CLASSIFYING CURRICULAR REASONING:
WAYS FOR CAPTURING TEACHERS' CURRICULAR DECISIONS

Shannon Dingman
University of Arkansas
sdingman@uark.edu

Dawn Teuscher
Brigham Young University
dawn.teuscher@byu.edu

Travis Olson
University of Nevada Las Vegas
travis.olson@unlv.edu

Amy Roth-McDuffie
Washington State University
mcduffie@wsu.edu

Mathematics teachers make numerous decisions that form lessons that in turn greatly influence what students learn. In making these decisions, teachers rely on their curricular reasoning (CR) to decide on what mathematics to teach, how to structure their lesson, and what problems or tasks to use to achieve their lesson goals. However, teachers differ with respect to the sophistication of their CR and the diversity of CR aspects used in their reasoning. In this paper, we detail two ways to classify teachers' CR: a leveled approach to capture the increasing sophistication of teachers' CR, and a heat map approach that highlights the extent to which teacher use various CR aspects in their planning. These methods provide stakeholders avenues by which CR can be studied and that teachers' CR abilities can be further developed.

Keywords: Curriculum; Instructional Activities and Practices; Instructional Vision

Mathematics teachers make innumerable decisions that shape their lessons and impact students' opportunity to learn. Past research has often focused on teachers, students, and the mathematical content as key classroom elements that drive classroom interactions (Cohen & Ball, 1999; Cohen et al., 2003), yet research has illustrated the essential role curriculum plays in influencing instruction (Stein et al., 2007; Rezat, 2006). Over the past 25 years, mathematics education researchers have extensively studied mathematics curriculum and how teachers use it, including how curriculum can be educative for both teachers and students. Lloyd et al. (2017) defines curriculum as the "written curriculum materials and textbooks...and the resources with which students and teachers work most closely in the mathematics classroom" (p. 824). We extend the definition of curriculum to include any materials (e.g., written or digital resources) that teachers use to plan and enact lessons that support students' mathematics learning.

Following current trends in the field, we have come to view teachers as designers who engage in a participatory relationship as they work with curriculum (Brown, 2009; Remillard, 2005). From this perspective, Roth McDuffie and Mather (2009) expanded the work of Shulman (1986) on curricular knowledge by exploring the cognitive work teachers engage in while working with curriculum. This cognitive work, termed as curricular reasoning (CR), encompasses the "thinking processes that teachers engage in as they work with curriculum materials to plan, implement, and reflect on instruction" (Breyfogle et al., 2010, p. 308). CR is heavily informed by teachers' knowledge, background, and teaching experience, and is used by teachers as they operationalize curriculum and enact decisions with different types of curricula. Building on Roth McDuffie and Mather's (2009) research, Dingman et al. (2021) identified five

CR aspects that teachers reason with as they use curriculum to plan and enact mathematics lessons. The five empirically identified CR aspects identified in our research are: 1) *Viewing Mathematics from the Learner's Perspective* (teachers' reasoning about the assessment and anticipation of student thinking and reasoning about the purpose of the task, given their prior knowledge about students' backgrounds and experience); 2) *Mapping Learning Trajectories* (teachers' reasoning about connections across content in the unit, across units, or across grades); 3) *Considering Mathematical Meanings* (teachers' reasoning about the mathematics for themselves or for the student); 4) *Analyzing Curriculum Materials* (teachers' reasoning about the curriculum, identifying strengths and limitations); and 5) *Revising Curriculum Materials* (teachers' reasoning about past teaching experiences to make changes to the task or the curriculum). Teachers vary in terms of how many CR aspects they reason with while planning and enacting lessons. Qualitative findings indicate that these differences in teachers' reasoning influence students' learning of mathematics (Dingman et al., 2021).

Much research on teachers' decisions is based on the Instructional Triangle introduced by Cohen et al. (2003), which is used to study interactions among teachers, students, and the content under study. However, some researchers (Rezat, 2006; Rezat & Sträber, 2012; Tall, 1986) suggest that this focus is too narrow, neglecting other resources that inform classroom interactions such as technology and curricula. Our findings demonstrate that curriculum is a critical element of teachers' practice, and that CR is inherent to teachers' work as they plan and enact lessons (Choppin et al., 2018, 2020, 2021; Dingman et al., 2021; Roth McDuffie et al., 2018). Given the diverse approaches to mathematics provided by various curricula, these findings make sense. Our findings also suggests that teachers' CR influences their decisions and students' learning opportunities. Further investigations should consider curriculum as well as the teachers, students, and the content under study.

To that point, Dingman et al. (2021) propose the Instructional Pyramid displayed in Figure 1 that expands the Instructional Triangle (Cohen et al., 2003) in order to represent and capture the myriad interactions that occur in the instructional environment. Figure 1 illustrates the interplay among these four key classroom elements and highlights the aspects of CR teachers employ as they reason with these elements. Teachers' curricular decisions are based upon the classroom *elements* (vertices) and the CR *aspects* (edges and faces) found in the Instructional Pyramid.

How and why teachers make these curricular decisions is important to understand, given the potentially limited role of written textbooks in determining what is taught, in favor of open-source and teacher-developed activities aligned (or purported to align) to standards (Banilower et al., 2013). Teachers' decisions regarding the use of these various resources holds considerable influence over students' opportunity to learn mathematics (Stein et al., 2007).

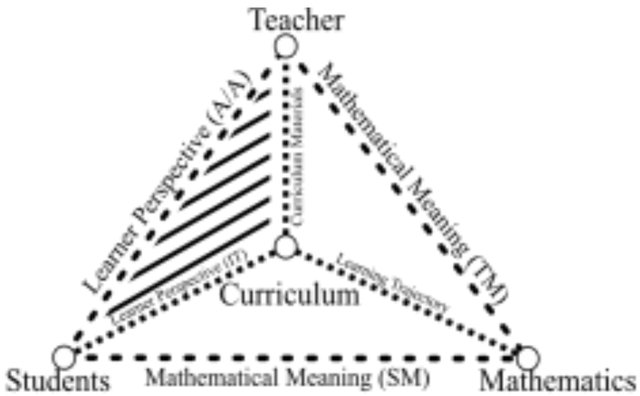

Figure 1: Instructional Pyramid for Curricular Reasoning

The importance of viewing teachers' decisions and reasoning through the Instructional Pyramid model for CR is based on data from our project that suggests that oftentimes teachers reason on more than one edge of the model. Investigating teachers' CR can elucidate why teachers make specific mathematical decisions, such as skipping or reordering lessons, revising tasks, or modifying definitions. While some decisions may appear to be inappropriate for a given situation, these decisions are likely multi-faceted and nuanced in ways that elude clear notions of correct and incorrect, or right and wrong. By determining how and why teachers come to these decisions and identifying the aspects of CR teachers reason with most frequently, professional developers, teacher leaders, and mathematics teacher educators can build teachers' capacity to reason differently and more robustly about these decisions.

Our working hypothesis is that teachers who coordinate multiple CR aspects in their decision-making provide different learning opportunities for students than those who reason with only one or two CR aspects. In the preliminary analysis of data from teachers within our project in relation to the Instructional Pyramid, it is apparent that some teachers tend to reason with certain CR aspects more than others. In fact, many teachers reasoned with *Viewing Mathematics from the Learner's Perspective* (Anticipating/Assessing) and *Considering Mathematical Meanings* (Teacher and Student Mathematics) yet used the other CR aspects less often.

However, some teachers reasoned with multiple CR aspects as they made individual decisions. This suggests that some teachers may need support to recognize ways in which they are reasoning, to understand what they are not attentive to, and to develop productive ways to learn how to reason with different CR aspects so as to create different learning opportunities for their students.

To this point, we propose two potential ways to characterize teachers' CR in terms of its sophistication and its diversity. In this paper, we detail these approaches and provide data from our work with middle grades mathematics teachers to illustrate ways to capture differences in teachers' CR as well as highlight CR aspects that are most/least used. Data discussed in this paper derive from our research question under investigation: What CR aspects do middle school teachers reason with as they plan and enact mathematical lessons? See Dingman et al. (2021) for detailed discussion of our overall research project.

A Leveled Approach to Characterizing CR

Our first approach to capturing differences in teachers' CR is a six-level framework that works to classify the varying levels of sophistication teachers incorporate as they reason about curriculum. This framework aims to capture the degree to which teachers use the various vertices and edges depicted in the Instructional Pyramid (see Figure 1) when making curricular decision. These six levels are:

- **Level 0:** A teacher reasons only with the Reflect and Revise CR aspect. In this case, *no vertices or edges* are used in the reasoning. In these instances, the teacher is implicitly reasoning with one or more elements but the reasoning is not explicit.
- **Level 1:** A teacher reasons with *any single edge* of the Instructional Pyramid. In doing so, the teacher reasons with one CR aspect and uses only <u>two elements</u> in the decision-making process. For example, a teacher reasons only with the Mapping Learning Trajectory CR aspect, which connects the two elements Curriculum and Mathematics.
- **Level 2:** A teacher reasons with *two edges connected by a common vertex* on the Instructional Pyramid. In doing so, the teacher reasons with two CR aspects that connect three elements in their decisions. For example, a teacher reasons with the Considering
- Mathematical Meaning (TM) and Mapping Learning Trajectory CR aspects, which incorporates the elements of Teacher, Mathematics and Curriculum (but no discussion of Students).
- **Level 3:** A teacher reasons with *three edges that form a face* on the Instructional Pyramid. In doing so, the teacher reasons with three CR aspects that connect three elements but does not incorporate the fourth element in their reasoning. For example, a teacher reasons with the Considering Mathematical Meaning (TM), Mapping Learning Trajectory, and Analyzing Curriculum Materials CR aspects, which connect the three elements Teacher, Mathematics, and Curriculum on a complete face.
- **Level 4:** A teacher reasons with *two unconnected edges* on the Instructional Pyramid. In doing so, the teacher incorporates all four elements that form the Instructional Pyramid but in a manner in which the two edges are not connected. For example, a teacher uses the Viewing Mathematics from the Learner Perspective-Anticipating/Assessing (A/A) and the Mapping Learning Trajectory CR aspects, which uses all four elements on two edges of the pyramid that are unconnected. Note that, even though a teacher is reasoning with fewer CR aspects in Level 4 in comparison to Level 3, all four elements are used in Level 4 reasoning, as opposed to only three of the four elements used in Level 3.
- **Level 5:** A teacher reasons with *three or more connected edges* on the Instructional Pyramid. In doing so, the teacher reasons in a manner that connects all four elements of the Instructional Pyramid. For example, a teacher reasons with the Viewing Mathematics from the Learner Perspective (A/A) the Viewing Mathematics from the Learner Perspective - Intentionality of Task (IT), and the Mapping Learning Trajectories CR aspects, connecting all four elements in a path around the edges of the pyramid.

As part of our research, we collected interview data from 15 middle grades teachers as they planned instruction for a unit on geometric transformations. These teachers were given the geometric transformations unit from the *UCSMP* series (Benson et al., 2009) and used this curriculum as the basis for planning their grade 8 unit pertaining to geometric transformations (reflections, rotations, translations, and sequences of transformations). This topic was chosen as a content area that had traditionally appeared in secondary mathematics but that had now emerged in the grade 8 curriculum after the widespread adoption of the *Common Core State Standards for Mathematics (CCSSM)*. The *UCSMP* unit on geometric transformations was chosen because of its unique approach to geometric transformations to construct the definition of congruence, which was rarely seen in past state standards but is now used in *CCSSM.* Teachers were interviewed before and after teaching lessons with the *UCSMP* curriculum. These interviews were partitioned in initial pass coding according to teachers' mathematical decisions that emerged during the pre- and post-interviews and then coded for the various CR aspects seen in Figure 1. The codes for each decision (N) were then analyzed and classified by the levels described above. Our analysis illustrated considerable differences in teachers' levels of reasoning when planning and reflecting upon their curricular decisions. We share the results from two teachers—Jill with 12 years of teaching experience, and Cathy with 8 years of teaching experience—to highlight these differences in Table 1.

Table 1: Breakdown by level of teachers' CR

Teacher	N	Level 0	Level 1	Level 2	Level 3	Level 4	Level 5
Jill	161	10 (6.2%)	104 (64.6%)	36 (22.4%)	3 (1.9%)	1 (0.6%)	7 (4.4%)
Cathy	281	3 (1.1%)	120 (42.7%)	91 (32.4%)	6 (2.1%)	15 (5.3%)	46 (16.4%)

As seen in table 1, Cathy used a greater percentage of higher-level CR (levels 4 & 5) when making decisions than Jill. Further analysis of the data revealed trends on which vertices and edges each teacher uses most when making decisions. To provide an illustration for one of these teachers, 58% of Cathy's Level 1 instances were on the *Viewing Mathematics from the Learner Perspective (A/A)* edge connecting the Student and Teacher elements, while nearly 70% of Cathy's Level 2 codes involved either the Student or Teacher element as *the connector* between the two CR aspects used in her reasoning. However, Cathy's Levels 3 and 4 instances saw an even balance across the four elements (student, teacher, mathematics and curriculum), while her Level 5 instances contained a heavy emphasis on either Mathematics or Curriculum. This led us to conclude that Cathy tended to reason more about the Student and Teacher elements in her instructional decisions, while her limited higher level reasoning involved her expanding her focus to the Mathematics and Curriculum elements.

Using Heat Maps to Identify Use of CR Aspects

Our second approach to analyzing teachers' use of CR aspects involved the utilization of heat maps. After collecting and analyzing the pre- and post-interview data, we compared the frequency of reasoning with each CR aspect to the total number of decisions made by individual teachers to calculate a percentage of use for each CR aspect by teacher. Because teachers used multiple CR aspects to make decisions or teachers did not provide reasoning for some decisions, the percentages do not add to 100%. These CR percentages were used to create heat-map models - a graphical representation where data values are labeled with cool and warm colors. Warmer colors signify that the teacher reasoned with the CR aspect often, while cooler colors signify that the teacher reasoned with the CR aspect less often. Table 2 displays the scale we used to design the data-generated heat-maps. We determined that five colors allowed us to differentiate among teachers more easily than three colors. We also note that green is the optimal range because if teachers reasoned with a CR aspect over 45% of their decisions, this often limited their use of other CR aspects. Using the Instructional Pyramid model of CR we developed a data-generated model for each teacher.

Table 2: Color Scale for Heat-Map Models

	Curricular Reasoning Percentage
Purple	0-15%
Blue	15.01-25%
Green	25.01-45%
Orange	45.01-55%
Red	55.01%+

Data-Generated Models

Our goal in creating the data-generated models for the teachers participating in our study was to highlight similarities and differences in reasoning and identify CR aspects that may not be used as often as others. This led to implications for teachers, teacher educators, and professional developers. A few notable patterns emerged when comparing the data-generated models across all teachers. First, we found that for 91% of the teachers in our study, the CR aspect *Viewing Mathematics from the Learner Perspective (IT)* was used the least-often in their decisions regarding their planning and enacting of mathematics lessons. These decisions connect the student and curriculum aspects of the Instructional Pyramid model of CR. Secondly, we found that 74% of our teachers used the CR aspect *Viewing Mathematics from the Learner Perspective (A/A)* the most often in their reasoning, with the remaining 26% of teachers using the CR aspect *Considering Mathematical Meaning (TM)* the most often. Both of these CR aspects are represented on the front face of the Instructional Pyramid of CR, connecting the elements teachers and students, and teachers and mathematics respectively. Importantly, the CR aspects with the highest use do not connect the elements of teacher, student, or mathematics to the curriculum while the CR aspect with the lowest use has the curriculum connection. In fact, across all these data, the majority of teachers reasoned with CR aspects on the front face of the Instructional Pyramid of CR (i.e., *Viewing Mathematics from the Learner Perspective (A/A)* and *Considering Mathematical Meaning (TM/SM)*) with greater frequency than the CR aspects connected to curriculum (i.e., *Viewing Mathematics from the Learner Perspective (IT)*, *Analyzing Curriculum Materials*, and *Mapping Learning Trajectory*). These patterns have potential implications for professional development of teachers and pre-service teachers.

Figure 3 displays data-generated models for two teachers in our study that highlight extremes to show the range in our data. Helen (left) was an example of a teacher in the mid-range with five of the six CR aspects in the green and the sixth being close to the less extreme mid-range (blue). Vance (right) was an example of a teacher who reasoned with fewer CR aspects. The majority of teachers' models were more similar to Vance than Helen, with more variety in the colors on the model. In fact, only 17% of the teachers' models had four or more CR aspects in the green mid-range. However, with all but one CR aspect outside of the mid-range, Vance displayed a more varied CR use with the more extreme-ranges (i.e., purple, red) present than many of the other models. We found that the majority of teachers (52%) had at least half of the six CR aspects in the green mid-range. The data were reasonably centered around the green mid-range.

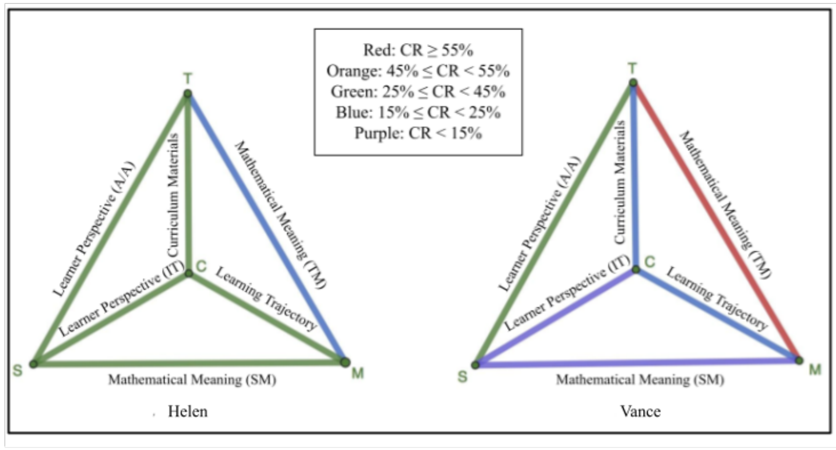

Figure 2: Two Teachers' Heat-Maps to Represent Their Curricular Reasoning

Implications/Conclusion

Using our two ways we can see how teachers are using their CR in two different ways. In the leveled approach, we found that teachers reasoned most often with CR aspects connecting the elements of teacher, student, and mathematics on the Instructional Pyramid, but reasoned much less frequently with those connected to the curriculum vertex on the pyramid. In the heat-map approach, we found that teachers reasoned most often with the *Viewing Mathematics from the Learner Perspective (A/A)* and *Considering Mathematical Meaning (TM)* as they planned and enacted lessons.

Both ways highlighted above provide different and significant approaches to characterize teachers' CR. The leveled approach allowed for the examination of the sophistication of teachers' reasoning. As the levels of reasoning increased, teachers incorporated more of the four classroom elements (Teacher, Student, Mathematics, Curriculum) represented as the vertices on the Instructional Pyramid as well as more of the CR aspects represented as the edges on the Instructional Pyramid. As stated previously, our working hypothesis is that teachers who coordinate multiple CR aspects and subsequently focus their attention on greater numbers of elements provide different learning opportunities for students than those who reason with only one or two CR aspects and subsequently fewer elements. The leveled approach provides a method to examine how often teachers reason with greater sophistication (more CR aspects and elements). On the other hand, the heat map approach allows for a detailed analysis of how often teachers use different CR aspects. In the process, teachers can see the CR aspects that figure most prominently in their reasoning as well as the CR aspects used least often. To that point, the heat map approach can be used to provide support to teachers in developing their abilities to diversify their reasoning in order to coordinate more CR aspects into their decision making.

While both approaches provide more information about a single teacher, these findings have implications for teachers, teacher educators, and professional developers. The first is that current professional development and pre-service education appear to be developing teachers' ability to reason with *Viewing Mathematics from the Learner Perspective (A/A*) and *Considering Mathematical Meaning (TM)* as those two CR aspects are more widely used by teachers in our study. In the heat-map models these were the CR aspects that were most often in the warm colors (i.e., red and orange) on teachers' models. This suggests that teachers are reasoning with these aspects often. In our leveled model, we also found that these CR aspects are the ones that connect the elements of teacher, student and mathematics. Second, teachers' reasoning with the other CR aspects that connect curriculum in the Instructional Pyramid: *Analyzing Curriculum Materials*, *Mapping Learning Trajectories*, and *Viewing the Learner Perspective (IT*) were not as widely used. This has implications for students as their teachers are generally reasoning less with the element of curriculum as they prepare and enact lessons. These results may not be surprising because professional developments have primarily focused on making decisions and reasoning from the front face of the Instructional Pyramid. We believe these ways of reasoning are important and should remain a key part of teacher education and professional development.

Therefore, we recommend that teacher educators and professional developers include explicit activities and task that encourage teachers to make decisions and reason with the other CR aspects that include curriculum as all CR aspects are important to provide the best possible learning opportunities for students.

These findings also have implications for teacher educators and professional developers, informing content to be taught. Based on initial work, we propose that the Instructional Pyramid model can be used to examine how teachers reason with the classroom elements and CR aspects discussed previously. This can allow stakeholders to analyze the factors and reasons that shape teachers' decisions as they plan and implement mathematics lessons. The Instructional Pyramid model can also be used as a self-assessment for teachers to identify strengths and weaknesses in their CR. Through the use of both the leveled approach and the heat map approach, teacher educators can support teachers' continuing development of the sophistication and diversity of their CR.

Acknowledgments

This material is based upon work supported by the National Science Foundation under Grant Nos. 1561542, 1561154, 1561569, 1561617. Any opinions, findings, and conclusions are recommendations expressed in this material are those of the authors and do not necessarily reflect the view of the National Science Foundation.

References

Banilower, E. R., Smith, P. S., Weiss, I. R., Malzahn, K. A., Campbell, K. M., & Weis, A. M. (2013). Report of the 2012 national survey of science and mathematics education. Horizon Research, Inc.

Breyfogle, M. L., Roth McDuffie, A., & Wohlhuter, K. A. (2010). Developing curricular reasoning for Grades pre-K-12 mathematics instruction. In B. Reys, R. Reys, & R. Rubenstein (Eds.), Mathematics Curriculum: Issues, Trends and Future Direction, 72nd Yearbook (pp. 307-320). Reston, VA: National Council of Teachers of Mathematics.

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann & G. M. Lloyd (Eds.), Mathematics teachers at work: Connecting curriculum materials and classroom instruction (pp. 17-36). New York: Routledge.

Choppin, J., Roth McDuffie, A., Drake, C., & Davis, J. (2018). Curriculum ergonomics: Conceptualizing the interactions between curriculum design and use. International Journal of Educational Research, 92, 75-85.

Choppin, J., Roth McDuffie, A., Drake, C., & Davis, J. (2020). The role of instructional materials in the relationship between the official curriculum and the enacted curriculum. Mathematics Thinking and Learning. DOI: 10.1080/10986065.2020.1855376
Choppin, J., Davis, J., Roth McDuffie, A., & Drake, C. (2021). The influence of features of curriculum materials on the planned curriculum. ZDM Mathematics Education, 53(6), 1249-1263.
Cohen, D., & Ball, D. L. (1999). Instruction, capacity, and improvement (Research Report Series No. RR-43). Philadelphia: University of Pennsylvania, Consortium for Policy Research in Education.
Cohen, D., Raudenbush, S., & Ball, D. (2003). Resources, instruction and research. Educational Evaluation and Policy Analysis, 25(2), 119-142.
Dingman, S. W., Teuscher, D., Olson, T. A., & Kasmer, L. (2021). Conceptualizing curricular reasoning: A framework for examining mathematics teachers' curricular decisions. Investigations in Mathematics Learning, 13(4), 267-286.
Lloyd, G. M., Cai, J., & Tarr, J. (2017). Issues in curriculum studies: Evidence-based insights and future directions. In J. Cai (Ed.), First compendium of research on mathematics teaching and learning (pp. 824-852). Reston, VA: National Council of Teachers of Mathematics.
Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. Review of Educational Research, 75(2), 211-246.
Rezat. S. (2006). A model of textbook use. In J. Novotna, H. Moraova, M. Kratka, & N. A. Stehlikova (Eds.) Proceedings of the 30th conference of the international group of the psychology of mathematics education (Vol. 4, pp. 409-416). Prague: Charles University, Faculty of Education.
Rezat, S. & Sträber, R.(2012). From the didactical triangle to the socio-didactical tetrahedron: Artifacts as fundamental constituents of the didactical situation. ZDM Mathematics Education, 44(5), 641-651.
Roth McDuffie, A., & Mather, M. (2009). Middle school mathematics teachers' use of curricular reasoning in a collaborative professional development project. In J. T. Remillard, B. A. Herbel-Eisenmann & G. M. Lloyd (Eds.), Mathematics teachers at work: Connecting curriculum materials and classroom instruction (pp. 302-320). New York: Routledge.
Roth McDuffie, A., Choppin, J., Drake, C., & Davis, J. (2018). Middle school teachers' orientations and noticing of features of mathematics curriculum materials. International Journal of Educational Research, 92, 173-187.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 15(2), 4-14. Stein, M. K., Remillard, J. T., & Smith, M. S. (2007). How curriculum influences student learning. In F. Lester Jr. (Ed.), Second Handbook of Research on Mathematics Teaching and Learning (Vol. 1, pp. 319-369). Reston, VA: National Council of Teachers of Mathematics.
Tall, D. (1986). Using the computer as an environment for building and testing mathematical concepts. In Papers in Honour of Richard Skemp (pp. 21-36). Warwick.