USING BIG IDEAS OF MEASUREMENT AS A FRAMEWORK TO EXPLORE PROFESSIONAL NOTICING OF STUDENTS' THINKING

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This paper presents Big Ideas of Measurement as a framework of students' thinking about measurement. Drawing from research-based evidence, the framework is a collection of key concepts that students must develop for a robust understanding of measurement and, as such, are key aspects of students' thinking teachers should learn to notice. In a case study with four mathematics teacher educators, this framework was utilized to design an instrument to measure their professional noticing and to analyze the results. Findings provided snapshots of professional noticing of participants with varied expertise in content knowledge and student thinking. Additionally, the choice of artifacts appears to have influenced participants' noticing.

Keywords: Teacher Noticing, Measurement, Teacher Educators

Noticing students' mathematical thinking is fundamental to reform-based instruction which emphasizes adaptive and responsive teaching (Jacobs et al., 2022). While current literature has explored preservice and practicing teachers' professional noticing of student thinking in various content areas, such as arithmetic (Fisher et al., 2018; Jacobs et al., 2010; Schack et al., 2013), fractions (Jacobs et al., 2022), algebraic thinking (Walkoe, 2015), and multiple representations (Dreher & Kuntze, 2015), research on this topic is scarce in the domain of measurement (Caylan Ergene & Isiksal Bostan 2022), a crucial topic in the mathematics curriculum, particularly at the elementary school level. The purpose of this study is to investigate ways to assess the construct of professional noticing of students' thinking about measurement and to capture snapshots of varying levels of expertise in noticing.

According to NCTM (2000), measurement has widespread practical applications in everyday life, in other domains of mathematics, and in areas outside of mathematics. Therefore, it is essential for students to develop a robust understanding of this content domain. Nevertheless, research indicates that many students hold fragile and shallow understanding of measuring various attributes, such as length, angle, and area (Smith & Barrett, 2017). For example, students may know how to use a ruler to measure the length of an object, but they may struggle when the ruler is not aligned at the conventional zero-point (Kamii & Clark, 1997). To help students overcome the challenges of learning about measurement, teachers must be able to notice and use students' thinking to make instructional decisions. However, given the complexity of classroom instruction, teachers cannot pay attention to everything. Hence, they must be selective in their noticing. This raises the first question:

> **Q1:** What are key aspects of students' thinking about measurement should teachers notice? This question was answered by conducting a literature review on students' thinking about measurement. Results are presented as a research-based framework entitled "Big Ideas of Measurement". This framework was then used to develop an instrument to explore teachers' professional noticing of students' thinking about measurement to answer the second question:
>
> **Q2:** How do teacher educators with various expertise in measurement content and students' thinking notice key aspects of students' thinking about measurement when engaging with

different instructional scenarios?A case study with four mathematics educators was conducted to investigate this research question.

Theoretical Framework

The foundational construct in this study is professional noticing of students' thinking about measurement. This section unpacks its essence by elucidating the two key terms "professional noticing" and "students' thinking about measurement".

Professional Noticing of Students' Mathematical Thinking

In this paper, I adapted Jacobs et al.'s (2010) concept of professional noticing of students' mathematical thinking, which comprises three interrelated skills: *attending to students' strategies details*, *interpreting students' understanding*, and *deciding how to respond on the basis of students' understanding*. This construct focuses on teachers' in-the-moment noticing when students explain their mathematical thinking verbally or in writing during classroom instruction rather than before or after instruction. For the purpose of this study, I focused on the first two component skills, attending and interpreting, because they serve as the foundation for the third.

Attending to students' strategies details pertains to the extent to which teachers identify mathematically significant details in students' strategies while *interpreting* students' understanding involves using these details to reason about their understanding in a way that is consistent with research on students' mathematical thinking.

Students' Thinking about Measurement

The term "student thinking about measurement" can encompass a wide range of students' conceptions related to various measurement quantities, and the development of their thinking regarding these quantities. This can include, for example, students' conceptions of angles, their struggles with distinguishing between area and perimeter, and the learning trajectory they follow in order to understand length. However, this study focuses on key concepts *across attributes* that researchers identified as foundational for students to develop a deep and robust understanding of measurement. This approach aligns with Smith and Barrett's (2017) call for prioritizing research on common concepts across measurement quantities.

Researchers have identified multiple lists of key measurement concepts that students should understand and named their lists using titles such as theories of measures (Lehrer, 2003), conceptual principles (Smith & Barrett, 2017), and essential understandings (Goldenberg & Clements, 2014). However, these lists are neither exhaustive nor unique; they contain overlapping and distinct ideas. To create a structured framework on students' thinking about measurement, I synthesized 16 key concepts from the literature and grouped them into five clusters. This research-based framework, called the *Big Ideas of Measurement*, is summarized in Table 1.[1] It is important to note that these big ideas are neither isolated nor "acquired in an all-or-none manner"; they are interrelated, forming a "web of connections" (Lehrer, 2003, p.182) that assist students in building a robust understanding of measurement across different attributes.

Methods

Based on the framework of *Big Ideas of Measurement*, a noticing instrument with three items, each featuring a different instructional scenario, was developed. A case study utilizing this instrument to measure professional noticing of four mathematics teacher educators was conducted in January 2023. The goal was to explore their professional noticing of students' thinking about measurement to answer Q2.

[1] I borrow the phrase *Big Ideas of Measurement* from Empson et al. (2006) who used it in course materials and interviews.

Table 1: Big Ideas of Measurement[2]

Clusters	Big Ideas	Description
The nature of measurement	*Assigning a number to an attribute*	Measurement can specify "how much" by assigning a number to attributes such as length, area, volume, angle, etc.
	Error & Precision	All measurement is inherently approximate.
Principles of Measurement	*Conservation*	Moving an object, or a shape does not change the measures of some attributes, for example, length, area, volume, angle, etc.
	Transitivity	Measures are transitive (e.g., If A>B and B>C, then A>C).
	Additivity	Units can be composed and decomposed (10 units is a composition of 6 units and 4 units or 10 groups of 1 unit). Most measures are additive (temperature is not).
Important Early Developmental Concepts/ Conceptions of Unit	*Identifying the attribute*	Does the child understand what they are measuring?
	Identical units	Is the same-sized unit used to create the measure?
	Exhaustive measure	All of the object has been measured without gaps between units or overlapping units
	Unit iteration	This includes making copies of units and arranging them, accumulating those units to obtain a measure, and eventually being able to reuse/copy a single unit
	Partitioning unit	Units can be partitioned into fractional amounts smaller than one unit
Standardization & Systems of Units	*Standard units*	Conventions/standard units facilitate communication
	Proportionality	The same attribute of an object can be measured with different units, and the different measurements produced are inversely proportional to the size of the unit.
	Composite units	Units of units—smaller units can be coordinated into a single larger unit. For example, measuring by groups of 100 centimeters is the same as a meter, seeing row and column structure in an array.
	Unit choice & Precision	The choice of unit in relation to the object determines precision.
The Use of Measurement Instruments	*Meaning of Instruments*	Instruments replace the need for use of multiple copies of units and move toward efficiency. Moreover, many big ideas of measurement are embedded in the design of instruments.
	Zero-point/ Origin	Each instrument has conventional zero point(s). Moreover, any point can serve as the origin or zero point on the instrument.

Note: These big ideas of measurement were drawn from and supported by existing literature on students' thinking about measurement including: Beckmann (2018); Bishop (2022); Dietiker et al. (2011); Goldenberg & Clements (2014); Jaslow & Vik (2006); Kamii (2006); Lehrer (2003); Lehrer et al. (2003); Lehrer et al. (2014); Lehrer et al. (1998); NCTM (2000); NGA Center & CCSSM (2010); Nitabach & Lehrer (1996); Piaget et al. (2013); Smith & Barrett (2017); Stephen & Clements (2003).

Participants

Purposeful sampling was used to select participants for this study. The intention was to capture a variety of noticing expertise. Four participants, Brielle, Luis, Hazel, and Mason (pseudonyms) are mathematics teacher educators who possess varied experiences in both the subject matter of measurement and students' thinking. They are enrolled in a doctoral program in mathematics education. Brielle has experience teaching developmental mathematics courses to

[2] My organization of the framework in this table was originated from Bishop (2022)'s MATH2312 course materials.

undergraduate students but has limited experience with regards to measurement content and students' mathematical thinking. Luis has taught a content course focusing on geometry and measurement for preservice teachers. He has expertise in measurement content but has little experience with students' thinking about measurement. Hazel has taught a content course on number systems and operations for preservice teachers and focused on students' mathematical thinking in her course. However, similar to Brielle, she also has limited exposure to measurement content. Lastly, Mason has experience with both measurement content and students' thinking about measurement, though he has not taught any courses for preservice teachers. He has, however, conducted clinical interviews with elementary students and analyzed their understanding of the big ideas of measurement for a course he was taking.

Noticing Instrument

This instrument was designed to measure professional noticing of students' thinking about measurement. It includes three items that prompt participants to engage with three instructional artifacts and respond to writing prompts to elicit their noticing. The artifacts were thoughtfully selected to represent a range of common scenarios in teaching practice and to cover multiple big ideas of measurement across quantities. Table 2 provides a summary of the key features of these three items.

Table 2: Descriptions of Three Items of the Noticing Instrument

	Item 1	**Item 2**	**Item 3**
Instructional Scenarios	Classroom Interactions	One-on-one Conversation	Students' Written Work
Artifacts	Video "Who's taller?: Measurement during circle time" (4 minutes)	Video "Helena measures Speedy with paper clips and a ruler" (5 minutes)	Three students' written work regarding finding area of a rectangle
Quantities	Length (Pre-measurement)	Length (Measurement)	Area
Description	During whole class instruction, the class engages in an activity to directly compare the heights of students and classify them into three sizes: Small, Medium, and Big	Helena, a first grader, measures Speedy, a string snake, with non-standard units (big and small paper clips), and standard units (inches, using a paper ruler)	Three students' written work shows how they draw tiles to cover a 6x5 rectangle to find its area.
Focused Big Ideas of Measurement	Identifying the attribute Conservation Transitivity	Identifying the attribute Identical units Exhaustive measure Unit iteration Partitioning unit Zero-point/ Origin Meaning of Instrument	Identical units Exhaustive measure Unit iteration Composite units

The writing prompts used in Item 2 (as an example) are:

Question 1: Please describe in detail what you think Helena did in response to each problem.

Question 2: Explain what you learned about Helena's understandings.

These prompts align with the two components, attending and interpreting, in the conceptualization of professional noticing in this study. The prompts for Item 1 and Item 3 are similar with a minor modification for Question 1 in Item 1. Specifically, this question was broader, "*Please describe in detail what you noticed when watching the video. Try to ask yourself "What else do you notice?" until you have nothing else to share*." This modification was made to investigate the potential impact of prompts on participants' noticing.

Data Analysis

To analyze participants' written responses for each item, I adapted the coding scheme developed by Jacobs et al. (2010, 2022). First, I created a rubric based on the framework of Big Ideas of Measurement. Particularly, I examined the artifact, identified big ideas of measurement which were present in students' strategies. I then identified significant details in students' strategies for each big idea and interpreted their understanding of the big idea (see examples in Row 2 of Table 3). Next, I looked at participants' responses for each item and assigned big idea codes whenever they appeared. If participants attended to and provided evidence for other significant mathematical details in students' strategies that were not included in the rubric, I assigned the code "Other." Finally, I gathered all details related to each big idea from participants' responses (see examples in Row 3 of Table 3) and compared them to the rubric, assigning a score of 0, 1, or 2 for their attending and interpreting skills based on the level of evidence in the participants' responses (see examples in Row 4 of Table 3).

Table 3: Example of Coding Process for Mason's Responses to Item 2

Big ideas of measurement	*Attending to details related to the big ideas in Helena's strategies*	*Interpreting Helena's understanding of the big ideas*
Use of identical units (Rubric)	Although there were small and big paper clips on the table, Helena consistently used the same-sized paper clips (5 big ones) until there was not enough space that she put a small one. However, she clearly named each unit in her measure (5 big and 1 small paperclips). She didn't say 6 paper clips long.	Helena mostly understood the big idea "use of identical unit". We can see that her understanding of identical units, exhaustive measure, and partitioned unit plays a role here: Helena understood that she needed to use the same-size (big) paperclips, but in the end, there was not enough space to fit a big paper clip. To maintain exhaustive measures, she used a small one.
Use of identical units (Mason's Response)	For the first problem, Helena started by taking big paper clips […]. After placing the last big paper clip, she realized a big paper clip would not fit in the small space left. She decided to try a small paper clip instead and found lining this next to big paper clips.	She also did not demonstrate the use of identical units and chose to instead, use a small paper clip to cover the last bit of the snake's length.
Use of identical units (Mason's scores & Reasons)	**Score: 1** Reason: Mason attended to some details related to this big idea in Helena's responses, but also missed some important details such as Helena clearly distinguished big and small paperclips as two different units.	**Score: 1** Reason: Although Mason was correct that Helena did not use all same-sized unit, she demonstrated some understanding of this big idea when consistently using big paper clips at the beginning.

For the attending component, a score of 0 indicated that participants did not attend to any details, or just very few details relevant to the big ideas in students' strategies. A score of 1 indicated that participants attended to some details in students' strategies but missed some important details. A score of 2 indicated that participants attended to almost all details related to the big ideas in students' strategies. Similarly, participants' interpreting component for each big idea was scored 0, 1, or 2 depending on how their interpretation aligned with research-based knowledge of students' measurement thinking. A score of 0 indicated that participants did not interpret students' understanding of the big idea, or their interpretation was misaligned with research findings. A score of 1 indicated that participants made inferences about students' understanding of the big idea, and their interpretation reflected students' thinking to some extent, but there were some minor misalignments with research, or the interpretation did not go in-depth. A score of 2 indicated participants' interpretation of students' understanding is thorough and consistent with research. After scoring participants' attending and interpreting components for each of the identified big ideas, I calculated their overall attending and interpreting scores for each item by taking the average.

For Item 1, participants' noticing was broader because the first question did not focus specifically on students' thinking. To further analyze their noticing, I used van Es and Sherin's (2008) coding framework. Specifically, I segmented participants' responses to Question 1 in Item 1 into smaller chunks based on shifts in the main ideas and assigned codes for four dimensions: *Actor* (codes: Teacher, Students, Other), *Topic* (codes: Mathematical thinking, Pedagogy, Climate, Management, Other), *Stance* (codes: Describe, Interpret, Evaluate), and *Specificity* (codes: General, Specific). The frequency of each code was then counted, and their percentages were calculated.

Findings

Table 4 summarizes participants' scores for attending (A) and interpreting (I) for three items in the instrument, along with their overall attending and interpreting scores. The last row shows the average scores for each item across all participants. It is important to note that these scores are not intended to classify participants as good or bad teachers, nor do they indicate what participants noticed to be good or bad. Rather, they measure the alignment of participants' noticing with the conceptualization of professional noticing in this study and help investigate varied expertise in noticing. In the following sections, I discuss what was learned about participants' professional noticing of students' thinking about measurement and how the choice of instructional artifacts influenced their noticing performance.

	Item 1	Item 2	Item 3	Participants' Overall Mean Score
Brielle	A: 1.2 I: 0.6	A: 1.25 I: 0.75	A: 0.75 I: 0.17	**A: 1.07** **I: 0.51**
Luis	A: 1 I: 0.6	A: 1.14 I: 0.86	A: 0.67 I: 0	**A: 0.94** **I: 0.49**
Hazel	A: 1.2 I: 0.6	A: 1.43 I: 1	A: 1.42 I: 0.83	**A: 1.35** **I: 0.81**
Mason	A: 1.4 I: 0.6	A: 1.43 I: 1.43	A: 1.58 I: 1.17	**A: 1.47** **I: 1.07**
Items' Overall Mean Score	**A: 1.2** **I: 0.6**	**A: 1.31** **I: 1.01**	**A: 1.11** **I: 0.54**	

Table 4: Summary of Participants' Noticing Scores

Snapshots of Varied Professional Noticing Expertise

Although the results of this case study with four participants cannot be generalized, there is a consistent pattern in Table 4: Mason has the highest performance in both attending and interpreting skills across three items, followed by Hazel, Brielle, and Luis. To gain more insight into their noticing, we can consider their responses to Question 2 in Item 3, where they interpreted Cassie's understanding from her work sample (See Figure 1).

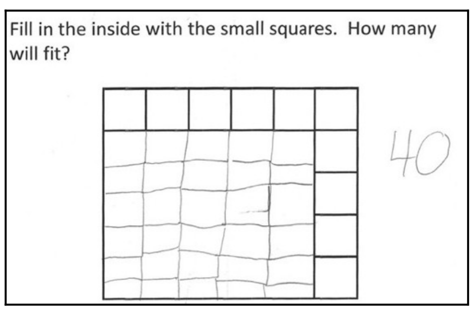

Figure 1: Cassie's Work Sample (Source: Empson et al., 2020)

Mason: Both Cassie and Eric demonstrate an exhaustive understanding of measurement, leaving no space inside the shape empty. Cassie demonstrates a partial understanding of identical units, as some drawn squares are the same, while others are not. […] Cassie demonstrates some understanding [of row and column structure] as her columns are lined up correctly.

Hazel: Cassie understands that the entire rectangle needs to be filled with squares. However, she does not understand completely that all the squares need to be the same size [..]

Brielle: Cassie uses **|** [*the vertical lines of squares in the first row*] as a reference point from the 1st row. Last row is much smaller than the rest but still counts each as a whole – Consistency.

Luis: Eric has a good understanding of partitioning…Cassie does as well, but something happened to where extra rows were made.

Mason's response showed that he paid attention to and interpreted Cassie' understanding of three big ideas of measurement: exhaustive measure, identical unit, and composite unit. While Hazel did not use the exact terminology from the framework like Mason, she did notice that Cassie filled the *"entire rectangles"* and did not use *"same size"* units, which showed evidence of Hazel's attention to and interpretation of Cassie's understanding of exhaustive measure and identical unit. However, she did not notice the big idea of composite unit like Mason did. Brielle attended to identical unit when she mentioned *"last row is much smaller than the rest"* while Luis paid attention to composite unit when he noticed the extra row in Cassie's work. Brielle and Luis received credit for attending to these big ideas, but their interpretations did not align with research on key concepts in students' thinking about measurement. Mason and Hazel's experience with students' thinking seemed to support their professional noticing. Especially for Mason, his knowledge of the framework on Big Ideas of Measurement seemed to give him a structure of what key aspects of students' thinking he should attend to and how to interpret their understanding. Moreover, Table 4 shows a pattern that all participants' attending scores are higher than their interpreting scores, which is consistent with existing literature in the field: participants cannot interpret something that they fail to notice.

Influences of Instructional Artifacts on Participants' Noticing

Notably, all participants had higher noticing scores for Item 2 compared to Items 1 and 3. It's worth considering that Item 2 utilized a video of a one-on-one conversation with a student, while Item 1 used a video of a whole-class interaction, and Item 3 used work samples from three different students. The video format and the focus on one student's thinking about measurement likely contributed to the participants' improved noticing performance in Item 2.

Comparing the performance of each participant in Items 1 and 3 in Table 4 reveals an interesting pattern: while Brielle and Luis had higher noticing scores for Item 1, Hazel and Mason performed better in Item 3. By looking at Table 5, which summarizes participants' more general noticing of Item 1 across four dimensions in addition to student thinking, we can get some interesting insight into this phenomenon. We can observe from this table that participants' focus for the two dimensions *Actor* and *Topic* differs. Brielle and Luis predominantly centered their attention on the students and mathematical thinking, while Hazel and Mason focused on the teacher and pedagogy. For instance, Mason initiated his response to Question 1 in Item 1 by noting, "*I noticed that the teacher was very good at using students' voice to center the discussion. Frequently, she would revoice what students said and asked."* The broadness of the prompt and the complexity of whole-class interactions led Hazel and Mason to attend to other actors and topics. This explains why their noticing scores for Item 1 were lower than their scores for Item 3.

Furthermore, the specificity of the prompts had an impact on the participants' inquiry stance towards their noticing. When the prompts were specific, such as in Items 2 and 3, participants took the corresponding inquiry stance of Describe for Question 1 and Interpret for Question 2. In contrast, when the prompt was broader, as in Question 1 in Item 1, participants' inquiry stance was more varied. For instance, in the previous example, Mason's noticing of the teacher's pedagogy took an evaluative stance, as he complimented the teacher's instruction.

Table 5: Participants' Noticing of Item 1 Across Four Dimensions

Dimensions	**Actor**			**Topic**					**Stance**			**Specificity**	
Codes	Tea	Stu	Oth	Mat	Ped	Cli	Man	Oth	Des	Int	Eva	Gen	Spe
Brielle	14	**100**	0	**100**	14	0	0	0	**71**	29	0	**71**	29
Luis	43	**86**	0	**71**	0	14	0	14	29	**57**	0	29	**71**
Hazel	**80**	40	0	20	**100**	0	0	0	40	**60**	0	20	**80**
Mason	**100**	14	0	43	**71**	14	14	14	29	29	**57**	14	**86**

Legend: Tea: *Teacher*; Stu: *Student*; Oth: *Other*; Mat: *Mathematical Thinking*; Ped: *Pedagogy*; Cli: *Climate*; Man: *Management*; Des: *Description*; Int: *Interpret*; Eva: *Evaluation*; Gen: *General*; Spe: *Specific*.
Note: Numbers in this table represent the percentage of segments getting a code out of the total segments

This analysis provided evidence that the instructional artifacts and the corresponding questions had an impact on participants' noticing. Specifically, the form of the artifacts, such as dynamic videos versus static student work samples; their nature, such as whole class interactions versus one-on-one conversations; and the level of specificity of the prompts used, all seemed to influence how participants noticed different aspects of the instruction. This finding highlights the importance of carefully selecting instructional artifacts and prompts in research on teacher noticing.

Brief Conclusions and Considerations

This paper makes three main contributions. First, it presents a synthesis of the *Big Ideas of Measurement* framework as a tool for understanding students' thinking about measurement across different quantities. Second, it introduces a noticing instrument and demonstrates how the framework can be used to analyze professional noticing of students' thinking about measurement. Third, a case study with four mathematics educators is presented to showcase the diverse expertise of professional noticing and to illustrate how the choice of instructional artifacts used in the instrument can influence noticing performance.

This case study suggests a potential relationship between participants' professional noticing and their prior experience with measurement content and students' thinking (both specific to measurement and a broader exposure and appreciation for student thinking in other content areas). Specifically, Mason's exposure to the framework appeared to support his attention and interpretation of students' thinking about measurement. However, further research with a larger sample size is needed to investigate this relationship more thoroughly. Additionally, future research could explore in a more systematic fashion how instructional artifacts influence noticing performance, and how the third component of professional noticing, deciding how to respond on the basis of student thinking, relates to attending and interpreting components in the context of measurement. These findings will be valuable for teacher educators and professional development facilitators to design effective courses and programs that support pre-service and in-service teachers in improving their professional noticing of students' thinking about measurement.

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References

Beckmann, S. (2018). Mathematics for elementary teachers with activities, 5th edition. Pearson.

Bishop, J.B. (2022). Handout "Big ideas of measurement" [Course curriculum for MATH2312, Texas State University]

Caylan Ergene, B., & Isiksal Bostan, M. (2022). Supporting Pre-service Mathematics Teachers' Professional Noticing of Students' Reasoning About Length. European Journal of Science and Mathematics Education, 10(1), 50–70. https://doi.org/10.30935/scimath/11384

Dietiker, L. C., Gonulates, F., & Smith, J. P. (2011). Understanding linear measure. Teaching children mathematics, 18(4), 252–259.

Dreher, A., & Kuntze, S. (2015). Teachers' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. Educational Studies in Mathematics, 88(1), 89–114.

Empson, S. B., Turner, E. E., & Junk, D. (2006). Formative Assessment Instruments for Children's Mathematical Thinking in English and Spanish [Set of seven problem-solving interviews covering 1) addition and subtraction; 2) base-ten concepts and place value; 3) multiplication and division; 4) fractions; 5) multidigit operations; 6) measurement; and 7) algebra].

Empson, S. B. (2020). Course materials. Austin, TX: The University of Texas.

Fisher, M. H., Thomas, J., Schack, E. O., Jong, C., & Tassell, J. (2018). Noticing numeracy now! Examining changes in preservice teachers' noticing, knowledge, and attitudes. Mathematics Education Research Journal, 30, 209–232. https://doi.org/10.1007/s13394-017-0228-0

Goldenberg, E. P., & Clements, D. H. (2014). Developing essential understanding of geometry and measurement in grades PreK-Grade 2. National Council of Teachers of Mathematics.

Jacobs, V. R. & Spangler, D. A. (2017). Research on core practices in K-12 mathematics teaching. In J. Cai (Ed.), Compendium for Research in Mathematics Education, (p. 766–792). Reston, VA: National Council of Teachers of Mathematics.

Jacobs, V. R., Empson, S. B., Jessup, N. A., Dunning, A., Pynes, D. A., Krause, G., & Franke, T. M. (2022). Profiles of teachers' expertise in professional noticing of children's mathematical thinking. Journal of Mathematics Teacher Education, 1–30.

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. Journal for Research in Mathematics Education, 41(2), 169–202.

Jaslow, L., & Vik, T. (2006). Using children's understandings of linear measurement to inform instruction. Teachers Engaged in Research: Inquiry into Mathematics Classrooms, 109–134.

Kamii, C. (2006). Measurement of length: How can we teach it better?. Teaching children mathematics, 13(3), 154–158.

Kamii, C., & Clark, F. B. (1997). Measurement of length: The need for a better approach to teaching. School Science and Mathematics, 97(3), 116–121.

Lehrer, R. (2003). "Developing understanding of measurement." In Jeremy Kilpatrick, W. Gary Martin, Deborah Schifter (Eds), A research companion to principles and standards for school mathematics (pp. 179–192). National Council of Teachers of Mathematics.

Lehrer, R., Jacobson, C., Thoyre, G., Kemeny, V., Strom, D., Horvath, J., Gance, S., & Koehler, M. (1998). Developing understanding of space and geometry in the primary grades. In R. Lehrer & D. Chazan (Eds.), Designing learning environments for developing understanding of geometry and space (pp. 169–200). Erlbaum.

Lehrer, R., Jaslow, L., & Curtis, C. (2003). Developing an understanding of measurement in the elementary grades. Learning and teaching measurement, 1, 100–121.

Lehrer, R., Slovin, H., Dougherty, B. J., & Zbiek, R. M. (2014). Developing essential understanding of geometry and measurement for teaching mathematics in grades 3-5. NCTM, National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2000). Principles and standards for school mathematics. Reston, VA: NCTM.

National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). (2010). Common Core State Standards for Mathematics. Washington, DC: NGA Center and CCSSO. http://www.corestandards.org.
Nitabach, E., & Lehrer, R. (1996). Research into practice: Developing spatial sense through area measurement. Teaching Children Mathematics, 2(8), 473–476.
Piaget, J., Inhelder, B., & Szeminska, A. (2013). Child's conception of geometry. Routledge.
Roth McDuffie, A., Foote, M. Q., Bolson, C., Turner, E. E., Aguirre, J. M., Bartell, T. G., ... & Land, T. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. Journal of Mathematics Teacher Education, 17(3), 245–270.
Schack, E. O., Fisher, M. H., Thomas, J. N., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers' professional noticing of children's early numeracy. Journal of Mathematics Teacher Education, 16(5), 379–397. https://doi.org/10.1007/s10857-013-9240-9
Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). Situating the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), Mathematics teacher noticing: Seeing through teachers' eyes (pp. 3–14). Routledge.
Smith, J. P., & Barrett, J. E. (2017). The learning and teaching of measurement: Coordinating quantity and number. In J. Cai (Ed.), Compendium for Research in Mathematics Education. (pp. 355–385): Reston, VA: National Council of Teachers of Mathematics.
Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. Journal of mathematics teacher education, 11(2), 107–125.
Stephen, M., & Clements, D. H. (2003). Linear and Area Measurement in Prekindergarten to Grade 2. Learning and Teaching Measurement, (August), 100–121.
van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers' "learning to notice" in the context of a video club. Teaching and Teacher Education, 24(2), 244–276.
Walkoe, J. (2015). Exploring teacher noticing of student algebraic thinking in a video club. Journal of Mathematics Teacher Education, 18, 523–550.