers (or not), as well as that provided by their teachers (or not). Other factors from the student’s culture and out-of-mathematics class experiences are also implicated here, but I have not included them in Figure 1 for clarity’s sake. These memories of mathematics past are used as evaluative templates to help the student determine their role, mode, requisite knowledge, and feelings about the task set before them.

There is a transition point between the introduction of a task, and its doing. In this transition, a student may assess their own ability with respect to the task requirements (task efficacy), their interest in the mathematics, and in the context (situational interest), and how they will fit in with their group mates (norms of participation). These things determined, even if tentatively, will color the student’s engagement-in-the-moment.

Then, at the task level, while engaged, there is constant monitoring of the cognitive, behavioral, social, and emotional characteristics of the task. As one is engaged, the initial model of the situation may be altered as one recognizes that they have knowledge to bring to bear on the problem, or that their group mate brings up an important point. So, the situational interest displayed, the situational efficacy attributed, the behaviors employed, and the person’s social

role play off each other dynamically. Peers and teacher provide potential support in the form of scaffolding, motivational talk, helping and feedback. The social acts teachers and peers proffer in carrying out the task can improve aspects of engagement (for some students) or inhibit them (for other).

Together, the initial model, its dynamic interaction with situational variables, and the support interjections of teacher and peers provide information for the learner to update portions of their long-term engagement attitudes. This feedback reinforces already developed notions of mathematics and the learner's ability and role within it and provides another episodic memory that can be called upon to direct future mathematics engagement in similar situations.

Over time, the stereotypical mathematics problems given to students, the routines of mathematics classes, and the kind of feedback typically provided students create robust, long-term attitudes towards mathematics that color students' future expectations of mathematical engagement, including their tendency to select mathematically intensive coursework and future occupational aspirations (Sullivan, 2013; Betz, 2023). We are all familiar with reports showing the steady decline of positive mathematics motivation from the middle grades onward. Most students leave compulsory education with less-than-positive long-term engagement patterns. So, what are the key levers we can employ to reduce the slope of this decline? Might we even be able to reverse it for more students? It is to this question I devote the remainder of this essay.

Implications for Practice

It has always baffled me why our field tends to focus its attention on the long-term manifestations of mathematics engagement instead of the short-term. It is only in the moment of learning where we can make an impact. Luckily there are a handful of practices that have been shown to improve patterns of engagement, particularly those associated with social support--the degree that students have opportunities to interact in a social environment such that their ideas are valued, and they feel welcome to be there. In other words, when students can feel like they matter, both their mathematical thinking and they as members of the learning community, they will tend to become more engaged:

Small Group Discourse. Discourse for engagement encourages nearly all participants in a group to share their thinking. Moreover, students listen to try to understand each other's thinking to build consensus around the mathematical idea.

Status Raising. The teacher *and students* regularly use language that magnifies *specific student's strengths* with respect to knowing and doing mathematics and assumes capability instead of shortcomings. This kind of feedback serves a reward purpose, enhancing the sense of task efficacy in the learner, and as information for others regarding what good mathematical thinking looks like. It is important that such status raising is distributed authentically to good ideas across each member of the classroom.

Motivational Discourse. When the teacher provides explicit language that supports student motivation using warm and welcoming language. These efforts can take several forms: 1) Focusing on the process of learning, challenging students, viewing errors as constructive, or supporting persistence; 2) Modeling positive affect to reduce anxiety and address emotional needs; and 3) Encouraging peer support and collaboration emphasizing joint goals & shared responsibilities.

Accountability/high expectations. When the teacher holds students accountable for engaging with learning of content such that all students are expected to participate in class (work hard and put forth effort, communicate their thinking, and listen to the teacher and each other). A number of strategies here have been shown to be productive including: 1) Cold calling or using

random calling to lower incidence of loafing; 2) circulating the room and, while doing so, pushing students to keep working on the math (not letting students get away with not trying hard in their groups); 3) Using a timer to let students know how much time they have; 4) Enabling students to share their thinking without being called upon; 5) Using structured protocols to help students learn to talk with one another – here, group work roles are explicitly referred to; and 6) ANY methods that don't allow students to hide—making explicit efforts to get more students involved in learning content.

Together, these practices have been shown to be related to increased situational interest and task efficacy in students and reduced negative emotions towards mathematics. Each of these outcomes is important, but the reduction of negative emotions directed towards mathematics has a multiplier effect. That is, negative emotions in mathematics are strongly, negatively associated with the following aspects of engagement: situational interest in the task, social engagement, task efficacy, and beliefs that the mathematics in the task is useful and important (Middleton, et al., 2023). So not only do these practices directly support the development of interest and mathematics self-efficacy, they also may indirectly reduce the impact of the negative emotions students exhibit in relation to challenging and difficult content.

In a kind of shocking finding, my colleagues and I found that academic support practices like selecting challenging tasks, scaffolding discussions, assisting, and providing feedback can actually reduce some important aspects of affective/emotional and cognitive engagement (Middleton, et al., 2023). When tasks are highly challenging, for example, students necessarily make more mistakes, become confused and potentially frustrated. Teachers' attempts to maintain a rigorous level in mathematics learning experiences can reduce students' sense of efficacy and interest, which, unchecked, may lead to less productive engagement, or engagement in other activities that are unrelated to learning mathematics. In our conversations with students, and in large scale quantitative assessment, students in our studies felt that challenge was negative, and that pressing them to explain was also detrimental in relation to their engagement.

The implications of these two seemingly contradictory findings, to me are this: It is the social support system in the classroom that undergirds rigorous work. The teacher, like the conductor of an orchestra, must always be pushing to improve the knowledge students are displaying. This can be difficult and disconcerting and may cause negative feelings such as frustration and embarrassment. However, with teacher and peers providing social support—i.e., status raising, motivational discourse, encouragement to keep level of effort high and others, the negative impacts, motivationally, of rigor may be overcome such that the value of the knowledge gained in pursuing and tackling hard problems takes a prominent role in promoting a strong sense of efficacy and interest (Ahmed, et al., 2010). My friend Gerry Goldin called this complex interplay between emotional content “affective structures” that can take negative emotional content and re-interpret it as positive when barriers are overcome, and when a person's contribution to the collective understanding is respected and valued (Goldin, et al., 2011).

Next Steps in Research

We have an opportunity at this time, to greatly enhance our understanding of mathematics learning by taking engagement seriously. In the past we have focused too exclusively on the cognitive or behavioral aspects of engagement to the detriment of its emotional and social counterparts. These two facets are pregnant with possibility, and I would like to end this essay with some directions I see as fruitful.

Emotional Object. The latest research on emotion shows quite convincingly that we experience multiple emotions simultaneously. Work done by Schukalow et al., (2023), and my

colleagues (Middleton et al., 2023) explain this phenomenon in part by the fact that emotions are not just positive or negative (valence), they also have intensity (e.g., very angry versus ticked off), they have activation (e.g., active such as enjoyment, or passive such as boredom), and they have object (e.g., I am ashamed of myself, or I am ashamed for my classmates). The combinations of these features create the vast majority of affective patterns we see in the classroom. Furthermore, they can be shown to associate with different manifestation of engagement. Mapping these emotional patterns to develop an understanding of productive emotional engagement and its role in interpreting and evaluating learning experience is a new and promising area of research.

Social Support. Our understanding of social support is being sped along by researchers focusing on culturally appropriate practices and identity, as well as others focusing on inclusion. It is time for the main body of mathematics education research to embrace this activity and try to incorporate their ideas into our body of work in engagement. As an example, I would like to point out Melissa Gresalfi and Vicki Hand's (2019) lovely model of identity construction that accounts for identity to be seen as building from norms of practice which exist within the frames and storylines within which students are positioned. These frames and storylines exist within larger cultural narratives that guide one's mathematics identity within one's larger set of identities related to race, class, gender, and intersectionality. Questioning dominant racial and economic narratives by which mathematics and its place in society have been positioned, and proposing intervening narratives has the potential for personal frames and storylines to become more supportive of diverse identities. But also, in the classroom, positioning, and redefining what it means to be mathematically competent, corresponds to the very social support practices I highlight in this article. They have the potential to scaffold students' playing a new kind of music in their mathematical experiences. Perhaps mathematics learning will become, like jazz, a beacon of inclusive artists and aficionados learning from each other and growing as new, diverse perspectives on mathematics and its role in our lives are shared.

Conclusion

The time is ripe for our community to step back and look at mathematics learning from a broader perspective: A perspective that accounts for the experience of learning—the engagement one has with the content, one's self, and one's community—as the fundamental unit of change in mathematics education. Those moments in which students persist in challenging tasks, supported by peers and teachers to showcase their ability, and where that ability is valued and respected will improve, ultimately, the affective responses learners have towards mathematics, and more importantly, change their identity with respect to mathematics in ways far more profound than focusing on a particular task sequence, or on teachers' practices isolated from how they impact engagement.

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