PROGRESSIONS IN GRADES K–1 STUDENTS' UNDERSTANDING OF PARITY ARGUMENTS

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Understanding how young learners come to construct viable mathematical arguments about general claims is a critical objective in early algebra research. The study reported here characterizes empirically developed progressions in Grades K–1 students' thinking about arguments concerning sums of evens and odds. Data are drawn from classroom lessons of an early algebra instructional sequence and interviews conducted at four timepoints during the implementation of the sequence. Overall, students transitioned from unfamiliarity with the concepts of even or odd prior to instruction in Kindergarten to making valid parity arguments at the conclusion of instruction in Grade 1. Results of this study align with other research that shows young learners can develop viable arguments to justify mathematical generalizations.

Keywords: Reasoning and proof; algebra and algebraic thinking; learning trajectories and progressions; early childhood education

Purpose of the Study

Research increasingly supports that engaging in proving in developmentally appropriate ways in the elementary grades can deepen students' conceptual understanding of mathematics as a sense-making activity (Carpenter et al., 2003; Stylianides & Ball, 2008; Van Ness & Maher, 2019). Moreover, when elementary students are supported through instruction, they can learn to use deductive—rather than empirical—reasoning to build mathematical arguments (Stylianides & Stylianides, 2008). However, intervention studies are needed to develop a "fine-grained conceptualization" (Stylianides, 2007b, p. 18) of appropriate forms of proving in early grades and to understand how curriculum can support students' construction of viable arguments.

The study reported here responds to this call by identifying learning progressions (Clements & Sarama, 2004) in how children construct mathematical arguments as they are taught an early algebra instructional sequence in Kindergarten (hereafter, Grade K) and Grade 1. The study focuses on the following question: *What levels of thinking do Grades K–1 students exhibit in their understanding of arguments about sums of evens and odds as they advance through an early algebra instructional sequence?*

Perspective

The study reported here is part of a suite of projects in which we use a learning progressions approach (Clements & Sarama, 2004) to build an effective early algebra intervention across

Grades K–5 and to identify progressions in students' thinking as they advance through the instructional sequence that forms the intervention. We organize early algebra around the practices of generalizing, representing, justifying, and reasoning with mathematical structure and relationships (Blanton et al., 2018). The focus on children's mathematical arguments here is a natural part of our early algebra research. In particular, in this study we are interested in justifying as a practice of building arguments for claims about general relationships, which can elevate the role of argumentation in the elementary grades.

Methods

Our research design involved the use of classroom teaching experiments (CTEs) with individual interviews (Cobb & Steffe, 1983) to identify progressions in children's thinking. Because CTEs incorporate instructional design with the ongoing analysis of classroom data, they serve as an important mechanism for developing empirically based conjectures about progressions in thinking as students advance through an instructional sequence (Clements et al., 2007; Lesh & Lehrer, 2000).

The K–1 early algebra instructional sequence in the study reported here consisted of eighteen 30-minute lessons for each grade. A subset of lessons within the instructional sequence (3 in Grade K, 2 in Grade 1) addressed the development of parity arguments. Lessons initially focused on preliminary concepts such as "pair," "even number," and "odd number," then transitioned to the development of representation-based arguments (Schifter, 2009) about the parity of the sum of two even numbers, two odd numbers, and then an even and an odd number. Additional lessons (8 in Grade K, 3 in Grade 1) reviewed parity concepts in lesson warm-ups.

Participants

Two Grade K classrooms (Year 1) and two Grade 1 classrooms (Year 2) in one school in the Northeastern US participated in the study. Effort was made to keep the initial Grade K cohort intact in Grade 1. Table 1 shows the number of participants by grade level. The school district's demographics consisted of 10% students of color and 16% students categorized as low SES.

Table 1. Participants by Grade Level

Grade	Total No. of Participants	Participated in Grade K only	Participated in Grade 1 only	Participated in Grades K–1
K	48	22		26
1	48		22	26

Data Collection

Grade-level early algebra lessons were taught approximately once per week during each school year by a member of the research team. All lessons were videotaped, and lessons or portions of lessons (e.g., lesson warm-ups) related to even and odd concepts were identified for analysis. A subset of students was selected at each grade for semi-structured, 30-minute, individual pre/post interviews. Classroom teachers helped identify students who fit a diverse academic range. An early math diagnostic assessment (EMDA™) was administered to students at the beginning of Grade K to further ensure academic diversity. In all, 17 students in Grades K–1 participated in video-taped interviews about parity concepts. Twenty-four full or partial classroom lessons and 40 interviews were analyzed.

Data Analysis

We used a grounded theory approach (Strauss & Corbin, 1990) that focused on the identification of progressions in students' thinking around the concepts of pair, parity of numbers, and arguments about parity of sums. Interview data were analyzed independently by three team members to identify preliminary codes (levels of thinking) for our core concepts. Theoretical memos (Glaser, 1998) were constructed to provide supporting evidence for the codes, or levels. Agreement among coders was determined by comparing coding decisions and negotiating any discrepancies around early codes/levels. New codes were identified as warranted and data were re-analyzed until subsequent coding did not change our emerging models. Video of classroom data were then analyzed for confirming or disconfirming evidence of emerging codes (levels). The levels-as-coding schemes were then organized based on our empirical findings vis-à-vis canonical understandings of the mathematics attempted (Battista, 2004).

Results

In Table 2, we report our findings on levels in students' thinking about the most complex interview task—parity arguments about the sum of arbitrary even and/or odd numbers. We then share some observations from our study.

Table 2. Parity Arguments for the Sum of Two Arbitrary Even/Odd Numbers

	Non-Structural Reasoning	
Level 1 *Empirical*	Students suggest a strategy that uses or implies the use of testing cases to determine parity for an arbitrary sum.	Student provides an argument based on empirical experience (counting): "If you add even and odd it's usually even because every time I count that, it's usually even. One time it wasn't."
	Structural Reasoning	
Level 2 *Generic Number*	Students use a pairs strategy applied to generic numbers to reason about parity of the sum. A pairs strategy involved finding if a number represented through cubes (for example) could be separated into pairs without a cube left over.	When asked if the sum of "a really big odd number" and a "really big even number" is even or odd, a student gave the following response: "Odd, because let's say you did it with [12 and 15]. Fifteen has a leftover and 12 doesn't have a leftover, so it can't combine." He indicates that by "can't combine" he means there would still be a leftover.
Level 3 *Arbitrary Number*	Without referencing any numbers, students use a pairs strategy to reason about parity of the sum.	When asked to explain why any even plus any odd is odd, a student argues, "Because an odd number has a leftover and an even number doesn't have a leftover and if you combine an odd number with an even number, it equals an odd number," confirming that there would still be a leftover.

There are important similarities between the levels of thinking students exhibited and taxonomies reported elsewhere. Empirical reasoning (Level 1), by which students examined randomly chosen cases to establish a conjecture's truth, reflects *naïve empiricism* (Balacheff, 1988) or an *empirical (inductive) proof scheme* (Harel & Sowder, 2007). In the example provided, the student reflects on prior experiences in "counting" (adding) particular even and odd numbers as the basis of her argument. Schifter (2009) similarly characterizes this type of argument as *inference from instances* in her analysis of Grade 3 students' parity arguments. We see Level 2 as consistent with the notion of *generic example* (Balacheff, 1988) or *deductive* *(transformational) proof scheme* (Harel & Sowder, 1998) because of the use of a specific number as a generic placeholder for a general number in students' arguments. Schifter (2009) refers to this form of argument as *reasoning from representation or story context*. Finally, Level 3 reflects the construct of *thought experiment* (Balacheff, 1988), where actions are dissociated from specific examples. With young children, these actions are based on "the language of the everyday (Balacheff, 1988, p. 228)" and not the transformation of formal symbolic expressions. Level 3 thinking seems to also reflect the emergence of *structure in thought* (Harel & Soto, 2017) in that students were able to present a sequence of arguments without the need to manipulate physical, visual, or symbolic representations. Our work extends that reported elsewhere in that our focus is on the genesis of these ideas at the start of formal schooling (Grades K–1). Identifying similarities in students' thinking across K–16 grade domains can help us better understand how to structure curriculum that connects and builds ideas over time.

We found students' ability to develop representation-based parity arguments (rather than empirical arguments) to be surprisingly robust. For example, at pre-interview Grade K students were unfamiliar with the concepts of pair and even and odd numbers, but by Grade K post-interview all students were able to define even and odd using a pairs strategy and many students routinely used a pairs strategy to reason about the parity of numbers represented in concrete, visual, and abstract forms. Moreover, although Grade K students were not asked to build representation-based parity arguments at pre-interview (given that they were unfamiliar with the concepts "even" and "odd"), by Grade 1 pre-interview no students used an empirical argument to justify why the sum of an even and an odd would be odd. Six out of 10 students were able to correctly use a structural argument involving a pairs strategy (three students could not build either type of argument; one student was not asked this question).

Although these students were capable of producing structural arguments (Levels 2–3), we do not claim that they yet appreciated the power of the generality of their arguments, nor did they yet understand the logical proof structure underlying their arguments. In truth, older students struggle with this as well (Stylianou et al., 2015). At this early, pre-symbolic point in their understanding, students' thinking was more intuitive than anticipatory or intentional. However, a goal of early algebra is to help students build on these intuitive ways of thinking so that their understanding can deepen over time. For young children to engage in Level 3 thinking, even in informal, non-symbolic ways, is a critical starting point.

Conclusion

The *Common Core State Standards* (NGA Center & CCSSO, 2010) maintains that elementary students should be able to "construct viable arguments and critique the reasoning of others." There is a vital need, however, for research-based, curricular pathways by which this goal can be met (Bieda et al., 2014), as well as studies that detail how young learners' argumentation progresses as they advance through such pathways (Stylianides, 2007a). The study reported here is intended to help address this by characterizing the emergence of students' understanding of viable parity arguments from an early algebra instructional sequence Understanding how young leaners develop robust ways of thinking and arguing mathematically can not only help avoid situations where students develop "a conception of proof in the elementary school that has to be undone or unlearned in high school (p. 4, Stylianides, 2007b)," but can also help realize the ambitious learning standards advocated in current reforms.

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