EXPLORING CHANGES IN MATHEMATICS TEACHER PRACTICE FROM PROFESSIONAL DEVELOPMENT ROOTED IN THE TRU FRAMEWORK

Victoria Bonaccorso	Helene Leonard	Amy Daniel
Montclair State University	Montclair State University	Montclair State University
bonaccorsov@montclair.edu	leonardh2@montclair.edu	daniela4@montclair.edu

Youngjun Kim	Joseph DiNapoli
Montclair State University	Montclair State University
kimy8@montclair.edu	dinapolij@montclair.edu

Equitable and accessible classrooms should engage all learners with mathematics content in meaningful ways. However, practicing teachers need support from professional development (PD) to learn to teach with this ambitious vision. Informed by sociocultural theory, we employed an evaluative case study methodology to describe, explain, and assess the experiences of one middle school mathematics teacher's longitudinal participation in a continuous PD model focused on the Teaching for Robust Understanding (TRU) framework. Based on classroom observations and interview data, our findings show evidence of TRU-aligned changes in teaching practice as a result of years of participation in the PD model. These findings strengthen the call for PD programs focused on equity and access and suggest design elements of such PD to support effectiveness.

Keywords: Professional Development, Instructional Vision, Instructional Activities and Practices

Recent reform movements have framed engaging all learners as an issue of equity and access to mathematics content (Larson, 2017). Teachers must create a classroom culture that empowers students to actively participate in mathematics lessons through productive struggle, collaboration, and explaining their reasoning, in contrast to lecture-based pedagogies (Baldinger & Louie, 2014; Porter et al., 2011). Thus, there is a strong need for professional development (PD) to support instructional shifts that help teachers develop practices aligned to this ambitious vision (Gallagher, 2016; Rosli & Aliwee, 2021; Sztajn et al., 2017). In this paper, we report on the changes in one middle school mathematics teacher's classroom practice based on her participation in a professional learning community (PLC) within a PD model (AIM-TRU) focused on creating engaging and powerful mathematics classrooms. This case study addresses the research question guiding our work: How does participation in a community of practice centered on the collective investigation of video cases grounded in high-quality instructional materials impact teachers' use of these materials and practice?

Theoretical Perspective and Review of Literature

Our work is framed by sociocultural theory in the ways we have studied the engagement of a PLC of mathematics teachers in PD and the impacts of those experiences on one middle school mathematics teacher's classroom practice. The following sections will describe socioculturalism and communities of practice (CoPs), tenets and examples of effective professional development, the Teaching for Robust Understanding (TRU) framework, and Formative Assessment Lessons (FALs). Literature in these areas helps preview the AIM-TRU PD model and its impacts on a middle school mathematics teacher's practice that we will report in this paper.

Sociocultural Theory and Communities of Practice

Sociocultural theory claims that learning and the activities, contexts, and cultures in which it takes place are inseparable (Brown et al., 1989; Collins et al., 1988; Lave & Wenger, 1991). Within socioculturalism, CoPs are groups of people who mutually engage in an activity, are connected by a joint enterprise, and engage with a shared repertoire of resources (Wenger, 1998, 1999). Within a CoP, evidence of learning can occur through changes in participation and reification. In teaching, the changes in the practice of one teacher in a PLC can demonstrate that individual's learning while engaging with a CoP through participation in the PD program.

Effective Professional Development

To support teachers' instructional shifts toward creating engaging and powerful mathematics classrooms, effective PD should be centered on coherent mathematics content, have sustained duration, and involve teachers in collective and active participation (Garet et al., 2001). Also, effective PD should be explicitly connected to classroom lessons to help facilitate changes in teaching practice (Desimone & Garet, 2015). When a PD program incorporates these features of effectiveness, research has shown that teachers can alter their instructional practice toward ambitious standards, develop their content and pedagogical knowledge, form productive beliefs about engaging all learners, and better support learning in their classrooms (Desimone, 2011).

PD research has also shown that PLCs focused on video case study can help teachers productively shift classroom practices. Borko et al. (2008) found that when a PLC of teachers was actively engaged with familiar mathematics lessons and collaborated to understand students' solution strategies in videos, they developed specialized content knowledge and pedagogical content knowledge (see Ball et al., 2008). This PD was also effective in helping teachers change their formative assessment practices in their classrooms. Santagata and Bray (2016) found that video-based error analysis helped teachers in a PLC develop new lesson-planning practices. Teachers started incorporating anticipations and responses to common student misconceptions about big mathematical ideas into lessons, providing more opportunities for student discourse during class. van Es and Sherin (2008) found that a PLC video club focused on professional noticing in teachers' classrooms helped them interpret and examine classroom interactions in new ways, which informed the teachers' implementation of a reform mathematics curriculum. In our own work, changes in participation and reification of TRU concepts were found to be associated with video case analysis in the AIM-TRU PD model (Leonard et al., 2022).

Teaching for Robust Understanding via Formative Assessment Lessons

Imperative to a CoP engaged in PD is the development of a shared repertoire built around best practices. In the context of the AIM-TRU PD model, this shared repertoire consists of the TRU framework and FALs. The TRU framework outlines an ambitious vision of mathematics classrooms that create engaging and equitable learning environments to support all students in becoming independent mathematical thinkers (Schoenfeld, 2015). The TRU framework details five interrelated dimensions: The Mathematics; Cognitive Demand (CD); Equitable Access (EA); Agency, Ownership, and Identity (AOI); and Formative Assessment (FA).

The Mathematics dimension refers to the rich, coherent mathematical content that forms the foundation of powerful mathematics classrooms (National Council of Teachers of Mathematics, 2000; National Governors Association, 2010; Schmidt et al., 2005). Engaging students with such content through mathematical tasks requiring high levels of *CD* with appropriate scaffolds creates the opportunity for the productive struggle necessary for developing conceptual understanding (Hiebert & Grouws, 2007). Such tasks are associated with improved opportunities to learn (Jackson et al., 2013; Stein et al., 1996), higher student achievement (Boaler & Staples,

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2008; Stigler & Hiebert, 2004), and the development of sophisticated solution strategies (Downton & Sullivan, 2017).

The *EA* dimension highlights instructional practices that can meaningfully engage all students with rich mathematical content. For example, selecting tasks with multiple entry points and solution strategies provides students with different ways of connecting their prior knowledge to new content, thus positioning all students as capable doers of mathematics (Boaler, 2016; Hodge & Cobb, 2019). *AOI* refers to the extent to which students are positioned with agency as creators of mathematical knowledge in the classroom, rather than passive recipients (Engle & Conant, 2002). When teachers establish classroom norms wherein students are responsible for making mathematical arguments and for evaluating the validity of those made by their peers, students are more likely to identify themselves as mathematically competent (Cobb et al., 2009). The use of *FA* in the classroom to elicit student thinking in order to inform instruction and provide feedback has been connected with improved student learning outcomes (Andersson & Palm, 2017; Black & Wiliam, 1998). In contrast to performance-based summative assessments (e.g., tests and quizzes), FA practices can foster intrinsic motivation for learning (Shepard, 2000), and can encourage students' development of a growth mindset and metacognitive habits (Darling-Hammond et al., 2020; Granberg et al., 2021).

Related to the TRU framework are FALs, which are high-quality, research-based lessons developed by Schoenfeld and his team to support teachers in creating TRU-aligned classroom experiences. Designed to be incorporated into existing curricula, each FAL includes structures to support teachers in formatively assessing student thinking (e.g., pre-assessment tasks) and in providing access to the mathematics for all students (e.g., a whole-class introduction wherein teachers can preview the context of the lesson). FALs also include high-CD small-group tasks designed to support students' AOI as they collaboratively construct mathematical knowledge.

Methodology

We used an evaluative case study methodology (Merriam, 1998) to describe, explain, and assess the experiences of one middle school mathematics teacher's longitudinal participation in a continuous PD model focused on creating engaging and powerful mathematics classrooms. This case study is part of a larger, multi-year PD research project involving over 150 teachers spanning multiple regions in the United States. To gain insight into learning within this CoP, we chose Ms. Chaves (pseudonym), a middle school teacher, to be the focus of our case study because of her active involvement in the AIM-TRU PD model as both a participant and facilitator throughout the four years of this study.

Ms. Chaves

At the beginning of her participation in the AIM-TRU PD model, Ms. Chaves had nine years of middle school mathematics teaching experience in a suburban district in the northeast United States, teaching primarily Math 8 and Algebra 1 classes. For the first year of her participation in this PD model, Ms. Chaves engaged as a teacher-participant in a PLC. Since then, she has led a PLC as a facilitator-participant. Prior to her involvement in this PD model, she had engaged in various PDs about classroom practices.

The AIM-TRU PD Model

The AIM-TRU PD model engages middle and high school mathematics teachers in a collaborative investigation of FALs and their enactment to deepen instructional knowledge and support shifts in practice aligned to the TRU framework. Grounded in tenets of effective PD (e.g., Desimone & Garet, 2015; Garet et al., 2001), this PD model provides opportunities for

teachers and facilitators to collectively generate professional knowledge for teaching and learning mathematics using the dimensions of ambitious instruction that are necessary and sufficient to produce equitable environments supporting deep learning opportunities for all students (Schoenfeld & the TRU Project, 2016). This PD model focuses on the following components: (a) unpacking the big mathematical ideas in a TRU-aligned FAL, (b) making observations about video cases demonstrating students' mathematical thinking while engaging in FALs, and (c) sets of video case reflective discussion questions based on the TRU framework (see Figure 1). During the reflective discussion of the video analysis, teachers often co-construct understandings about TRU-aligned teaching practices through dialogue (Leonard et al., 2022).
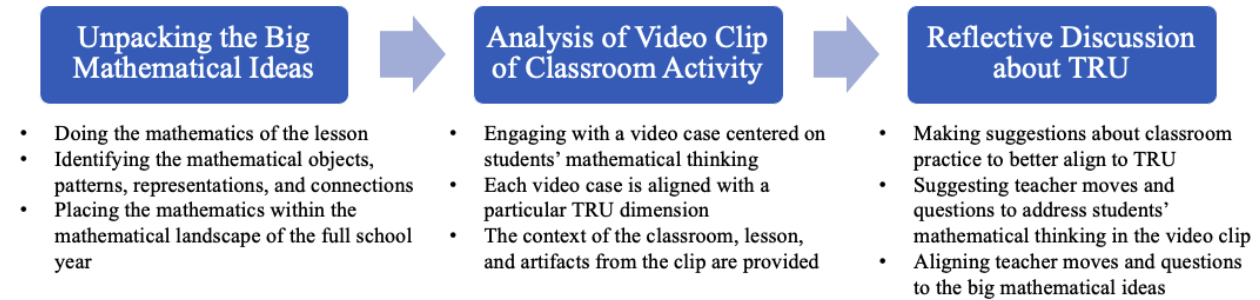

Unpacking the Big Mathematical Ideas	Analysis of Video Clip of Classroom Activity	Reflective Discussion about TRU
• Doing the mathematics of the lesson • Identifying the mathematical objects, patterns, representations, and connections • Placing the mathematics within the mathematical landscape of the full school year	• Engaging with a video case centered on students' mathematical thinking • Each video case is aligned with a particular TRU dimension • The context of the classroom, lesson, and artifacts from the clip are provided	• Making suggestions about classroom practice to better align to TRU • Suggesting teacher moves and questions to address students' mathematical thinking in the video clip • Aligning teacher moves and questions to the big mathematical ideas

Figure 1. The AIM-TRU PD Model

Data Sources and Analysis

Data collected and analyzed for this case study included one classroom observation video from each Year 1 and Year 4 and a follow-up interview. We watched the classroom video data, segmented it by class activity structure (whole class, set-up, small group, etc.), and used thematic analysis to describe Ms. Chaves' teaching practices. We used an observation rubric (see Schoenfeld et al., 2014) to assess her classroom practices in Year 1 and Year 4 relative to the pedagogical TRU dimensions to assess the alignment of teaching practices with the TRU framework. On a 1-3 half-point scale, with 3 being the highest alignment, we assessed pedagogical alignment with the following guiding questions:

- CD: How are students supported in productive struggle?
- EA: How is access to the content supported for all students?
- AOI: How are students the source of ideas and discussion?
- FA: How is students' thinking surfaced and built upon?

We also conducted a semi-structured follow-up interview with Ms. Chaves to gather details about the practices captured in her classroom observation video data and to learn from her perspective on the impact that the AIM-TRU PD model had on changes to her practice.

Findings

One teacher's journey with the AIM-TRU PD model provides a window into the ways that teacher learning manifests in a CoP through participation and how that learning can be reified with changes in a teacher's practice. In Year 1, Ms. Chaves had little prior knowledge of the TRU framework, relied on lecture-based practices, was influenced by institutional expectations, and was hesitant to implement FALs with fidelity. After prolonged participation in this PD model, we found evidence of shifts in her classroom practices and implementation of FALs. Her exposure to and immersion in the TRU framework and our analysis of her teaching practices can help explain these shifts.

Year 1: Ms. Chaves' Classroom at the Start of the AIM-TRU PD Model

In this section, we describe Ms. Chaves' teaching practices and her implementation strategies for FALs during Year 1 of her involvement in the AIM-TRU PD model.

Classroom practices. In the video of Ms. Chaves' classroom from Year 1, she was observed facilitating a whole-class homework review. Students were expected to self-check solutions by comparing their answers to a posted key. Following the self-check, Ms. Chaves asked, "Does anyone have any questions?" Seeing none, she moved on to the next part of the lesson. During her interview, Ms. Chaves reflected on her motivations for homework:

> But it's because I was supposed to, and everyone in our district, starting in sixth grade does this sort of homework, and this is how much it is. And all of the teams give the same. And it's due on this day. And this is how we grade it.

This indicates that Ms. Chaves was conforming to the institutional expectations for assigning, reviewing, and grading homework. This excerpt was coded using the observation rubric as CD: 1.5, EA: 1, AOI: 1, and FA: 1. Within the CD dimension, her homework assignments provided students an opportunity to productively struggle through problem solving, but this opportunity was not fostered or built upon through classroom practices. Her standardized homework practices created differential access for students because some students may not have had the background knowledge needed to enter the tasks (EA). In addition, students were limited to individually accessing solutions and could not engage in student-to-student discussion (AOI). This practice also limited assessment to purely corrective feedback on student solutions (FA).

Another classroom practice we noted through our analysis was the use of accountability talk. Ms. Chaves prompted discussions in small groups by reminding the students that they had structures of accountability talk. She indicated during her interview that she supported student accountability talk by hanging a poster of prompts on the wall for students to reference. This practice supported student engagement and discourse, but the placement of the resource may have limited students' access to these supports.

Implementation of FALs. Observations from Year 1 also provided a baseline for how Ms. Chaves implemented FALs. In one observation, she made significant changes to the format of the FAL, the tasks involved, and the questions suggested. For instance, Ms. Chaves chose not to use the FAL's pre-assessment task, eliminating a critical opportunity for her to formatively assess students' prior knowledge. She also eliminated the whole-class introduction, which would have increased students' access to the mathematics by providing them opportunities to engage with the content prior to small group work. When implementing the FAL's card sort activity, Ms. Chaves removed a pair of matches from the card sort and provided students with information about the number of matches, lowering the CD of the activity. Using the observation rubric, this episode was coded as CD: 2, EA: 2, and AOI: 2. These scores show that Ms. Chaves attended to CD, EA, and AOI, yet productive struggle was scaffolded away, access was inhibited, and means of fostering student agency were not promoted to the fullest extent.

Ms. Chaves shared in her interview that in Year 1, she was unfamiliar with FALs and used them to piece together her existing classroom practices and new practices related to FALs: "[I was] looking at those [FAL activities] like, oh, this would be good. And like I would just pull it and plop it in and like, trying to figure it out as I went." Ms. Chaves also expressed that she felt compelled to implement only the main task from the FAL because she was under institutional pressure to cover content. She shared that she needed to "keep pace" with other teachers, even if her students needed more time with particular lessons.

Year 4: The Impact of the AIM-TRU PD Model on Ms. Chaves' Teaching

In this section, we describe Ms. Chaves' practices and her implementation strategies for FALs demonstrated after four years of involvement with the AIM-TRU PD model.

Classroom practices. In the Year 4 observation video, we observed changes to Ms. Chaves' classroom practices around homework. In her interview, she shared that she thought her Year 1 homework practices were inequitable because "whatever these kids are going home to may or may not be conducive to them doing [home]work" and to "then penalize a child [for not doing homework] … seems like a one-two punch." She explained that she has adjusted her intentions regarding homework since Year 1, assigning less homework, but increasing the emphasis on making connections between mathematical ideas. She viewed this as a more equitable practice to promote students' retention of mathematical understandings. Furthermore, if students report struggles with homework assignments, she finds time during class for students to collaborate to explore mathematical ideas rather than posting an answer key, as in Year 1. This practice fosters discussion among students around concepts and connections and encourages students to evaluate their own mathematical thinking. She described this in her interview by stating, "I'm letting them come to those conclusions by themselves now." This practice shows her alignment with the TRU framework: she attended to EA by recognizing that conditions at home may not be favorable to completion, raised CD by having students productively struggle to form conclusions, and provided opportunity for students to develop their AOI by having the students work on the problems together to construct mathematical truths, and introduced more opportunities for FA by eliciting student thinking.

Supporting students to come to their own mathematical conclusions was also observed in the classroom observation from Year 4. During her launch of the FAL, Ms. Chaves asked students to determine if 0.123 was a terminating, non-terminating repeating, or a non-terminating non-repeating number. After a student shared their choice, Ms. Chaves asked the student, "Why did you choose that one?" The student gave their justification, and Ms. Chaves then asked another student, "Is that what you are thinking, would you like to add on?" She then asked if students had "any argument" for the other two choices. Ms. Chaves proceeded to ask if the number was rational or irrational. After giving time for students to think individually, she solicited student responses. Students responded with various ideas, including "both," and when the students did not agree on a choice, she did not disclose the correct answer. She instead told the class, "we will be figuring this out in our task." We coded this setup and exchange as FA: 3 and CD: 3 because she used students' emerging understandings to build on student thinking and engaged and supported students in productive struggle by not scaffolding away challenges, respectively.

In her interview, Ms. Chaves contended that her years of participating in this PD model helped her reify the dimensions of the TRU framework. When confronted with outside curricular materials or resources, she now critically analyzes them with a TRU framework lens:

> [How can I] make sure all kids have access to the lesson, but also make it cognitively demanding, and also give the kids agency? If someone comes up to me and says, I want you to teach like this now, I'm going to naturally throw that up against TRU in my mind.

This shows that Ms. Chaves changed her evaluation of classroom experiences, considering whether they raise CD, provide EA for all students, promote student AOI, and allow her the opportunity to formatively assess students' thinking effectively (FA).

Classroom observation of Ms. Chaves from Year 4 also revealed classroom practices that differed from those observed in Year 1. She reminded students to use the "accountability-talk stems" on their desks. When interviewed, she explained that these stems contained prompts for responding to peers, asking peers for clarifications, and sharing new ideas with peers. By using these prompts, students were supported in engaging in conversations by challenging others and justifying their own mathematical thinking. Unlike Year 1, these accountable-talk stems were placed on student desks instead of the wall, supporting student engagement with these practices. This teaching move attended to EA by helping more students to engage in mathematical conversations and to AOI because students were supported in sharing their ideas and building on others' understandings.

Implementation of FALs. In our classroom observation from Year 4, we observed Ms. Chaves implementing an FAL with fidelity and more closely aligned with TRU. In Year 4, she used the pre-assessment on definitions of decimals as recommended in the FAL rather than omitting it as in Year 1. Then, she used multiple approaches to formatively assess students' understandings. She prompted students to work on whiteboards and display them so she could assess their thinking. Next, she facilitated a class discussion based on some perceived misunderstandings, prompting students to justify their thinking and reasoning (FA: 3). Each portion of the FAL was implemented with fidelity, which was a stark difference compared to her Year 1 observation. In her interview, Ms. Chaves attributed this change, in part, to the work done within this PD model:

> There were always discussions … my kids can't do this, but if I edit it, maybe they can get it and then we would talk about, what does that do to the lesson if you edit it? If you make this easier, or if you scaffold this up, because you want to increase your access. But are you simultaneously lowering your cognitive demand? How do you do both? The more we would talk about editing the FALs, the more you question if you should be editing the FAL at all.

In her interview, Ms. Chaves referenced discussions from previous PD sessions in which teachers debated the impact of altering the format and structure of an FAL. Through these learning experiences in the PLC, which were marked by changes in participation and reification in the discussion, she was able to make shifts in her implementation of FALs in her classroom. She also noted that student engagement with this FAL has shifted over multiple years:

> I've done one lesson … three or four years, I finally feel like I let it breathe enough. And all of a sudden, these kids figured out things throughout the lesson that they had never done in previous years. It was like, oh, my gosh, what just happened? The answer is I gave them more time. I didn't try to rush.

Ms. Chaves now gives students time, space, and structures to make connections and persevere and has seen students making better mathematical connections than in previous years. She stated in her interview that this PD "is the only one that I've done that's been long term, sustained." The sustained duration of her involvement in this PD afforded her the opportunity to enact lessons multiple times, reflect on them with others, and improve her practice.

Discussion

Ms. Chaves' participation in the AIM-TRU PD model motivated changes in her teaching practices and the fidelity with which she now implements FALs in her classroom. Through her learning experiences shared across the CoP, she was able to demonstrate clear changes in

teaching practices aligned with the TRU framework. Also, the increased fidelity to FALs positioned Ms. Chaves to attain a closer alignment with the TRU framework. These changes in practice help answer recent calls to engage all learners with mathematical content in an equitable way (Larson, 2017). Our analysis of Ms. Chaves' changing classroom practices and her own reflections suggest that it was her continued participation in this PD model and the design of the PD itself (Desimone & Garet, 2015; Garet et al., 2001) that provided her the opportunity to reify ideas about powerful mathematics teaching and the implementation of high-quality materials.

We noted specific shifts in classroom practice regarding homework expectations and student discussion strategies. Ms. Chaves' altered her homework practices due to recognizing inequities in her prior practices. Through sustained duration and collaborative interrogation of teaching moves related to equity, Ms. Chaves chose to alter her practice to create a more equitable space for students. Related to Borko et al. (2008), Ms. Chaves leveraged her PD experiences to allow for more FA opportunities in her classroom as students discussed homework and sought to make their own connections between mathematical ideas to overcome any challenges. Additionally, while Ms. Chaves' change in structures for accountable-talk stems may seem small, shifting from a whole-class anchor chart to individual small-group reference sheets provided additional support for more students to engage in mathematical discourse. The teaching move of providing students with individual prompts was present throughout Ms. Chaves' participation in the PD. Drawing from Cobb et al. (2009), this practice situated students to view themselves as more mathematically competent. There were tangible and available resources for small groups to access and enter mathematical discourse and build on each other's thinking. The FAL instructions for student small-group work are also intentionally designed to promote student discourse. Akin to Santagata and Bray (2016), Ms. Chaves planned for more student discourse by drawing on discussions from PD sessions about the importance of setting and maintaining the FAL expectations for student talk structures; she fostered these interactions among students by making the structures clear and providing reminders.

The ways in which Ms. Chaves implemented FALs changed dramatically, due in part to her involvement in this PD model. Desimone and Garet (2015) stressed the importance of sustained PD with active learning experiences that can connect to teacher practice. Ms. Chaves' participation in the PD model provided her with sustained time reading, analyzing, and reflecting on the implementation of FALs within a CoP. Through discussions in the PLCs, she shifted her perception of how she can use FALs in her classroom and her opinions of the impact of altering the resource. Aligned with these tenets of effective PD, Ms. Chaves also shifted the way she adopts other materials for her classroom by analyzing the alignment of the materials with the TRU framework. This provided her the opportunity to push past institutional norms to adopt TRU aligned teaching practices. It was through her prolonged investigation of teaching practices using the TRU framework that she was able to take this resource and use it to select materials and moves for implementation that leads to ambitious teaching practices.

Conclusion

In this paper, we used sociocultural theory and an evaluative case study methodology to describe, explain, and assess Ms. Chaves' longitudinal experiences in the AIM-TRU PD model. Our findings show that a well-designed PD program focused on the TRU framework can inform shifts in classroom practice toward engaging all learners in ambitious learning opportunities with mathematics. A next step in our research is to broaden the scope of our methodology to study the impact of the AIM-TRU PD model on the collective learning of the entire PLC.

Acknowledgements

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