EMBODIMENT AS EVIDENCE FOR STUDENT ENGAGEMENT IN AN INQUIRY-ORIENTED MATHEMATICS CLASSROOM

Kaylee Fantin-Hardesty
Colorado State University
Kaylee.fantinhardesty@colostate.edu

Rachel Tremaine
Colorado State University
rachel.tremaine@colostate.edu

Jocelyn Rios
Colorado State University
Jocelyn.rios@colostate.edu

Hortensia Soto
Colorado State University
Hortensia.Soto@colostate.edu

Student engagement is an impactful component of student experience in mathematics classrooms and can shape academic and affective outcomes. The measurement of engagement in classroom settings has been limited to self-report measures or observational frameworks which privilege verbal participation. By conducting a microanalysis of two students' engagement with a meaningful mathematics task in an inquiry-oriented discrete mathematics classroom, we provide evidence for embodiment as a lens through which to effectively observe behavioral, affective, and cognitive engagement. We emphasize embodiment's potential to (a) illuminate engagement which may have gone unnoticed under observational scales which privilege verbalization, (b) account for the multimodal nature of utterances, and (c) consider the broader classroom context in which students engage with mathematical tasks.

Keywords: Embodied Cognition, Research Methods, Undergraduate Education

Student engagement is an impactful component of students' experiences and outcomes in mathematics classrooms. Increased levels of student engagement support students' academic achievement (Carini et al., 2006; Pilotti et al., 2017) and mediate against negative shifts in mathematics identity (Voigt et al., 2022). Alongside the study of the consequences of student engagement comes the predicament of how to measure it; Fredricks et al. (2016) write that "developing valid and reliable measures [for student engagement] is especially important in math and science because engagement in these subjects is so critical to academic achievement and career choices related to STEM" (p. 14). In this paper, we propose embodied cognition as a lens through which to observe students' engagement with a meaningful mathematics task. We provide evidence for this proposal via a microanalysis of two students' engagement with a meaningful mathematics task and discuss our findings.

We draw upon Fredricks et al.'s (2004) definition of student engagement which comprises of three domains: *behavioral* engagement, *affective* engagement, and *cognitive* engagement. *Behavioral* engagement consists of on-task, participatory classroom actions that indicate a student is performing classroom tasks in the way that the cultural and institutional authority in that classroom expects. *Affective* engagement encompasses emotional responses to and emotional investment in the task at hand and its associated discoveries, and *cognitive* engagement involves students' self-regulation of their learning and thoughts, and their perseverance in the face of cognitive challenge. While additional domains of engagement have been proposed and utilized (Joshi et al., 2022; Quintero et al., 2022; Veiga, 2016), we utilize Fredricks et al.'s (2004) domains due to their specific prevalence in mathematics engagement literature, and the ways in which they provide a bases for other methods of observing engagement in the classroom.

Comprehensive pictures of student engagement are best painted when they account for the interplay between the three domains. Fredricks et al. (2004) note that the three domains are necessarily "dynamically interrelated" (p. 61), and students' experiences of engagement in one domain are correlated with experiencing engagement in others (Böheim et al., 2020; Joshi et al., 2022). Measuring engagement as a multifaceted construct in the mathematics classroom most often utilizes self-reported data (Fredricks & McColskey, 2012), which is valuable to studies of engagement (Appleton et al., 2006; Garcia & Pintrich, 1991), and is also limited. Survey items are often broad enough to be interpreted in several ways and may not be answered with fidelity to the researchers' interpretation. This limits what can be gleaned from self-reports regarding students' engagement in the classroom (Fredricks & McColskey, 2012; Hodgson et al., 2017).

Consequently, scholars have investigated how one might observe student engagement in the classroom. For example, hand-raising or designated on/off-task behaviors have been proposed as observable measures of behavioral engagement (Böheim et al., 2020; Hodgson et al., 2017). While these do provide methods through which to observe student engagement, verbal participation is often privileged in such scales. Women, students of color, and individuals of lower academic status may have less opportunities to engage in verbal participation (Black & Radovic, 2018; Civil, 2014; Fink, 2022). For example, Böheim et al.'s (2020) notion of hand-raising as an observable indicator of student engagement is rooted in the assumption that *all* students in a classroom are equally comfortable contributing vocally to a whole-class discussion. If we are to study student engagement in the classroom, there exists a need to develop, refine, and advocate for the use of observational tools which meaningfully incorporate engagement beyond verbalization in the classroom. This need for mechanisms to document classroom engagement that do not privilege but incorporate verbalizations led us to explore embodiment as a lens through which student engagement could be effectively observed.

Theoretical Framing: Embodied Cognition

In this paper, we use the terms "embodiment" and "embodied cognition" interchangeably and draw from Radford's (2009) definition of embodied cognition as the notion that "thinking does not occur solely in the head but also in and through a sophisticated semiotic coordination of speech, body, gestures, symbols, and tools" (p. 111). This notion insinuates that evidence of students' internal learning processes—including their engagement—goes beyond inner cognitive dialogue and expresses itself in a myriad of observable ways in the classroom. Within an embodied cognition paradigm, we focus on the concept of "utterances" as a categorical lens through which to conceptualize embodiment, as presented by Nemirovsky and Ferrara (2009). Utterances are body activities and actions involved in a conversation. They include gesture, gaze, body poise, body motion, facial expressions, eye motion, and more. Further, when a speaker includes an object in the conversation, the object becomes part of an utterance.

An embodied cognition lens has been used in the classroom for a variety of purposes, including to study how students collectively discover mathematics (Nemirovsky et al., 2012), and to understand the role that tools and materials have on students' cognition (Radford, 2014). Embodied cognition is a lens that pays mind to verbal participation but it also, by its very nature, incorporates many other forms of classroom participation. While some scholars study how incorporating embodiment into task design can enhance engagement with those tasks (Georgiou & Iannou, 2020; Lindgren et al., 2016), we intentionally studied embodiment in the context of an activity not explicitly designed to be embodied to effectively assess whether embodiment can provide evidence for student engagement in the mathematics classroom. Our investigation

centers around the research question: *In what ways can embodiment provide evidence for students' engagement with meaningful mathematical tasks?*

Methodology

Setting and Data

This study took place in Fall 2022 at a large research university in the Rocky Mountain region of the United States. The class selected for this observation is an undergraduate Discrete Mathematics course, within which an experienced instructor (Dr. A) follows an Inquiry-Based Mathematics Education (IBME) teaching paradigm. In IBME, instructors encourage students to participate in the classroom in ways which reflect actions of expert mathematicians such as exploring patterns, generating conjectures, proving theorems, (re)inventing definitions, and comparing solutions (Laursen & Rasmussen, 2019). Such classrooms are an optimal setting to investigate student engagement, as one of its four pillars is student engagement with meaningful mathematics tasks as one (Laursen & Rasmussen, 2019).

Over the course of two class periods, we videotaped a total of 2.5 hours of data. Early in the research process, we narrowed our focus to a particular task Dr. A set forth for the students: the "Locker Problem." We refined our unit of analysis by focusing on students who engaged with the task using two-color counters, provided by Dr. A. When asked to split into groups, four students from two groups– Rebecca, Lauren, Carly, and Jason– all chose to use the two-color counters to tackle the problem. While Carly and Jason immediately began working together, Rebecca and Lauren worked individually for several minutes before coming together to enhance their understanding of the problem. It is for this reason– the presence of both individual and social interactions with the task– that we narrowed our unit of analysis to be the videotaped data of Rebecca and Lauren.

Analysis

Video data of Lauren and Rebecca engaging in the Locker Problem task totaled approximately 13 minutes of our collected data. We transcribed Lauren and Rebecca's verbal utterances and performed three phases of coding. The initial phase entailed descriptive writing through an *ethnographic microanalysis of interaction* (microanalysis) (Garcez, 1997). Garcez's work details this style of analysis as one in which communication necessarily "involves conversationalists contained in physical bodies, occupying space in simultaneously constraining and enabling social situations, who must reflexively make sense of each other's actions as they act" (p. 187). Microanalysis has a rich history in embodied cognition literature (Alibali et al., 2019; Nemirovsky et al., 2020; Kelton & Ma, 2020; Walkington et al., 2019) and entails transcribing participants' second-by-second utterances for the entirety of the 13 minutes of data.

Once we comprehensively described the data in this way, we selected smaller snippets of 1-3 minutes, and within those episodes, smaller excerpts of 10-25 seconds (as modeled by Nemirovsky et al., 2020). These clips exemplified instances in which Lauren and Rebecca needed to "reflexively make sense of each other's actions as they act" (Garcez, 1997, p. 187). The interactions chosen were not exclusively between Lauren and Rebecca; Episode 1 centers around Rebecca making sense of Dr. A's verbalizations, Episode 2 centers around Rebecca and Lauren reaching a shared understanding, and Episode 3 involves Rebecca and Lauren grappling with another group's verbalized conclusion. In each of these chosen episodes, we position Rebecca and Lauren as agentic bodies in a wider classroom space, one rich with embodiment in interactional ways.

Upon selecting these episodes, we engaged in two rounds of thematic analysis (Braun & Clarke, 2006) using *a priori* codes from the embodiment and engagement literature. The first round entailed coding the specific types of utterances as aligned with those provided in Nemirovsky and Ferrara (2009): sound production, eye motion, facial expression, gaze, body motion, body poise, tone of voice, and hand gesture. As we knew Lauren and Rebecca worked with the counters as necessary materials in their learning process, we also included "materials" as an *a priori* code. Because Nemirovsky and Ferrara explicitly acknowledge that additional types of utterances other than those they specify exist, we remained open to acknowledging and describing other forms of embodiment. During our second round of coding, we coded the type of engagement evidenced by the embodiment of the participants: behavioral, affective, or cognitive. This was done with careful consultation of Fredricks et al.'s (2004) definitions and descriptions. Organized into episodes, below we provide context for each selected episode, include details of our microanalysis, and provide screenshots from the episode which illustrate the students' engagement in embodied ways.

Results

Just prior to the beginning of our first selected episode, Lauren and Rebecca worked individually on the locker problem. They both used the two-colored counters and quietly worked through their worksheets. The two students did not talk with each other, but Rebecca leaned in towards Lauren, indicating that she seemingly wanted to engage with Lauren. Dr. A walked over to the two students and began conversing with Lauren.

Episode 1: Responsive Resetting (04:58 – 05:25)

When Dr. A walked over to Lauren and Rebecca, she prompted them to explain their thinking. Lauren verbally discussed what she had written on her worksheet, and she and Dr. A engaged in a verbal conversation about Lauren's thought process. In this episode, Dr. A attempted to reorient Lauren toward the problem, but simultaneously elicited non-verbal engagement from Rebecca.

Rebecca was leaning in towards Lauren and Dr. A, apparently listening to their conversation and thus behaviorally engaging. After several seconds, at time 05:04, Rebecca leaned back and reached for the counters in front of her. As Dr. A said, "If I rephrase that question as, we have the lockers of ten students" (Timestamp 05:10 – *Figure 1*), Rebecca began resetting her counters to their original "closed" position. This counter-resetting action appeared to be in direct response to Dr. A's verbalized notion of "rephrase[ing] the question." While Dr. A's verbalization was directed toward Lauren, Rebecca clearly thought about Dr. A's remarks in an on-task and participatory way and evaluated her own prior notions of the problem. Rebecca was behaviorally engaged in her attention and response to Dr. A's verbalization as well as cognitively engaged as evidenced by her resetting of the counters.

After she reset her counters, Rebecca laid her hand flat with fingers spread and started rearranging her counters (Timestamp 05:15 – *Figure 1*). Her use of gesture and her continued interaction with materials indicated her ongoing behavioral engagement. She then used her right pointer finger to point to the counters one-by-one, and in doing so extended her material environment to a line of 20 "closed" counters. After Rebecca had rearranged her counters and added more in front of her, she pulled away from her counters and looked over at Lauren's worksheet, where Dr. A was pointing (Timestamp 05:24 – *Figure 1*). Her body position and eye gaze were demonstrative of further, continued behavioral engagement.

Episode 1 - 05:10   Episode 1 - 05:15   Episode 1 - 05:24

Figure 1: Selected Screenshots from Episode 1

Between Episodes 1 and 2

Fourteen seconds after the conclusion of Episode 1, Dr. A left the students. For twelve seconds, the two sat in silence, until Lauren turned to Rebecca and verbally speculated on pattern existence present in their counters. Lauren listed off the "open" counters in front of her– counters one, four, and nine– while Rebecca pointed to those "open" counters using the counters in front of her. This did not seem to lead them toward any conclusive thoughts, so Rebecca began counting something else: the number of "open" counters between the "closed" ones.

Episode 2: Collaborative Conjecturing (01:33-01:48)

This episode begins when Rebecca started counting the "closed" counters in between the "open" counters, both verbally and by using spread-out fingers of her left hand. She drew Lauren's attention to the numeric value of these gaps, and in doing so, they came to a shared understanding of the pattern they saw without verbally expressing it. At 01:35, Lauren's abrupt change in body posture, and deliberate reaching toward Rebecca (*Figure 2*) indicated that Lauren had made a relevant realization. Without verbalizing the realization, she gained confirmation from Rebecca that they were on the same page cognitively from Rebecca's clapping and pointing toward Lauren (Timestamp 01:37 – *Figure 2*). The ways in which they both positioned themselves and reached toward each other with their gestures indicated to the other collaborator that they arrived at a similar conclusion on the pattern they saw (namely, that each "open" counter was separated by an increasing multiple of 2 "closed" counters). Rebecca's reiteration of the counts confirmed their shared understanding of the pattern at timestamp 01:41. Lauren's verbalization of "Dude, that's so cool!" and her brief eye contact with Rebecca seemed to confirm that their understanding of this pattern was a shared understanding as developed by Rebecca's verbal counting and gestures and a source of shared excitement.

Rebecca's gestures and associated verbalizations during her initial verbal and gestural counting was evidence of on-task participation and thus of behavioral engagement. The point at which Lauren abruptly shifted her body posture and extended her arm indicates a realization; we infer that this was the point when Lauren identified the pattern and mentally made a conjecture about how future "closed" lockers will be separated. Her extended arm gesture, body position, and facial expressions demonstrated behavioral and cognitive engagement associated with coming to a conjecture. Both students' smiling and laughing indicated that they were affectively engaged with the activity.

Figure 2: Selected Screenshots from Episode 2

Between Episodes 2 and 3

After making the conjecture detailed in Episode 2, the students sat in silence for approximately six seconds (1:49 - 1:55). Then Lauren proposed that they try the problem using thirty counters, and she and Rebecca pushed their desks together, end-to-end, and started combining their counters into a continuous line spanning both of their desks. This resulted in one long material environment with 30 counters. However, they did not reset their counters, leading them to neglect to account for integers 11-20. This left them with the conclusion that 1, 4, 9, 16, 22, 24, 25, 26, 27, and 30 are "open," breaking their earlier conjecture. They grew quiet, seemingly recognizing that their conjecture did not work and stared silently at Lauren's worksheet for 13 seconds. We continue with the embodied narrative in Episode 3.

Episode 3: Second-guessing Squares (00:44-00:55)

In this episode, Lauren and Rebecca attempted to reconcile the materials in front of them with what they heard from the classroom context around them. This episode centers on the embodiment they illustrated as they listened to a group that was positioned in front and to the right of where they were seated in the classroom. A student member of that group, Anthony, told Dr. A that he believed that any locker with a number that is a square should be closed. This contradicted the counters on display in front of Lauren and Rebecca.

Lauren and Rebecca's observable embodiment was influenced by their classroom surroundings. In front of them, counters 1, 4, 9, 16, 22, 24, 25, 26, 27, and 30 were "closed," making Anthony's claim that all squares should be closed contradictory to the cues they received from their counters. Their acknowledgement of this contradiction started at the beginning of the episode (Timestamp 00:49), where they both expressed confusion and focused on Dr. A and Anthony's conversation. At 00:50, they shared this confusion with each other via expressive eye contact. Both embodied uncertainty of what to make of Anthony's claim and Dr. A's validation of it via their facial expressions. At 00:52, they turned back to their respective materials (the worksheet for Lauren and the counters for Rebecca), possibly attempting to reconcile the notion of squares being "open" with what they had observed in their materials or had written down. *Figure 3* showcases this sequence of events. While Anthony in the front group began formalizing his group's conjecture, Lauren and Rebecca continued to express confusion. Lauren looked at the counters and shook her head and Rebecca brought her left hand to her forehead in an apparent show of deep thought about the counters in front of them.

In this episode, Lauren and Rebecca's embodiment provided evidence for cognitive and affective engagement. The affective engagement was well-evidenced by the facial expressions they individually adopted as they listened to Anthony's conjecture and the facial expressions they wore as they surveyed their own counters which contradicted Anthony's assertion. They also illustrated their individualized affective engagement in their body movements and positionings. At 00:52 Lauren shook her head as she looked at her worksheet; this movement indicative of "no" seemed to apply to the mismatch between Anthony's words and the pair's

observations. At the end of the episode, Rebecca brought her left hand to her forehead, indicating that she was deep in thought and thus cognitively engaged with the material. The students' cognitive engagement was also evidenced by their facial expressions as their confusion indicated that they grappled seriously with Anthony's conjecture, which led to the cognitive dissonance associated with their materials illustrating something else.

Episode 3 - 00:49   Episode 3 - 00:50   Episode 3 - 00:52

Figure 3: Selected Screenshots from Episode 3

In each episode, a multitude of utterances were observed. Further, no domain of engagement existed purely on its own; embodiment evidenced cognitive and behavioral engagement in Episode 1, all three domains in Episode 2, and affective and cognitive engagement in Episode 3. *Table 2* below provides documentation of which utterances were coded as evidencing which forms of engagement throughout the three episodes.

Table 1: Types of Utterances which Evidenced Domains of Engagement

Behavioral	Affective	Cognitive
Gaze	Gaze	Gaze
Gesture	Gesture	Gesture
Body Poise	Facial Expression	Body Poise
Materials	Body Movement	Facial Expression
Verbalization	Sound Production	Materials
Environment alteration	Verbalization	

Discussion

To begin the discussion, we restate our research question: *In what ways can embodiment provide evidence for students' engagement with meaningful mathematical tasks?* We saw that various types of utterances provided evidence for Lauren's and Rebecca's collaborative (Episode 2 and Episode 3) and individual (Episode 1) engagement with the Locker Problem task. *Table 1* is a summary of the types of utterances exhibited by the participants for each of Fredricks et al.'s (2004) domains of engagement. Through our microanalysis, we assert that embodiment provides evidence for students' engagement in ways which (a) illuminate engagement which may have gone unnoticed under observation scales which privilege verbalization, (b) account for the multimodal nature of both utterances and engagement as a construct, and (c) account for the broader classroom context in which the students engage with the task.

In each episode, studying embodiment as evidence for student engagement revealed domains of engagement which may have gone unnoticed had we purely looked at the students' verbalizations. This is particularly clear in Episode 1 via Rebecca's embodied responses to Dr.

A's verbal prompting. In addition, the cognitive engagement associated with conjecturing in Episode 2 may have gone unnoticed without an embodied lens, as Lauren and Rebecca *never* directly verbalized or wrote down their shared conjecture, but rather communicated and verified its existence in an embodied way. Further, the only verbalization made in Episode 3 was Lauren's rhetorical "What?" at timestamp 00:50, but additional realms of embodiment (e.g., gaze and facial expression) indicate that Lauren and Rebecca were continually engaged with the task and attempted to reconcile Anthony's assertion with their own materials.

Further, the utterances which provided evidence for the three domains of engagement enabled us to describe the students' engagement in a way reflective of their interrelated nature. Much of the embodiment that we observed did not fall squarely into one category of engagement or utterance, but rather provided evidence for multiple domains and types respectively. For example, in Episode 2, Rebecca laughed and clapped her hands, and then pointed to Lauren as she cognitively constructed her conjecture. This was coded as Facial Expression, Sound Production, and Gesture (types of utterances), and Affective and Cognitive (domains of engagement). The multiple assignment of *a priori* codes is unsurprising, given what Nemirovsky and Ferrara (2009) refer to as the "multimodality" of any given utterance, as well as the interactional nature of Fredricks et al.'s (2004) three types of engagement. Much of the embodiment we observed was built from several simultaneous utterance types, and illustrated multiple different domains of engagement, as is reflective of the nature of these constructs.

Embodiment, as an evidential lens for student engagement, enabled consideration of the classroom context. This claim has been supported by literature indicating that the social, physical, and technological classroom context influences student engagement (Hodgson et al., 2017; Kahn, 2014; Keith, 2016). In Episode 3, the cognitive and affective engagement the students experienced as they tried to reconcile their counters with the conclusion of the group in front of them was evidenced by the embodied ways in which they responded to the other groups' verbalizations. Their confused facial expressions, eye gaze between their materials and the other group, and eye contact between each other served as evidence of their attempts to make sense of this finding, and thus be cognitively and affectively engaged in the continuation of the task. Our microanalysis of embodiment allowed for this by mediating the students' responsive utterances with the broader classroom context in which they were situated.

Both triangulation of our data with other observational and self-report measures, as well as any comparative analysis with other mathematics courses, other majors, or other institutional settings, is an important topic for future work. We also note that embodiment, as is the case with other observational scales for student engagement, may be limited in regards of what we are seeing. For example, the embodiment Lauren and Rebecca exemplified in Episode 3 illustrated affective and cognitive engagement. If we consider the linkage between cognitive and behavioral engagement established by Böheim et al. (2020), it is likely that they were also behaviorally engaged, although that wasn't evidenced by their utterances. We further recognize the need to triangulate these data with other measures of engagement, such as self-reports. This is particularly relevant for cognitive engagement, as it foundationally relies on the notion of students self-regulating their own learning (Fredricks et al., 2004), something which an observer can only speculate on without concrete knowledge of that students' thought process.

Conclusion

This paper has set forth an evidence-based argument for the potential of embodiment, particularly Nemirovsky and Ferrara's (2009) notion of utterances, as a novel way through which

to observe behavioral, affective, and cognitive engagement in a mathematics classroom. In proposing the lens of embodiment as an observation tool, we broaden who is seen as engaging in classroom mathematics tasks, as we assert that engagement happens in embodied ways which are *not* purely verbal. We know that students experience high levels of engagement within embodied tasks (Georgiou & Iannou, 2020; Lindgren et al., 2016); our work builds upon this notion by asserting that embodiment in and of itself is indicative of student engagement. This study demonstrates its potential to provide researchers and instructors a novel way to view their students' engagement with meaningful mathematical tasks in the classroom.

References

Alibali, M. W., Nathan, M. J., Boncoddo, R., & Pier, E. (2019). Managing common ground in the classroom: Teachers use gestures to support students' contributions to classroom discourse. *ZDM*, *51*(2), 347-360. http://dx.doi.org/10.1007/s11858-019-01043-x

Appleton, J. J., Christenson, S. L., Kim, D., & Reschly, A. L., (2006). Measuring cognitive and psychological engagement: Validation of the Student Engagement Instrument. *Journal of School Psychology, 44*(5), 427-445. http://dx.doi.org/10.1016/j.jsp.2006.04.002

Black, L., & Radovic, D. (2018). Gendered positions and participation in whole class discussions in the mathematics classroom. In U. Gellert (Ed.), *Inside the Mathematics Class* (pp. 269-289). Springer. http://dx.doi.org/10.1007/978-3-319-79045-9_13

Böheim, R., Urdan, T., Knogler, M., & Seidel, T. (2020). Student hand-raising as an indicator of behavioral engagement and its role in classroom learning. *Contemporary Educational Psychology, 62*. https://doi.org/10.1016/j.cedpsych.2020.101894

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3*(2), 77-101. http://dx.doi.org/10.1191/1478088706qp063oa

Carini, R.M., Kuh, G. D., & Klein, S. P. (2006). Student engagement and student learning: Testing the linkages. *Research in Higher Education, 47*(1). http://dx.doi.org/10.1007/s11162-005-8150-9

Civil, M. (2014). Guest editorial: Musings around participation in the mathematics classroom. *The Mathematics Education, 23*(2), 3-22.

Fink, H. (2022). Obscuring racial inequities through gender-focused discourse: An analysis of student participation in a calculus class. In Lischka, A. E., Dyer, E. B., Jones, R. S., Lovett, J. N., Strayer, J., & Drown, S. (Eds.), *Proceedings of the 44th Annual Meeting of the North American Chapter of the International Group for the Psychology in Mathematics Education* (pp. 387-395). Nashville, Tennessee, USA.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59–109. http://dx.doi.org/10.3102/00346543074001059

Fredricks, J. A., & McColskey, W. (2012). The measurement of student engagement: A comparative analysis of various methods and student self-report instruments. In S.L. Christenson, A. L. Reschly, & C. Wylie (Eds.), *Handbook of Research on Student Engagement* (pp. 763-782). Springer. http://dx.doi.org/10.1007/978-1-4614-2018-7_37

Fredricks, J. A., Wang, M., Shall Linn, J., Hofkens, T. L., Sung, H., Parr, A., & Allerton, J. (2016). Using qualitative methods to develop a survey measure of math and science engagement. *Learning and Instruction, 43*, 5-15. http://dx.doi.org/10.1016/j.learninstruc.2016.01.009

Garcez, P. M. (1997). Microethnography. In N.H. Hornberger & D. Corson (Eds.), *Encyclopedia of Language and Education* (pp. 187-196). Springer. http://dx.doi.org/10.1007/978-94-011-4535-0_18

Garcia, T., & Pintrich, P. R. (1991). Student motivation and self-regulated learning: A LISREL model. Office of Educational Research and Improvement, United States Government.

Georgiou, Y., & Ioannou, A. (2020). Developing, enacting and evaluating a learning experience design for technology-enhanced embodied learning in math classrooms. *TechTrends, 65*(1), 38– 50. http://dx.doi.org/10.1007/s11528-020-00543-y

Hodgson, T. R., Cunningham, A., McGee, D., Kinne, L. J., & Murphy, T. J. (2017). Assessing behavioral engagement in flipped and non-flipped mathematics classrooms: Teacher abilities and other potential factors. *International Journal of Education in Mathematics, Science, and Technology, 5*(4), 248-261. http://dx.doi.org/10.18404/ijemst.296538

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1). University of Nevada, Reno.

870

Joshi, D. R., Adhikari, K. P., Khanal, B., Khadka, J., & Belbase, S. (2022). Behavioral, cognitive, emotional, and social engagement in mathematics learning during COVID-19 pandemic. *Plos one, 17*(11). http://dx.doi.org/10.1371/journal.pone.0278052

Kahn, P. E. (2014). Theorising student engagement in higher education. *British Educational Research Journal, 40*(6), 1005-1018. http://dx.doi.org/10.1002/berj.3121

Keith, H. (2016). *The effects of community-building on achievement, motivation, and engagement in undergraduate mathematics* [Thesis, Western Kentucky State University].

Kelton, M. L., & Ma, J. Y. (2020). Assembling a torus: Family mobilities in an immersive mathematics exhibition. *Cognition and Instruction, 38*(3), 318-347. http://dx.doi.org/10.1080/07370008.2020.1725013

Laursen, S. L., & Rasmussen, C. (2019). I on the Prize: Inquiry approaches in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education, 5*(1), 129–146. https://doi.org/10.1007/s40753-019-00085-6

Lindgren, R., Tscholl, M., Wang, S., & Johnson, E. (2016). Enhancing learning and engagement through embodied interaction within a mixed reality simulation. *Computers & Education, 95*, p. 174-187. http://dx.doi.org/10.1016/j.compedu.2016.01.001

Nemirovsky, R., & Ferrara, F. (2009). Mathematical imagination and embodied cognition. *Educational Studies in Mathematics, 70*(2), 159-174. http://dx.doi.org/10.1007/s10649-008-9150-4

Nemirovsky, R., Rasmussen, C., Sweeney, G., & Wawro, M. (2012). When the classroom floor becomes the complex plane: Addition and multiplication as ways of bodily navigation. *Journal of the Learning Sciences, 21*(2), 287–323. http://dx.doi.org/10.1080/10508406.2011.611445

Nemirovsky, R., Ferrari, G., Rasmussen, C., & Voigt, M. (2020). Conversations with materials and diagrams about some of the intricacies of oscillatory motion. *Digital Experiences in Mathematics Education, 7*(1), 167-191. http://dx.doi.org/10.1007/s40751-020-00073-5

Pilotti, M., Anderson, S., Hardy, P., Murphy, P., & Vincent, P. (2017). Factors related to cognitive, emotional, and behavioral engagement in the online asynchronous classroom. *International Journal of Teaching and Learning in Higher Education, 29*(1), p. 145-153.

Quintero, M., Hasty, L., Li, T., Song, S., & Wang, Z. (2022). A multidimensional examination of math anxiety and engagement on math achievement. *British Journal of Educational Psychology, 92*(3), 955-973. http://dx.doi.org/10.1111/bjep.12482

Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics, 70*(2), 111–126. http://dx.doi.org/10.1007/s10649-008-9127-3

Radford, L. (2014). Towards an embodied, cultural, and material conception of mathematics cognition. *ZDM, 46*(3), 349–361. https://doi.org/10.1007/s11858-014-0591-1

Veiga, F. H. (2016). Assessing student engagement in school: development and validation of a four dimensional scale. *Procedia: Social and Behavioral Sciences, 217*, 813-819. http://dx.doi.org/10.1016/j.sbspro.2016.02.153

Voigt, M., Hagman, J. E., Street, C., Guglielmo, J., Martinez, A., & Tremaine, R. (2022). A quantitative critical analysis of instructional practices and math identity. In A. E. Lischka, E. B. Dyer, R. S. Jones, J. N. Lovett, J. Strayer, & S. Drown (Eds.), *Proceedings of the 24th Annual Conference on Research in Undergraduate Mathematics Education*, p. 666-674. Boston, Massachusetts, USA.

Walkington, C., Chelule, G., Woods, D., and Nathan, M. J. (2019). Collaborative gesture as a case of extended mathematical cognition. *Journal of Mathematical Behavior, 55*. http://dx.doi.org/10.1016/j.jmathb.2018.12.002