DILATING PERSPECTIVES: A COMPARISON OF TEACHER- AND STUDENT-FACING TEXTS ON A UNIT ON SIMILARITY

Soobin Jeon
University of Michigan
sbjeon@umich.edu

Patricio G. Herbst
University of Michigan
pgherbst@umich.edu

Mathematics curricula are often the primary resource used in the teaching and learning of mathematics–in particular, they represent mathematical knowledge for teachers and students. Yet, there seems to be a discrepancy between the curricular materials written for teachers and students (e.g., more variety in the semiotic choices in text provided for teachers than for students). This paper illustrates a way to compare the language written for the teacher and the student in curricula. Drawing on the social semiotic theory known as systemic functional linguistics (SFL), we examine what meaning potentials can be associated with the exchanges of knowledge proposed in mathematical text. Examining a Unit on Similarity in an online mathematics curriculum widely used in the United States, the paper explores how the semiotic systems in text presented for the teacher and student may impact the students' opportunity to learn and pose challenges in the instructional exchange between teacher and student.

Keywords: Curriculum, Geometry and Spatial Reasoning, Instructional Activities and Practices, Communication

Background and Theoretical Framing

Brousseau (1997) proposed that the relationship among teacher, students, and mathematics is mediated by a didactical contract, a set of tacit responsibilities that bind the teacher, students, and content. For example, the contract makes the teacher responsible for students' acquisition of knowledge at stake and the students responsible for engaging in the work the teacher organizes for them to acquire that knowledge (Herbst & Chazan, 2012). Similarly, in an instructional system, "a teacher has the responsibility to organize and sustain activities for students in which the work that students do can be described by appealing to a culturally current representation of the knowledge at stake" (Herbst & Chazan, 2012, p. 605). Among the work a teacher needs to do is manage instructional exchanges, which involves equating or reconciling seemingly unequal representations of knowledge, such as the work the students did on one or more tasks and the knowledge at stake in those tasks. The management of an instructional exchange requires teachers not only to apply the didactical contract to the task at hand but also identify what the students need to do and figure out what aspects of the mathematical task point to the knowledge at stake (Herbst & Chazan, 2012). The statement of the knowledge at stake in an instructional exchange makes use of various elements of the mathematical register but managing instructional exchanges also "requires the teacher to engage in serious interpretation of much more opaque signs in the realm of the students' work" (Herbst & Chazan, 2012, p. 606). Mathematics curricula are one important medium that guides teachers on how to engage in instructional exchanges.

Mathematics curricula play a crucial role in the teaching and learning of mathematics. Textbooks are often the primary resource used by teachers to plan and deliver instruction and students also rely on textbooks as a reference and study guide (Reys et al., 2004). Mathematical concepts in textbooks are written in a voice that is appropriate for students (Remillard, 2000) and teachers are often regarded as mediators of the text (Love & Pimm, 1996). Given the prominent role of mathematics curricula, "making a wise selection is crucial because it determines the scope of mathematics that students experience and, to some extent, how teachers present the material and how students learn" (Reys et al., 2004, p. 65). While studies on the use of textbooks or curricula focus on the teacher or student, there is a lack of attention to the interactions between the teacher and student when using mathematical texts (Rezat, 2011) and the meanings construed in texts addressed to the teacher and students. As mathematical curricula often provide separate instructions for the teacher and student, semiotic resources in the teacher-facing and student-facing instructions may differ, allowing for different experiences, meaning-making, and shaping how the teacher enacts their responsibility in an instructional exchange. Specifically, the language choices made in the text addressed to students may differ from those made in the text addressed to teachers and both of them may mediate when and how the teacher affects instructional exchanges. An analysis of the different language choices in these texts can provide us a first approximation at what work might exchange for what knowledge claims in instruction. The goal of this paper is to illustrate this way of using curriculum resources to analyze a priori the instructional exchanges that might take place in classes in which these resources are used.

Methods

Context

We analyze a Geometry Unit on Similarity from the Illustrative Mathematics (IM) curriculum for grades 9-12 (Illustrative Mathematics Certified, 2023). Illustrative Mathematics is an independent nonprofit that provides high-quality K-12 instructional materials, rigorous, standards-aligned content, and engagement in mathematical discussion. The Unit on Similarity has students learn the definition of similarity in terms of dilations and rigid transformations. Students prove that if triangles have three pairs of congruent corresponding angles and three pairs of corresponding sides in a proportional relationship, the triangles are similar. They also draw conclusions about figures they have proven to be similar, such as, corresponding angles are congruent and corresponding sides are in a proportional relationship.

In terms of structure, the unit consists of 16 lessons organized into four sub-sections: (A) properties of dilations, (B) similarity transformations and proportional reasoning, (C) similarity in right triangles, and (D) putting it all together. There is also a modeling prompt, called Scaling a Playground, that follows the first lesson. The modeling prompt provides students with an opportunity to choose and use appropriate mathematics and statistics to analyze empirical situations, helping students understand that they can use mathematics to better understand things they are interested in. During the task, students formulate a model, which can be a geometric, graphical, tabular, algebraic, or statistical representation that describes the relationships between variables in the situation.

The IM curriculum provides text that is for the teacher (e.g., labeled under "teacher-facing") and text that is for the student (e.g., labeled under "student-facing"). Each of the 16 lessons has a set of teacher-facing learning goals and student-facing learning goals. The teacher-facing learning goals describe mathematical, pedagogical, and language goals of the lesson for a teacher audience. While these goals are intended to be read by teachers, the actor they allude to is the students. For example, if a goal reads, "Comprehend that dilations take angles to congruent angles" (see Figure 1a), it construes for the teacher the meaning that students are ultimately to be able to comprehend that dilations take angles to congruent angles. In contrast, the student-facing learning goals are written in the first person, often starting with "let's" and "I can," and invite students to the lesson of the day. Likewise, the modeling prompt distinguishes sections for the intended audience, calling sections “teacher instruction” or “student facing statements.”

Data Analysis

We conducted an analysis of discourse that seeks patterns in linguistic data by drawing on a meaning-focused theory of language, *systemic functional linguistics*. In particular, we draw on Halliday and Matthiessen’s (2004) system of Transitivity in lexicogrammar, that states a clause, as a token of ideational meanings (i.e., goings-on in the world) can be analyzed by identifying processes, participants, and circumstances, which are “organized in configurations that provide the models or schemata for construing our experience of what goes on” (p. 175). The ideational metafunction of text construes meaning through the different types of processes within the Transitivity system. Analyzing the domain of experiential meaning in mathematics is helpful in understanding how teachers and students can be engaged with and connect to the learning experience (O'Halloran, 2005).

Ideational meanings are construed through choices of processes, which can be *Mental*, *Material*, *Relational*, *Existential*, *Verbal*, *Behavioral*, or *Operational*. Processes are typically realized with verb choices in simple clauses but in more complex clauses they may be nominalized. *Mental* (sensing) processes may be realized with verb choices such as consider or think, material (doing) processes with verb choices such as draw or project. *Relational* clauses relate two separate entities, they “characterize and identify” (Dimmel & Herbst, 2015, p. 166) as in the statement of a mathematical definition. *Operational* processes, a process type specific to mathematical discourse identified by O'Halloran (2005) are often realized by choices of arithmetic and algebraic operation symbols or by the “words that in mathematics mean an operation” (Dimmel & Herbst, 2015, p. 167) or symbols that represent a mathematical action or function (e.g., a raised number or letter next to a letter or number indicating the operation “raised to the … power”). *Existential* processes show that “phenomena of all kinds are simply recognized to ‘be’” (Halliday & Matthiessen, 2004, p. 171). *Existential* processes stipulate the existence of something (Dimmel & Herbst, 2015). For example, in a proof context, the word ‘let’ in “let O be the center of a dilation” tells the student that there exists an entity, O, that they can utilize in doing the proof.

The system of Transitivity “offers a range of options for ideational (content) meaning that is comprehensive of the ways language varies in presenting experience: as doing, sensing, saying, or being” (Schleppegrell, 2013, p. 22). For example, *Verbal* processes are enacted in the form of ‘saying.’ *Behavioral* processes are enacted in the form of partly ‘doing’ and partly ‘sensing,’ projecting the outer reflection of the consciousness (e.g., observe). Identifying the ideational meaning that can be represented in the form of processes, we compare teacher-facing text and student-facing text presented in (1) the lesson goals and (2) the unit’s modeling prompt. The lesson goals span across 16 lessons, allowing us to see patterns in the text across lessons that vary in the ways they position the intended reader (the teacher or the student). Looking at the modeling prompt offers another perspective on how text is presented to the teacher and the student, how their interaction is supported, and how printed language may stand on its own or be supported by other modalities (e.g., visual, technological). In particular, we identify meanings in the text construed through choices from the transitivity system. In our analysis, we first determine the form and quantity of processes in the teacher-facing text and student-facing text within the lesson goals and modeling prompt. Subsequently, we compare the processes in each lesson or prompt between the teacher- and student-facing texts, determining whether the processes are more or less aligned with similar meanings. Through the analysis of transitivity patterns, we consider the following questions:

1. To what extent do the teacher-facing text and student-facing text draw on processes with similar meanings?
2. What patterns can we observe in the processes used within and across the teacher-facing and student-facing texts?

Findings

The analysis of transitivity patterns displayed a contrast between the teacher- and student-facing texts in the learning goals: More use of the *mental* (cognizing) processes in the teacher-facing goals and more use of the *material* and *verbal* processes in the student-facing goals. The teacher-facing goals are represented in statements that start with verbs that indicate what the students should be able to know by the end of the lesson. In terms of instructional exchanges, these instructional goals refer to the claim that the teacher may be able to make on behalf of the student based on what the teacher sees the students doing. If a teacher-facing goal says, "Comprehend that dilations take angles to congruent angles" (Figure 1a), the teacher is expected to ensure that students mentally understand this idea, as comprehend realizes a *mental* process. At the same time, the corresponding student-facing goal says, "Let's dilate lines and angles" (Figure 1b), representing the work students will do as taking action in the *material* world, doing the dilation. The distinction between the *mental* process in the teacher-facing goal and the *material* process in the student-facing goal is also highlighted in the difference between the words "dilation" and "dilate". These are key technical terms in this unit that identify an important concept students will use to justify that triangles are similar. We can see a distinction between "dilation" in the teacher-facing goal and "dilate" in the student-facing goal by how often each form (verb or its nominalization) is used and which transitivity process co-occur with their use.

Comprehend that *dilations* take angles to congruent angles. **Prove** that a *dilation* takes a line not passing through the center of the *dilation* to a parallel line, and leaves a line passing through the center unchanged.	Let's **dilate** lines and angles. I can **explain** what happens to lines and angles in a *dilation.*
Figure 1a: Teacher-facing Goals	Figure 1b: Student-facing Goals

Figure 1: Learning Goals for Lesson 4: Dilating Lines and Angles

"Dilation" appears five times in the teacher-facing goals across the unit. For all five clauses, "dilation" is introduced by a *mental* process (i.e., comprehend, prove, Figure 1a), emphasizing to the teacher that they should be supporting students' understanding of and perception on the concept of dilation. In contrast, "dilation" appears only once in the student-facing goal when students are asked to verbalize (i.e., explain, Figure 1b) what happens in a dilation. "Dilate," the verb form and *material* process, is more commonly shown in the student-facing goals, appearing four times while appearing not once in the teacher-facing goals. In a general sense, the teacher is being asked to focus on having students sense what a dilation is while the student is being asked to do the dilation, or to dilate, or to narrate what they did (or verbalize). It then becomes the teacher's responsibility to not only ensure that the students dilate different properties of figures but also to interpret such actions as comprehending the properties of dilation what it means for dilations to work. Whereasthis is evidence of how nominalization supports the development of mathematical knowledge that is more concise and advanced, it packs within it material actions that may but also may not always entail all the properties attributed to dilations.

Relational processes play the important role of describing the qualities and characteristics of entities. In the context of a geometry curriculum, relational clauses are essential for making meaning as they provide a way to describe and establish the properties of and relationships among geometric objects. The *relational* processes in the teacher-facing and student-facing goals convey similar meaning but are represented differently. In particular, the *relational* clauses in the teacher-facing goal are presented as complex noun phrases in the student-facing goals. For example, a *relational* clause in the teacher-facing goal has an embedded clause that is linked by 'are similar to' the noun group, the original right triangle: "the two smaller right triangles formed when a right triangle has an altitude drawn to its hypotenuse are similar to the original right triangle" (Figure 2a). In the student-facing goal, two complex noun phrases describe what is to be explored and found: "right triangles with altitudes drawn to the hypotenuse" and "similar triangles formed by the altitude to the hypotenuse in a right triangle" (Figure 2b). We see that the complex noun groups in the student-facing goals are more concise and simplified. Yet, they are more complicated in parsing the mathematical meaning, requiring more work from the students to unpack them.

The *material* processes in the student-facing goals (i.e., explore, find, Figure 2b) provide a concrete basis for understanding what is presented to the teacher in a *relational* process that would otherwise be difficult without an example or a visual support. Through exploration, students can physically draw altitudes on right triangles and see how they relate to the hypotenuse. In addition, by finding similar triangles, students can see how similar triangles are formed by right triangles with altitudes drawn to the hypotenuse. As the *material* processes support students' concept of similar triangles in a more tangible way, the teacher-facing goal presents the same meaning but highlights the cognitive action through the use of the *mental* process (i.e., justify, Figure 2a). The *mental* process is important because it involves cognitive processes such as critical thinking and reasoning, deepening the students' understanding of the properties of similar triangles. The *mental* process also shows that the teacher supports students' learning from doing to sensing the relationship between the original right triangle and the two smaller triangles.

Justify that the two smaller right triangles formed when a right triangle has an altitude drawn to its hypotenuse are similar to the original right triangle.	Let's **explore** right triangles with altitudes drawn to the hypotenuse. I can **find** similar triangles formed by the altitude to the hypotenuse in a right triangle.
Figure 2a: Teacher-facing Goals	Figure 2b: Student-facing Goals

Figure 2: Learning Goals for Lesson 13: Using the Pythagorean Theorem and Similarity

Operational processes are helpful in elaborating the *material* process because the *operational* processes articulate steps to carry out a particular action. For example, the *operational* process of multiplication in the teacher- and student-facing goals (Figure 3) supports the elaboration of the *material* process of dilating. While the relationship between the *operational* process of multiplication and *material* process of dilation remains the same, it is represented differently in the teacher-facing and student-facing goals. In the teacher-facing goal, the *operational* process of multiplication is directive; the teacher may instruct the students to determine that dilating results in multiplying all lengths (Figure 3a). The focus is on the process of understanding how dilating works and recognizing that dilation is equivalent to the sequences of operations. On the other hand, in the student-facing goal, the operational process of multiplication will come to be known and the student will understand the concept, knowing that when figures are dilated by a scale factor of k, all lengths in the figure are multiplied by k (Figure 3b). The operational process in both teacher- and student-facing goals is projected by the *mental* process (i.e., determine, Figure 3a; know, Figure 3b), comprehending that dilate is multiply. The *mental* process reports that students are able to apply multiplication when thinking of dilations. For example, students can understand that by multiplying all lengths in the figure by a scale factor of k, dilating is able to change the size and dimensions of the figure but maintain its shape and proportions.

Determine that *dilating* by a scale factor of k *multiplies* all lengths by k	I **know** that when figures are *dilated* by a scale factor of k, all lengths in the figure are *multiplied* by k
Figure 3a: Teacher-facing Goals	Figure 3b: Student-facing Goals

Figure 3: Learning Goals for Lesson 3: Measuring Dilations

In the modeling prompt, the *existential* process especially plays an important role in setting the context. The *existential* processes in the teacher- and student-facing sections introduce the diagram (teacher is to tell students “this is a diagram of a playground”, Figure 4a; students are given “Here’s a playground for a school of 360 children in Springfield”, Figure 4b). The purpose of the *existential* process here is to introduce the diagram as a model of the actual playground (Figure 4a).

In the student-facing section, there is no specific goal set for students that indicates the processes they will engage in. Instead, they are given a scenario about the playground with the diagram and asked to answer questions (i.e., “how many?”, “how much?”, Figure 4b) that prompt their solving the problems and making sense of the modeling task. The teacher-facing section (Figure 4a) guides teachers to engage in *material* and *verbal* process (i.e., display, ask, invite, tell, clarify) so that students can engage in the *material* process (i.e., use) and ultimately the *mental* processes (i.e., wonder, notice) as they “share” (*verbal* process). Teaching-facing components also emphasize the *mental* processes, such as considering how students will experience something when it happens (e.g., “notice”). In the student-facing instruction, details on the hypothetical context are provided, placing more emphasis on the *existential* processes.

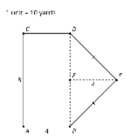
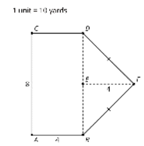
Display this image of the original playground for all to see:  **Ask**, “What do you notice? What do you wonder?” After some quiet think time to **notice** and **wonder**, ask students to **share** with a partner. Then **invite** students to **share** what they noticed and wondered with the class, and record the responses for all to see. Then **display** this diagram:  Tell students that *this is a diagram of a playground,* and they will **use** it to design a new playground to fit different constraints. If needed, **clarify** that the fence of the playground only goes around the perimeter, not along the segments.	*Here’s a playground for a school of 360 children in Springfield.*  1. The fence around the playground costs 90 dollars per 50-foot roll. Laying the grass across the area costs 400 dollars per 500 square feet. How much did the fencing and grass for this playground cost to build? 2. How many kids per square yard can Springfield’s playground hold? 3. There’s a new playground going up in nearby Wintermeadow. Wintermeadow has a budget about 3 times greater than Springfield’s. Recommend a playground shape and size that would fit Wintermeadow’s budget and hold at least 3 times as many kids as Springfield’s playground at the same density of kids per square yard. How many kids can your playground hold?
Figure 4a: Teacher Instruction	Figure 4b: Student-facing Statement

Figure 4: Modeling Prompt: Scaling a Playground

Discussion

The different semiotic resources used in the teacher- and student-facing components of curricula provide opportunities for researchers to anticipate how teachers and students make meaning of mathematical concepts and whether any differences in the meanings construed can be expected. The Illustrative Mathematics curricular texts showcase an intriguing relationship between the articulation of goals for teachers and students. While teachers are directed to approach these lessons with the *mental* processes students are to develop through the unit foregrounded, students are focused on the practical aspect of doing, from a *material* standpoint. Even when the student’s goal is to know something, the expression of what they are to know focuses on the *material* processes involved in dilation. The focus on the *doing* process, although it is emphasized in the students’ learning goals, is only a partial understanding of their learning experience. This raises questions about whether the objectives for students’ learning should involve more reflection around the concepts of dilation, and whether teachers require a better understanding of how they can connect the *material* processes students engage in with the cognitive outcomes that are expected. The discourse facing students often fails to represent their *mental* activities, making it challenging for them to explain their approach to a task. This issue is especially important in current approaches to mathematics, where a focus on developing students’ thinking is critical for advanced learning. While the field recognizes the importance of justification and proof in mathematics, mathematics teachers need guidance in supporting students to articulate the ways they are thinking through meaningful classroom discourse.

Effective mathematics instruction requires a continuum of modes, including spoken, written, stand-alone, and visual support, to construct activities that meet students' diverse needs. Collaborative interaction, allowing for meaning to be construed between individuals rather than within, is a key way teachers can support students' movement between doing and thinking through talk (Gibbons, 2003). Teachers can model this by thinking aloud themselves to show how *material* processes can be articulated in what students achieve in more abstract language, as implying a cognition of sorts. This movement between the observed and the inferred, encoded in language by the choice of *material* and *mental* processes can support students to communicate their thinking for others to engage with (Herbel-Eisenmann et al., 2013). Through such efforts, teachers can help their students develop a deeper understanding of mathematical concepts and communicate their thinking effectively. The instructional exchange between the teacher and students involves building on the language students bring and adding the technical and abstract languages unique to mathematics to enhance students' understanding of mathematical concepts.

Addressing the discrepancy in texts between the teacher- and student-facing goals can improve the quality and specificity of curriculum materials and equip teachers to better support students. The teacher can report what the student is doing using the *material* processes and align instructional goals with *mental* processes, enabling the teacher to manage the instructional exchange. The management of an instructional exchange not only complies with the didactical contract but also highlights how students are expected to demonstrate the specific knowledge at stake. The teacher has the responsibility to engage students in actions that instantiate the processes in the instructional goals, regardless of what students actually think, and help develop concrete understanding of abstract mathematical concepts, but then the articulation of what students have learned needs to be further supported. The next logical steps in capitalizing on this understanding would be to conduct research that addresses how teachers can become more aware of the role of language in positioning them and their students to engage in discourse. By conducting such research, researchers can work towards ensuring that curricular texts effectively support instructional exchanges and high-quality mathematics education that meets students' individual needs and builds on their languages.

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