

Language and Mathematics Learning: A Comparative Study of Digital Learning Platforms

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Abstract

This paper presents a conceptual exploration of how Digital Learning Platforms (DLPs) can be utilized to investigate the impact of language clarity, precision, engagement, and contextual relevance on mathematics learning from word problems. Focusing on three distinct DLPs—ASSISTments/E-TRIALS, MATHia/UpGrade, and Canvas/Terracotta—we propose hypothetical studies aimed at uncovering how nuanced language modifications can enhance mathematical understanding and engagement. While these studies are illustrative in nature, they provide a blueprint for researchers interested in leveraging DLPs for empirical investigation so that future investigators gain a better understanding of the emerging infrastructure for research in DLPs and the opportunities provided by them. In highlighting three distinct implementations of the same core research question, we reveal both commonalities as well as differences in how different educational technologies might build evidence, offering a unique opportunity to advance the field of math education and other education research fields.

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Introduction

Within the SEERNet network of Digital Learning Platforms (DLPs), nonprofit organizations, and universities, we've begun to explore the unique affordances and constraints that researchers may encounter while conducting research in DLPs. In this paper, we investigate how DLPs can be used to study improvements in mathematics learning by way of nuanced language modifications in how problems are presented to the learner. The hypothetical studies outlined below aim to illustrate the potential of conducting research within these three distinct DLPs. The studies are designed to open avenues for generating insights about the role of language in mathematics education, with the prospect of such findings being applicable across educational technologies, pending empirical validation.

Our goal is to both enrich readers' understanding of the research capabilities and infrastructure supporting education research within these platforms and to motivate researchers to employ these DLPs for investigating their own educational interventions. By showcasing the hypothetical studies, we intend to inspire empirical research that further explores digital interventions in education, thereby advancing our collective knowledge of effective educational practices in digital environments.

Context and Significance of Language in Mathematics Learning

The intersection of language and mathematics in educational settings is emerging as a critical area of study, underscoring the nuanced role language plays in facilitating children's mathematical development (Purpura & Reid, 2016). This exploration is marked by diverse perspectives, illustrating that the relationship between language and mathematical understanding is complex and multifaceted. Research by Fuchs et al. has elucidated that computational abilities and word problem-solving skills may be underpinned by distinct cognitive processes, suggesting that the influence of language on mathematics learning is not uniform but varies depending on the specific mathematical domain (Fuchs et al., 2006; Fuchs et al., 2008; Swanson, 2006; Swanson & Beebe-Frankenberger, 2004). Concurrently, Viesel-Nordmeyer et al. (2022) contribute to this complex landscape by highlighting how language fundamentally supports children's ability to grasp and manipulate abstract mathematical symbols. This perspective enriches our understanding by positioning language not merely as a tool for communication but also as an integral component of mathematical reasoning and conceptualization. Such a synthesis of views underscores the importance of considering both the distinct and interconnected roles of language in the broader matrix of mathematics education.

Emerging research suggests that struggles in mathematics may often be rooted in language difficulties rather than in an inherent inability to grasp numerical concepts (Paetsch, Felbrich, & Stanat, 2015). This insight has profound implications for educational practice, as it calls for a reevaluation of the traditional approaches to teaching and assessing mathematics, emphasizing the need to integrate linguistic considerations into mathematical pedagogy and assessment.

The manner in which mathematical concepts are communicated is crucial to student comprehension and engagement (National Council of Teachers of Mathematics, 2000). For instance, the linguistic structure of word problems (a common device in mathematics where a mathematical problem is posed as narrative text and the learner is challenged to apply abstract mathematical knowledge and procedures to the problem) in mathematics has been a significant barrier to student understanding, often overshadowing the mathematical concepts these problems are intended to convey (Cummins et al., 1988). Moreover, the challenge

is magnified for multilingual students, who must navigate mathematical language in a non-native language (Barwell, 2005, 2009). Additionally, research indicates that students with dyslexia often find mathematics particularly challenging, further complicating the interplay between reading comprehension and math learning (Willcutt et al., 2013). Among students with autism, known for their heightened visuospatial skills, complexities arise in tackling math word problems that present intricate social scenarios (Wei et al., 2023).

The potential for enhancing linguistic clarity, precision, and contextual relevance as a means to facilitate math learning has been a topic of significant interest (Abedi & Lord, 2001; Pongsakdi et al., 2020). Notably, Abedi and Lord (2001) found that linguistic simplification of math word problems resulted in improved performance, particularly among English language learners and students from low-socioeconomic backgrounds. This finding is echoed in culturally responsive pedagogical approaches, such as the development of math modules tailored for Alaskan Native students, which demonstrated substantial improvements in mathematics performance across diverse student groups (Kisker et al., 2012).

The Evolving Role of Digital Learning Platforms

In the rapidly evolving landscape of digital education, DLPs emerge as transformative tools redefining the learning experience, with some DLPs offering new avenues for integrating linguistic considerations into mathematics education, assessment, and learning. DLPs can not only facilitate personalized and adaptive learning experiences but also provide a platform for systematic research, such as investigation into the interplay between language and mathematics. For instance, the MATHia platform has demonstrated how DLPs can offer valuable insights into the role of language in mathematics education, showcasing the potential of these platforms for empirical research (Almoubayyed et al., 2023). The adaptability of DLPs allows for controlled experimentation with language variations in math problems, providing a fertile ground for research that could lead to significant educational advancements.

This paper delves into how three distinct DLPs—ASSISTments/E-TRIALS, MATHia/UpGrade, and Canvas/Terracotta—can be leveraged as innovative tools for research in mathematics education. It's important to note that the selection of these platforms was intentional, focusing on those with significant application and impact within the K-12 sector. The other two DLPs considered, ASU and OpenStax, were not included in our study due to their less direct focus on K-12 education. Each chosen platform presents its unique design and methodological strengths, offering diverse approaches to a shared research question: How does language influence mathematics learning?

ASSISTments/E-TRIALS aids K-12 students in mastering mathematics through timely hints and explanations, focusing on immediate learning support. For researchers, ASSISTments/E-TRIALS facilitates the testing of various educational hints and explanations within unobtrusive math interventions in an online learning environment, along with analyzing the resultant learning outcomes in ASSISTments. UpGrade/MATHia integrates Carnegie Learning's adaptive tutoring system and facilitates field trials and experiments in real classroom settings, offering a robust environment for empirical investigation. Terracotta, in collaboration with Canvas, enables flexible, rigorous, and responsible randomized experiments across learning activities within a learning management system course site.

Contribution and Scope of This Paper

In proposing hypothetical studies across these DLPs, this paper aims to exemplify how nuanced language modifications can significantly enhance mathematics education. In preparing for this paper, Digital Promise researchers, researchers and/or representatives from the DLPs, and a math practitioner from the SEERNet Practitioner Advisory Board had conversations to ensure the topic reflects the perspectives, experiences, and needs of students and teachers. These illustrative studies are designed to demonstrate the utility of DLPs in educational research and their potential to contribute to a broader, more generalized understanding across different educational technologies.

By delving into the precise and contextually relevant use of language within DLPs, this paper seeks to bridge a critical gap in current research. Our exploration is pivotal for advancing academic discourse in the field and has substantial implications for practical applications in digital education. Language plays a fundamental role in shaping student learning experiences, and understanding its impact within the context of DLPs is essential for developing effective and engaging educational strategies in our increasingly digital world.



ASSISTments/E-TRIALS

The [E-TRIALS](#) platform is a free learning science A/B testing platform that allows researchers to conduct randomized controlled trials to examine research questions. It integrates with ASSISTments, a free online K-12 math learning platform supporting millions of students. Teachers use the platform to assign students math problems from popular open educational resources such as Kendall Hunt Illustrative Mathematics and EngageNY/Eureka Math. By using the platform, researchers can investigate different types of student supports (hints or explanations) for math problems or study how different types of math problem content may impact learning.

Design of Contrasting Conditions in ASSISTments/E-TRIALS

Researchers could develop a range of experimental conditions to explore the impact of language in math problem-solving using E-TRIALS/ASSISTments, including creating different types of hints or explanations with varied language and strategies for math problems to support students when they need help. For example, to help students solve math problems, researchers can examine the effects of social emotional language (SEL) and worked examples that use an animal context. The four conditions could be explanation, SEL + explanation, worked example, and SEL + worked example. In the **explanation** condition, students get an explanation on how to solve the problem. In the **SEL + explanation** condition, before the explanation, students get a message that says, "It's great that you're actively working on this problem and wanting to understand it better. Let's solve this problem together." In the **worked example** condition, students get a worked example that uses a dolphin context and then work on the original problem. In the **SEL + worked example** condition, students get the SEL message and then the worked example. Figure 1 shows an example of the design of this study.

Figure 1. Four Conditions of Student Support to Help Students Solve the Math Problem

Condition 1: Explanation	Condition 2: SEL + Explanation
<div style="border: 1px solid #ccc; padding: 10px;"> <p>Explanation REPORT THIS EXPLANATION</p> <ol style="list-style-type: none"> 1. First, let's add the miles up: $9/10 + 7/10$ 2. The denominators are the same, so we add the numerators: $9/10 + 7/10 = (9 + 7) / 10 = 16/10$ 3. Simplify the fraction by dividing both the numerator and denominator by 2: $16/10 = 8/5$ 4. Convert to mixed number: $8/5 = 1\ 3/5$ </div>	<div style="border: 1px solid #ccc; padding: 10px;"> <p>Explanation REPORT THIS EXPLANATION</p> <p>It's great that you're actively working on this problem and wanting to understand it better! Let's solve this problem together! :)</p> <ol style="list-style-type: none"> 1. First let's add the miles up: $9/10 + 7/10$ 2. The denominators are the same, so we add the numerators: $9/10 + 7/10 = (9 + 7) / 10 = 16/10$ 3. Simplify the fraction by dividing both the numerator and denominator by 2: $16/10 = 8/5$ 4. Convert to mixed number: $8/5 = 1\ 3/5$ </div>
Condition 3: Worked Example	Condition 4: SEL + Worked Example
<div style="border: 1px solid #ccc; padding: 10px;"> <p> Hint ^</p> <p>First, let's see how to solve a similar problem. REPORT THIS HINT</p> <p>Nemo the fish swam $5/8$ mile on Wednesday and $7/8$ mile on Thursday. How many miles did Nemo swim in the 2 days?</p> <ol style="list-style-type: none"> 1. First, let's add the miles up: $5/8 + 7/8$ 2. The denominators are the same, so we add the numerators: $5/8 + 7/8 = (5 + 7) / 8 = 12/8$ 3. Simplify the fraction by dividing both the numerator and denominator by 4: $12/8 = 3/2$ 4. Convert to mixed number: $3/2 = 1\ 1/2$ <p>After learning how to solve this problem, try the problem you are asked to solve.</p> </div>	<div style="border: 1px solid #ccc; padding: 10px;"> <p> Hint ^</p> <p>It's great that you're actively working on this problem and wanting to understand it better! Let's solve this problem together! :) REPORT THIS HINT</p> <p>First, let's see how to solve a similar problem.</p> <p>Nemo the fish swam $5/8$ mile on Wednesday and $7/8$ mile on Thursday. How many miles did Nemo swim in the 2 days?</p> <ol style="list-style-type: none"> 1. First, let's add the miles up: $5/8 + 7/8$ 2. The denominators are the same, so we add the numerators: $5/8 + 7/8 = (5 + 7) / 8 = 12/8$ 3. Simplify the fraction by dividing both the numerator and denominator by 4: $12/8 = 3/2$ 4. Convert to mixed number: $3/2 = 1\ 1/2$ <p>After learning how to solve this problem, try the problem you are asked to solve.</p> </div>

Implement the Experiment

Researchers who opt to utilize ASSISTments' pre-existing math problem content library can add custom hints or explanations for the math problems. This allows for the seamless integration of the study into ASSISTments' platform. Teachers and students who use ASSISTments form the participant base for the study. Teachers access the ASSISTments content library and assign their students the problem set(s) used by the study. When students do the assignment with the problem set, ASSISTments will randomly assign them to different study conditions as depicted in Figure 1, as well as the best-so-far condition that consists of existing hints or explanations in ASSISTments. In this case, researchers benefit from the existing user base and are not required to independently recruit participants. Of note is that the randomization in the backend of the platform will allow studies to run unobtrusively, as students will just be doing the assignments in a natural online learning environment.

In addition to hints or explanations, researchers can also design their own math problems for their studies. However, if researchers prefer to design their own math problems, they must undertake the recruitment of student participants for their studies. Despite this additional step, these custom studies can still be executed online, leveraging the digital capabilities of ASSISTments for data collection and analysis.

Data Collection and Analysis

To facilitate these research efforts, E-TRIALS offers a structured workflow, which starts with the researcher selecting the study type, developing the study in the E-TRIALS platform, and submitting for deployment. The E-TRIALS administrator receives and reviews the submission and then approves and deploys the study if it is properly developed. Once the study is deployed, the content will be accessible through ASSISTments. Teachers go to the ASSISTments content library and assign the problem set(s). When students do the problem sets, ASSISTments will randomly assign them to a condition of the study. When finishing data collection, the researchers can request data of their study for analysis and disseminate their findings.

Before the ASSISTments/E-TRIALS team shares the entire dataset, the researchers will be advised to pre-register their study on the Open Science Foundation (OSF) website to promote transparent research. Once the study is properly pre-registered, the data will be shared with researchers and a data dictionary will be provided to help them understand it. The data encompass students' prior learning with ASSISTments, students' learning performance (correctness and scores) and requests for hints or explanations in the experiment, log data on students' actions, students' post-learning data on the same skill covered in the experiment, as well as class-level student learning data and school-level demographics, including school area (rural, urban, suburban), school type (public or other), title 1 status, race and ethnicity, and free and reduced lunch percentages. This comprehensive suite of data allows for a nuanced analysis of the study's impact, enabling researchers to draw meaningful conclusions about the effectiveness of their educational interventions.

MATHia/UpGrade

UpGrade is a free and open source platform for conducting field trials in EdTech software. This platform has been integrated into Carnegie Learning’s MATHia software. MATHia is an adaptive, intelligent tutoring system for learning math that is currently used as part of a blended curriculum by over 600,000 students in over 3,000 schools each year. Researchers can conduct content, feature, and sequence-based experiments in diverse, real classrooms that use MATHia, leveraging the power of UpGrade to manage experimental design and to collect data for evaluating learning outcomes.

Design of Contrast Conditions in MATHia/UpGrade

There are a number of ways to address the question of how best to develop clear, precise, engaging, and contextually relevant language to support math learning in the context of MATHia. This includes removing passive voice and simplifying language in math word problems. Additionally, to address contextual relevance or engagement, math problems could be thoughtfully rewritten to include novel scenarios or contemporary topics. This approach aims not only to make math problems more relatable but also to stimulate students’ interest and involvement in learning.

Figure 2. Enhanced Clarity and Relevance in Math Design using MATHia/UpGrade

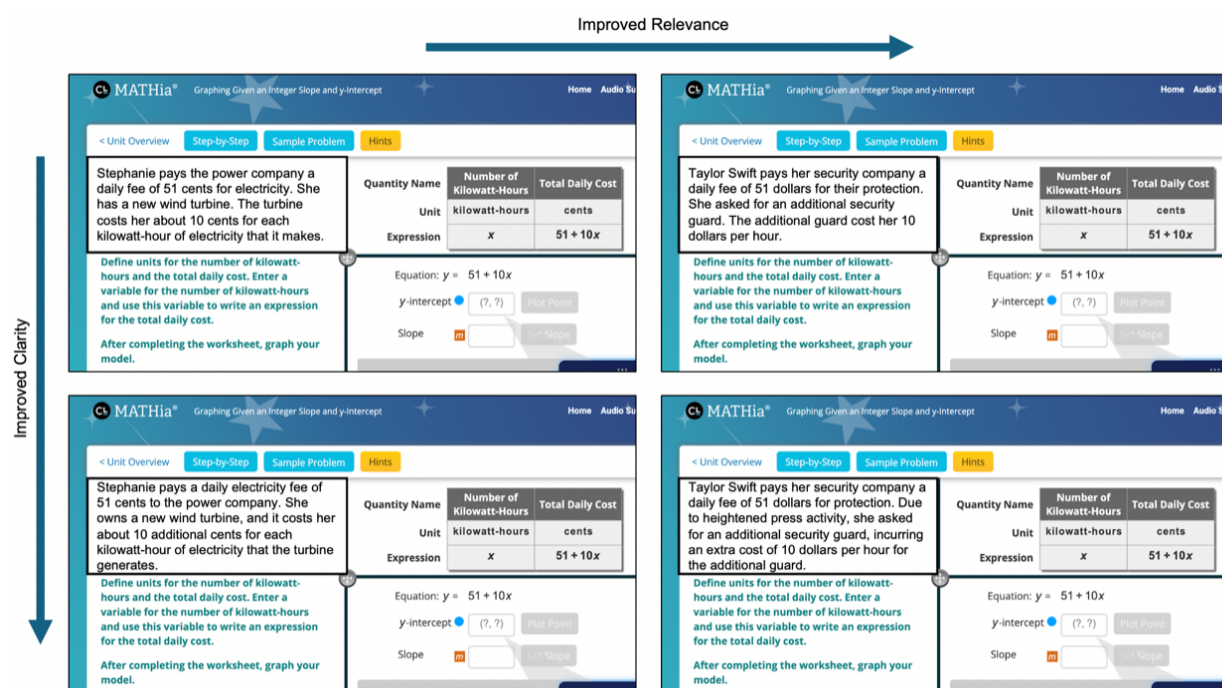


Figure 2 illustrates an approach for crafting math items that elevate both clarity and relevance through the use of MATHia. Despite testing the same mathematical skills across all word problems, the item positioned in the lower right corner is distinctively crafted. It utilizes clearer language and introduces a relatable scenario involving Taylor Swift hiring security guards. This intentional design aims to boost student comprehension and engagement when tackling such math word problems.

Implement the Experiment

Collaborating with the Carnegie Learning team is a critical step. Researchers will work with them to identify specific math topics or problem types to use in conducting the intervention(s) and discuss solutions for implementing ideas in the software.

Once the experimental design is finalized and the content changes have been published in the MATHia software, setting up the experiment in UpGrade takes minutes. The researcher can decide whether to conduct multiple simultaneous (or sequential) experiments, using simple weighted randomization for condition assignment, or opt for a factorial design such as a 2x2 approach to assess interaction effects between the conditions.

Outcome metrics can be defined on a by-experiment basis. Examples include maximum time to complete a math topic, average number of hints used, or mastery rate of a math topic associated with the intervention.

Data Collection and Analysis

While the experiment is running, participant enrollment data and metrics are collected as students progress through MATHia in their usual curriculum sequence. In addition to viewing a “snapshot” of outcomes via the UpGrade dashboard, researchers can export a csv file of experiment data from the UpGrade dashboard, as well as work with the Carnegie Learning team if more detailed, transaction-based data is needed for analysis.

Through this integrated approach, MATHia/UpGrade stands as a powerful tool for researchers aiming to enhance math learning through language-focused interventions. The platform’s capabilities allow for a rich exploration of how language clarity, precision, and contextual relevance can significantly influence students’ mathematical understanding and performance.

Canvas/Terracotta

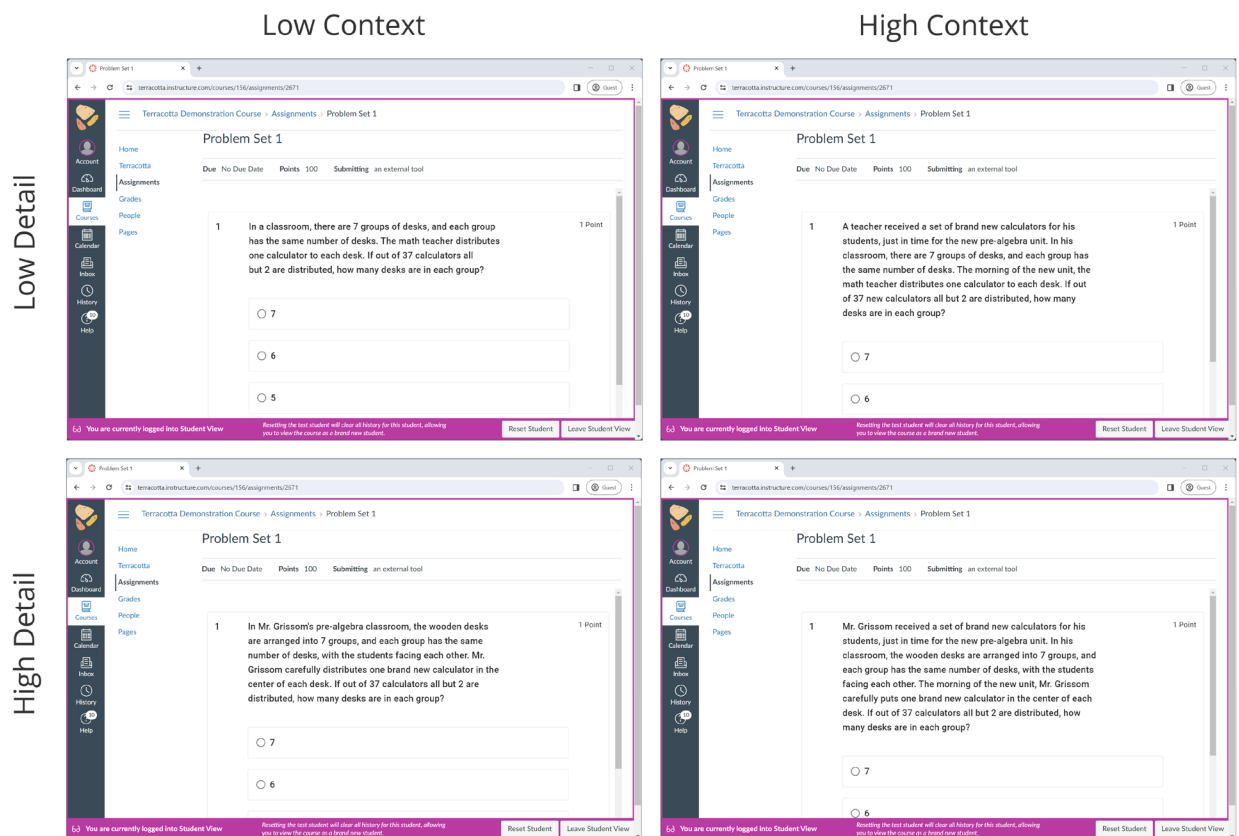
[Terracotta](#) is a digital learning platform that integrates with the Canvas learning management system (LMS) and makes it possible to embed controlled experiments in Canvas assignments (Mutz et al., 2023). With an eye toward enabling a variety of research questions and designs, Terracotta allows teachers and researchers to create original content for their experiments and deploy them within the context of regular instructional practice in the LMS. As such, Terracotta is well suited to support a research study that involves manipulation of the language presented in word problems.

In addition to enabling diverse experimental designs, Terracotta automates features that are fundamental to responsible experimental research but difficult to implement in classroom settings. For instance, potential participants should have the opportunity to opt into a study; Terracotta creates an informed consent assignment (or assent assignment, in the case of research with minors) during experiment setup and gives students credit for having completed the assignment without revealing students’ responses to their teacher. Once students have opted in, Terracotta randomly assigns consenting participants to different experimental conditions, collects data automatically as students complete their assignments, and provides de-identified data exports to the user.

Design of Contrast Conditions in Terracotta

The focal research question—How does language influence mathematics learning?— could be addressed by examining a range of language-related features of mathematics instruction that may have distinct influences. For example, language that is more contextually relevant may be more engaging, but when providing this context, additional linguistic details are often added that may impact the clarity and precision of instruction (Fyfe et al., 2014; Day et al., 2015). Given these possibly separable influences, the researcher could use a factorial design to investigate this question. Terracotta makes it possible to create an experiment with two different factors, each with two different levels: context (low/high) and detail (low/high) (see Figure 3 below). This 2x2 design would have four conditions: low context/low detail, high context/low detail, low context/high detail, and high context/high detail.

Figure 3. Systematically Manipulating Context and Detail in a Word Problem in Terracotta



Implement the Experiments

To conduct the experiment, the researcher would recruit a cohort of math teachers from schools where Terracotta has been integrated into the Canvas LMS to collaborate. Weekly practice question sets from their current math curriculum (for simplicity, imagine these as multiple-choice problems) would be modified to create four different versions of each question set, corresponding to the four experimental conditions. For example, in the high context/high detail condition, questions would include vivid word problems designed to immerse the student in relevant context, but in the low context/low detail condition, questions would be simple, clear, and straightforward. This would be set up in Terracotta as a four-condition experiment with a within-subjects design, where students switch between conditions for each unit in the math class. Multiple submissions are allowed on the practice sets, and Canvas stores the students' highest earned grade in the gradebook. Additionally, a demographic survey (where students get full credit regardless of their responses) and an assent assignment would be set up in Terracotta and administered at the start of the term. Terracotta automatically randomizes students who provide assent into different sequences of conditions between the different units, and students' scores on common (unmodified) end-of-unit assessments would be imported from Canvas into Terracotta as experimental outcomes.

Data Collection and Analysis

Once the term has completed and the experiment has concluded, at the touch of a button, Terracotta produces a de-identified data export. This export contains a set of csv files that include metadata about the experiment and the manipulated assignments, students' condition assignments, students' responses to all questions in Terracotta (both the survey and the manipulated question sets), timestamped clickstream data describing when students start and submit each assignment, and students' outcome scores on the end-of-unit assessments. Each class's data export is in the same standardized format, with numeric codes to identify classes, assignments, and student responses, and the export files are easily stacked across classes to create a combined data set.

Among the range of possible ways of examining the data, several analytical approaches stand out for potential prioritization. The first involves an investigation into treatment characteristics by looking at assignment submission rates, time on task, and student performance on the manipulated weekly practice sets. A second analytical approach focuses on student performance in end-of-unit outcome assessments depending on their condition assignments, directly addressing the core research question regarding the effect of context and detail on math learning. Third, moderator analyses could be employed to examine whether the effects of context and detail generalize across classes and student demographics.

Conclusion

In our three hypothetical studies, we identify a broad spectrum of capabilities that highlight the significant potential of DLPs to answer our research questions. ASSISTments/E-TRIALS, MATHia/UpGrade, and Canvas/Terracotta, each with their specific methodologies and attributes, provide opportunities to experimentally manipulate student learning activities and collect data on student behavior and achievement. While they have distinct approaches, the three studies unite in a common goal: to enhance the confluence of language and mathematics for improved student learning. The practical value of this research lies in offering insights on how modified language can facilitate math learning in a digital environment, thereby providing guidance to researchers, DLP and assessment designers, and teachers on improving practices, experiences, and outcomes of math learning.

ASSISTments/E-TRIALS' method of offering on-demand hints and explanations in rigorous-but-unobtrusive online math learning experimental settings, MATHia/UpGrade's blend of adaptive learning and experimental field trials, and Terracotta's utilization of Canvas for conducting flexible randomized experiments in diverse learning environments, all underscore the distinctive features and innovative approaches of each platform. Despite their distinct strategies, these platforms share a unified purpose: to leverage digital environments for probing critical research questions poised to influence the future of education. This synergy suggests that findings from these studies could potentially be generalized to other DLPs, offering insights that may enhance the educational experiences of teachers and students across various digital learning landscapes.

For researchers and educators, the challenge and opportunity are in identifying how best to use each platform for their specific research goals. How can one capitalize on E-TRIALS' hint and explanation systems? How might MATHia's adaptiveness be central to a study? Or in what ways can Terracotta's flexibility facilitate innovative experimental approaches?

Looking ahead, the field of DLPs is poised for continued innovation and growth. Emerging trends, such as the integration of artificial intelligence and advanced analytics, promise to further personalize learning experiences and provide deeper insights into educational strategies. Moreover, as these platforms evolve, they offer the potential to reshape how we teach and learn.

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