Adding family math to the equation: Promoting Head Start preschoolers' mathematics learning at home and school

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Abstract

Differences in children's mathematics knowledge are evident at kindergarten entry, favoring children who have greater access to economic resources. Fostering preschoolers' mathematics learning at home and in classroom settings, through games and other developmentally appropriate activities, is of great interest to educators, early childhood leaders, and policymakers. This cluster randomized trial examined the effects of a naturalistic, game-based mathematics intervention implemented in Head Start classrooms and examined whether including a family math component added value. A total of 573 children (64% Hispanic; 60% multilingual) were included from 66 classrooms which were randomly assigned to Classroom Math (CM), Classroom Math + Family Math (CM+FM), or business-as-usual (BAU). Results indicated that the family math component did add value to the classroom-based intervention as CM+FM resulted in a significant positive impact on children's mathematics knowledge relative to BAU, but CM did not. For preschoolers age 50+ months, both interventions had significant effects on children's mathematics knowledge relative to BAU, but CM+FM had a stronger effect (d = .36). The number of math games played was significantly associated with higher mathematics scores and the number of family math mini-books returned had a significant impact on children's spring scores, over and above the number of games played. The CM+FM intervention also had a significant effect on teachers' instructional practice (d =.79). Adding a family math component to a game-based classroom intervention resulted in positive impacts for preschoolers and seems to be an effective, ecologically valid intervention that fosters early mathematical competencies.

Keywords: Mathematics; Head Start; Early Childhood Education; Family Engagement; School Readiness; Families; Instructional Support

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Early mathematics knowledge strongly affects and predicts future academic outcomes and success through high school (Clements et al., 2020; Dumas et al., 2019; Duncan et al., 2007; Duncan & Magnuson, 2011; McCoy et al., 2017; Watts et al., 2014) and is considered a core component of young children's foundational cognitive skills (Clements et al., 2020). High-quality early mathematics experiences at home and in preschool also promote children's social–emotional and cognitive development (Clements et al., 2020; Dumas et al., 2019; Sarama et al., 2012), setting them on a path for success. Unfortunately, systemic opportunity gaps can create unequal access to high-quality mathematics learning experiences and result in disparities in educational opportunities and achievement. There is evidence that young children with limited access to economic resources may start kindergarten up to a full year behind in their mathematics skills (DeFlorio & Beliakoff, 2015; Garcia & Weiss, 2017) with these gaps in mathematics skills persisting throughout the course of schooling (Cross et al., 2009; Garcia & Weiss, 2017). However, early intervention with preschoolers could help to narrow this gap and have important longer-term implications as a growing body of evidence demonstrates that investing in early childhood programs and supporting families as education partners can narrow education gaps, particularly in mathematics (Bivens et al., 2016; Daucourt et al., 2021; Garcia & Weiss, 2017; Susperreguy et al., 2020).

If schools were to include mathematics in their family engagement programming, this could be an avenue for reducing long-term educational disparities (Harris et al., 2017), as researchers have suggested that connecting children's mathematics learning experiences across home and school environments may be an effective way to promote school success, particularly for children from families who have been historically underserved (Daucourt et al., 2021; Lange et al., 2020; Sonnenschein et al., 2021). However, to date, the evidence for the efficacy of family engagement interventions in mathematics or “family math” is limited (Eason et al., 2020), and there is a need for more rigorous family engagement studies to examine effects on children’s outcomes (Hoffman et al., 2020; Van Voorhis et al., 2013). Furthermore, understanding how to bolster children’s mathematics learning at home and in classroom settings, through games and other developmentally appropriate activities, is of great interest to educators, early childhood leaders, and policymakers (Eason et al., 2020). The present study aims to add to the research evidence by measuring the effectiveness of a naturalistic cross-context intervention designed to foster Head Start preschoolers’ mathematics knowledge by leveraging the role of families in children’s mathematics learning and the interconnectedness of children’s learning environments (Bronfenbrenner, 1979, 1986).

1.1. Children’s Mathematics Learning

Differences in children’s mathematics knowledge are evident before the start of formal schooling, favoring children who have greater access to economic resources (Harris & Petersen, 2019). This results in educational learning gaps that are persistent and pernicious, as children who start kindergarten behind in mathematics may struggle to catch up to their peers (Garcia & Weiss, 2017; Schoenfeld & Stipek., 2011). However, supporting children’s mathematics learning in early childhood presents an opportunity to promote educational equity, as mathematics skills in early childhood predict later academic achievement in mathematics, reading, and science (Claessens & Engel, 2013; Duncan et al., 2007, Purpura et al., 2017, Watts et al., 2014).

Preschoolers learn more mathematics when they experience more mathematics learning activities in the classroom (Ginsburg et al., 2008; McCray & Chen, 2012) and more mathematics-related interactions at home (Daucourt et al., 2021; Huntsinger et al., 2016; Mutaf-Yildiz et al., 2020). However, there is significant variability in children's mathematics learning environments both in preschool classrooms (Farran et al., 2007; Fuhs et al., 2013) and at home (Daucourt et al., 2021; Susperreguy et al., 2020). Importantly, in a recent study, parents,[1] including highly educated parents, reported a lack of confidence in supporting children's mathematics and would like more information from their children's teachers about what they should be doing to support learning, particularly mathematics learning (Sonnenschein et al., 2021). Based on this research, Sonnenschein and colleagues (2021) recommend that preschool teachers and parents collaborate on home-based activities to support young children's mathematics learning. Developing effective preschool mathematics interventions that bridge home and school learning environments may provide a key for all children to have a strong start in mathematics before elementary school.

Many preschool mathematics interventions, however, focus on implementing high-quality full-year mathematics curricula in classrooms, such as *PreK Mathematics* (Klein et al., 2004), a curriculum that was recently re-evaluated for impact (Starkey et al., 2022) and found to be effective at promoting mathematics learning for children from families with less access to economic resources. The implementation of the curriculum intervention was intensive, including 45 hours of professional learning, access to additional apps and software, and onsite coaching for each classroom by project trainers one or two times per month. The intervention also included a family component with home materials and activities, although the unique contribution of this aspect of the program was not identified. Other high-quality, research-based preschool mathematics curricula, such as *Building Blocks* (Clements & Sarama, 2007) and *Big Math for*

[1] We use the term "parent" to refer to anyone in a caregiving role, which may include guardians, grandparents, and foster parents.

Little Kids (Ginsburg et al., 2003), have also been shown to improve children's mathematics learning (Clements & Sarama, 2007/2013; Lewis Presser et al., 2015), and are available for purchase. However, while proven effective, many preschool programs are resistant to implementing full-year mathematics curricula because of competing priorities for classroom time, cost of the materials, and limited budgets to provide professional development. In addition, many early childhood educators believe that published mathematics curricula are not appropriate for young children because they are too prescribed (Chen et al., 2013), with early educators endorsing play-based approaches with games over curriculum-based approaches (Vogt et al., 2018). Two of the most widely used public preschool curricula are *Creative Curriculum*® for Preschool (Teaching Strategies, LLC, n.d.) and *HighScope* (Epstein & Hohmann, 2012). Both cover a wide range of child development domains and have been evaluated for impact but have not been found effective for promoting mathematics learning (Howard & Weinberg, 2021; Preschool Curriculum Evaluation Research Consortium, 2008).

Historically, early childhood teachers have not been trained to teach subject-specific content (Isenberg, 2000) and are not prepared to provide rich mathematics experiences that support school readiness (Brenneman, 2014). As a result, most early childhood teachers are not only underprepared to teach mathematics but are afraid of it (Ginsburg et al., 2008; Parks & Wager, 2015). This presents a significant challenge in which many teachers do not have high-quality mathematics resources to use in their classrooms and have limited training in how to plan and implement mathematics lessons on their own. Preschool classroom instruction could be augmented by mathematics teaching strategies and materials that are not linked to a specific curriculum, do not spark teachers' fear of math, but do provide developmentally appropriate and mathematically challenging experiences that could be used to promote family engagement and alignment across school and home settings (Eason et al., 2020; Lange et al., 2020). The intervention described in this study is an attempt to fill this gap, so teachers have low-cost instructional materials that they consider developmentally appropriate with implementation supports and a complementary family-engagement component designed to align home and school learning environments.

1.2. Engaging Children in Mathematics Games at Home and School

Children enter the world curious and intrinsically motivated to learn from mathematics-rich interactions (Ginsburg et al., 2006; Ramani & Siegler, 2015), naturally engaging in mathematics during unstructured play (Geist, 2009; Ginsburg, 2006; Seo & Ginsburg, 2004) and playful learning activities such as games (Hassinger-Das et al., 2017; Lange et al., 2020; Skene et al., 2022). Children's home mathematics environments have been associated with children's achievement, with researchers suggesting that to promote children's mathematics skills, it may be beneficial to support parents in providing positive home mathematics experiences for their children (Daucourt et al., 2021). A growing body of evidence indicates that when parents interact around mathematics and provide their children with more mathematics-related activities and talk, children have higher mathematics outcomes regardless of their families' levels of income or education (DeFlorio & Beliakoff, 2015; Galindo & Sonnenschein, 2015; Levine et al., 2010, McCormick et al., 2020) although the findings depend on the types of home activities, how they are defined and measured (Eason et al., 2020), and their mathematics content (Daucourt et al., 2021). This is also true for research investigating the relation of family game play to young children's mathematics learning, with differences emerging depending on the types of games played and the learning outcomes assessed (Ramani & Scalise, 2020; Scalise et al., 2022; Sonnenschein et al., 2016). Yet, mathematics games provide a context for playful learning that is not only fun, but also challenging, balancing difficulty and skill level, thus fostering motivation and engagement among young children (Hassinger-Das et al., 2017). Indeed, when parents and children interact around simple games like mazes and connect-the-dots activities at home, this helps to promote children's mathematics learning (Daucourt et al., 2021; Ramani & Siegler, 2015; Skwarchuk, 2009). Traditional games, such as board games, card games, dominoes, and dice games, also encourage adult–child co-play with mathematical ideas (Hirsh-Pasek, 2014) and fosters more math talk in families (Scalise et al., 2022) while providing an authentic and approachable context to support young children's mathematics learning. Dice games, for example, systematically repeat simple counting and adding procedures (Kreilinger et al., 2021), and card games provide information about number symbols and number words (Niklas et al., 2016), magnitude comparison (Scalise et al., 2018; 2020), and geometry (Scalise et al., 2022). Importantly, children who play mathematical-thinking games more frequently in preschool show higher mathematics achievement later in school (Niklas & Schneider, 2014). Additionally, game play can foster important school readiness skills, such as self-regulation, through the act of following rules and taking turns while also offering opportunities for children to practice their skills in communication, empathy, and conflict resolution (Hassinger-Das et al., 2017), skills valued by teachers and parents alike.

Games are also an inexpensive tool that could be the basis of a cross-context intervention that aligns home and school learning while providing families with something that they say they want—concrete examples of the mathematics preschoolers can learn through daily activities (Cannon & Ginsburg, 2008; Lange et al., 2020) and specific activities they can do at home that complement children's school-based learning (Van Voorhis et al., 2013). Both families and teachers can support children's mathematics development through simple activities: by engaging in math talk, involving children in everyday mathematics activities, and playing games that involve math (Lange et al., 2020; Levine et al., 2010; Leyva, et al., 2017; Ramani & Siegler, 2015). Engaging children in game-based learning also provides opportunities for teachers to observe children's choices and strategies and then provide feedback about specific mathematics concepts while implementing developmentally appropriate instructional strategies. By aligning mathematics learning opportunities across home and school contexts, children have multiple chances to practice their growing mathematics skills. Furthermore, while many educators raise concerns about the over-academization of early childhood, when implemented well, games are both fun and developmentally appropriate and can be easily integrated into classroom routines. For parents, particularly those who are intimidated by the prospect of "doing math" or simply aren't familiar with early mathematics development (Sonnenschein et al., 2021), the game context is more approachable, as many families welcome incorporating games as part of their family routines and see games as a natural way to play and interact together. There is also some evidence to suggest that a game context may naturally prompt families to engage in math talk. For example, when Sonnenschein and colleagues (2016) compared two board games (one with numbers and one with colors), parents were told not to count when playing the "color" game, but there was evidence to suggest that they may have anyway. In a study by Ramani and Scalise (2020) they found that parents used number words and engaged in math talk during play with card games, whether the cards had numerals or not. Importantly, studies have linked family math talk to children's mathematics knowledge (Gunderson & Levine, 2011; Levine et al., 2010) so providing a context where families naturally engage in more math talk may be an easy way to support young children's learning.

Building on Bronfenbrenner's theory (1979, 1986), a potentially robust strategy to promote children's mathematical learning may be to link children's learning environments of school and home through mathematics games. Family support of children's school learning has positive academic benefits, including increased attendance and higher grades, and families often support their children's school learning at home in ways not always recognized by teachers and schools (Henderson & Mapp, 2002; Zarate, 2007). In addition, researchers have argued that the missing link in educational equity is family engagement and that the most successful programs promote strong school–family partnerships and parent–child communication (Larocque et al., 2011; Sheldon & Epstein, 2005) to help parents extend their children's learning (Nitecki, 2015). In fact, several studies found that adding a family engagement component to an existing intervention enhanced child outcomes beyond what the interventions achieved on their own, such as in a literacy intervention (Anthony et al., 2014) and in an intervention targeting childhood obesity (Quattrin et al., 2014). Whereas these studies suggest that adding a family engagement component to a mathematics intervention may be a promising approach, there is a need for more research investigating the effectiveness of family math interventions (Eason et al., 2020).

In a study by Lange et al. (2020), the researchers used number games based on a linear board game developed by Ramani and Siegler (2008, 2011; Siegler & Ramani, 2008, 2009; Ramani et al., 2012) to support mathematical learning in preschool classrooms and in home environments but under more naturalistic conditions than originally developed and tested. The intervention resulted in positive impacts for numeral identification but not for the other mathematical domains assessed. The number of math games played was positively associated with verbal counting, but no impact of family game play was found. However, implementation challenges could have influenced this finding (Lange et al., 2020). This study provides evidence of promise for cross-context mathematics interventions, although the researchers suggest that to increase sustainability of a mathematics game intervention for classrooms and homes, additional inexpensive games that target different mathematical skills could be used to keep children interested and to expand the skills they are practicing. There are many challenges that can impact family math interventions and efficacy studies, such as high attrition (e.g., Scalise et al., 2022; Sonnenschein, et al., 2016), high variability in children's home math experiences (Daucourt, et al., 2021; Ramani & Scalise, 2020; Susperreguy et al., 2020), and variability in family's mathematics guidance during game play, even when given uniform directions (Ramani & Scalise, 2020; Scalise et al., 2022; Sonnenschein et al., 2016). Therefore, more studies of family math interventions are needed, but they need to be carefully constructed.

1.4. Mathematics Learning and Instructional Support

To enhance opportunities for all children, regardless of background, teachers and parents need to provide children with learning opportunities that meet their diverse needs (Vogt et al., 2018). Research shows that the most important aspects of instructional quality in preschool education are stimulating and supportive interactions between teachers and children, and effective use of curricula (Yoshikawa et al., 2013). However, children's mathematics experiences in school are highly variable (McCray & Chen, 2012), with educators spending more time teaching mathematics concepts that may not be sufficiently challenging (Engel et al., 2016). Interventions that aim to impact preschoolers' mathematics learning in the classroom, therefore, should include a focus on increasing the quality of the instruction children receive. Game-based mathematics activities may support this aim by promoting extensive adult–child interactions that offer multiple opportunities for teachers to support children's learning through guidance and feedback (Hassinger-Das et al., 2017). Interventions that strengthen teachers' abilities to provide high-quality instructional support could help maximize the benefits of games and other interactive activities designed to promote mathematics learning.

Given the need for low-cost preschool mathematics interventions and the dearth of evidence on the value of family math and cross context interventions (e.g., Lange et al., 2020, Scalise et al., 2022), this study specifically teases apart the unique contribution of adding a family-engagement component to an early childhood mathematics classroom intervention. Partnering with Head Start programs whose mission is to provide high-quality early education and comprehensive services to families living at or below the poverty level, this study examined the effectiveness of a relatively low-cost, scalable game-based intervention designed to be implemented at home and in school settings.

1.5. Study Purpose

The primary aim of the present study was to answer the following research questions: *1. What is the impact of a naturalistic, game-based classroom mathematics intervention, with or without a family math component, on preschoolers' mathematics learning? 2. What is the added value of including a family math component in a naturalistic classroom-based mathematics intervention?* Our first hypothesis was that both interventions, with and without a family math component, would support greater mathematics achievement compared to a comparison condition. Second, because of the expected benefit of engaging in mathematics at home, we hypothesized that the classroom condition with an added family math component would support greater mathematics achievement compared to the classroom-only condition.

A secondary aim of the study was to explore the conditions that might influence learning outcomes, including learner characteristics such as child age and dosage of the intervention (indicated by the number of games played in the classroom and the number of family math activities). Additionally, with a subsample of teachers, we explored the effect of the intervention on instructional quality and the classroom learning environment through teacher–child interactions. We hypothesized that both experimental conditions would have a positive effect on instructional quality relative to a comparison condition.

2. Method

2.1. Research Design

The current study was a cluster-randomized trial (CRT) comprising three conditions: a *business-as-usual* (BAU) group, a *classroom math* (CM) group, and a *classroom math + family math* (CM+FM) group with a pre-test, intervention, post-test design. To address the research questions, the first experimental condition, *classroom math* (CM), included a set of classroom mathematics games and instructional materials, professional development (PD) support, and resources for teachers. The second experimental condition, *classroom math + family math* (CM+FM), comprised the same set of classroom mathematics games and instructional materials, PD support, and resources for teachers but also included a family math component. The family math component included PD support in family engagement, a set of mathematics games and activities for the home that complemented the classroom games, and additional family-engagement resources.

2.2. Participants

Classroom teachers (n = 66) were recruited from three Head Start programs from two states in the U.S. Northeast. Prior to randomization of the classroom clusters, all children in the research sample were identified. Thus, no children joined the sample after randomization. Teachers gave their consent at the beginning of the year to ensure compliance with randomization procedures. Classrooms were sorted into one of six blocks based on each participating teacher's years of experience (0–7, 8–15, or 16 or more) and fall mathematics score (above or below the mean). Within each block, classrooms were sorted using a random-number generator and then assigned to one of three conditions, resulting in an initial sample of 22 classrooms in the BAU condition, 22 in the CM condition, and 22 in the CM+FM condition.

All treatment teachers (n = 44) received continuing education units or PD hours (depending on state licensure) and received classroom and family math materials. Comparison teachers received classroom and family math games at the end of the study. Teachers and their programs received no other incentives. Sixty-five teachers self-identified as female, and one self-identified as male. Sixty-three of the teachers reported their race/ethnicity, education, and years of experience. Among these teachers, 4.8% were Asian, 38.7% were Hispanic; 56.5% were white. Forty-six percent had an associate degree; 46.0% had a bachelor's degree, and 8.0% had a master's degree. Twenty percent of teachers had been teaching in early childhood for 0–7 years, 33.8% had been teaching for 8–15 years, and 46.2% had been teaching for 16 years or more. Teachers in all three programs used *The Creative Curriculum*® for Preschool (Teaching Strategies, LLC, n.d.). The three Head Start programs involved in the study did not use a mathematics-specific full-year preschool curriculum.

Parents filled out a short family survey that included information on the child and caregivers' background including child age, home language(s) spoken, family members in the home, caregivers' education, attitudes toward math, and types and frequency of home learning activities. Children met income eligibility for Head Start, which requires that at least 90% of children come from families who either have an income below 130% of the poverty line or are homeless (Improving Head Start for School Readiness Act of 2007). Children were considered eligible for the study if (1) the child was at least 3 years and 5 months old by September 1 of that school year, (2) a parent or guardian provided consent, (3) the child was proficient in English or in Spanish, and (4) the child did not have a disability that precluded valid one-on-one assessment (e.g., nonverbal or severe behavioral issues). We constrained eligibility for the study based on age, according to the lower limits of available validity evidence for the mathematics outcome measure. Language proficiency was based on an English-language screener (Duncan & De Avila, 2000) administered if the child did not speak English in the home or if indicated by the teacher. Children who scored at least 14 out of 20 were considered proficient in English and received the English-language assessment (Vogel et al., 2008). Children who spoke Spanish in the home and scored 13 or below were assessed with the Spanish-language assessment. In the fall, 573 children (50.2% girls, M = 51.2 months, SD = 5.32) were assessed in mathematics, 87 (15%) of whom were assessed in Spanish. Parents indicated that among the child participants, 64.0% were Hispanic or Latino, any race; 26.8% were white, non-Hispanic; 4.8% were non-Hispanic and non-white; and 4.5% reported more than one race; 60.5% of children spoke a language other than English in the home. Mother's highest level of education varied: 19% indicated they had less than a high school education; 38% indicated they had a high school diploma or GED; 36% indicated they had some college or an associate degree; and 7% indicated having a bachelor's degree or higher. Over the course of the study, no classrooms dropped out, but there was attrition of children from classrooms. Seventy-four (13%) children who were assessed in the fall were not assessed in the spring, either because they left the classroom or because of absences. Four hundred ninety-nine of the children assessed in the fall (87%) were assessed again, 79 children (16%) were assessed in Spanish.

2.3. Measures

2.3.1. Mathematics Knowledge

Children's knowledge of number and geometry was assessed using an abbreviated 19-item version of the full-length *Research-Based Early Maths Assessment* (Clements et al., 2008) called the *REMA Brief* (Weiland et al., 2012) and includes both English and Spanish versions of all items. The *REMA Brief* is a standardized measure that uses pictures and manipulatives to assess children's mathematics knowledge and was developed and validated using a Rasch model designed to represent the full range of pre-kindergarten and kindergarten mathematics competencies (verbal and object counting, comparing number and sequencing, recognition of quantity and subitizing, recognition of numerals, number composition, arithmetic, shape recognition and composition, and patterning). Validity evidence is available using samples of children in pre-kindergarten and kindergarten. While exact age data are not provided for these validation samples, the authors report that some 3-year-olds were included (Clements et al., 2008). *REMA Brief* was validated and refined based on multiple economically and racially diverse samples. Standardized scores are based on an average score of 50 and a standard deviation of 7, with scores ranging from 5 to 98. For the current study, Cronbach's alpha was .72.

2.3.2. Instructional Interactions

We hypothesized that for teachers in the intervention conditions, we would see an increase in the quality of their instructional interactions because of the training they received on supporting children's persistence at challenging tasks and deepening their own professional learning of mathematics teaching. As this was a secondary hypothesis, it was measured using the *Classroom Assessment Scoring System, Pre-K* (CLASS Pre-K; Pianta et al., 2008) in a subsample of classrooms (44 of 66) from two of the three participating sites. This manualized observation protocol is conducted by observing classrooms for 20-minute intervals and rating the quality of 10 dimensions of teacher–child interactions on a 7-point scale. Dimension scores were averaged across cycles, and these were averaged to create domain scores. The *Instructional Support* domain is an average of three dimensions (Concept Development, Quality of Feedback, and Language Modeling) and captures the degree to which teachers promote children's higher-order thinking skills, use feedback to deepen learning and encourage persistence, and support language development.

2.4. Procedure

Fall data collection took place shortly after the school year began but before the intervention was introduced. The data collectors had experience working with young children and were from local communities but were not affiliated with project development. Thirty-two data collectors completed approximately 6 hours of training over 2 days on administering the mathematics assessment. After the training and practice period, research staff assessed data collectors during a live mock assessment to ensure that each data collector's test administration and scoring skills were reliable and adhered to the standardized administration manuals. Data collectors were blind to intervention condition and completed a refresher training prior to the spring data collection. Child participants met one on one with a trained data collector for 20–30 minutes in a quiet area of the classroom. The fall assessments were conducted between October and mid-November, and spring assessments were conducted between April and May. Fall and spring assessments occurred on average 6 months apart (M = 6.3, SD = 0.38), ranging from 4.8 to 7.3 months apart.

Classroom observers were hired and trained to reliability by a project staff person who was a certified CLASS-PreK trainer. Four observers completed a 2-day training on the CLASS Pre-K and passed an online certification test. Observers were blind to study condition and recruited and trained separately from the child-data collectors. Observations occurred during a typical morning in each classroom (four or five 20-minute cycles, approximately 2–3 hours total) in the fall and spring. Fall observations took place before PD (October–November), and spring observations took place after teachers had completed at least five of seven PD sessions (during March and April). Observers double coded 20% of classroom observations and scored within 1 point of each other on 86% of codes in the fall and 82% of codes in the spring.

To minimize contamination across conditions, teachers participating in each intervention were instructed not to share materials or talk about the mathematics activities outside of the PD. Teachers in all three programs did not plan curricula together; this typically occurred within classrooms and only with the co-teacher or teaching assistant. In addition, the education supervisors were advised about this constraint and the importance of minimizing contamination across the groups, and they ensured that no program-wide training or staff meetings addressed mathematics topics that could interfere with the group distinctions prior to spring data collection.

2.4.1. Pilot Study

The initial development of the CM intervention was supported by an exploratory grant investigating mastery motivation or persistence at problem-solving as a key variable relating to children's mathematics learning. As part of the study, we designed and developed mathematics games and a teacher PD course using the data and feedback from the pilot study. With guidance from Head Start teachers and mathematics education experts, we made further revisions. As part of the development of the CM+FM family math component, we conducted a landscape scan of existing family-engagement interventions and conducted interviews to investigate the key elements that early childhood programs believed should be included in a family-engagement intervention focused on supporting mathematics. The pilot study and landscape scan informed the design, methods, and procedures for the larger-scale experimental study, such as using blocked randomization to establish baseline equivalence between conditions, carefully ordering the presentation of games in PD and to children, assessing children in English and Spanish, texting families "math tips," and creating mini-books to support engagement in family math.

2.4.2. Classroom Math (CM) Intervention

All treatment teachers participated in the CM PD. This consisted of seven 90-minute in-person PD sessions (10.5 hours total) aimed at strengthening teachers' understanding of early mathematics concepts and positive attitudes toward mathematics through the implementation of games collectively focused on number, operations, geometry, and patterns. Teachers learned about early mathematics concepts, supporting children's mathematical thinking and persistence while problem-solving. Teachers learned how to play, scaffold, and modify each game to meet the right level of challenge for different children; they watched videos of children playing games and then played the games themselves. This method is based on empirical evidence that learning mathematics through practice is more effective than learning content alone (Zaslow, 2014).

The instructional materials included seven mathematics games, six mathematics picture books, and a teacher guide. The games were designed to be educative and included supports to foster adults' understanding of early mathematics concepts and to encourage interactions around mathematics and engaged children in solving problems, puzzling, and discussing solution strategies. Teachers were not given uniform language to use; rather, they were encouraged to pay attention to children' thinking, ask open-ended questions, and prompt children to describe their thinking. Teachers were encouraged to play the games with small groups of children; aiming to play the games at least six times with each child. Additionally, teachers often introduced a game to the whole class and then included the game at the math center. Teachers recorded each time a child played a game on the sticker chart—we did not distinguish between playing independently, with another child, or with a teacher. This helped teachers keep track of children's participation to meet the target number of games played and provided an index of intervention fidelity. Teachers were supported in using their understanding of early mathematics development to adapt the games and extend or scaffold children's learning. The games were designed to have multiple entry points depending on children's mathematical development; while accessible to 3-year-old children, the cognitive demand extends to early elementary-level mathematics. Five games targeted early mathematics skills in number and operations: dot card and finger play games, games with cards and dice (like "Shut the Box"), hiding games with counters, and the number path board game *Jumping on the Lily Pads* (see Figure 1). This board game is played with a 1–5, 1–10, or 1–20 number path and a homemade 1–3 die and it promotes counting, cardinality, one-to-one correspondence, comparing numbers, and number composition. Children take turns rolling a die and moving a "frog" a specific number of spaces on the number path trying to reach the pond first. This game is similar to the linear number board game developed by researchers (Ramani & Siegler, 2008, 2011; Siegler & Ramani, 2008, 2009) with teachers encouraging children to use the strategy of "counting on," however, the number path was presented vertically rather than horizontally and uses dice rather than a spinner.

Figure 1

Jumping on the Lily Pads game as observed in a classroom

The games in the geometry strand included pattern block puzzles and games that focus on children's knowledge of shapes, such as describing and comparing the attributes of shapes, composing and decomposing shapes, and spatial relationships. They also offer vocabulary practice with shape names, age-appropriate geometric language (e.g., *corner*, *sides*, *length*, *same*, *longer*, *shorter*), and practice describing spatial relationships in context (e.g., *in front of*, *behind*, *over*, *under*, *next to*). The patterning games support children to playfully create, copy, extend, fix, and transfer patterns. With the shape card games, children pay attention to mathematical attributes such as shape and number as well as practice executive function skills by taking turns and remembering and matching cards.

2.4.3. Classroom Math + Family Math Intervention (CM+FM)

Teachers in the CM+FM intervention condition attended the CM PD and used all the same games and materials as the CM condition; however, the CM+FM condition included an additional 3.5 hours of PD focused on family-engagement strategies (total PD time = 14 hours). Teachers received an additional five picture books for the classroom and family math materials to send home with children, including four family math games, a set of 15 bilingual Spanish–English mini-books (13 focused on mathematics and two focused on promoting persistence while problem-solving (Reed & Young, 2017; Young & Reed, 2017). To support children's mathematics learning at home and to enrich teacher–family interactions around mathematics through common experiences, the family materials closely paralleled the classroom mathematics games. The family games were designed to (1) allow families a window into the rich mathematics happening in the classroom, (2) invite open-ended questions and rich math talk, (3) promote parent–child co-play with mathematical ideas, and (4) be fun and adaptable enough to play repeatedly. Teachers sent the games and accompanying directions home with children and were encouraged to talk to parents about the games and other mathematics learning strategies. To support implementation in a family-friendly and developmentally appropriate way, families received bilingual "mini-books" to read together that included Spanish and English on the same page and are referred to as "mini-books" because they are physically small and easy for young children to hold. The stories included games and activities like the classroom games and included a short paragraph for parents about the key mathematical concept and ways to extend children's learning. The books were printed in black and white so they could be easily printed and children could color them. Teachers sent home a new mini-book each week and were encouraged to introduce them during classroom activities. They asked children to read them with their families and then bring them back to school to receive a stamp. Families were invited to opt in to weekly "math tips" via text message and received reminders to return the mini-books. As an index of the family math intervention fidelity, teachers used a chart to keep track of when children brought back their mini-books with evidence of having engaged with it (coloring or trying the activity). This worked well; however, teachers indicated that children sometimes talked about reading a book with their family but did not return it for a stamp.

2.5. Data Analysis

We first assessed attrition and found an overall attrition rate of 13% and a maximum differential attrition of 5%. A differential attrition analysis found that the 74 children who were not assessed at follow-up were roughly equally distributed across the three groups, with 22 in CM (12%); 31 in CM+FM (16%); and 21 in BAU classrooms (11%), $\chi^2(2) = 2.78$, $p = .250$. Children who were not assessed in the spring did not differ from children who were assessed in the fall and spring by age, $t(614) = 1.19$, $p = .236$, or by fall mathematics scores $t(571) = 0.29$, $p = .771$). To retain the 74 children with missing spring data in the analytic sample and reduce the risk of bias in our estimation of intervention effects, we conducted multiple imputation using SPSS Version 25 following What Works Clearinghouse (2020) guidelines for any missing outcome data. We generated 10 sets of imputed spring mathematics scores. We used a rich set of variables to estimate the imputations, including all covariates used in the models described below and other background characteristics from the family survey.[2] We imputed the scores separately for each of the three intervention conditions (WWC, 2020).

To account for the nesting of children in classrooms, we estimated a series of two-level hierarchical linear regression models using HLM Version 8 (Raudenbush et al., 2019) with restricted maximum likelihood, with children at Level 1 and classrooms at Level 2 (see Appendix B for Eq. B.1). To address the primary research questions, we used the HLM multiple imputation function, which estimates parameters for each imputed dataset and averages these values (Raudenbush et al., 2019). For exploratory analyses, we used complete-case, non-imputed datasets. During the model-building stage of the analysis, we incrementally added student-level

[2] The variables included fall math score, language, home language other than English, DLL status, race, ethnicity, age, gender, maternal education, paternal education, presence of mother/father in home, presence of other children in the home, program, parent in texting program, fall and spring parent survey scales: parent's role in school readiness, math anxiety, confidence helping child with math, home math activities, and math as a goal for learning

predictors to the models and evaluated the significance and magnitude of the regression coefficients along with whether the relation between the outcome and those predictors varied randomly across classrooms. We evaluated the reliability and significance of the random slope variance. Random components that were significant were retained; those that were not, were fixed for parsimony. We then added classroom-level covariates and intervention-condition dummy codes to assess the impact of the interventions (see Appendix B for Eq. B.2). Note that the number of programs ($N = 3$) was too small to model as a third level, so we acknowledge that the between classroom variability is confounded with the between program variability (Maas & Hox, 2005). To examine differences in outcomes across the three participating Head Start programs in the study, programs were also modeled at Level 2 as two dummy codes. To confirm that the blocking variables we used pre-random assignment led to equivalent groups, we included them in the model; when they were not significant, we removed them for parsimony. To answer our exploratory research questions, we investigated whether the treatment effect was moderated by learner characteristics by adding cross-level interaction terms (see Appendix B for Eq. B.3). For an effect size, we calculated the ratio of the beta coefficient for the treatment group indicator (numerator), controlling for the other variables in the model, divided by the pooled standard deviation of the student-level spring math scores. To examine the effect of the intervention on the quality of teachers' interactions, we conducted an ordinary least-squares (OLS) regression with the interventions modeled as two dummy codes predicting spring *Instructional Support* controlling for fall *Instructional Support,* and BAU as the reference group.

3. Results

3.1. Descriptive Statistics

Descriptive statistics for the analytic child sample and for each condition, CM+FM, CM, and BAU are in Table 1. Using ANOVA, we established baseline equivalence among the three groups; there were no significant differences across the conditions in fall math scores or age. A paired *t*-test using the full analytic sample indicated that children had significantly higher math scores in the spring than in the fall, $t(498) = -23.1$, $p < .001$.

Table 1

Descriptive Data as a Function of Condition

	Full sample	CM+FM	Classroom Math	BAU		
	N = 573	*n* = 198	*n* = 186	*n* = 189		
Female	51%	49%	52%	52%		
Spanish assessment	15%	18%	9%	19%		
	M (SD)	*M (SD)*	*M (SD)*	*M (SD)*	*F (df)*	*p*
Age (in months)	51.2 (5.32)	50.8 (5.43)	51.2 (5.26)	51.6 (5.26)	1.31 (570)	.270
Fall math score*	40.4 (7.54)	40.5 (7.81)	40.8 (7.08)	40.0 (7.71)	0.61 (570)	.546
Spring math score*	48.5 (7.41)	48.5 (7.83)	47.3 (7.55)	46.4 (6.72)		
Total games played	*	29.9 (15.2)	30.6 (13.0)	*		
Total mini-books returned	*	8.51 (4.78)	*	*		

**Note.* Standardized scores are based on an average score of 50.

3.1.1. Classroom Math Game Play

Across both the CM and CM+FM conditions, children varied in the number of times they played each game, from zero times to 16 times. Children played each game, on average, 4.7 times (SD = 1.88) across the seven games (approaching the target number of six play sessions for each child). The average numbers of games played by children in the CM and CM+FM conditions were not significantly different, $F(371) = 0.259$, $p = .611$. Teachers reported some implementation challenges such as difficulty playing at least six times with children who were frequently absent.

3.1.2. Family Math Play

As an index of the CM+FM intervention fidelity, we asked teachers to keep track of the number of mini-books children returned with evidence of having engaged in the activity. Return of the mini-books varied, but on average children returned 8.4 books (SD = 4.8; range 0—15) or slightly more than half of the books, 10% of children did not return any mini-books, and 12% returned all 15 books.

3.2. Research Question 1: What is the impact of a naturalistic, game-based classroom mathematics intervention, with and without a family math component, on preschoolers' mathematics learning?

Based on an unconditional two-level model predicting children's spring scores (Model 0), 6.2% of the variance in spring math scores was associated with classroom-level differences (see Table 2). We tested the associations between spring scores and child-level covariates: fall scores, age, gender, and language of assessment. Fall scores and age were centered on their grand means; gender and language were entered into the models uncentered. We examined the significance of the variability in the slopes for each covariate, but none varied significantly across classrooms, so each was fixed in subsequent models. Fall scores and age were significantly associated with spring math scores (see Table 2, Model 1). Gender was not significantly associated with spring math scores, $B(SE) = -0.74(0.53)$, $t(317) = -1.40$, $p = .162$, nor was language of assessment, $B(SE) = 0.03(0.89)$, $t(503) = 0.04$, $p = .971$. Although these two covariates did not explain additional variance in spring scores above and beyond other covariates, we retained them in subsequent models because they were variables of interest. We tested associations between intervention conditions and spring scores, controlling for program sites using two dummy variables (Table 2, Model 2). Program sites were not significantly associated with spring scores, Program 1: *B(SE)* = 0.51(0.80), *t*(61) = 0.64, *p* = .526, Program 2: *B(SE)* = -0.21(0.64), *t*(61) = -0.33, *p* = .742. To address the first research question, we examined the regression coefficients associated with the CM and CM+FM interventions predicting children's mathematics outcomes compared with the BAU group (see Table 2). The CM condition was not significantly associated with spring scores relative to BAU, *B(SE)* = 0.64(0.66), *t*(61) = 0.97, *p* = .336. However, the CM+FM condition was significantly associated with spring scores relative to BAU, *B(SE)* = 1.47(0.64), *t*(61) = 2.31, *p* = .024, representing a standardized effect size of *d* = .20. The pattern of results was the same using the complete-case dataset (see Appendix C).

Table 2

Hierarchical Linear Regressions of Spring Math Scores; Coefficient (Standard Error)

Parameter	Model 0	Model 1	Model 2	Exploratory Model
Intercept	47.4 (0.41)***	47.7 (0.38)***	46.9 (0.63)***	47.0 (0.65)***
Child-level variables				
Age		0.35 (0.05)***	0.36 (0.05)***	0.24 (0.07)**
Fall math score		0.50 (0.05)***	0.50 (0.05)***	0.50 (0.05)***
Female		–0.74 (0.53)	–0.72 (0.52)	–0.73 (0.51)
Spanish assessment		0.03 (0.89)	–0.27 (0.96)	–0.25 (0.94)
Classroom-level variables				
Program 1			0.51 (0.80)	0.53 (0.80)
Program 2			–0.21 (0.64)	–0.22 (0.65)
CM intervention			0.64 (0.66)	0.60 (0.67)
CM+FM intervention			1.47 (0.64)*	1.44 (0.64)*
Cross-level interactions				
CM * Age				0.20 (0.10)†
CM+FM * Age				0.17 (0.10)

† $p < .10$, * $p < .05$, ** $p < .01$, *** $p < .001$.

3.3. Research Question 2: What is the added value of including a family math component in a naturalistic classroom-based mathematics intervention?

To assess the effectiveness of the CM+FM intervention compared with the CM intervention we had all the same Level-1 and Level-2 covariates as Model 2 but only included children from the two intervention conditions (n = 398) and used one indicator variable for the CM+FM condition, with CM as the referent group. CM+FM was not significantly associated with spring scores relative to CM, *B(SE)* = 0.98(0.65), *t*(40) = 0.14, *p* = .151, indicating that there was no difference between the two intervention groups. Using the case-complete data (n = 331), however, CM+FM was marginally associated with spring scores relative to CM, *B(SE)* = 1.14(.66), *t*(40) = 1.73, *p* = .09, indicating that children who experienced both the CM+FM games had marginally higher spring scores than children who only experienced CM games, representing a standardized effect size of *d* = .15.

3.4. Exploratory Analyses of Learner Characteristics: Treatment Effects and Child Age

Using the full sample, we explored whether the intervention differentially impacted children based on their age. We included all the same Level-1 and Level-2 covariates as Model 2. There was a positive marginal interaction between the CM condition and age, *B(SE)* = 0.20(0.10), *t*(429) = 1.92, *p* = .055) and a non-significant interaction between the CM+FM condition and age, *B(SE)* = 0.17(0.10), *t*(206) = 1.63, *p* = .105 (Exploratory Model; Table 2). For comparison, the pattern of results using the complete-case dataset was very similar, but the interaction terms were significant and marginal, respectively (see Appendix C). Taken together, and because the direction of the interaction term was positive, this provided some evidence that the strength of the effect of the intervention increased with the age of the children. To explore this effect further, we split the sample into younger and older children (below and above 50 months old). We noted some important differences between the age groups (see Table 3). A significantly larger percentage of the younger children required assessment in Spanish (24%) compared with the older children (10%), *t*(497) = 4.36, *p* < .001. Also, among children in both intervention conditions, older children played significantly more games (*M* = 33.8, *SD* = 13.9) compared with younger children (*M* = 30.1, *SD* = 11.7), *t*(329) = 2.54, *p* = .012. Older and younger children did not differ significantly in the number of mini-books that were returned, *t*(186) = 0.146, *p* = .703.

Table 3

Descriptive Data for Younger and Older Children

	Younger children			Older children		
	CM+FM	CM	BAU	CM+FM	CM	BAU
	n = 73	*n* = 69	*n* = 71	*n* = 94	*n* = 95	*n* = 97
Female	48%	54%	65%	45%	48%	46%
Spanish	27%	17%	27%	11%	3%	15%
	M (SD)	*M (SD)*	*M (SD)*	*M (SD)*	*M (SD)*	*M (SD)*
Age	45.6 (2.47)	46.1 (2.42)	46.2 (2.44)	55.1 (2.88)	55.3 (2.75)	55.4 (2.93)
Fall math	37.3 (6.93)	38.6 (6.15)	37.2 (7.64)	43.7 (6.97)	42.0 (7.35)	41.8 (6.91)
Spring math	44.5 (7.19)	44.1 (7.19)	44.3 (6.79)	51.5 (6.87)	49.6 (6.96)	48.0 (6.24)
Total games	30.3 (11.1)	30.0 (12.4)	NA	33.7 (16.0)	33.9 (11.7)	NA
Total mini-books	8.20 (4.79)	NA	NA	8.48 (4.88)	NA	NA

Among the younger children (*n* = 213 with case-complete data), the CM condition was not significantly associated with spring scores, *B(SE)* = -0.85(0.97), *t*(56) = -0.87, *p* = .386, nor was the CM+FM condition, *B(SE)* = 0.28(0.82), *t*(56) = 0.35, p = .729. For the older children (*n* = 286 with case-complete data), both the CM condition, *B(SE)* = 1.67(0.71), *t*(59) = 2.35, *p*

= .022, and the CM+FM condition, *B(SE)* = 2.59(0.81), *t*(59)= 3.19, *p* = .002, were significantly associated with spring scores with standardized effect sizes of *d* = .24 and .36 respectively.

3.5. Exploratory Analyses of Learner Characteristics: Treatment Effects and Dosage

Next, we examined the total number of games children played in the classroom as recorded by teachers as a proxy for dosage of the intervention. Among children who participated in the CM or CM+FM conditions (*n* = 331 with case-complete data), we examined dosage of the classroom math component based on the total number of games children played in the classroom as recorded by teachers. We included all the same Level-1 and Level-2 covariates as Model 2, as well as an indicator for the CM+FM condition. Total games played was significantly associated with spring scores, *B(SE)* = 0.06(0.03), *t*(282) = 2.07, *p* = .039, indicating that the more games children played in the classroom in both intervention conditions, the higher the spring mathematics scores. The CM+FM condition was marginally significant, *B(SE)* = .1.25(.72), *t*(40), = 1.73, *p* = .091 relative to the CM condition, indicating that the family math component may explain additional variance in spring scores above and beyond the number of games played in the classroom and the age of children. Given the significant intervention-by-age interaction effect (Exploratory Model, Table 2) and the significant difference in game play by older and younger children, we explored whether the association between game play and spring scores was moderated by age. To test this, we added an interaction term between game play and age; this was not significant, *B(SE)* = 0.00(0.02), *t*(281) = 0.96, *p* = .340, indicating that the effect of game play on spring mathematics scores did not differ based on age.

Among children who participated in the CM+FM condition (*n* = 167 with case-complete data), we examined dosage of the family math component based on the total number of completed mini-books that were returned as recorded by teachers. We included all the same

Level-1 and Level-2 covariates as Model 2, but also included the total number of games played. The number of completed mini-books was significantly associated with spring scores, *B(SE)* = 0.22 (0.07), $t(139) = 3.25$, $p = .001$, indicating that the more completed mini-books children returned, the higher the spring mathematics scores, above and beyond the effect of games played in the classroom. The number of games played in the classroom was no longer significantly associated with spring scores, *B(SE)* = 0.04 (0.03), $t(139) = 1.14$, $p = .257$.

3.6 Teacher-level Intervention Effects.

To explore whether the interventions were effective in promoting high-quality instructional practice, we examined the *Instructional Support* domain of the CLASS Pre-K observation tool (see Table 4). Because this was an exploratory analysis, only a subset of teachers, those in Program 1 and Program 2, were observed using the CLASS Pre-K tool and are subsequently referred to as the Cohort sample ($n = 44$).

Table 4

Descriptive Statistics for Instructional Support; Mean (SD)

	Cohort Sample ($n = 44$)	Classroom + Family Math ($n = 14$)	Classroom Math ($n = 16$)	BAU ($n = 14$)
Fall Instructional Support	2.62 (0.80)	2.53 (0.83)	2.73 (0.80)	2.58 (0.82)
Spring Instructional Support	2.63 (0.74)	2.92 (0.91)	2.62 (0.67)	2.35 (0.52)

Note: Teachers from two of the three sites were observed using the CLASS Pre-K observation tool.

While on average, *Instructional Support* did not change substantially from fall (M = 2.62) to spring (M = 2.63), across all teachers there were significant differences among the intervention conditions (see Table 5). Whereas there were no significant effects found for the

CM condition on *Instructional Support*, there was a significant effect of the CM+FM condition on instructional quality compared with BAU with a large effect size of $d = .72$.

Table 5

OLS Regression of Spring Instructional Support

	Coefficient (SE)	*t*	*p*
Intercept	1.60 (0.38)	4.16	.000
Fall Instructional Support	0.29 (0.13)	2.21	.033*
Classroom Math	0.23 (0.25)	0.93	.358
Classroom Math + FM	0.59 (0.26)	2.28	.028*

* $p < .05$.

We also investigated the association between spring *Instructional Support* and children's spring math scores using the same procedures as described above for testing child-level intervention effects. Teachers' spring *Instructional Support* score was significantly associated with students' spring scores, controlling for fall math scores, age, gender, and language of assessment at Level 1, *B(SE)* = 0.92(0.33), $t(42) = 2.74$, $p = .009$. Once Level-2 covariates (program site and intervention conditions) were included, however, this effect was no longer significant, *B(SE)* = 0.53(0.37), $t(40) = 1.45$, $p = .154$. This additional analysis indicates that variation in *Instructional Support* did not explain children's mathematics learning above and beyond the effect of the intervention.

4. Discussion

Disruptions due to the COVID-19 pandemic continue to have cascading effects across the education field. Now more than ever, there is a need for effective early childhood interventions that bolster home and school learning environments that can be scaled and implemented in typical Head Start classrooms. This study addresses that need by investigating an innovative approach aimed to improve the mathematics learning environments of young children, with a particular focus on Head Start programs that serve children from under-resourced communities.

To do this we evaluated two preschool mathematics interventions and their effects on child outcomes, relative to a business-as-usual condition: a classroom math (CM) intervention and a classroom plus family math (CM+FM) intervention.

4.1 Research Question 1: What is the impact of a naturalistic, game-based classroom mathematics intervention, with and without a family math component, on preschoolers' mathematics learning?

Given that teachers received the same classroom mathematics supports, we hypothesized that both interventions would promote children's mathematics learning relative to a BAU condition. Our hypothesis was partially confirmed, as the results showed that in mixed-age (3- to 5-years) Head Start classrooms, the CM+FM condition was significantly associated with spring mathematics scores relative to BAU (effect size of d = .20) but the CM condition was not. This finding suggests that the CM+FM intervention has potential as an effective means to fill a gap in early childhood instructional practice, providing preschool teachers with reproducible instructional materials that are developmentally appropriate and playful, and can be implemented at scale without substantial investments in curriculum, PD, or coaching support. A simple fidelity measure (number of games played) can support teachers' implementation which increases the replicability and sustainability of the intervention under realistic implementation conditions. In fact, interventions that can be qualified as "ecologically valid" (i.e., naturalistic, conducted by the teacher with the whole class, targeting more than one mathematics skill) are scarce, and knowing under which circumstances such interventions might be effective for the different children of a classroom is essential (de Chambrier et al., 2021). Importantly, the design of the current study demonstrates the value of combining a family-engagement component with a classroom mathematics intervention. For instance, if we had only compared CM+FM to BAU, we would have masked the key role of the family component in promoting children's mathematics learning, as the CM condition was not significantly related to spring scores.

4.2 Research Question 2: What is the added value of including a family math component in a naturalistic classroom-based mathematics intervention?

We hypothesized that relative to the CM condition, the CM+FM condition would have a stronger impact on children's mathematics outcomes. This hypothesis was partially supported. When comparing the CM condition and the CM+FM condition to each other, without the BAU group, the effect was marginally significant (d = .15) but only in the complete-case data, although the direction of the effect was positive in both analyses. Given that the classroom supports were the same in both conditions, it is not surprising that the effect was marginal. However, when taking into consideration the overall pattern of results from RQ1 and RQ2, it suggests that the family math component did add value to the classroom-based intervention as the effect of CM+FM was significant (d = .20), and relative to CM, the CM+FM condition also had a small but positive effect.

4.3 Exploratory Analyses of Learner Characteristics

To better understand how learner characteristics might influence mathematics outcomes, we investigated whether age moderated the relation of the intervention to spring scores and found some evidence that the strength of the effect of the intervention increased with the age of the children. When splitting the sample (<50 months; 50 months+), we found that the CM intervention led to improvements in mathematics skills for older preschoolers (d = .24), but the greatest impact came from the CM+FM intervention (d = .36) for older children, indicating that the family math component added value to children's learning beyond the classroom activities.

Several factors may have contributed to the age-related differences. There were significant differences between the younger and older preschoolers; younger preschoolers were more likely to take the assessment in Spanish and play fewer games than older children. Teachers mentioned that they tended to invite kindergarten-bound children more than younger children to play mathematics games and encouraged older children to stay and persist at game play for longer; focusing more of their mathematics instructional time on the children who would soon enter kindergarten.

Since teachers mentioned that they tended to play more games with the older children in their classrooms, we investigated whether game play influenced children's mathematics outcomes. We found that the number of mathematics games children played in the classroom did matter, providing an advantage for children who played more games in both interventions compared to control group children. A possible explanation for why older children benefited more from the intervention than younger children is because they may have had more opportunities to learn mathematics, as the effect of game play on spring mathematics scores did not differ based on age.

In addition, teachers may have been intuitively aware of age-related differences in executive function and mathematics. Based on this awareness, teachers may have provided more math-learning opportunities to children with greater EF skills, many of whom were likely older. Research has suggested that children with stronger executive function skills demonstrate a greater response to mathematics input compared with children with less developed skills (Silver et al., 2021). Teachers also mentioned that older children were typically able to play for longer and often enjoyed the mathematics games more than younger children. While we considered the number of games played, we did not have teachers monitor the length of time children played the games. Thus, another factor contributing to greater mathematics scores for older children may have been that the total time they engaged with mathematics was greater based on playing more games *and* playing those games for longer than younger children. Finally, it is also possible that measurement issues may have contributed to the age-related differences in intervention effects. Although the assessment we used was intended for preschoolers, we suspect that it may not have included enough items that could be sufficiently sensitive to detect variation at the beginning levels of mathematics ability, which may have limited our ability to detect effects of the intervention for younger children.

Given the pattern of results suggesting that the family math intervention was adding value above and beyond the classroom intervention, we also investigated the total number of completed mini-books as an indication of dosage of the family math component. Interestingly, there was a strong effect found for family math mini-books on children's mathematics outcomes. Both the number of games played in the classroom and the number of completed mini-books were associated with higher spring mathematics scores, but once dosage of family math was taken into account, the number of games played was no longer significantly associated with spring scores. While this effect could only be explored among children in the CM+FM condition, it provides additional evidence that the family math component was adding value above and beyond the CM condition. This finding supports the idea that cross-context learning—intentionally coordinating classroom-based learning with home-based learning activities to help parents support children's mathematics learning and development—is a critical component of effective early childhood mathematics interventions.

While we were able to measure some aspects of dosage for both components of the intervention, we were not able to collect data on whether specific classroom games or specific mini-books were more effective than others, nor were we able to collect data on how frequently families played the games or read and played the activities from the mini-books. Therefore, we do not know whether or how families differed in their engagement with each type of activity. We also do not know whether the family math supports by themselves (mini-books, take-home games) would have had an impact on children's learning without a corresponding classroom mathematics intervention. While we were not able to observe parent-child interaction or explore whether the family math games and mini-books influenced families' math talk and engagement at home, this is a fruitful area for further exploration because the amount and kind of math talk parents engage in with children has been shown to influence young children's mathematics knowledge (Susperreguy & Davis-Kean, 2016). Researchers have found that young children's knowledge of the cardinal meanings of number words are related to the amount of number talk they hear from their primary caregiver at home (Levine et al., 2010), and certain kinds of number talk are particularly predictive of this knowledge (e.g. counting and labeling sets of objects and large number talk) (Casey et al., 2018; Elliot, et al., 2017; Gundersen & Levine, 2011). Future work could explore how family math games and family math mini-books influence specific types of math talk at home.

We suspect that the playful, engaging nature of the activities supported families to engage with mathematics more frequently, while maintaining positive relationships. In family literacy interventions, researchers have underscored the importance of maintaining positive social and emotional relationships between parents and children and not disrupting this with the pressure that might come from a teaching situation (van Steensel et al., 2011). Anecdotal evidence from this study suggests that children's interest in mathematics may have been heightened because of the additional importance of the family math materials being sent home. Families mentioned that their children often asked to play with the math games, and teachers reported that some families that had never returned any other "homework" did engage with, and return, the mini-books. Additionally, the mini-book format may have been particularly approachable for families; Berkowitz et al. (2015) showed that parents are less likely to participate in math and complex problem-solving activities with children than they are to read to their children. Future research on family-engagement interventions that serve as a complement to classroom-based learning should capture details about the specific ways that families engage with home materials to identify the specific aspects of these interventions that promote children's learning.

4.4 Teacher Instructional Support

We explored the impact of the interventions on teachers' instructional practice as an additional potential benefit of providing these interventions. We found a very strong effect of the CM+FM intervention on the quality of teachers' instructional support for children's learning. This is a key finding, as teachers who provide higher-quality instructional support may promote a broad range of school-readiness skills, including language skills and mathematics skills (Mashburn et al., 2008). Moreover, improving the quality of early childhood instruction is a primary goal of many state and federal initiatives and is often the target of school readiness interventions (Early et al., 2017). Contrary to our hypothesis, we found that the CM intervention did not have a significant effect on instructional support or teacher–child interaction, this was surprising, especially since the teacher-focused PD was very similar in both interventions. It is unclear why the CM+FM condition promoted high quality instructional practice, but the CM condition did not. We suspect that the family math component of the CM+FM condition supported greater integration of learning across multiple settings, which is a component of instructional support. For example, teachers may have referenced the mathematics experiences children had at home which may have supported deeper learning. In addition, children's engagement in the mathematics activities may have had a positive influence on the learning environment and contributed to a higher level of instructional quality.

We suspect that the family math intervention materials also facilitated communication between teachers and families and provided scaffolding for family caregivers to do more math and talk more about mathematics, while becoming more familiar with developmentally appropriate early mathematics activities, something families have reported wanting (Sonnenschein et al., 2021). In particular, the mini-books were designed to include both English and Spanish on the same page so that teachers didn't have to spend time sorting and organizing different versions of the same book. However, this may have been beneficial for families, as some of the Spanish-speaking families mentioned that they liked having the English and Spanish on the same page so that they could learn the English words for the mathematics concepts and support their child in their language of instruction (mainly English). We postulate that this alignment across home and school and in culturally sensitive ways may have contributed to the additional benefits provided by the CM+FM. Additionally, the transactional model of child development (Sameroff & Fiese, 2000) posits that the child is both product and producer of their own development, such that just as the environment influences the child's development, the child also influences their environment. So as children's mathematics support increased at home, their skills may have improved such that they became more deeply involved in the mathematics games at school, which was also reinforcing to the teachers and elicited from the teachers a greater level of instructional support.

While instructional support was associated with child outcomes, it did not explain variation beyond the effect of the intervention condition. Given the small sample of classrooms included in this analysis, we suspect that we did not observe sufficient variation in instructional support within the interventions to detect an effect beyond that of the intervention itself. While changes in the quality of teacher–child interactions may have been one important mechanism for supporting children's mathematics learning in participating classrooms, many other aspects of instruction were likely important as well, for example, the strategies teachers used to engage children in the games, math-related talk, and time spent engaging in mathematics learning.

4.5 Synthesis and Limitations

While the effect sizes for mathematics outcomes are in the small range (.20–.36; see Cohen, 1992), they are close to or surpass what has been considered a "substantive" effect (.25 or greater; WWC, 2020). Typically effect sizes in mathematics interventions are larger when studies use a researcher-made assessment and address only one content strand (Wang et al., 2016); however, this study used an externally validated assessment of generalized early mathematics skills that addressed several content strands. Further, the effect sizes are meaningful when the intervention is put into context. This intervention was implemented in a naturalistic setting, such that teachers played mathematics games in their classrooms and engaged families as part of their regular practice. While some interventions have also taken this approach (e.g., Lange et al., 2020), substantive effects have not been clearly shown for broader math skills. While several high-quality full-year comprehensive preschool mathematics curricula have found effects ranging from .35 to .69 (Starkey et al., 2022; Lewis Presser et al., 2015; Sarama, Clements, et al., 2012), many preschool programs are hesitant to purchase an additional mathematics curriculum. Furthermore, while most early childhood educators agree that early mathematics instruction is important for preschoolers (96.6% in one study), very few of the same educators (19.2%) favored the use of a published early mathematics curriculum (Chen et al., 2013), and when comparing teachers' attitudes toward a play-based approach and a curriculum-based approach to teaching mathematics, early childhood educators preferred the play-based intervention that included card and board games, indicating that they felt it was better suited to children's diverse needs (Vogt et al., 2018). Thus, a program like the one described in this study, that is supplemental to an existing holistic curriculum, is light-touch, and play-based, may encourage more uptake and be easier to implement for in-service Head Start teachers. Both the classroom games and the family math resources involved low-cost materials and easily adaptable activities, making this intervention replicable and sustainable for Head Start contexts under realistic implementation conditions.

While we explicitly called attention to ways to adapt the games for younger children and asked teachers to play the games equally with all children, there were differential effects of the intervention for younger children. In future work, it will be important to investigate ways to support younger children's game play with adaptations that may foster that play. For example, some researchers have suggested that instructional format (small group, whole group, one on one) may be an important way to adapt math games based on ability level (de Chambrier et al., 2021). The same pattern of engagement with the materials may have contributed to age-related differences in outcomes. In addition, measurement issues may have contributed to the age-related differences in intervention effects. Although intended for preschoolers, we suspect that the assessment may not have included enough items with lower difficulties to be sufficiently sensitive to detect variation at the lower levels of math ability of the younger children. Alternatively, because younger children were more likely to be assessed in Spanish, allowing children to answer questions in either English or Spanish might have better reflected their knowledge.

This study took place within the specific context of Head Start; therefore, we must be careful not to generalize the findings to the general population. While race and ethnicity did not have a significant statistical effect on child outcomes, the specifics of these demographic characteristics may further limit the generalizability of our findings; more than half (64%) the children in the study were Hispanic or Latino; less than one-third (27%) were white, and more than half (61%) the families in the study spoke a language other than English in the home. We also should not generalize the findings to childcare settings outside of Head Start. It is also worth noting that the PD was taught by the intervention developers. This investment and familiarity with the materials represent ideal conditions for PD implementation. A next step in the development of the intervention could involve creating a PD model for teacher educators to implement and continuing to improve the resources for teachers and families as the mathematics content addressed here were limited to number and geometry, but there may be an additional benefit in creating mathematics games that engage children in a broader range of skills such as spatial relationships, measurement, and data.

5. Conclusion

To improve children's early mathematics learning, strengthening home—school connections in family math should be part of the equation. This study demonstrates that teachers can effectively engage Head Start families in early mathematics activities, and it highlights the value of coordinating mathematics learning across home and school contexts to support children's mathematics development. In addition, this study establishes that a cross-context mathematics intervention that supports both teachers and families in engaging in effective mathematical interactions can be implemented in a way that is relatively inexpensive and sustainable. Moreover, including a family math component in a classroom intervention by providing family math resources that bridge home and school, results in positive impacts on preschoolers' mathematics knowledge, and teachers' instructional practice, and seems to be an effective, low-threshold intervention.

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Appendix A

Description of the mathematics games

Games for Young Mathematicians

	Description and learning goals
Finger Play and Dot Card Games	These games promote children's understanding of early mathematics concepts from counting and cardinality to composing and decomposing numbers. When children use their fingers to count, they are strengthening their number knowledge and their abilities to visualize numbers in their minds. For dot cards, children use cards that have 0–10 black dots arranged in different configurations—linear, rectangular, dice pattern, circular, and scattered. Differing dot arrangements help children develop different mental images of quantity. Children can play with these cards in a variety of ways, and these games help children practice subitizing, counting, and cardinality.
Hiding Games with Counters	Games with counters help children practice counting, subitizing, cardinality, and composing and decomposing numbers. For example, in the game *How Many Are Hiding?*, children count the total number of objects (e.g., fingers, tokens, playing cards) and then close their eyes while the adult "hides" some. Children figure out how many are hiding.
Games with Cards and Dice	Adults can choose to use the cards that are best for the developmental level of their children (e.g.,1–3, 1–6, 1–10, or 0–12) and use regular 6-sided dice or homemade dice that may only have configurations of 1, 2, or 3 dots. While playing these games, children practice recognizing numerals, composing and decomposing numbers, number order, and using a number path.
Number Path Games	Number path games, when played like board games, build children's understandings of early mathematics concepts such as counting and comparing numbers, while giving them experience with a valuable mathematical tool—a number path.

Pattern Games	These pattern games invite children to playfully create, copy, extend, fix, and transfer patterns. They help children see that patterns repeat in a regular way—once you recognize a pattern structure, you can predict what comes next.
Pattern Block Puzzle Games	These puzzles use pattern blocks, and whereas they are a mainstay in preschool classrooms, many adults are unaware of the complex mathematical thinking involved in solving these puzzles. Children develop knowledge of spatial relationships and composition of shape; they are also exposed to more advanced concepts, such as part–whole relationships, fractions, and area.
Shape Card Games	With specially designed shape cards, children play card games where they pay attention to mathematical attributes such as shape, number, and color. They practice executive function skills by taking turns and remembering and matching cards.

Appendix B

Equation B.1

Level 1: $Y_{ij} = \beta_{0j} + \beta_{1j}(\text{Age}_{ij}) + \beta_{2j}(\text{Fall Math}_{ij}) + \beta_{3j}(\text{Female}_{ij}) + \beta_{4j}(\text{Spanish Assessment}_{ij}) + r_{ij}$

Level 2:
$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10}$
$\beta_{2j} = \gamma_{20}$
$\beta_{3j} = \gamma_{30}$
$\beta_{4j} = \gamma_{40}$

Combined: $Y_{ij} = \gamma_{00} + \gamma_{10}(\text{Age}_{ij}) + \gamma_{20}(\text{Fall Math}_{ij}) + \gamma_{30}(\text{Female}_{ij}) + \gamma_{40}(\text{Spanish Assessment}_{ij}) + u_{0j} + r_{ij}$

Equation B.2

Level 1: $Y_{ij} = \beta_{0j} + \beta_{1j}(\text{Age}_{ij}) + \beta_{2j}(\text{Fall Math}_{ij}) + \beta_{3j}(\text{Female}_{ij}) + \beta_{4j}(\text{Spanish Assessment}_{ij}) + r_{ij}$

Level 2:
$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{Program 1}_j) + \gamma_{02}(\text{Program 2}_j) + \gamma_{03}(\text{CM}_j) + \gamma_{04}(\text{CM+FM}_j) + u_{0j}$
$\beta_{1j} = \gamma_{10}$
$\beta_{2j} = \gamma_{20}$
$\beta_{3j} = \gamma_{30}$
$\beta_{4j} = \gamma_{40}$

Combined: $Y_{ij} = \gamma_{00} + \gamma_{01}(\text{Program 1}_j) + \gamma_{02}(\text{Program 2}_j) + \gamma_{03}(\text{CM}_j) + \gamma_{04}(\text{CM+FM}_j) + \gamma_{10}(\text{Age}_{ij}) + \gamma_{20}(\text{Fall Math}_{ij}) + \gamma_{30}(\text{Female}_{ij}) + \gamma_{40}(\text{Spanish Assessment}_{ij}) + u_{0j} + r_{ij}$

Equation B.3

Level 1: $Y_{ij} = \beta_{0j} + \beta_{1j}(\text{Age}_{ij}) + \beta_{2j}(\text{Fall Math}_{ij}) + \beta_{3j}(\text{Female}_{ij}) + \beta_{4j}(\text{Spanish Assessment}_{ij}) + r_{ij}$

Level 2:

$\beta_{0j} = \gamma_{00} + \gamma_{01}(\text{Program 1}_j) + \gamma_{02}(\text{Program 2}_j) + \gamma_{03}(\text{CM}_j) + \gamma_{04}(\text{CM+FM}_j) + u_{0j}$

$\beta_{1j} = \gamma_{10} + \gamma_{11}(\text{CM}_j) + \gamma_{12}(\text{CM+FM}_j)$

$\beta_{2j} = \gamma_{20}$

$\beta_{3j} = \gamma_{30}$

$\beta_{4j} = \gamma_{40}$

Combined: $Y_{ij} = \gamma_{00} + \gamma_{01}(\text{Program 1}_j) + \gamma_{02}(\text{Program 2}_j) + \gamma_{03}(\text{CM}_j) + \gamma_{04}(\text{CM+FM}_j) + \gamma_{10}(\text{Age}_{ij}) + \gamma_{20}(\text{Fall Math}_{ij}) + \gamma_{30}(\text{Female}_{ij}) + \gamma_{40}(\text{Spanish Assessment}_{ij}) + \gamma_{11}(\text{CM}_j * \text{Age}_i) + \gamma_{12}(\text{CM+FM}_j * \text{Age}_i) + u_{0j} + r_{ij}$

Appendix C

C.1 Complete Case Results (Non-Imputed)

*Complete Case Hierarchical Linear Regressions of Spring Math Sco*res; *Coefficient (Standard Error)*

Parameter	Model 0	Model 1	Model 2	Exploratory Model
Intercept	47.4 (0.42)***	47.8 (0.37)***	46.8 (0.67)***	46.9 (0.69)***
Child-Level Variables				
Age		0.35 (0.05)***	0.36 (0.05)***	0.23 (0.06)***
Fall Math Score		0.50 (0.05)***	0.49 (0.05)***	0.49 (0.05)***
Female		–0.74 (0.51)	–0.66 (0.50)	–0.73 (0.51)
Spanish Assessment		–0.09 (0.92)	–0.56 (1.04)	–0.51 (1.04)
Classroom-Level Variables				
Program 1			0.88 (0.81)	0.91 (0.81)
Program 2			–0.13 (0.62)	–0.15 (0.63)
CM			0.58 (0.63)	0.56 (0.64)
CM+FM			1.59 (0.65)*	1.58 (0.65)*
Cross-level Interactions				
CM * Age				0.21 (0.09)*
CM+FM * Age				0.18 (0.09)†

† $p < .10$, * $p < .05$, ** $p < .01$, *** $p < .001$.

C.2 Complete Case *Hierarchical Linear Regressions of Spring Math Scores; Coefficient (Standard Error)*

Parameter	Younger Children (n = 213)	Older Children (n = 286)
Intercept	44.7 (0.98)***	48.5 (0.73)***
Child-level variables		
Age	0.44 (0.20)*	0.49 (0.11)***
Fall Math Score	0.49 (0.08)***	0.50 (0.06)***
Female	–0.74 (0.83)	–0.84 (0.70)
Spanish Assessment	0.14 (1.25)	–1.57 (1.20)
Classroom-Level Variables		
Program 1	0.19 (1.14)	1.48 (0.94)
Program 2	0.27 (0.93)	–0.31 (0.71)
CM	–0.85 (0.97)	1.67 (0.71)*
CM+FM	0.28 (0.82)	2.59 (0.81)**

* $p < .05$, ** $p < .01$, *** $p < .001$.