A COMPARATIVE ANALYSIS OF FRACTION PROBLEMS WITHIN THE IRANIAN CURRICULUM AND GO-MATH TEXTBOOKS

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Textbooks play an important role in teachers' instructional decisions (Jones & Tarr, 2007), which consequently affects students' learning. This paper reports on a comparison of the elementary mathematics textbooks used in Iran and the United States, the Go-Math textbook. I analyzed topic sequences, frequency of the tasks, and cognitive demands of the fraction task in second and third-grade textbooks, employing the framework developed by Smith and Stein (1998) regarding the Levels of Cognitive Demands (LCD). Findings showed that Iran's textbooks devoted more percentage of pages to fractions in second grade than Go-Math textbooks. LCD of the tasks in second grade in both courtiers were in lower levels. Also, the presentation of the fraction concepts varied in different countries and Go-Math covered more fraction concepts in third grade. Recommendations for future research were offered.

Keywords: Textbook analysis; Fraction; Levels of cognitive demand; Curriculum; Content analysis.

Fractions are one of the topics in elementary school textbooks used in many countries (Edwards et al., 2023) and methods and the presentation of this concept vary in different countries. In Iran, there is only one mathematics textbook published by the Ministry of Education (ME) for each grade, and teachers and students rely on the textbooks published by ME. Thus, the underlying rationale for focusing on the concept of fractions in textbooks was that the analyzed Iran's textbooks were used nationwide and reflected a good picture of the nature of tasks implemented by teachers. This study was part of a larger study that compared the type of questions in mathematics textbooks in different countries and discuss the result of the study about comparing fraction tasks in the textbook series in Iran and a commonly used textbook in the U.S., Go-Math textbook.

Literature Review

Learning and teaching fractions is one of the difficult areas in mathematics (Edwards et al., 2023; Getenet & Callingham, 2019). Research showed that students struggle with the concept of fractions and these difficulties with fractions have been observed across all levels (Charalambous & Pitta-Pantazi, 2006). Furthermore, some research shows a relation between differences in students' performance and the approaches of the textbooks to mathematics topics (e.g. Li (2000)). In their study of 340 fifth graders and 306 sixth graders Charalambous and Pitta-Pantazi (2006) argued that "the differences in students' performance on the five interpretations of fractions mirror the imbalance in the emphasis placed on them during instruction" (p. 309). For example, students' performance in part-whole subconstruct of fractions was better than other five interpretations of fractions, and it was the subconstruct that students met more frequently in their mathematics textbooks. Furthermore, the research emphasizes that textbooks in many ways can contribute to improved learning outcomes (Milligan et al., 2018) and the nature of the mathematics tasks in textbooks can influence students' thinking and also can limit or broaden their views (Henningsen & Stein 1997).

Lamberg, T., & Moss, D. (2023). *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1). University of Nevada, Reno.

In mathematics education, textbooks have a great influence on the education scene (Fan et al., 2013; Milligan et al., 2018). Textbooks play an important role in teachers' instructional decisions (Jones & Tarr, 2007), the materials they use in the classroom (Chang & Silalahi, 2017), and the concepts that are imparted in the classroom (Jones & Tarr, 2007). Chang and Silalahi (2017) stated that analysis of textbooks can aid in understanding the effectiveness of specific approaches and identify what is required in terms of teaching and identify areas for improvement in curriculum development. Thus, this study aimed to analyze Iran's mathematics textbooks in different grades.

Iran's elementary textbooks, published by the ME, are used nationwide, and national assessments use these textbooks as the main sources of learning material. There is only one mathematics textbook published by the ME for each grade, thus they reflect a good picture of the nature of the tasks that are implemented by teachers in the classrooms. This paper sets out to analyze the introduction to the fraction concept used in elementary textbooks to establish the frequency and problem types of the tasks. Furthermore, students need opportunities "to engage with tasks that lead to deeper, more generative understandings about the nature of mathematical concepts" (Stein et al., 2001, p. 15). Thus, this study also reports on the result of the analysis of the level of the cognitive demands required by tasks in Iran's elementary textbooks and compares it with a commonly used textbook in the U.S., Go-Math.

Research Questions

In this study, I addressed the following research questions:

1- What are the cognitive demands of the fraction tasks in Iran's textbooks in comparison to a commonly use textbook in the U.S., Go-Math?
2- What are the nature and frequency of the fraction tasks in second and third grades mathematics textbooks in Iran and Go-Math in the U.S.?

Theoretical Framework

This study draws from a framework developed by Smith and Stein (1998) in considering what kind of thinking tasks the textbooks demand of the students. Smith and Stein (1998) categorized mathematical tasks in terms of four cognitive features. They provided a scoring rubric that can be applied to all kinds of mathematical tasks and helps to rate the tasks based on the required cognitive demand. Characteristics of the tasks in the Lower-level Memorization (LM) were "reproducing previously learned facts, rules, formulas, or definitions" (Smith & Stein, 1998, p. 348). This type of task cannot be solved by using procedures (Smith & Stein, 1998). Lower-level Procedure without connections (LP) task is defined as a task that requires "use of the procedure either is specifically called for or is evident from prior instruction" (p. 348) and requires limited cognitive demand. Higher-level Procedures with connections (HP) refers to a task that focuses students' attention on the use of procedures, suggests explicit or implicit pathways, and requires some degree of cognitive effort. Higher-level Doing Mathematics (HDM) task requires "students to explore and understand", and requires "complex and nonalgorithmic thinking" and considerable cognitive effort (p. 348). In this study, I used the list of the characteristic of the tasks at each level and examples provided by Smith and Stein (1998) to analyze questions related to fraction tasks in second and third-grade elementary textbooks, which will be elaborated more in the next part.

Methods

This study was a part of research in analyzing mathematics textbook questions in different countries and intended to examine the extent that second and third-grade mathematics textbooks in Iran and the U.S. attend to fraction concepts. The subjective data of this study was all questions related to the fraction concept in two textbook series in the U.S. and Iran. Two grade-level textbooks published under the approval of the ME in Iran for grades two and three, the tenth edition published in 2022, were selected and compared to the Go-Math textbook series in the U.S., the student e-book 2015 Common Core. The selection of the second and third garde was done purposefully, because subconstructs of the fractions were not covered in earlier grades.

Tasks that were explored in this study were all of the questions from both textbook series, Iran's textbooks and Go-Math textbooks, related to the fraction concept, and included questions listed as activities, introduction, problem-solving, application, practice and review questions. In counting the number of the tasks for both textbook series, a set of exercises that build on one another were considered as one task, as illustrated by an example in Figure 1 part A. Also, a set of questions that attend to the same topic but had different question numbers, and can be answered in isolation, was considered as a single task (see Figure 1 part B).

A B

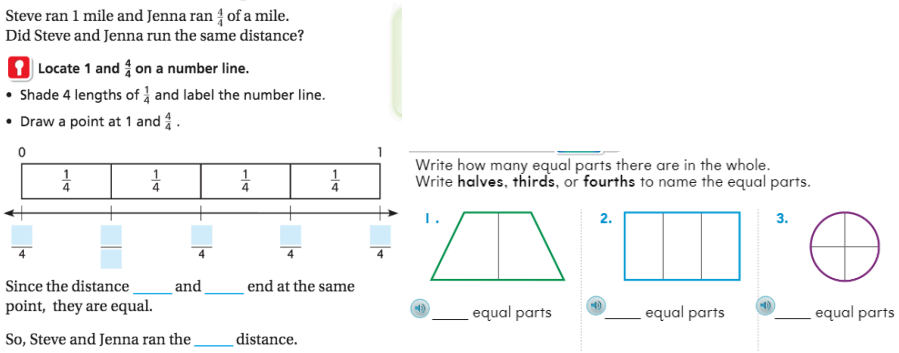


Figure 1: Example of Multiple Questions Considered as One Task in Coding Process

In the first stage, the topic sequences, frequency of fraction questions, and page position of the questions were compared in both textbook series. In the second stage, the level of cognitive demands of all the questions was analyzed employing the framework developed by Smith and Stein (1998). Finally, the contents of selected textbooks were compared in their pedagogical approach to their focus on learning fractions.

In the second stage, I used the list of characteristics in Smith and Stein (1998) to assign a level for each task. Figure 2 shows examples of the tasks coded in each LCD. In this process, if a task appeared for the first time in the textbook that required using a procedure and represented the concept in multiple ways or made connections among multiple representations, it was coded at HP. Similar tasks that were observed later in the textbook were coded as LP because they reproduced previously learned facts (Smith & Stein, 1998). For example, finding a fraction on the number line that was introduced in the third-grade Go-Math textbook was coded as HP in the first appearance in the textbook and as LP later because it required limited cognitive demand (Figure 2). To ensure the validity of the data, I asked an expert college (Creswell & Miller, 2000) to do coding independently for random questions, then we compared assigned codes.

Lower-Level Demands

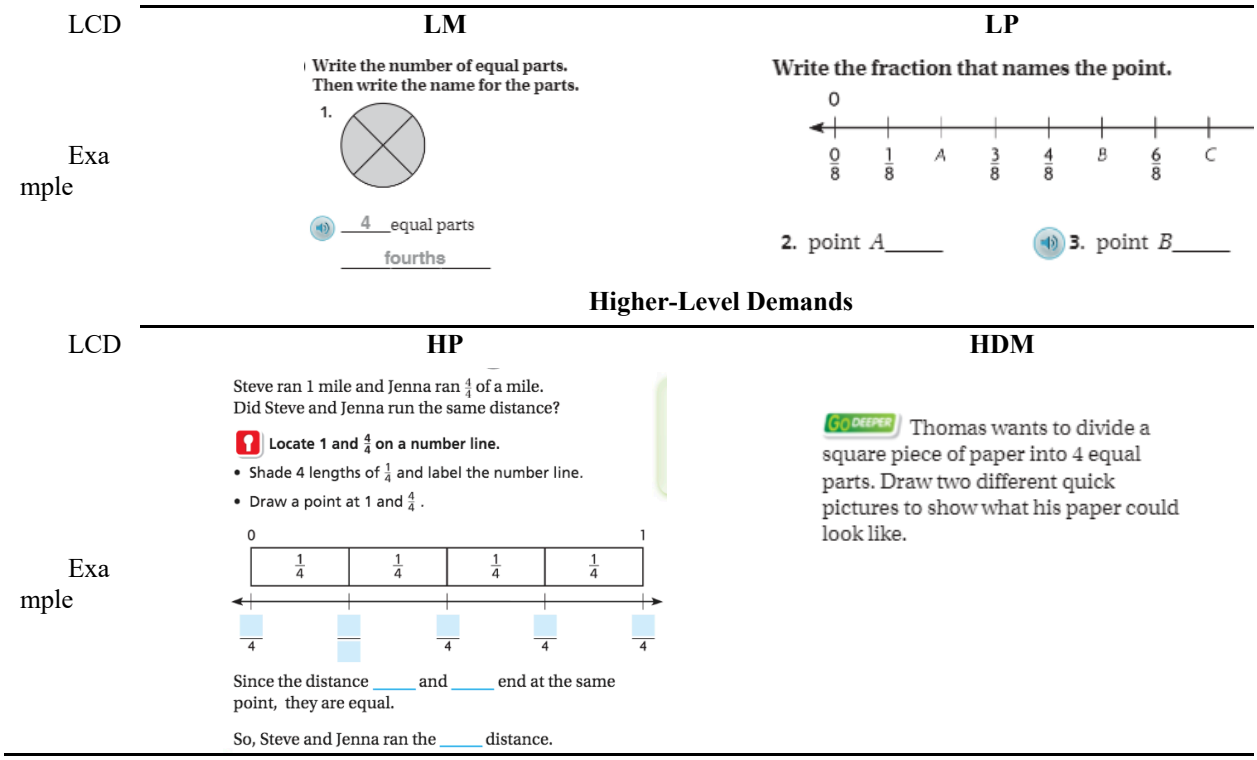


Figure 2: Example of the Tasks Coded in Each LCD

Findings

Comparison of the Number of Pages

In general, the total number of pages in Iran's textbooks was less than the total number of pages in Go-Math textbooks (145 pages in comparison to 774 pages in second grade and 151 pages in comparison to 758 pages in third grade). The number of pages that included fraction tasks in the third-grade Go-Math textbook was higher than the number of pages in second grade (117 and 28 respectively), while in Iran's textbook, a relatively equal number of pages were allocated to fraction tasks in both grades (15 pages in third grade and 17 pages in second grade which were 11% of the total pages in the both grades' textbooks). Furthermore, in comparing the percentage of the pages that were allocated to the fraction tasks in second grade, in the Go-Math textbook fraction tasks were 3% of the total number of gages in the textbook, while this percentage was 0.11 in Iran's textbook. In third grade, the percentage of the pages that were allocated to the fraction tasks in Go-Math textbooks was higher than in Iran's textbook (15 % in comparison to 11%). Table 1 shows the number of pages and page position in fraction tasks in both textbook series.

Table 1: Number of Pages and Page Position in Fraction Tasks

Textbook	Grade Level	Number of Pages	Number of Fraction Pages	% of pages with Fractions	First Fraction Page Position (Unit/ch/pg)	Final Fraction Page Position (Unit/ch/pg)
Go Math! 2015	2nd	74	28	3%	p. 747 ch 11out of 11 Lesson 8, 9, 10, 11	p. 774 ch 11 out of 11 Lesson 8, 9, 10, 11

	3rd	758	117	15%	p. 440 ch. 8 Out of 12	p. 556 ch 9. Out of 12
Iran's Math Textbook 2022	2nd	145	15	11%	p. 110 ch 7 out of 8	p. 125 ch 7 out of 8
	3rd	151	17	11%	p. 41 ch 3 out of 8	p. 57 ch 3 out of 8

Comparison of the LCD

The findings of this study indicated that the levels of the tasks related to fraction concepts in both textbook series in second grade required low cognitive demand. In Go- Math textbook only 6 tasks that required students to compare different parts of a whole or to find different ways of showing equal parts were coded as HDM and 86% of the tasks required low-level cognitive demand. All of the tasks in Iran's second-grade textbook required low-level cognitive demand; there was not any task that required high-level demand (0%, see Table 2). Also, the Go-Math textbook included more memorization tasks than Iran's textbook (81% in comparison to 60%).

Both third-grade textbook series included more high-level tasks in comparison to the second grade; 40% of the tasks in third-grade Iran's textbook in comparing to 0% in second grade, and 27% of the tasks in third grade in Go-Math in comparison to 14% in second grade (see Table 2) were in HP or HMD levels. Table 2 shows the number of tasks at each level.

Table 2: LCD of the Tasks in Go-Math Textbooks and Iran's Textbooks

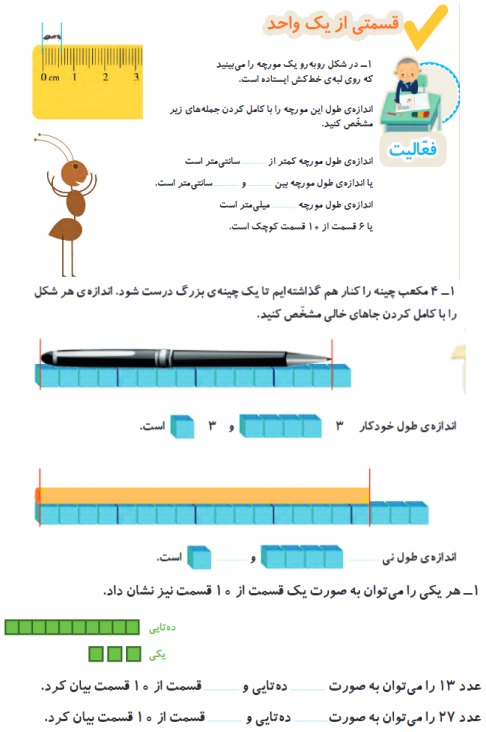
Textbook	Grade Level	Number of Fraction Pages	Number of Tasks	LM	LP	HP	HDM
Go Math 2015	2nd	28	43	35	2	0	6
				86%		14%	
	3rd	117	358	88	173	84	13
				73%		27%	
Iran's textbook 2022	2nd	15	25	15	10	0	0
				100%		0%	
	3rd	17	35	3	18	7	7
				60%		40%	

Pedagogical Analysis

Grade 2. Both textbook series introduced the concept of fractions by dividing the whole unit to equal parts in second grade. Go-Math introduced the concept of equal parts by partitioning geometric shapes in lesson eight in chapter 11, which is the last chapter of the textbook (see Table 1), and related this concept to the geometric figures that were discussed earlier in this chapter. Iran's textbook introduced this concept in chapter seven (out of eight chapters) by referring to the concept of measurement, discussed in chapter five, and also referring to the ten's and one's place values in two-digit numbers, discussed in chapter two of the textbook. Alajmi (2012) found that Japan's textbook similar to Iran's books introduce fractions with referring to the concept of measurement.

Furthermore, while in the Go-Math textbook, equal parts of a whole is the last chapter of the textbook, Iran's' textbook followed this concept with probability questions; connecting the likelihood of the occurrence to the number of equal parts that were colored in a shape. Figure 4 shows examples of how these textbook series approached the concept of equal parts in connection to previously introduced concepts.

Iran's Textbook



Go-Math Textbook

Figure 4. Introduction to The Concept of Fraction in Iran's Textbook and Go-Math Textbook, Grade 2

Grade 3. The presentation of the fraction concepts varied in different countries. The number of concepts that were covered in the Go-Math textbook was higher than in Iran's textbook (see Table 3). However, Iran's Textbook included Part of a Part of a Whole, writing a fraction for 3D figures, and Estimated Fraction that were not discussed in Go-Math (see Figure 4). Figure 3 shows the example of the Part of a Part of a Whole in which one part of a whole/unit where considered as part of a bigger whole.

The Go-Math textbook, introduced the fraction concepts with LM and LP tasks while the first pages of the fraction chapter in Iran's textbook had more HP and HDM tasks. For example, Figure 3 shows the questions on the first page of the chapter about fractions in which students were required to work on Part of a Part of a Whole and Comparing Parts of a Whole, in HDM tasks. The concept of Comparing Parts of a Whole, as presented in Iran's Textbook in the form of dividing a whole to different pieces and comparing created parts (see Figure 3) was discussed on page 519 (the first page of the chapter is 440) in the concept of Compare Fractions with the Same Numerator in Go-Math in a form of HP task and later on in the form of LP tasks.

Table 3: Concepts Related to The Fraction in Iran's Textbook and Go-Math Textbook

	Go-Math	Iran's Textbook
2nd	Equal Parts (the whole, halves, thirds, and fourths).	Equal parts of a whole unit (cm as a unit and mm as parts, equal parts in a geometric shape, equal parts of a segment)
3rd	Equal Parts of a Whole Equal Shares Unit Fractions of a Whole (First fraction p. 455) Fractions of a Whole Fractions on a Number Line Relate Fractions and Whole Numbers whole number and a fraction greater than 1(such as 2=8/4) Fractions of a Group Find Part of a Group Using Unit Fractions Problem Solving Find the Whole Group Using Unit Fractions Compare Fractions with the Same Denominator Compare Fractions with the Same Numerator Compare Fractions with Different Numerators and Numerators (Missing pies strategy) Compare and Order Fractions (order fractions with same numerator) Model Equivalent Fractions	Equal Parts of a Whole , Model Equivalent Fractions Compare Fractions with the Same Numerator Compare Fractions with the Same Denominator Compare Fractions with Different Numerators and Denominator (Same whole and different parts) Two or Three Whole units and fraction (greater than 1, Ex: 2 wholes and 1/3 of a whole)

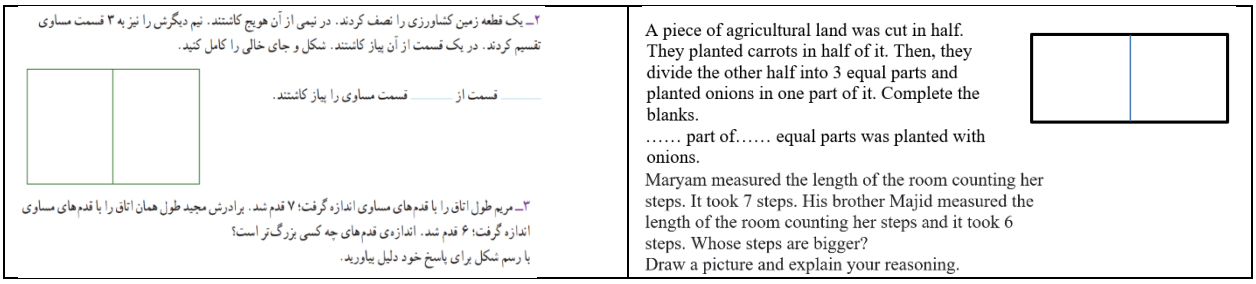


Figure 3: Example of the Questions in the First Page of chapter 3 in Iran's Textbook, Grade3

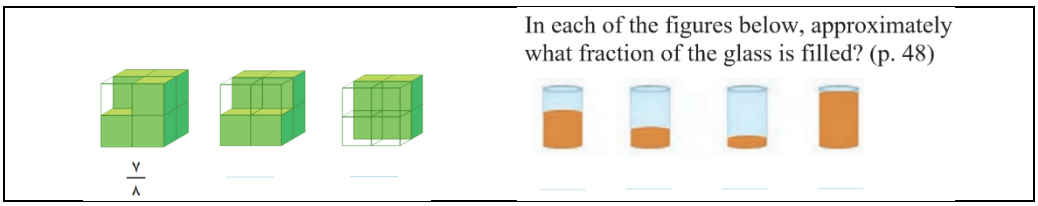


Figure 4: Fraction for 3D Figures and Estimated Fraction in Iran's Textbook

Discussion and Implications

This study contributes to the field of research on elementary school mathematics curricula with respect to fraction concept analysis.

As mentioned in the data analysis section, the total number of textbook pages allocated to fractions in Iran's textbook was considerably less than the U.S. textbooks. A study by Alajmi (2012) showed a similar result in comparing Kuwait, Japan, and U.S. mathematics textbooks. Further studies are required to study textbooks from different countries in an attempt to gauge how textbooks approach the concept of fractions in elementary grades.

Students should have the opportunity to engage with higher-level tasks that lead them to a deeper understanding of the concept (Stein et al., 2000). On the other hand, research showed that the LCD of tasks might declines throughout teachers' implementation in the classroom (Stein & Smith, 1998; Stein et al., 2001). Considering the findings of the current study that showed a higher number of lower-level tasks in both countries, and the result of similar studies such as Bütüner (2021) in Turkish and Singaporean textbooks, including more high-level tasks is required in the school's curriculum. Furthermore, according to Smith and Stein (1998) setting up a high-level task does not guarantee students' engagement at a high level. Thus, further studies are required to analyze and compare how teachers implement these tasks in different countries. Moreover, professional developments are avenues that could prepare teachers to implement lessons with high cognitive demands.

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