UTILIZING ASYNCHRONOUS NUMBER TALKS AS A WAY TO ENGAGE ALL LEARNERS

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Due to the sudden and unexpected move to remote learning in 2020 influenced by COVID-19, both mathematics teacher educators (MTEs) and prospective elementary teachers (PTs) faced a new challenge in creating a productive remote teaching and learning environment. In this study, we used Parrish's (2014) addition strategies and Thanheiser's (2009) conception of multidigit numbers to analyze 41 prospective elementary teachers' responses in Number Talks (NTs) from two online asynchronous mathematics content courses for prospective elementary teachers. We found that (1) the order in which NTs are posed influences the strategies used, (2) some PTs identified the sameness of strategies differently than us (MTEs), and (3) PTs' conception of digits developed over time. In online asynchronous NTs, all PTs are asked to share their strategies before they see strategies provided by other students. This allowed all students to contribute and for instructors to observe and trace every PT's use of strategies over time. Therefore, we argue that asynchronous NTs can be a way to pursue the engagement of all learners in both face-to-face and online learning environments.

Keywords: Preservice Teacher Education, Online and Distance Education, Number Concepts and Operations. Equity, Inclusion, and Diversity.

Purpose of the Study

Discourse and participation are essential in mathematics learning (Boaler, 2002: Ernest, 1994; Lampert, 1990; Sfard, 1998; Staples, 2007; Wood, 1999) for two reasons: 1) To allow all students to share their own and engage with each other's mathematical thinking, and 2) to have a way for instructors to access their students' mathematical thinking (Carpenter et al., 1996; Franke et al., 2001; Kazemi et al., 2016). Focusing on students' mathematical thinking supports both students' learning of mathematics and instructors' learning of students' mathematical thinking (Durkin et al., 2017; Jacob & Spangler, 2017; Yackel & Cobb, 1996). Discourse-rich practices like Number Talks can support instructors in making sense of student thinking (Han & Thanheiser, 2021; Laustgarten & Matney, 2019; Stott & Graven, 2015). However, Number Talks are typically synchronous activities so implementing them asynchronously presents a challenge. Due to the sudden move to remote learning in 2020 influenced by COVID-19, both mathematics teacher educators (MTEs) and prospective elementary teachers (PTs) faced a new challenge in creating a productive remote teaching and learning environment.

To address this new challenge, we developed and implemented an online asynchronous version of Number Talks (NTs) with PTs in mathematics content courses for elementary teachers. We (Han & Thanheiser, 2021) demonstrated how NTs can be successfully enacted in online asynchronous learning. We argued that NTs can serve as a tool for more equitable learning and as a formative assessment in online asynchronous learning. In this study, we build on this prior work and argue that online asynchronous NTs can serve as a tool to engage all learners (in both face-to-face and online learning) by providing space for all learners to share their mathematical thinking. We also argue that the teachers can have a better understanding of their students' mathematical thinking since teachers can attend to all learners' mathematical thinking (rather than just a few students who share). In this study, we examined 1) the impact of purposeful sequencing of NT problems, 2) the strategies that sequencing elicited, 3) how PTs view their strategies as the same and different, and 4) PTs' conceptions of digits. Our research questions were:

1. Do the different sequences of NTs affect PTs' use of strategies?
2. How do PTs view each other's strategies as the same and different?
3. Can NTs support PT's development of conceptions of digits?

Literature Background / Theoretical Framework

Number Talks

Number Talks (NTs) are typically five to fifteen minutes of whole-class discussions on mental computations and/or mental problem-solving (Gerstenschlager, & Strayer, 2019; Han & Thanheiser, 2021; Humphreys & Parker, 2015; Johnson & Partlo, 2014; Okamoto, 2015; O'Nan, 2003; Parrish, 2011, 2014; Parrish & Dominick, 2016; Sun, et al., 2018; Woods, 2018, 2021, 2022). Parrish (2014) suggested five key components of NTs: a) classroom environment and community, b) classroom discussions, c) the teacher's role, d) the role of mental math, and e) purposeful computation problems. The teacher poses a purposeful computation problem or investigates students' strategies, elicits specific strategies, and/or makes connections between similar or different strategies. Students engage in private-think time to mentally solve the problem first and then the teacher facilitates a whole-class discussion. Parrish (2014) categorized addition strategies, which can be elicited during NTs, into eight categories (*counting all, counting up, breaking each number into its place value, making landmark or friendly number, doubles/near doubles, making tens, compensation, and adding up in chunks*).

NTs serve to engage students in mental math so that they strengthen their number sense and computation skills (O'Nan, 2003; Parrish, 2011; Johnson & Partlo, 2014; Parrish, 2014; Okamoto, 2015) by practicing mental math through purposefully selected computation problems followed by classroom discussions to debrief various strategies. NTs can play an essential role in developing accuracy, flexibility, and efficiency for computation and number sense (Han & Thanheiser, 2021; Okamoto, 2015; O'Nan, 2003). In NTs, students are asked to share their mental strategies and justify their thinking. Justification leads to better math understanding (Parris, 2011; Staples et al., 2012). NTs can also provide more equitable mathematics classrooms (Han & Thanheiser, 2021; Sun, Baldinger, & Humphreys, 2018) by allowing all students to participate and valuing all ideas. In online asynchronous NTs, every participant gets to share their thinking and see everyone else's thinking (Han & Thanheiser, 2021). NTs can help students to develop ownership of their mathematics learning (Parish, 2014) by helping them recognize what they can make sense of (Han & Thanheiser, 2021). Also, NTs serve to establish a classroom community in which all student thinking is valued and students are given time to complete their thinking (Parish, 2014; Woods, 2018, 2022). Students' private thinking time is essential to make sense of mathematical tasks (Anthony & Walshaw, 2009; Kelemanik et al., 2016; Staples, 2007). Establishing a classroom culture of mutual respect is essential for creating a safe environment for effective NTs (Parrish, 2014). This includes sharing incorrect answers, invalid strategies, unfinished strategies, etc. In face-to-face classrooms, making various strategies public on the board can elicit that there is more than one way to solve a problem (Lustrgarten & Matney, 2019). Also, NTs allow for comparing across strategies to determine their similarities and differences. This kind of reflection allows students to develop a deeper understanding of mathematics. (CCSS, 2010; Durkin et al., 2017; Jacobs & Spangler, 2017; NCTM, 2000; Yackel & Cobb, 1996). However, despite the wide use of NTs among teachers, schools, professional development, and teacher education, there is little rigorous research about if and how Number Talks support students' development of whole number conceptions and operations (Matney et al., 2020). Woods (2018, 2022) demonstrated that the Math Talk community (questioning, explaining mathematical thinking, the source of mathematical ideas, and responsibility for learning; Huffered-Ackles, 2004) was improved through NTs in third-grade classrooms. Woods (2021) studied PTs learning through the implementation of NTs through video simulations. PTs reported that they noticed students' different ways of thinking and strategies. They also learned the importance of public records, opportunities for students to participate, and support students to expand and revise their thinking.

Prospective Elementary Teachers' Conceptions of Whole Number and Operation

PTs typically enter mathematics content courses for teachers being able to perform addition and subtraction algorithms but unable to explain why they work (Browning et al., 2014; Thanheiser, Browning, et al., 2014; Thanheiser, 2009; 2010; 2018; Thanheiser, Whitacre, et al., 2014). This is because PTs are typically taught how to use the standard algorithms but do not connect the digits in the number to the values. For example, the number 321 would be interpreted as 3 next to a 2 next to a 1 (*concatenated digits only*) or 300 ones and 2 next to a 1 (*concatenated digits plus*) rather than as 3 hundreds, 30 tens, or 300 ones combined with 2 tens or 20 ones, combined with 1 one (*reference-units* or *groups-of-ones)* (Thanheiser, 2009; 2010; 2018). The concatenated digit conception restricts PTs' ability to explain regrouping conceptually and leaves them with a view of math that is not based on sense-making. Thus, NTs can be used as one way to allow PTs to rediscover the sense-making of digits and using numbers meaningfully.

Methods

We analyzed 41 PTs' responses in NTs from the two online asynchronous courses. Both courses were the first in a sequence of three mathematics content courses for elementary teachers in the United States. The courses were taught by the same instructor in Spring 2020 (23 PTs) and Fall 2020 (18 PTs). In Spring 2020, the course was suddenly moved to remote learning due to COVID-19. Since the school is on a quarter system this means that the whole course was moved from the first day. So, the PTs and the instructor participated in an online, asynchronous course despite never intentionally signing up for that format. The online asynchronous class was delivered through Google Slides and an Online Discussion Forum (in our case, D2L). The instructors did know in advance of Fall 2020 that the course would be online and asynchronous. Students were aware in Fall 2020 that they were registering for an online format of the course. No PTs in the study were exposed to different types of addition strategies (Parrish, 2014) or conceptions of whole number (Thanheiser, 2009; 2010; 2018) prior to the NTs.

We followed the part of the Online Asynchronous Collaboration (OAC) model of instruction, which supports teachers to develop their mathematical knowledge for teaching (MKT) (Clay et al., 2012; Silverman & Clay, 2009). The OAC model consists of six sequenced activities: "1) reviewing an expert model, 2) creating initial responses to the task, 3) listening to/viewing others' responses, 4) reviewing and commenting on others' responses, 5) discussing, and 6) revisit initial responses" (Clay, Silverman, & Fischer, 2012, p. 767). We did not use the first activity, reviewing an expert model, since NTs do not begin with sharing the instructor's strategies. OAC allows a slow pace of learning, makes all participants' ideas public and permanent, increases access to others' various thinking and understandings, and enables participants to return to their and others' previous thoughts to reinforce their understanding.

We analyzed the first two NTs from Spring 2020 (S1 and S2) and the first three NTs from Fall 2020 (F1, F2, and F3). The difference between the sequences is that in the Fall we started the sequence purposefully with F1, while the remaining NTs were the same, S1 = F2 and S2 = F3 (see Table 1). For each NT, we provided the problem through a Google Slide and asked PTs to solve the problem without writing anything down. Then we asked the PTs to share their strategies (written explanation on how they computed the NT problem) through D2L. To ensure every PT shared their own strategies, we utilized the function: *Users must start a thread before they can read and reply to other threads in each topic* in D2L. Once PTs posted their responses, they were able to see everyone else's responses. After their initial sharing, we asked PTs to look for other solutions. In this way, all PTs had to share their strategies to participate in the NTs. In both Spring 2020 and Fall 2020, we asked the PTs to look for strategies that were (a) the same as their own and (b) different from their own and to use the comment function in D2L to respond to the solution and to explain how their solutions were the same and/or different.

Table 1: Number Talks Problems

Term	Spring 2020		Fall 2020		
	S1	S2	F1	F2	F3
Problems	13+18	99+98	15+16	13+18	99+98
Timing	Week 4	Week 5	Week 1	Week 2	Week 4
Number of Participants	22	23	18	18	18
Number of Total Strategies	23	24	23	22	18

Analysis to answer the first research question and the third research question focused on each PT's use of strategies in each NT and across NTs. We had specific goals and anticipated strategies for each NT (see Table 2). Although Parrish (2014) did not include the standard algorithm as a category, we anticipated the algorithm as a strategy that PTs would use based on their experiences with school mathematics (Thanheiser, 2014). Analysis to answer the second research question focused on how PTs' use of strategies in F2 and F3 are similar and/or different compared to S1 and S2, respectively.

Table 2: Goal and Anticipated Strategies (Parrish, 2014) for Each NT

NT	Goal	Anticipated Strategies
S1 13+18	Introduce NT to the PTs. Observe PTs' initial strategy use.	*the standard algorithm*, *breaking each number into its place value, adding up in chunks*,
S2 99+98	Will PTs use the same strategies as S1 or different? Can 99+98 elicit *making landmark or friendly numbers* strategy?	the s*tandard algorithm, breaking each number into its place value, making landmark or friendly numbers, compensation,*
F1 15+16	Introduce NT to the PTs. Obverse PTs' initial strategy use. Can 15+16 elicit *doubles/near doubles* and *making landmark or friendly numbers* strategies, which were not used in S1?	*the standard algorithm, breaking each number into its place value, making landmark or friendly numbers,* *doubles/near doubles*,

F2 13+18	Will PTs use the same strategies as F1 or different? Will *doubles/near doubles* and *making landmark or friendly numbers* show up based on the inclusion of F1? Will *compensation* strategy emerge?	*the standard algorithm*, *breaking each number into its place value, adding up in chunks*, *making landmark or friendly numbers, doubles/near doubles*, *compensation,*
F3 99+98	Will PTs use the same strategies as F1 and F2 or different? Will *doubles/near doubles* and *making landmark or friendly numbers* show up based on the inclusion of F1?	*the standard algorithm, breaking each number into its place value, making landmark or friendly numbers, doubles/near doubles*, *compensation,*

We analyzed PTs' strategies through two different lenses: addition strategies by Parrish (2014) and conceptions of digits (Thanheiser, 2009). Through our initial analysis by using Parrish's (2014) categorization, we realized that some PTs used *concatenated concepts of digits* (Thanheiser, 2009). Thanhesier's (2009) original framework requires three-digit whole numbers to differentiate *reference units* and *groups of ones*, *concatenated digits plus* and *concatenated digits only*, respectively. Since Thanheiser (2009) found that the *concatenated digits plus* conception "arises only in numbers with three or more digits," (p. 279), that conception was left out of our analysis. Since we asked for two-digit addition problems, we simplified the categories in Thanheiser's (2009) framework into *sufficient* (reference units or groups of ones) and *limited* (concatenated) categories referring to the ability to explain regrouping.

Two researchers categorized PTs' strategies individually and reconciled them. For example, in F1, one PT responded, "*When I first look at this problem, my brain immediately added 15+15=30. Then, because one of the numbers is actually 16, instead of 15, I added a 1 to my answer to get 31.*" We highlighted 15+15=30, 16=15+1, 30+1=31, and matched the response to *doubles/near doubles* strategy since the student began with 15+15=30. We matched this student's response to *sufficient* conception of digits because the digits were clearly referred to as their place value. Responses were coded as the *standard algorithm* if the response described the standard algorithm without mention of place value, but as *breaking each number into its place value* if the response used the standard algorithm in conjunction with mention of place value to explain regrouping. For example, one response to F2 13+18 was *"I know 1+1= 2, then I added 3+8 = 11. I took the 1 from 11 adding it to 2 = 31."* Since there is no evidence that the student understood the "1 from 11" as one 10, this response was coded as the *standard algorithm* and *limited* conception of digits. Another response for F2 included *"My mind automatically places the 13 above the 18... 3+8 is equal to 11. Since we are working on the ones side, and your sum is greater than 10, you're going to subtract 10 from the 11, leaving you with 1. The 10 will be added in the tens place, making the problem 1+1+1 in the tens place."* This response includes a specific reference to place value to justify the regrouping in the *standard algorithm*. As such, this response was coded *breaking each number into its place value*. However, this response showed a *limited* conception of digits since the PT referred to the numbers in tens place as 1 and 3.

Results

Most of the PTs used the instructor's anticipated strategies (shaded in Table 3) in all NTs. Three exceptions emerged: we did not anticipate using the *compensation* strategy in S1 but one

PT used it, we did anticipate using the *compensation* strategy in F2 but no PT used it, and we did anticipate using the *standard algorithms* in F3 but no PTs used it. We think these exceptions were important. Unexpected use of strategies showed why instructors need to make sense of PTs' mathematical thinking and how NTs allowed PTs to safely bring their own way of sense-making to the class discussion. More specifically, the emergence of unexpected strategies showed the importance of carefully examining instructors' anticipated strategies. Instructors can reflect on the emergence of unexpected strategies and use this information in their future NT planning (as we included *compensation* strategy as an anticipated strategy in F2). When instructors do not observe their anticipated strategies, they can then think about how to elicit those strategies.

We did not expect drastic differences between S1 & S2 and F2 & F3. However, we anticipated some differences, including the use of *doubles/near doubles* and *making landmark or friendly number*s strategy in F2 and the use of *doubles/near doubles* in F3. We were able to observe some meaningful differences. First, some PTs successfully used the *doubles/near doubles* strategy for F2 (3 PTs) and F3 (4 PTs) and the *making landmark or friendly number*s strategy for F2 (1 PT), which were not the strategies used in S1 and S2 (shaded in dark gray in Table 3). Among 4 PTs who used the *doubles/near doubles* strategy in F3, two PTs used the *doubles/near doubles* strategy in F1 and the other two PTs did not. Second, no PT used *the standard algorithm* in F3 (shaded in dark gray in Table 3) while 1 PT used the *standard algorithm* in S2. Third, no PT used the *compensation* strategy in F2 whereas one PT used it in S1. As a result, the NTs S1 and F2 and the NTs S2 and F3 showed different distributions of strategy use. Our hypothesis is that the *doubles/near doubles* strategy in F1 influenced PTs to find a strategy other than the *standard algorithm, breaking each number into its place value, or compensation* strategies in F2 and F3. Some PTs explained their strategies in F2 by referring to F1, such as *"When I began to solve the problem, I thought about the fact that I could take 3 away from the 8, and add it to the 3, making it 16+15, like last week's problem,"* and *"I took 2 away from 18 and added it to the 13. Making it 15+16. I know that 15+15 is 30, so I added one more to arrive at 31."*

Table 3: Prospective Teachers' Use of Strategies for Each Number Talk

Number Talks Strategies	S1 13+18	S2 99+98	F1 15+16	F2 13+18	F3 99+98
Standard Algorithm for Addition	3	1	1	4	
Breaking Each Number Into Its Place Value	16	9	12	13	4
Making Landmark Or Friendly Numbers		9	1	1	9
Doubles/Near Doubles			9	3	4
Compensation	1	5			1
Adding Up In Chunks	3			1	
Total Strategies	23	24	23	22	18

Most PTs well identified the same and different strategies in both Spring 2020 and Fall 2020. However, we found a few examples that PTs identified as the same strategies, but we viewed them as not the same strategies (see Table 4). In S1, we categorized provided strategy as the *compensation* strategy and another strategy as *breaking each number into its place value* strategy. Although both PTs started by making 20, they used different strategies to make 20.

However, the second PT identified the two strategies as the same strategies. In F1, although we categorized both strategies as *breaking each number into its place value* strategy, one PT started with adding tens and another PT started with adding ones. We have more instances where the PTs identified as the same strategies regardless of the order of adding tens and adding ones. Since the PTs did not know about the strategy categories by Parrish (2014), their reasoning for the sameness did not rely on it. Our hypothesis is that some PTs captured some parts of the strategies and compared them and determined that those are the same strategies.

Table 4: Strategies Provided by a PT and Strategies Identified as the Same by another PT

NT	Strategy Provided by a PT	Identified as the Same by another PT
S1 (13+18)	18+2=20; 20+10=30; 30+1=31	10+10=20; 3+8=11; 20+11=31
F1 (15+16)	10+10=20; 20+6+5=31	10+6+10+5; 5+6=11; 10+10=20; 20+11=31

PTs' use of limited conceptions of digits decreased over time in both Spring 2020 and Fall 2020 (see Table 5). Also, the number of PTs who drew on limited conceptions of digits and their percentages in F2 and F3 were lower than S1 and S2, respectively. Again, our hypothesis is that this difference is due to the different sequences of NTs. It is possible that F3 had a lower number of PTs drawing on the limited conception of digits than S2 because F3 was the third NT of the term. So, PTs in F3 were more familiar with other strategies than *the standard algorithm* or *breaking each number into its place value* strategies compared to S2.

Table 5: Prospective Teachers' Conceptions of Digits Across the Number Talks

Conceptions of Digits	S1 (13+18)	S2 (99+98)	F1 (15+16)	F2 (13+18)	F3 (99+98)
Sufficient	15	19	16	16	16
Limited	8 (34.8%)	5 (20.8%)	7 (30.4%)	6 (27.2%)	2 (11%)
Total	23	24	23	22	18

In response to the first research question, "do the different sequences of NTs affect PTs' use of strategies?", we found that different sequences of NT do, indeed, affect PTs' use of strategies. As we discussed earlier, PTs' use of strategy in F2 and F3 showed slightly different distributions compared to S1 and S2. We observed *doubles/near doubles* and *making landmark or friendly numbers* strategy in F2 and *doubles/near doubles* strategy in F3. Moreover, in F3, no PTs used *the standard algorithm*, and the *making landmark or friendly numbers* strategy was used the most (9 out of 18 PTs). Throughout the five NTs, F3 was the only case where *breaking each number into its place value* was not the dominant strategy. We interpreted this difference as due to the fact that a) F1 elicited strategies that were not observed in S1 and S2 (*doubles/near doubles* and *making landmark or friendly numbers* strategy) b) PTs in F2 and F3 were more familiar with the strategies other than *the standard algorithm* or *breaking each number into its place value* strategies compared to S1 and S2 since PTs in Fall 2020 had one prior NT (F1) before they were asked to solve F2 and F3.

In response to our second research question, "how do PTs view each other's strategies as the same and different?", we found that most PTs successfully identified the sameness and differences between the strategies. However, as we explained earlier, there were some instances where the PTs identified their strategy as the same as other strategies, but we viewed them differently. We interpreted that these discrepancies occurred because we focused on the details of the strategies whereas some PTs captured and compared some parts of the strategies.

In response to the third research question, “can NTs support PT’s development of conceptions of digits?”, we found that the number of PTs who used limited conceptions of digits (*concatenated*) decreased in both Spring 2020 and Fall 2020. We acknowledge that we do not have enough evidence to argue that NTs were the main cause of PTs’ improved conception of digits. However, participating in online asynchronous NTs required all PTs to explain their strategies. PTs had to share their strategies in a way that made sense not only to them but also to their peers and instructor. This process can allow PTs to reflect on their thinking and explanation so that they seek ways to improve their explanations. Therefore, we think NT can be one way to support PTs’ development of the conception of digits.

Discussions/ Conclusions/ Implications

Online asynchronous NTs allowed instructors to observe every PT’s use of different strategies over time, how different sequences of NTs can elicit different strategies, how PTs identified the same and different strategies, and how NTs can support the development of PTs’ conception of digits. We found that in online asynchronous NTs, all PTs can share their strategies and compare and contrast their strategies with others, which is related to higher participatory equity (Reinholz & Shah, 2018). These aspects are hard to be examined in face-to-face NTs due to time constraints. As such, we were able to have a more comprehensive understanding of PTs’ use of strategies and conception of whole numbers and digits. This result aligns with our previous study (Han & Thanheiser, 2021). Therefore, we argue that online asynchronous NTs can be a way to pursue the engagement of all students in both face-to-face and online learning environments. In a face-to-face or synchronous online classroom, the instructor can provide asynchronous NT through Google slides and Online Discussion Forum functions. Then, the instructor can attend to all students’ strategies and their conception of digits. Also, as we mentioned earlier, examining whether some students used the strategies that the instructor did not anticipate could be crucial for making sense of students’ strategies and planning future NTs.

We do not argue that NTs directly caused PTs’ use of different strategies and the development of conceptions of digits. However, we think online asynchronous NTs did shed light on possible strong relationships because we could not assume these relationships in face-to-face or synchronous online NTs. In future research, we can have pre and post-tests on PTs’ flexibility (use of different strategies) and conception of digits, and conduct interviews to elicit these relationships in detail.

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