

HOW INSTRUCTORS OF UNDERGRADUATE MATHEMATICS COURSES MANAGE TENSIONS RELATED TO TEACHING COURSES FOR TEACHERS?

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For centuries, there has been a debate about the role of undergraduate education in society. Some have argued that universities should focus on practical skills and knowledge to prepare students for the workforce, while others have supported the idea that universities should prioritize providing a broad understanding of disciplinary knowledge and practices. In this paper, we leverage data collected from 32 interviews to explore how instructors of the undergraduate geometry course for teachers (GeT) talk about the various tensions they experience in their work. Three distinct ways of talking about tensions emerged from the data: the tension as a dilemma that needs to be managed, the tension as a place to take sides, the tension as an opportunity to reframe aspects of the work. In closing we draw connections between these patterns in the data and the two perspectives about the role of undergraduate mathematics courses in preparing PTs for the work of teaching.

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This research has been founded on a reasonable conjecture that more knowledgeable teachers would be better prepared to lead instruction of higher mathematical quality and that the latter outcome should result in better mathematics learning from students. Efforts to flesh out that conjecture have produced substantial progress in refining conceptualizations of the phenomena involved, particularly teacher knowledge and its indicators. While at one time, teacher knowledge of mathematics was thought of as only disciplinary knowledge and indicated with degrees achieved or courses taken, research using those operationalizations of the construct have shown conflicting results (Begle, 1979; Monk, 1994). Also, a mathematics degree or the accrual of mathematics credits have not been reliable predictors of success in teaching (Hill, 2012; McDiarmid & Wilson, 1991). This has led scholars to promote a conceptualization of the mathematical knowledge needed for teaching based on an analysis of the recurrent work of teaching (Ball, Lubienski, & Mewborn, 2001). Relatedly, there has also been groups of scholars reconsidering the ways that undergraduate mathematics courses might be adapted to better meet the needs of prospective teachers (PTs) by equipping them with such knowledge (Adler et al., 2014; Appova et al., 2014; Speer & Wagner, 2009; Wasserman et al., 2022). While these two bodies of scholarship have done much to advance the field's conceptualizations of the knowledge needed for teaching and the role that undergraduate mathematics courses might potentially play in developing that knowledge, there has yet to be an adequate accounting of the perspectives of instructors who teach these undergraduate mathematics courses regarding these conceptualizations (Lai, 2019).

In this report, we build on previous work in which we drew on interviews conducted with 32 GeT instructors (Herbst et al., 2023) to report on how instructors perceive their position and the work they do in the GeT course in relation to institutional stakeholders. In those previous reports, we characterized the tensions that we detected beneath instructors' descriptions of the GeT

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courses they teach. The 5 tensions that emerged in those reports can be understood as the dilemmas that can result from an instructor's consideration of the following questions:

1. The **content tension**: What is the content students need to learn in the GeT course?
2. The **experience tension**: What experiences can support students' learning in the GeT course?
3. The **students tension**: Who are the students that populate the GeT course and what do they need and want from the course?
4. The **instructor tension**: Who am I and how have my experiences prepared me to teach the GeT course?
5. The **institutions tension**: Which institutions might benefit from, and support GeT improvement?

In the section entitled *Prior Results*, we elaborate on these tensions by illustrating ways that the answer to these questions can present instructors with a dilemma (Berlak & Berlak, 1981/2011; Lampert, 1985; Elbow, 1983)—forcing the GeT instructors to choose between two different courses of action, both of which are problematic for different reasons. These two distinct courses of action help illustrate the poles of the tension (or horns of the dilemma). More than simply identifying and defining those tensions according to their poles, our prior work accounts for these tensions as emerging from two distinct ways of organizing the work specific to teaching undergraduate mathematics courses for teachers: undergraduate mathematics education versus secondary mathematics teacher preparation.

In those prior reports, our focus was primarily on identifying the tensions, in terms of their poles, for the group of instructors. In this report, we leverage the same data corpus to explore the variability in the ways that different instructors experience these tensions—as a *dilemma that needs to be managed*, as a place to *take sides* by identifying with one pole of a particular tension, or as an opportunity to *reframe aspects* of their work by satisficing the demands from both poles of the tension. The study was guided by the following questions: What are the different ways that instructors relate to the tensions in their work that can be detected in the ways that instructors talk about those tensions? How can these differences be accounted for in terms of the ways of relating to the ongoing debates regarding the role of undergraduate mathematics courses in preparing PTs for the work of teaching?

Prior Results

In our previous work (Herbst et al., 2023), we have detailed five tensions that emerged from our interviews with instructors. Here, we provide a very brief description of those tensions to provide the necessary background for engaging with the results of this paper. The **content tension** describes the kinds of challenges an instructor might face when it comes to making decisions about the content addressed within a GeT course. On the one hand, instructors must consider the type of knowledge that future high school geometry teachers need to acquire to be effective in their roles. On the other hand, these courses also serve as an opportunity to expose mathematics majors to centuries of geometric research, including advanced ideas and evolving approaches to posing and addressing geometric questions. This creates a tension in course design as instructors must navigate the expectations of both groups of students and find a way to provide meaningful and relevant instruction to all.

The *experiences tension* relates to the types of practices students enrolled in the course are expected to be apprenticing into and the ways that their experiences in the course accommodate that apprenticeship. As an undergraduate mathematics course, students enrolled in the GeT course could reasonably be expected to be engaged substantively in mathematical practice valued in the disciplines: thinking about mathematics, doing mathematical activities (such as conjecturing, proving) and learning to communicate about mathematics according to disciplinary conventions (such as the process of publishing or writing in LaTeX). However, since these courses are also service courses for PTs, it is reasonable to expect that students will have the opportunity to apprentice into the work of teaching (such as learning how to address the entire class, pose questions to students, and respond to student contributions and inquiries). Similar to the content tension, our data suggests that attending to both kinds of practices may be challenging for instructors of the GeT course.

The *students tension* arises from the instructors' need to consider the diverse group of individuals enrolled in the course and how the instruction can cater to their individual needs. While the course is often purportedly offered to satisfy the needs of PTs, it also includes students pursuing different majors, such as pure mathematics, physics, or engineering. PTs have unique needs and expectations for the course. For instance, it has been suggested that they require explicit discussions on how the mathematical ideas and practices learned in the course are connected to the high school geometry course (Kilpatrick, 2019). Furthermore, it has been recommended that PTs would benefit from clear discussions on the pedagogical practices used to support mathematics teaching and learning (Wasserman et al., 2022). However, other students have conflicting expectations for the course, such as the hope that it would provide opportunities to learn new geometric content useful beyond the requirements of any specific profession.

The *instructor tension* highlights the challenges that instructors face in preparing to teach a course that requires expertise in multiple domains. In the case of the GeT course, instructors need to have a solid understanding of both mathematics education and mathematics research, which are two distinct but related areas of expertise. On the one hand, formal education and practical experience in mathematics education can help instructors develop the pedagogical skills and knowledge needed to effectively teach high school geometry. This might include experience teaching the subject, writing textbooks, supervising student teachers, or conducting research on effective teaching practices. On the other hand, the GeT course also requires instructors to have a deep understanding of the theoretical aspects of mathematics, particularly as they relate to proof and mathematical reasoning. This might involve experience conducting research in mathematical fields or engaging in other activities that require advanced mathematical skills. However, few instructors are experts in both of these areas of knowledge. This can create challenges in their work, particularly when they are required to make decisions or provide guidance that draw on both kinds of expertise.

The *institutions tension* stems from the complex set of demands placed on instructors by various institutions that support and oversee the course. Instructors are expected to navigate and reconcile the sometimes-conflicting expectations of both the mathematics department and teacher education programs. On the one hand, instructors rely on the mathematics department for guidance on the content and structure of the course. They must ensure that the course meets the department's standards and expectations for an upper-level undergraduate mathematics course.

On the other hand, instructors must also take into account the requirements and expectations of teacher education programs. In many institutions, the GeT course was originally designed to fulfill a programmatic requirement related to preparing PTs for the unique challenges of teaching

high school geometry, and as such, must meet the standards and expectations of these programs. Instructors must ensure that the course provides opportunities for pre-service teachers to develop the knowledge and skills necessary to effectively teach the subject, as well as to meet any program-specific requirements. While some GeT instructors recognize the importance of balancing the demands of both institutions, in practice, it can be challenging to reconcile the sometimes-divergent expectations and requirements.

To be clear, the nature of all five of these tensions are such that they are not easily resolved, even if instructors recognize them and avail themselves of resources that might help them manage those tensions. Furthermore, our data reveals that not all instructors experience these tensions in the same way, as evidenced by the ways they speak about the tensions. In this paper, we hope to elaborate on these differences and help account for them as drawing crucially from longstanding debates about the societal role of universities, in general, and university mathematics departments, in particular.

Literature Review/Theoretical Framework

For centuries, there has been a debate about the role of universities in society. Some have argued that universities should focus on practical skills and knowledge to prepare students for the workforce (Eliot, 1869; Cappeli, 2015), while others have argued that universities should prioritize providing a broad understanding of disciplinary knowledge and practices (Newman, 1891; Roth, 2014). The debate is relevant to mathematics departments, with some arguing that undergraduate mathematics courses should provide practical skills for future teachers (Hill et al., 2007; Leikin et al., 2018; Wasserman et al., 2018), while others prioritize educating individuals about the broader knowledge and practices drawn from the discipline of mathematics (Yacek & Kimball, 2017, see also Kimball, 2015; Silverman & Thompson, 2008). These ongoing debates have meaningful connections to the tensions that we will elaborate on in the closing remarks.

Methods

As part of a multi-year project aimed at developing an inter-institutional network for instructors of Geometry for Teachers (GeT) courses, data was collected through online video-conferencing interviews with GeT instructors. The purpose of the interviews, conducted over the first two years of the project, was to gain a better understanding of the problem space individual instructors may identify as worthwhile for the community to address. Audio and video records of the interactions were captured and transcribed for analysis. The interviews followed a semi-structured protocol with three sections. The first section, which is the exclusive focus of this analysis, consisted of 16 questions aimed at understanding the nature of the course, such as the profile of students taking the course and how faculty in the mathematics department came to teach the geometry for teachers' course. In addition, instructors were asked about the role they saw the course playing in improving capacity for high school geometry teaching, including how students' mathematical experiences in secondary schools could be influenced by the course.

We conducted interviews with a total of 32 instructors (21 men and 11 women) from 30 universities across the United States. All of the participating institutions were public, but they varied in size and focus, with some being primarily undergraduate-focused and others being doctoral-granting. All of the interviewees had recently taught a geometry course for prospective secondary teachers. Of the 32 interviewees, 30 were faculty members at various ranks, and the remaining two were graduate students in mathematics or mathematics education programs located in mathematics departments.

Analysis

Initially, the transcript data from the interviews was initially analyzed for the tensions alone. Because our goal was to identify these tensions for the group of instructors, rather than describe the variability in how individuals experience these tensions, we used the instructors' testimonies (regardless of form) to enrich the description of each of the poles in tension. That said, in that round of coding, we did notice that the data was not uniform, there were different ways that the tension was emerging in the data. In our second round of coding, we went back to the data and coded the instructors' speech about the tensions according to these distinctions, which we illustrate in the following section.

Results

In this section, we describe the three different ways that we observed GeT instructors relate to the five tensions that could be detected in the ways that instructors talk about those tensions. To be clear these distinctions showed up across the data drawn from all five tensions, but due to space constraints, we elect to illustrate them across only two of the five tensions: the *content tension* and the *instructor tension*.

In some cases, instructors talked about the tension as a *dilemma that needs to be managed*—making it clear in their speech that they were explicitly aware of the tension in their work. In these cases, instructors included more explicit expressions regarding the challenges they faced in managing their own or others' expectations about how to handle the two, often conflicting, poles of the tension. For example, in the course of describing the GeT course, one instructor talked about the *content tension* in the following way:

One of our algebraists, we don't have any geometers like no one who has geometry as their area, so one of the algebraists started borrowing a set of notes from someone down at [blinded university] and kind of made a book out of it. The book is slow in the sense that they tried to make half the semester about trying to prove how points are arranged on a line. Then eventually halfway through the semester we have two lines it's—it's really, really basic and very, very axiomatic and very formal. And it's probably really not the optimal out of arrangement for math for teachers. So, there are benefits to taking that approach but I think there are a lot of fall backs

In this quote we see an instructor acknowledging the tension by noting the ways that a GeT instructor might find themselves in a situation in which they are assigned to teach a course that has been previously designed in ways that are less than optimal for PTs. That said, while identifying this kind of organization (with a focus on a rigorous set of axioms, such as those formulated by Hilbert, 1899) as suboptimal for PTs, the instructor was also able to admit that this organization has its benefits, perhaps the ways that such an organization highlights disciplinary concepts such as the importance of consistency and completeness in axiomatic systems.

This way of talking about the tension also surfaced in the ways that some instructors talked about the *instructor tension*. For example, when responding to the question "In what ways is it important for mathematicians to be involved in teaching and improving this course?", one instructor talked about the *instructor tension* in the following way:

So, I do think it's important for mathematicians to teach this course. I think it's important for them to teach the course with the guidance from math educators and from the knowledge in the field. You know, it's not like I think mathematicians should take this and just say we're

going to do this our way, you know, come what may. But um, so I think it's important that it is done with the guidance of math educators, but ultimately it's a surprisingly mathematically sophisticated course. I think that it's mathematically sophisticated enough that mathematicians should be teaching it. Like it's, you know, even if you're not going to teach non-Euclidean geometry, the appreciation for the connections to and the differences to non-Euclidean geometry and just all these kind of very subtle mathematical things that come up in the context of the course I think do require a pretty high level of mathematical education.

Here we see an instructor acknowledging the tension by recognizing the importance of the knowledge and experiences of both the mathematician and mathematics educator as playing a crucial role in shaping the course. And while this individual ends up noting the importance for mathematicians in teaching the course, they recognize the unique knowledge that mathematics educators bring to the table with regards to the design of the course.

In other cases, instructors talked about the tensions as a place in which they had *taken a side*—identifying with one pole of a particular tension. In these cases, instructors failed to represent the tension as something they experienced as tensionful, and instead used the opportunity to provide descriptions and sometimes justifications related to their personal alignment or misalignment with one or the other poles in tension. For example, when asked about the importance for the field that the GeT course be taught to PTs, one instructor who identifies as a mathematician talked about the *content tension* by saying:

Um, I would hope that um, it is not just the material that they would teach in a high school, uh, but that it does include more. So, I guess in my case, the non-Euclidean geometry I'm doing, they would not do that in high school, but you need to have a little bit more about the idea of what else is out there other than just let's prove side, side, side criteria and for triangles or whatever it is that they're going to be doing in high school.

Different from the prior quotes, this instructor provides less evidence for an awareness of the *content tension*. Instead, the instructor takes sides by sharing details about the decisions they have made to align themselves with the side of the tension that argues for the need to focus on geometry from a more advanced perspective, rather than the knowledge needed by teachers.

Similarly, when asked about the organization of the course another instructor, who had been a high school teacher, talked about the *content tension* by saying:

So, when I was in undergrad, I took a college geometry course. And we did nothing that looked like high school geometry in that course. So, I did not feel like it prepared me for being a high school geometry teacher that I became. We need to make a decision about what are the components that will be important or valuable for future teachers.

Like the prior quote, this way of talking about the *content tension* provides little assurances that the instructor recognizes the tension. Instead, the instructor takes sides by aligning themselves with the other pole of the tension, namely the need to focus the course on supplying the knowledge needed to teach geometry.

We also observed instructors prone to talk about the *instructor tension* in ways that revealed a propensity to *take sides*. For example, when asked why it's important for mathematicians to be involved with the GeT course, one instructor talked about the *instructor tension* by saying:

Well, you know mathematicians know what proof is. They just have a broader perspective of – and I mean who else would [teach the course] if it wasn't us. I guess it would be math

education specialists. Math education specialists just don't have the same perspective that we do. Even when they're teaching calculus, they sometimes don't see how it's put together

When comparing this way of talking about the instructor tension with those from the previous subsection, we can see an individual who seems to lack the sensitivity necessary to recognize and acknowledge competing perspectives about the kind of knowledge and expertise needed to design and teach the GeT course—taking the stance that those trained as mathematicians are really the only appropriate choice for staffing the course.

Lest we begin to think that mathematicians might be alone in this lack of sensitivity, here we offer another example, from an instructor who identifies as a mathematics educator. When asked questions about how the course is staffed and the expectations for the course are made, this instructor brought up the *instructor tension* by saying:

The one issue that I have with it at a school like mine is that outside of me, the other people don't have a direct connection to K12 education. They are math faculty members who think they know what happens in schools, but they do not. They'll say, schools do this and this but I taught high school geometry for six years and I would beg to differ.

Like the previous quote, the instructor here seems equally unaware or unconcerned with the competing perspectives about the requisite expertise needed by instructors of the GeT course. Here the individual seems to have decided that having a direct connection to K12 education and knowing what happens in schools is the only or most important kind of knowledge to highlight.

Finally, we also observed instructors handling the tension by *reframing aspects of their work* in ways that satisfied one or both of the poles of the tension—electing to settle for a suboptimal solution to lessen the sense that there is a salient tension to wrestle with. In these cases, instructors had ways of talking about one or more of the poles of the tension in ways that were quite different from the ways that others had talked about the same tension.

Related to the *content tension*, some instructors found ways to reframe, and therefore minimize, the challenges related to the *content tension* by redefining one of the two bodies of knowledge. For example, to justify the choice to focus on more advanced mathematical topics, one instructor said,

So I want them to get depth and breadth of topics which are related to geometry that they will be teaching at the secondary level but also beyond that; in the sense that we are touching on non-euclidean geometry, we are explaining the big ideas behind the axioms. This course is supposed to introduce them to axiomatic structure ... I don't feel that it's okay for a teacher to graduate without even seeing what non-euclidean geometry is. I think it's just a part of their general education.

That is, rather than frame non-euclidean geometry as part of the more advanced perspectives of geometry (as so many instructors do), this instructor identifies this topic as belonging to the “general education” needed for all secondary mathematics teachers—suggesting non-euclidean geometry to simply be part of the canon of knowledge needed by all undergraduates.

Related to the *instructors' tension*, we also observed instructors engaged in this kind of reframing. For example, when asked about the needed characteristics of individuals that teach the GeT course, one instructor said, “*There's only two of us [in the mathematics department] that like really give a sh*t about geometry.*” Here, we see an instructor taking the attention off of the typical poles of the argument related to the *instructors' tension* by naming a purportedly more

pressing concern about staffing the course, namely instructors are generally unwilling to teach it. This way of talking about the *instructors' tension* draws the listeners attention away from issues related to the educational and practical experiences of an instructor and reduces the conversation about faculty qualifications to one about willingness, or perhaps motivation to teach the course.

Discussion/Conclusion

In this paper, we have described the different ways that instructors talk about the various kinds of tensions that exist within their work. We see these three ways of talking about the tension as potentially having some meaningful differences not only in the ways that a given individual might experience a tension, but also in the ways that an individual participates in the broader arguments about the role of the university. To begin, the way in which an instructor positions themselves relative to the longstanding debates about the role of the university in society could play a substantial role in the way they make sense of the *content tension* and the *instructor tension*. If the priority for universities and mathematics departments is on preparing students for the workforce, then it follows quite naturally that the purpose of the GeT course is to prepare PTs for the work of teaching by supplying PTs with an instructor who is uniquely qualified to support them in gaining the knowledge needed for teaching; and for this, no one is more uniquely qualified than a mathematics educator well versed in the work of a geometry teacher. If on the other hand, the priority for universities and mathematics departments is to prepare well rounded students with a broadly defined education that can be used flexibly in a variety of contexts, then the purpose of the GeT course is to avoid the trap of focusing too narrowly (on a single profession) and instead focus on supporting students in gaining the canon of disciplinary knowledge that has been built up across many centuries; and for this, no one is more qualified than a mathematician well versed in such knowledge. These two examples illustrate how an individual with those views might find themselves prone to relating to the *content* and *instructor tensions* by *taking sides*. Of course, the source of the variation here does not rest solely with the individual, as instructors work in different kinds of institutions (e.g., liberal arts, technical schools, land grant institutions) which have historical ties to these larger arguments that have led them to organize their programs in ways that might make these competing perspectives more salient. With that as a background, we make sense of the instances in which an instructor elects to deal with the tension by *reframing aspects of their work* as somehow the opposite of *taking sides*. That is, by reframing the poles of the tension, we see ways that the instructor can avoid the need to take sides with colleagues by settling for a less than optimal solution. We think such a technique could be a useful skill set for an instructor needing to navigate and (perhaps) avoid difficult or contentious conversations with colleagues seeking to come to agreement about challenging aspects of the work. Finally, we see those places in which an instructor talks about the tension as a *dilemma to be managed* as neither one of *taking sides* or *reframing*, but as a decision to hold the poles of the tension, in tension. In his work *Embracing Contraries*, Elbow (1983) argues for the importance of undergraduate instructors coming to see tensions in the work as inherent dilemmas stemming from the very nature of teaching. In this way, Elbow himself conceives of tensions as objects deserving not only our careful attention, but deserving an embrace that holds together the integrity of the work of teaching in all of its complexities. While we are still mulling these difference over, in terms of what they might mean for instructors' participation in larger discourses, our intuition leads us to believe that when it comes to the work of an instructor, the propensity to perceive of the tensions as *dilemmas to be*

managed as the more productive than the propensity to treat the tensions as a problem to be solved (by *taking sides* or *reframing*).

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References

- Adler, J., Hossain, S., Stevenson, M., Clarke, J., Archer, R., & Grantham, B. (2014). Mathematics for teaching and deep subject knowledge: voices of Mathematics Enhancement Course students in England. *Journal of Mathematics Teacher Education*, 17(2), 129–148. <https://doi.org/10.1007/s10857-013-9259-y>
- Appova, A. & Taylor, C.E. (2019). Expert mathematics teacher educators' purposes and practices for providing prospective teachers with opportunities to develop pedagogical content knowledge in content courses. *Journal of Mathematics Teacher Education* 22(2), 179–204 (2019). <https://doi-org.proxy.lib.umich.edu/10.1007/s10857-017-9385-z>
- Aveni, A. (2014). *Class not dismissed: Reflections on undergraduate education and teaching the liberal arts*. University Press of Colorado.
- Cappelli, P. H. (2015). Skill Gaps, Skill Shortages, and Skill Mismatches: Evidence and Arguments for the United States. *ILR Review*, 68(2), 251–290. <https://doi.org/10.1177/0019793914564961>
- Clinebell, S. K., & Clinebell, J. M. (2008). The tension in business education between academic rigor and real-world relevance: The role of executive professors. *Academy of Management Learning & Education*, 7(1), 99-107.
- Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers I*. American Mathematical Society/Mathematical Association of America.
- Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II*. American Mathematical Society/Mathematical Association of America.
- Elbow, P. (1983). Embracing contraries in the teaching process. *College English*, 45(4), 327-339.
- Herbst, P., Brown, A.M., Ion, M. et al. (2023). Teaching Geometry for Secondary Teachers: What are the Tensions Instructors Need to Manage?. *Int. J. Res. Undergrad. Math. Ed.* (2023). <https://doi.org/10.1007/s40753-023-00216-0>
- Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L. & Ball, D.L. (2008) Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, 25(4), 371-430.
- Jackson, B. S., Napier, D., Newman, B., Odom, S., Ressler, J., Ridgeway, S., ... & Spector, N. (2008). Nursing faculty qualifications and roles. National Council of State Boards of Nursing (NCSBN) Faculty Qualifications Committee.
- Lai, Y. (2019). Accounting for mathematicians' priorities in mathematics courses for secondary teachers. *The Journal of Mathematical Behavior*, 53, 164-178. <https://doi.org/10.1016/j.jmathb.2018.08.001>
- Leikin, R., Zazkis, R. & Meller, M. (2018). Research mathematicians as teacher educators: focusing on mathematics for secondary mathematics teachers. *J Math Teacher Educ* 21, 451–473. <https://doi.org/10.1007/s10857-017-9388-9>
- Lenz, L. (2015). Active learning in a math for liberal arts classroom. *Primus*, 25(3), 279-296.
- Levine, A., & Van Pelt, S. J. (2021). *The great upheaval: Higher education's past, present, and uncertain future*. JHU Press.
- Marcus, R., Fukawa-Connelly, T., Conklin, M., & Fey, J. T. (2007). New thinking about college mathematics: Implications for high school teaching. *The Mathematics Teacher*, 101(5), 354-358.
- Mounce, P. H., Mauldin, D. S., & Braun, R. L. (2004). The importance of relevant practical experience among accounting faculty: An empirical analysis of students' perceptions. *Issues in Accounting Education*, 19(4), 399-411.
- Newman, J. H. (1891). *The idea of a university defined and illustrated, 1852*. Newman Reader.
- Roberts, J., Propsom, P., & Tobin, W. (2018). Developing Shared Vision: A Case Study Documenting a STEM General Education Change Process at a Small Liberal Arts School. *Journal of college science teaching*, 47(6).
- Roth, M. S. (2014). *Beyond the university: Why liberal education matters*. Yale University Press.

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1). University of Nevada, Reno.

- Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education*, 40(5), 530-562. <https://doi.org/10.5951/jresmetheduc.40.5.0530>
- Smith, J. (2013). Amateur hour? Experience and faculty qualifications in US intelligence courses. *Journal of Strategic Security*, 6(3), 25-39.
- Wasserman, N.H., Fukawa-Connelly, T., Weber, K., Pablo Mejia-Ramos, J., Abbott, S. (2022). *Understanding analysis and its connections to secondary mathematics teaching*. Springer
- Wasserman, N., Weber, K., Villanueva, M., & Mejia-Ramos, J. P. (2018). Mathematics teachers' views about the limited utility of real analysis: A transport model hypothesis. *Journal of Mathematical Behavior*, 50(1), 74–89.