FASCINATED, SURPRISED, CURIOUS: THE ENGAGEMENT OF ELEMENTARY PRESERVICE TEACHERS IN OPEN MATHEMATICS TASKS

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Whole-class discussion of open mathematics tasks is an instructional practice K-12 mathematics educators report has the potential to engage all learners. Because this practice has not been extensively and systematically researched, this study aims to describe and analyze the engagement and experience of learners in open mathematics tasks. Drawing on a holistic conceptualization of engagement with behavioral, cognitive, affective, and aesthetic dimensions, the study specifically analyzes the engagement of three elementary preservice teachers as they participated in tasks and accompanying discussions in their elementary mathematics methods course. The three preservice teachers were selected because of their varied mathematical identities. While the engagement of the three focus preservice teachers varied, results suggest the openness of the tasks was an important factor in making their engagement possible.

Keywords: Instructional Activities and Practices; Affect, Emotion, Beliefs, and Attitudes; Preservice Teacher Education

In the effort to engage all learners in mathematics, a vital source of knowledge is the experiences of mathematics educators in the field. This knowledge includes instructional practices for which mathematics educators report anecdotal benefits for engagement across learners. While a practice may not yet have been formally and extensively researched, the fact that practitioners 'on the ground' are noticing a practice making an impact is reason to further explore the practice's potential. One such practice that has gained popularity in recent years is the use of open mathematics tasks for whole class discussion. These tasks—including "Which One Doesn't Belong?" (WODB, Danielson, 2016), "Notice and Wonder" (N&W, Fetter, 2021; Ray-Riek, 2013) and "How Many?" (HM, Danielson, 2018)—stand out in that they not only have multiple entry points, possible strategies, and solutions, but have ways learners can approach, interact with, and solve the tasks that an educator may not even anticipate. Practitioners and practitioner resources report that working with and discussing these tasks have positive impacts for the engagement and mathematical identities of a wide range of learners (Danielson, 2016; 2018; Illustrative Mathematics, 2021; Newell & Orton, 2019; Ray-Riek; Rumack & Huinker, 2019).

However, the field of mathematics education has yet to systematically examine these impacts and to identify characteristics of the tasks that may contribute to the impacts. In this regard, open mathematics tasks are a "black hole" of research, meaning they are "an instructional practice for which there is a scarcity of blind-peer-reviewed research evidence supporting its efficacy, yet has attained critical gravity in the teaching field" (Matney et al., 2020, p. 247). My study aims to cast light on the practice by reporting and analyzing a specific population's engagement with and experience of the tasks: preservice elementary teachers (PSTs). PSTs are a population of interest because they were recently K-12 students and because elementary PSTs often require intervention to address the narrow and sometimes negative mathematical experiences of their past (Ball, 1990). Two research questions guided my study: "How do PSTs engage with and experience open mathematics tasks?" and "What aspects of PSTs' engagement and experience can be attributed to the tasks' openness?"

Theoretical Framework

In addressing the two research questions, it is necessary to consider theories that inform how tasks play a role in influencing learners' engagement and those that identify innate characteristics with which these specific tasks may make a connection. Three theories inform this work: sociocultural positioning theory (Hand & Gresalfi, 2015; Langer-Osuna & Esmonde, 2017), a theory of aesthetic experience (Sinclair, 2001, 2004; Wong, 2007) that stems from the work of Dewey (1934), and self-determination theory (SDT, Ryan & Deci, 2017, 2020). Sociocultural positioning theory provides a framework for considering the tasks' role, while aesthetic experience and SDT offer ways to think about what it means to be human.

Sociocultural positioning theory highlights the role context plays in shaping events in a classroom. This context includes the way members of the classroom position one another, which, for the teacher, includes the selection and implementation of tasks. In essence, positioning theory asserts that "what someone does in a particular activity is always done in relation to what one has opportunities to do" (Hand & Gresalfi, 2015, p. 191). Tasks are one way the environment structures a learner's opportunities.

While positioning theory provides a foundation from which to consider the role of tasks, aesthetic experience and SDT identify innate aspects of being human with which tasks may connect. Recent theoretical literature on aesthetic engagement (e.g., Sinclair, 2001, 2004; Wong, 2007) builds on ideas from Dewey (1934), who asserted that aesthetic engagement is necessary in order to live a fulfilling life. Sinclair (2001) specifically addressed educational implications of aesthetics, explaining that "aesthetically rich learning environments…legitimize students' expressions of innate sensibilities and subjective impressions—they 'work with' such perceptions rather than exclude or deny them" (p. 26).

SDT's robust empirical research base is a helpful complement to aesthetic theory's more philosophical foundation. SDT asserts that all humans have basic psychological needs in addition to physical needs: autonomy, competence, and relatedness. Autonomy—or the extent to which a person feels regulation over their own experiences and behavior—is particularly relevant for this study. People who act autonomously engage in behaviors "whole-heartedly" (Ryan & Deci, 2017, p. 10).

Conceptual Framework

As conceptualizations of engagement vary widely, it is necessary to specify the definition of engagement I am using for this study. I draw upon the conceptualization of engagement put forth by Middleton et al. (2017):

> The in-the-moment relationship between someone and her immediate environment, including the tasks, internal states, and others with whom she interacts. Engagement manifests itself in activity, including both observable behavior and mental activity involving attention, effort, cognition, and emotion. (p. 667)

Particularly important to highlight in this definition is the consideration of engagement in the moment, the acknowledgement of internal and external factors, and the understanding of engagement as having behavioral, cognitive, and emotional components. As elaborated below, I also include aesthetic experience as a component of engagement, meaning engagement is not only a ‘taking up of’ but a being “caught up in” (Wong, 2007, p. 209).

It is also important to define what it means for a mathematics task to be open. Literature (e.g., Leikin, 2018; Mitchell & Carbone, 2011; Silver, 1995) has proposed ways to classify openness in mathematics tasks. These frameworks typically address two dimensions: number of possible strategies or approaches and number of possible solutions, with some reference to entry points and problem posing. Drawing on each of these frameworks as well as elaboration made necessary by recently emerging tasks, I propose that tasks can be open along three dimensions: multiple entry points, multiple strategies, and multiple solutions. There is also a range of extent to which tasks are open along each dimension. Figure 1 depicts this relative openness along dimensions, including the approximate openness of the four tasks used in this study (WODB, HM, N&W, and a word problem [WP] of high cognitive demand).

Entry Points		Strategies		Solutions	
WP **HM** **WODB** **N&W**	One entry point Multiple but limited entry points Unbounded entry points	**WP** **HM** **WODB** **N&W**	One strategy Multiple but limited strategies Unbounded strategies	**WP** **WODB** **HM** **N&W**	One solution Multiple but limited solutions Unbounded solutions

Figure 1: Dimensions of Openness in Mathematics Tasks

Methodology

When examining an existing practice that has anecdotally reported benefits, it makes sense to draw on multiple relevant methodologies rather than being narrowly confined to one. This pragmatic approach (e.g., Coyle, 2010; Frost & Nolas, 2011; Frost et al., 2010; Morgan, 2007) allows for strategic choice of specific methods that best capture the phenomenon in question. This study draws on three methodologies: case study, ethnography, and grounded theory. Case study (Merriam, 1998) is most strongly represented in this report, as case study involves thorough consideration of a clearly defined phenomenon such as individual PSTs’ engagement with open mathematics tasks. Ethnography (Macgilchrist & Van Hout, 2011) is relevant because of its emphasis on the participant’s perspective, which is necessary to emphasize when creating a holistic representation of engagement. Grounded theory (Glaser & Strauss, 1967) allows for new ideas to emerge rather than relying solely on previously defined constructs, which is particularly useful when considering a practice that is itself relatively new.

Context

This study was conducted at a large Mid-Atlantic public university in a semester-long undergraduate elementary mathematics methods course for PSTs. I worked with three sections of the course, all taught by the same instructor. This report focuses on the cases of three PSTs from one section of the course. There were 12 PSTs enrolled in the course. The PSTs were majority White and female; one PST was male.

Methods

The instructor of the course and I collaborated to choose four tasks for the PSTs to engage with as part of their elementary mathematics methods course. Three of the tasks (WODB, HM, N&W) did not have pre-determined answers and were of the most interest for this study. The fourth task was a word problem (WP) of high cognitive demand that allowed for multiple strategies but was implemented for the purpose of comparison as it only had one solution. Figure 1 above provides an approximate sense of the openness of each task in terms of entry points, strategies, and solutions.

The images and/or prompts for each task are included in Figure 2. The WODB task presented PSTs with four addition expressions and asked PSTs to choose which one did not belong and to explain why. Any of the four options was justifiable, and there were many possibilities as to why each one might not belong. In the HM task, PSTs were asked to consider an image of a large box of chalk. PSTs were prompted to choose what to count in the image and to be prepared to share what they counted (including the unit) and their counting strategy. For the N&W task, PSTs were shown three bulk packages of different brands of toilet paper and were asked to share what they noticed about the image and what they wondered. For the WP, PSTs needed to figure out the number of boxes of two types of cupcakes given the total number of cupcakes, boxes, and parameters as to how many cupcakes of each type could fit in a box.

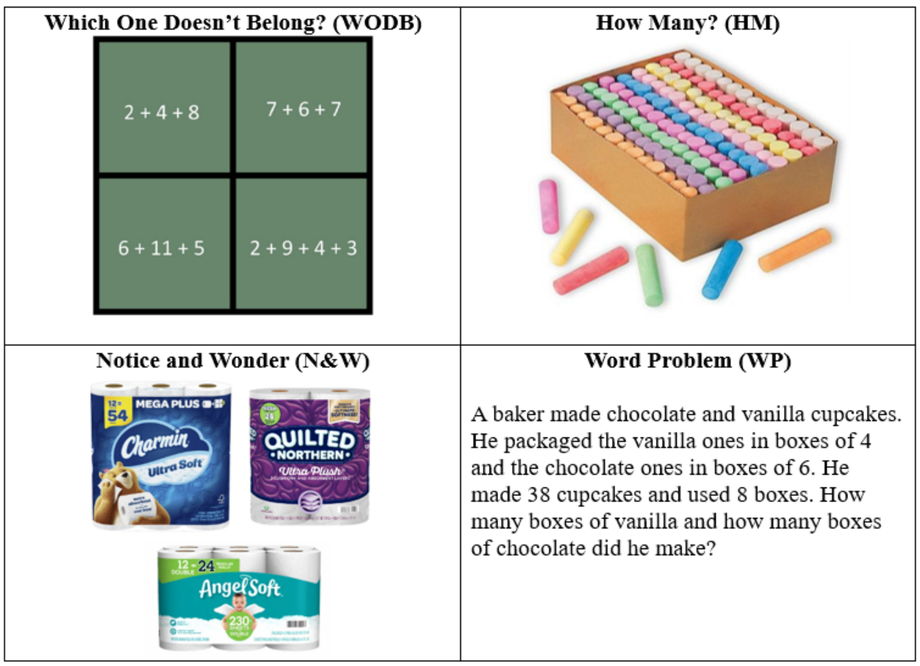

Figure 2: Open Tasks Used in Study
WODB from Heon (2013); HM image from Color Splash (n.d.); N&W images from Charmin (n.d.), Quilted Northern (n.d.), Angel Soft (n.d.); WP task from Hallman-Thrasher and Spangler (2020). N&W images representative, may not match exact task.

The instructor facilitated the tasks during four separate class sessions. I attended each of these sessions to video-record and observe. For each task, the instructor presented the task and accompanying prompt. The PSTs were given 3-5 minutes to work individually. Then, the instructor facilitated a discussion of the task. The length of the discussions ranged from approximately 10-24 minutes. Following each task, participants responded to a 5-item questionnaire that asked them about their strategies and emotions during the task and discussion, the appeal of the task, and how the task compared to how they typically thought about and experienced mathematics. I also solicited volunteers to participate in a semi-structured interview at the conclusion of the semester, during which I asked participants about their overall questionnaire responses as well as their specific impressions of the tasks. During the interviews, I used video-stimulated recall to ask participants about their experience of specific parts of the task sessions. In addition to the task session recordings, questionnaire responses, and interview transcripts, I also collected identity webs PSTs completed at the beginning of the semester. For the identity web assignment, PSTs described their identity as a mathematics learner by sharing and reflecting on memories of their experiences with mathematics prior to the course.

Because practitioners have reported that these tasks engage a wide range of learners, I began my analysis with PSTs' identity webs to get a sense of their relationship with mathematics. For the analysis for this report, I aimed to select three PSTs who varied in terms of the valence of their past experiences with mathematics. I also considered variation in the number of contributions PSTs made to discussions (since this is also information that would be immediately available to practitioners). If possible, I prioritized PSTs who had participated in an interview, since the most information was available in terms of their engagement.

After selecting three PSTs, I examined their questionnaire responses—and, if available, interview transcripts—for evidence of cognitive and emotional engagement as well as aesthetic experience. I coded deductively (Maxwell, 2013) according to established constructs as well as inductively (Glaser & Strauss, 1967), using constant comparison to identify themes. Generally speaking, items coded as cognitive engagement involved how students were thinking and affective engagement involved how students were feeling (Fredricks et al., 2004; Middleton et al., 2017). Aesthetic experience included items related to curiosity, creativity, and exploration (Sinclair 2001, 2004; Wong, 2007), as well as the opportunity to express oneself (Ryan & Deci, 2017). I also made note of any reports of behavioral engagement (i.e., a PST describing what they were doing). Then, I returned to the videos and transcripts of the task sessions to code for behavioral engagement, paying attention to PSTs' actions throughout the task sessions, not only the numbers of times they contributed to the discussion or to the content of their contributions. I made note of any evidence of cognitive, affective, and aesthetic engagement as well.

Because I did not collect PSTs' individual written work or notes, I did not include these items in my analysis. These artifacts may have offered additional insight; however, as demonstrated in the results, I was able to glean considerable information about PSTs' engagement from their own reports and from my observations.

Identifying themes across each individual PST's engagement and experience from the collected data, I crafted profiles of the three PSTs, provided in the results below. Finally, I examined all of the coded items for instances that could be clearly linked to the openness of the tasks.

Results

Out of 12 participating PSTs, 10 were present at every task session. From those 10, I identified three who ranged from negative to positive valence in terms of their past experiences with mathematics and also ranged in the number of contributions they made during class discussions. The three PSTs were Andrew, Faith, and Katie (pseudonyms). Andrew presented as male, and Faith and Katie presented as female. Andrew did not choose to participate in a follow-up interview, while Faith and Katie did. The profiles of their engagement are presented below.

Andrew: Mixed Past, High Verbal Participation, Engagement Task-Dependent

Andrew's identity web reported a mix of past experiences with mathematics. He described his elementary and middle school experiences as negative—that he was "always one class behind" and one year was "awful"—but that high school math was "awesome" and in college he encountered his first math class that "made sense."

Andrew contributed verbally in three of the four task discussions and had the largest number of contributions overall (15) in comparison to his peers. In the WODB, his contributions included building on others' strategies as well as offering his own ideas both simple and complex. When not speaking, he continually looked towards the board or occasionally towards his device. During the WP discussion, he leaned back in his chair as he looked towards the board or towards who was speaking. He offered insight into what the variables would represent in solving the task with a system of equations. He did not speak during the HM discussion, and was often on his device, an activity which at times may not have been related to the task. However, he spoke frequently during the N&W discussion, offering several observations and wondering about the implications of the numbers in the image.

After all four tasks, Andrew reported that his strategy involved starting with something "easy" or obvious. Otherwise, his engagement seemed to take on a different quality in the WP and HM tasks as opposed to the WODB and N&W tasks. In his questionnaire responses following the WP and HM tasks, he mentioned evaluating others' strategies as too complicated, and, for the HM task, losing interest. He also essentially reported no emotions for these tasks. For the WODB and the N&W tasks, he reported continuing past his initial responses to consider possibilities that were less obvious and to listen to others' strategies in order to build or add on. It was also during these tasks that he reported emotions directly related to the task, feeling "challenged" by the WODB and "involved" with and enjoying the N&W task. His reports ventured into the aesthetic, explaining that the WODB was appealing because of "the ability to have your own answer" and for the N&W, that he "honestly could've kept going for awhile." During the N&W discussion, he shared that he had looked up more about the information in the task because he was "really curious," and exclaimed, "I'm fascinated by this."

Faith: "Overall" Positive Past, Low Verbal Participation, Engagement Informed by Peers' Ideas

On Faith's identity web, she stated explicitly that she "overall enjoy[s] math," using adjectives like "amazing" and "great" in describing past experiences and even recalling that she "loved" tests (including timed ones). Any negative aspects she mentioned had to do with specific teachers or struggling with emphasizing grades.

Faith contributed verbally to two out of the four task discussions and contributed to each of those two discussions once. In the HM task, she shared her strategy for counting and multiplying the rows and columns of chalk. In the N&W, she asked about the definition of a term and asserted the importance of an additional consideration. Despite relatively low verbal contributions, however, her behavior suggested she was paying attention, as she usually looked toward the board or at the person speaking, reacted by nodding or laughing, and occasionally spoke with others at her table about the task. Faith shared that she usually participates more actively but did not feel the need to speak up once someone shared the same idea that she had been thinking or when the conversation had already moved in another direction. In other words, her behavioral engagement was impacted by her peers' sharing of ideas.

Her cognitive and emotional engagement were also influenced by her peers; after the WODB, HM, and N&W tasks, she mentioned being surprised and interested by others' strategies. Her detailed recall of the strategies suggested she was cognitively as well as emotionally engaged with them. After the N&W discussion she explained she was uninterested initially but became interested once she heard how others viewed the task differently.

Faith felt most comfortable with the WP, stating she tends to think more "logically." In this regard, she set her math identity in contrast to the openness of the other three tasks. However, she acknowledged that this perception of math "as one-dimensional right or wrong" was "how it always has been presented" to her. She contrasted this description with social studies, which she described as "full of ideas and engaging conversation or different opinions." She was open to shifts in her perceptions and identity, however, as she mentioned the WODB causing her to question her "previous beliefs about math" and, in general, shared that being able to determine her own answer was "very weird and eye opening for people… like me who think logically with one set answer to like expand ideas and be like… there is no right answer."

Katie: Somewhat Negative Past, Moderately High Participation, Freed to Engage by the Absence of "Right" and "Wrong" Answers

On Katie's identity web, she made the general statement that she "always had to try harder" at math, and that it "doesn't come easily." She did not remember much about elementary school math but mentioned struggling and studying hard in middle and high school. In her interview, she explained she must be an "English and people person," as a family member had a "math and science brain" and never seemed to have to work at it like she did. She feels more of a connection with language-oriented subjects because they were more creative and open-ended.

Katie's engagement was fairly consistent across all four tasks. She raised her hand several times during all four discussions. She contributed verbally to each discussion, with a total of seven contributions. When she was not actively participating, she usually looked towards the board or at the task on her own computer screen. She sometimes wrote in her notebook, and occasionally spoke with others at her table about the task. In her interview, Katie stated that this high level of participation was not typical for her in math-related classes in general but was probably typical of her participation in this class.

Katie's cognitive engagement was evident in her verbal contributions and on her questionnaire. Her contributions included pointing out patterns in the WODB, asking about how to use variables to correctly represent the objects in the WP, sharing counting strategies and pointing out differences in numbers of colors in the HM, and sharing information she noticed in the N&W. She described her strategies in detail on her questionnaire responses and mentioned listening to others' ideas both out of interest and to compare them with her own.

Katie reported similar emotions across all four tasks, feeling motivated, content, and curious, as well as interested and excited to hear what others had to say. She initially felt confused about the WP but felt excited after "figuring it out." A key for Katie seemed to be the opportunity to share without expectation of being right or wrong and the ability to be assured of her own answer because she was the one who determined it. After the WODB, she stated, "I have always experienced math as right or wrong. This is much better and more encouraging." Even though she assessed the WP to be "a pretty average word problem," she did feel "freedom" in being able to solve the problem in her own way. She mentioned freedom to "do what [she] wanted" with the HM and that she could just "speak [her] mind" in the N&W. She specifically identified the desire to share something unique in the N&W as different from her past experiences, as the expectation to do math in a prescribed way prevented creativity.

Discussion

While each of the PSTs' engagement varied and for different reasons, the openness of the tasks played a role in how each of them engaged with and experienced the tasks.

The role of the tasks' openness can most plainly be seen in the PSTs' own references. All three PSTs mentioned having a different experience because of the ability to choose their own answer or being freed from the pressure of finding one right answer. Andrew identified this aspect as the most appealing part of the WODB task, adding that "it would have improved how [he] saw math if [he] had seen something like this" in his previous experiences. Katie explained that the N&W task was "appealing because it didn't stress [her] out or include anything [she] couldn't do." Sociocultural positioning asserts that the teacher's acts—including choice of tasks—can make certain identities available to students and not others (Langer-Osuna & Esmonde, 2017). It seems the openness of these tasks afforded the opportunity for PSTs to identify as mathematically capable. The tasks also seemed to allow PSTs to be the "arbiter" of their own mathematical experience rather than not being able to have their own "personally felt emotion guiding the selecting and assembling of the materials presented" (Dewey, 1934, p. 68).

All three PSTs mentioned the wide range of possibilities of answers, which both gave them multiple opportunities to engage with the task and made them interested to hear and engage with ideas that others would share. When I asked Faith how typical it was for her to be surprised by what others had to share in a math-related class, she stated typically she was not. Math for Faith had always been that "you have a right answer, and you have all these wrong answers," communicating that usually in math, there was no room for anything to surprise her.

The way the tasks positioned PSTs is also evident in the opportunities that PSTs had that they would not have otherwise had if the task were not open. Andrew, Katie, and Faith all talked about looking for unique or challenging ways to interact with the tasks. While this desire could be a goal as part of a math task that allows for multiple strategies (a freedom Katie felt with the WP), the multitude of possibilities in being able to determine one's own answer was a significant shift for the PSTs. In the discussion of the HM task, Katie was inspired to find something different to count after hearing another PST's idea of counting chalk that looked used versus chalk that looked unused. For Faith, the opportunity to share with such wide parameters was so unusual that she worried that the idea she shared in the HM task was somehow "cheating." In the N&W task, Andrew described his fascination with the image and took the initiative to look up more information, an action that likely would not have taken place had the task been aiming for one correct answer that could be solved with the information given. It seems that Andrew's sensibilities were awakened (Sinclair, 2001) through the process of engaging with these open tasks.

In essence, in response to the first research question regarding engagement and experience with open mathematics tasks, PSTs demonstrated engagement in all dimensions (cognitive, behavioral, affective, aesthetic) to degrees that varied by PST and to some extent by task. The WODB and N&W tasks seemed to be the most engaging across PSTs and dimensions. With regards to the second research question, PSTs' engagement with and experience of the tasks was linked to the tasks' openness. PSTs' own reports support this claim, as well as the fact that aspects of the experience to which PSTs responded would not have been possible if the tasks were not open. It is also striking that the WODB and N&W tasks—which were the most open in terms of entry points and strategies and open to a high degree in terms of number of solutions—saw considerable engagement from three PSTs who differed from one another in terms of their past experiences with mathematics.

Implications

Of course, the openness of the tasks in this study was only one of a myriad of intertwined factors shaping PSTs' engagement—aspects like the PSTs' characteristics, other PSTs, the instructor's prompts, and the content of the tasks were also influential—but the openness was, nevertheless, a factor. The possibility that the openness of these tasks prompted PSTs to fully engage in a way that contrasts with their previous experiences suggests the benefits of further research on these tasks. More generally, these results suggest that the openness of tasks in mathematics is a characteristic to be more consistently considered in task design. Finally, the fact that these results are consistent with mathematics educators' reports about learners' engagement with these tasks highlights the importance of listening to and following up on what practitioners are noticing in the field.

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