Promoting Geometry Learning in Middle School through Ethno-Mathematics

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Abstract: In response to the challenge of geometry's abstract and less engaging nature for students, this study explored the potential of connecting geometry to cultural elements, specifically geometric ornaments found in various cultures worldwide. Geometric ornaments, laden with cultural and spiritual significance, serve as a bridge between mathematics education and cultural contexts. The current research focused on in-depth interviews with 10 school students chosen randomly from two ninth-grade classes from an Arab middle school who were engaged in activities involving analyzing of geometric properties of ornaments, constructing ornaments using a compass and straightedge, and problem-solving exercises related to these ornaments. The qualitative data were analyzed to answer how the students perceived the learning of geometry in the context of geometric ornaments. The findings revealed a significant positive view of the students towards geometry after engaging in these activities.

Keywords: Geometry, Ethno-mathematics, geometric Ornaments, Culture, Middle School.

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Introduction

Geometry learning allows students to solve problems in everyday life (Abdullah & Leow, 2017). It contributes to students in developing skills in visualization, critical thinking, intuition, perspective, problem-solving, estimation, deductive thinking, logical reasoning, and proof (Jones, 2002). However, it's one of the subjects in mathematics that school students consider as a challenging. The difficulties among school students are expressed mainly in solving geometric problems and developing formal proofs (Juman et al., 2022), and the fact that geometry requires students to understand abstract concepts (Andika et al., 2020). Difficulties in geometry are not limited only to the students, but also to mathematics teachers. They tend to have a very low understanding of geometry (Mashingaidze, 2012), when difficulties begin with the in-service preschool teachers. Markovits and Patkin (2021) found that among in-service preschool teachers almost lack of knowledge to name shapes and solids and have challenges in defining them by properties but more by the visual characteristics of these geometric shapes. It was found that pre-service mathematics teachers have difficulties with geometric concepts (Sunzuma & Mahara, 2019). Studies (Arabacı, & Kanbolat, 2023) pointed out to a lack of awareness

among teachers of their students' difficulties and in realizing in what level they are of understanding geometry, as Van Hiele's geometric thinking levels. The research literature offers different ideas to overcome difficulties in learning geometry. Geometry should be considered a practical subject that requires learners to use a variety of resources to investigate and explore the properties of shapes and geometry facts (Jones, 2002). Researchers in mathematics education call for connecting mathematics teaching to the culture of the learner (Gerdes, 1988; Bishop, 1988; Rosa & Orey, 2011; D'Ambrosio, 1985), and see culture as a rich resource for teaching mathematics (Presmeg, 1998). The cultural context allows students to explore mathematics more realistically (Nur et al., 2020a). Ornaments are part of the heritage of different cultures in the world that have a spiritual and cultural meaning. Researchers (Massarwe et al., 2010) pointed out that practice geometric constructions of ornaments, by compass and straightedge in geometry class through analyzing the properties of the shapes and solving geometric problems related to them, was found as a highly enjoyable experience, and fostering an understanding of the importance of ethno-mathematical learning activities related to the students' own cultures and those of others (Massarwe et al., 2012). It helps in understanding geometric concepts and contributing raise motivation among students and developing positive attitudes towards geometry. Geometric ornaments have a direct connection to art and culture ever since throughout the history (Cenani & Cagdas, 2006). Their cultural context can influence the students' thinking as well as the construction of mathematical knowledge among them (Nassar & al-Muheisen, 2013) which affect their achievement (Masgoret & Gardner, 2003). The current study is part of an ongoing research on Ethnomathematics which started together with the author and colleagues in the academy at the Technion. Following the research, a course in ethnomathematics was developed for graduate students in the Education in Science faculty at the Technion in 2008, which was taught by the author and the colleagues. Then, the course was invited and adapted to be taught by the author at the Academic Arab college for Education in Israel, where the author teaches and which is intended for master's degree students who are actually in-service mathematics teachers. Some teachers applied the content they learned in the course to their students at schools. The current study aimed to examine how the 9th grade students perceive learning of geometry in the context of geometric ornaments.

Theoretical Framework

Difficulties in Learning Geometry

In the perception of students, geometry is often regarded as a difficult and abstract discipline. The study of geometry necessitates the acquisition of new concepts and their application in various proofs, involving the manipulation of properties of different geometric shapes (Karimah et al., 2018; Nagubandi, 2018). These abstract concepts pose challenges to understanding geometry at the middle school level, primarily in the form of fundamental cognitive difficulties in grasping geometric concepts, or difficulties related to visual perception. Mathematics teachers, sometimes are unaware of these challenges or lack the tools to effectively address them (Adhikari, 2019; Karimah et al., 2018). In the middle school, the formal study of geometry commences, demanding the mastery of cognitive skills that go beyond the visual (Nagubandi, 2018; Regmi, 2019). At this stage, students are required to engage in logical and deductive thinking, which poses challenges for them

(Adhikari, 2019; Karimah et al., 2018). They meeting new concepts, like definitions and theorems, and learn to substantiate claims while also refuting incorrect claims through the use of counter examples (Adhikari, 2019; Regmi, 2019). The acquisition of geometric concepts, a fundamental component of learning, involves various aspects, including knowledge of the concept's perimeter, comprehension of its defining characteristics, an understanding of the concept's terminology, and similar elements (Karimah et al., 2018; Nagubandi, 2018). Geometric concepts can be acquired at four distinct levels: the concrete level, the level of identity, the classification level, and the formal level. In mathematics education, the emphasis primarily lies in the higher classification and formal levels (Adhikari, 2019; Karimah et al., 2018). At this stage, students encounter challenges in articulating concepts, differentiating between them, and providing accurate definitions (Adhikari, 2019; Karimah et al., 2018). The challenges associated with learning geometric concepts can be attributed to several factors, including instructional methodologies, limited access to educational materials, and the inherent complexity of these concepts (Ubi & Igiri, 2018). Another studies (Juman et al., 2022) pointed out to the fact that students encountered more significant difficulties in learning geometry, such as drawing diagrams for a given geometric problem and applying more than one theorem to solve a problem.

Learning Geometry through Cultural Context

The main idea of the current research is about the notion that the learning geometry through analyzing and constructing ornaments rooted in the learner's culture and other cultures, can potentially enhance the student's ability to engage with mathematical content. Utilization of tangible objects, like ornaments, in mathematics instruction contributes to enhance learning outcomes (Ahmad & Chang, 2017; Holtzman & Susholtz, 2011; Kohlhase, 2017; Lavinia, 2011). By employing physical objects, students engage their senses and thus acquire knowledge in an optimal manner (Murray, 2001). Employment of tangible means, such as ornaments, as part of mathematical instructional methods is commonly perceived as the application of manipulations in purpose to render abstract mathematical concepts meaningful to the student (McNeil & Jarvin, 2007). This requires the teachers to guide their students towards practical implementation, translating the tangible object into a tool for achieving the pedagogical objective (Lehtinen & Hannula, 2006). Learning with ornaments by engaging the students in the learning process can facilitate them connecting abstract concepts in geometry and the practical knowledge in the real life, and thus enhancing their comprehension in the subject (McNeil & Jarvin, 2007).

The Ethnomathematics Approach

Ethnomathematics describes the mathematical practices inherent within cultural groups, and inquiring of mathematical ideas rooted in each culture (D'Ambrosio, 1985). It can be described as the manner in which individuals from a specific cultural use to solve problems in everyday life (Rosa & Orey, 2011). This approach in learning mathematics validates the mathematical experiences of peoples, as it enlightens the contribution of every culture to mathematics. Over the past four decades, ethnomathematics become a pedagogical approach in various topics. It is based on the constructivist methodology, which consider the student as an active learner constructing knowledge through experiential and social interaction facilitated by the teacher. Guiding such

activities necessitates the teacher's specialized content and pedagogical knowledge expertise, and ability to engage with students, recognize their cultural backgrounds. Through engagement in mathematical activities rooted in diverse cultures, students can learn to appreciate mathematics and develop greater respect for those who differ from them (D'Ambrosio, 2007). success in mathematical learning can be better realized when considering the cultural background of the student. The cultural context allows deepen engagement of the students in mathematics and make the study of the subject more realistic (Nur et al., 2020a). Researchers in ethnomathematics pointed out to various benefits of learning mathematics based on the ethnomathematical approach as the students can grab the lesson easily and acknowledge their own cultures (Mania & Alam, 2021), and that it has a positive impact on increasing students' geometry spatial abilities (Sukestiyarno, 2023), and the realistic mathematics learning with ethnomathematics orientation can improve mathematical problem-solving skills and fosters the learner abilities (Nur et al., 2020b).

The Study

The current study accompanied the process of learning geometry in a cultural context based on the ethnomathematical approach, through engaging 9th grade school students in activities of analyzing geometric ornaments and constructing them by compass and straightedge, and solve geometric problems related to these ornaments. The study aimed to examined how school students perceive learning geometry in the context of ornaments. This paper focus on the qualitative part that included in-depth interviews with 10 school students to present their perceptions towards learning geometry through experience in analyzing and constructing of geometric ornaments.

Participants

Two 9th classes that included 70 school students were engaged in activities of learning geometry in the context of analyzing and constructing ornaments from different cultures. The students learn in a private school in the Arab sector of Israel in one of the Arab cities in the north of the country. This paper focus on in-depth interviews with 10 students chosen randomly from the two classes. They learned the subject with a mathematics teacher that studied at The Academic Arab College for Education for her master degree in where the author teaches. The teacher herself studied with the author a professional development course based on the ethnomathematics approach and applied the subject to the 9th graders. In the course she learned to adapt the material to school students, so adjustment was made to the contents learned that would be suitable for 9th grade students. For this purpose, an instructional unit was developed on the subject.

The Study Tool

The research tool included semi-structured interviews with the school students after learning geometry with ornaments to present their perceptions towards learning geometry through experience in analyzing and

constructing by compass and straightedge of geometric ornaments. The semi-structured interview was chosen because it allows enough flexibility for the interviewees to raise additional issues that are important to them besides the purpose to investigate the issue. The interviews were conducted in Arabic as the native language of the participants. After every closed question the students were asked to explain and give examples. The students' answers were written down in full. The closed questions of the interviews were: 1) What is your opinion about the activity of analyzing and constructing geometric ornaments in class?; 2)What did you like about the lesson?; 3) Do you prefer to learn more topics in geometry with the same method?; 4) How did the geometry activity contribute to you?; 5) What is your opinion on learning geometry in the context of geometric ornaments?; 6) Did the activity of analyzing and constructing ornaments contribute to understand geometric concepts or to solve geometric problems?.

The Study Process

The 9th grade students studied geometry combining analysis and construction of geometric ornaments. They learned for seven lessons of forty-five minutes each. At the beginning the students learned basic construction principles using a compass and straightedge to provide a basis for constructing the ornaments. The topics they studied were those necessary for constructing the specific ornaments they learned like: bisecting a segment, bisecting an angle, constructing an isosceles triangle, constructing an equilateral triangle, constructing a perpendicular to a straight line. Then the teacher presented the students with ornaments from different cultures and an introduction was made regarding the topic. The students were introduced to ornaments from the Islamic culture which is the culture of the students and other cultures. The teacher chose two geometric ornaments suitable to be constructed with the students: A typical Islamic ornament as symbolizes the cultural context and the Seed of Life ornament which relates to the Ancient Egypt culture and other cultures in the world. The last ornament was chosen so that it would be suitable for teaching the subject of the circle through the curriculum and to practice the theme through analyzing geometric ornament. Guided by the teacher, the students analyzed the properties of the ornaments and solved geometric problems related to them. The geometric problems were developed by the teacher to be suitable for 9th grade and appropriate for the material they learn in the school curriculum. The instructional unit is described in Table 1 below.

Table 1: The Instructional Unit

	The learning stages	Explanation
1	Acquisition of basic knowledge of geometric construction.	Acquiring a basic knowledge about constructing principles with the help of a compass and straightedge, which is necessary for constructing a specific ornament. The students analyzed every step in the construction and analyzed the properties of the geometric shapes created at each stage of the construction.
2	Presenting a background and a gallery of ornaments from the	Preparing a short introduction of a specific culture and a presentation of ornaments from the culture and different cultures

	The learning stages	Explanation
	culture.	through taking into account examples of ornaments from the students' cultures.
3	Geometric inquiry of an ornament.	Analyzing of the geometric properties of an ornament and building series of the construction of the ornament by compass and straightedge. The students discussed every step in the construction and analyzed the geometric shapes created at each stage of the construction.
4	Problem solving related to the ornament.	Engaging the students in solving different geometric problems related to the ornament.

At the end of the study unit, the students were randomly selected and after their consent to participate in an interview at the end of the process with the aim of having them interpret their interpretations, perceptions, opinions and attitudes towards the activity of analyzing and constructing geometric ornaments in class. Also, the participants were asked to explain what they like in a class that combines geometric ornaments. The categories were produced from the answers of the participants. The students' results were interpreted and were drawn conclusions from the contents emerging from the products. Examples of the activities the students were involved in are presented below.

Constructing an Ornament by Compass and Straightedge

Fig. 1 describes the construction of the Seed of Life ornament from the Ancient Egypt culture by students. All the circles have the same radius.

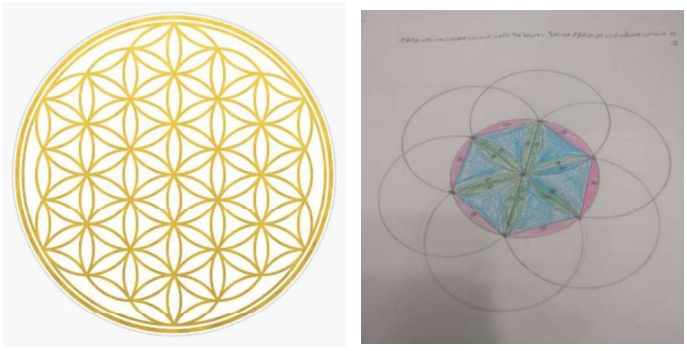

Figure 1. Constructing the Seed of Life ornament.

Solving Geometric Problems Related to the Ornament

Problem 1

For the constructed ornament assume that the radius of every circle is R as seen in Fig. 2. Prove that the hexagon AEFDBC is a regular hexagon.

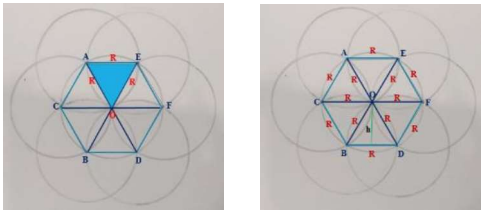

Figure 2. Problem Solving Related to the Ornament.

Solution:

A regular hexagon is built from 6 equilateral triangles whose side length is R (The left of Figure 2). There are 6 triangles like the OAE Triangle. A regular hexagon is a hexagon whose sides are all equal to each other; AE=EF=FD=DB= BC=CA= R

Problem 2

Find the area of hexagon AEFDBC using R. See Figure 2. (The right ornament).

Solution:

Each side is of length R. To find the area of the hexagon, find the area of each equilateral triangle $\frac{\sqrt{3}}{4}R^2$.

And the area of the hexagon is $6 \cdot \frac{\sqrt{3}}{4}R^2$.

Problem 3

In the ornament in Figure 3 find the area of the flower using R.

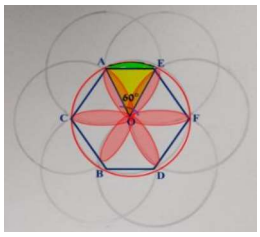

Figure 3. Find the area of the flower in the ornament.

Solution:

Each branch of the six branches of the flower is divided into two identical parts by the sides of the triangle OE and OA. i.e., the area of the flower consists of 6 whole branches or 12 half branches.

The area of one-half branch is equal to the area of the segment AE. multiply it by 12, and get the area of the flower. The area of the circle is πR^2. The area of the section AEO is the area of the circle divided by 6. So, the area of the flower is: $12 \cdot (\frac{\pi R^2}{6} - \frac{\sqrt{3}R^2}{4})$.

Findings

The findings from analyzing the responses of the school students in the interviews revealed five categories that characterizes their experience in learning geometry through engagement in activities of analyzing and constructing geometric ornaments and problem solving. The categories related to different directions as presented below: High level of interest in the geometric lesson; Implications for learning other subjects in geometry; the contribution of learning geometry with ornaments; positive attitudes of the students regarding learning geometry in the context of geometric ornaments; and the contribution of the activities with ornaments for understanding geometric concepts and enhancing geometric problems skills.

High Level of Interest in the Geometric Lesson

All the students pointed out that they liked the activities emphasizing the experience of constructing geometry shapes with compass and straightedge that based on circles, as the students clarified that "it was a nice activity, I especially liked the activity of using a compass and straightedge, I liked drawing the circle"; as other student similarly explained "I really liked how I can draw a circle with a compass and straightedge ". even if some found the experience as challenging but indeed liked it as student claimed that "at first it was difficult but I loved it". Students found the lesson of geometry with ornaments as interesting and that it facilized their learning: "The class was very interesting, I liked it, not boring"; "the activity is beautiful, I liked the activity of drawing the circle and the Dalton"; other student believed that it was "a good activity for understanding geometry, I understood better with the constructing"; others pointed out that "the activity helped me understand geometry in a different and interesting way".

Based on the above findings, it can be stated that the students did enjoy and that the activity of analyzing and constructing geometric ornaments in class is a joyful activity, clearly unusual, not boring like the regular class, satisfying, and increases the motivation to learn geometry in an interesting and out-of-the-box method.

Implications for Other Geometric Subjects

The study participants were asked if they prefer to learn more subjects in geometry through analyzing and constructing geometric ornaments. Based on the students' responses, it can be discerned that indeed these responses are aspects of the subjects under consideration and that they indeed prefer to study other geometric fields with the same method. The students indicated their preference for studying topics in geometry that seemingly pose challenges for them or that the activities contributed to their comprehension, as they mentioned: "It helps in understanding geometry, and concepts like triangle, Dalton, circle"; "It was easier for me to understand mostly the features of the Dalton". Two students indicated student expressed reservation and noted that it would be difficult and challenging for him "No, it will be difficult for me, but we can try"; and the other

said "doesn't know if it is possible. It depends on the subject". Other students expressed enthusiasm for the idea as they explained: "Yes, it seems interesting to me".

The responses of the students indicated that they were interested in the teaching method combining geometric ornaments, and therefore they think that the integration of this teaching method in other geometric fields can lead to optimal results, and in particular that it is a method that increases the dimensions of interest and motivation in learning.

Contribution of Learning Geometry with Ornaments

The participants were asked to elucidate the contribution of geometric ornaments to their learning outcomes. Based on their statements, it becomes evident that geometric ornaments serve as a highly pragmatic and tangible catalyst for enhancing their geometric products. Students claimed that the method "contributed to my knowledge of geometry, I could remember properties"; other clarified that the this experience has contributed to realize that they capable of adopting an alternative perspective when it comes to comprehending geometric forms: "contributed to my understanding that I could understand a different way of seeing geometric shapes"; and that it "helped me see how it is possible to understand geometric concepts through drawing"; and "helped me see a different method for understanding features in geometry". Other students pointed that learning geometry with ornaments enriched the comprehension and fostered their creativity within the classroom: "contributed to my understanding and being creative in class". Students provided an explanation that the contribution of the approach lay in the enhancement of one's orientation towards geometry and the cultivation of a deeper affection for the subject as they clarified: "It helped me love geometry".

Based on the aforementioned findings, it can be deduced that the incorporation of ornaments in the geometry class significantly enhanced their learning outcomes and fostered a heightened sense of enthusiasm and motivation. This effect can not only encourage active engagement with the subject matter but also can promote the attainment of satisfactory achievements within geometry.

Positive Attitudes regarding Learning Geometry in the Context of Ornaments

The participants construed their attitudes towards learning geometry within the context of geometric ornaments. Their statements indicated that their attitudes are unequivocally positive. As they claimed that it was an "interesting subject, you can learn a lot and remember geometric properties"; and "interesting, easy to understand". The students emphasized that the subject matter was intriguing despite its difficulty, and they derived enjoyment from it and that the topic was "interesting, despite the difficulty, but I enjoyed it"; and "Interesting, I really enjoyed learning geometry". They highlighted the method as one that facilitated their geometric learning as they claimed: "interesting, it was easier"; and that the subject was "interesting, it helped me a lot to see geometric properties in depth".

The participants' statements convey highly favorable attitudes toward learning geometry within ornaments. They particularly tense that this method substantially and distinctly eases the learning process.

Enhancing Problem Solving Skills

The research participants were required to clarify whether and how the activity of analyzing and constructing ornaments contributed to their ability to understand geometric concepts and solve geometric problems. They explained that learning in this way significantly facilitated their ability to solve geometric problems as they pointed that it's "helped me a lot, it was easier to solve geometric problems". Some of them explicitly mentioned the contribution in specific topics within geometry as "helped me understand mainly the features of the Dalton", and assisted in perceiving an alternative approach to comprehending geometric properties as "helped me see a different method for understanding geometric properties". Some of them went further and linked their professional affection with a deeper understanding of the subject matter when the activity contributed as it "helped me love geometry and understand more". This learning method opened up a new approach for them to study geometry and helped in " realizing that I could understand another way of seeing geometric shapes". Furthermore, the students reflected on their performance during the activity, emphasizing its creativity as "contributed to my understanding and being creative in class".

Based on the interviewees' statements, a distinct and unequivocal conclusion emerges: the impact of learning geometry with ornaments on enhancing the comprehension and resolution of geometric problems is highly substantial, as perceived by the participants. Consequently, they consider it as a beneficial method for application in the context of teaching geometry.

Discussion

The aim of the present study was to examine how the 9th grade students perceive learning of geometry in the context of geometric ornaments. The approach relies on the analysis of geometric ornament properties and their integration with the subject matter. Moreover, the act of physically constructing the ornament using a compass and straightedge facilitates the observation of the construction process and the geometric shapes comprising the ornament. This, in turn, affords the learner the opportunity to examine the features of the geometric shapes within the complex pattern and offers a pathway to resolving geometric problems associated with a tangible object. The students were involved in activities of analyzing and constructing geometric ornaments and problem solving. The findings from analyzing the responses of the school students in the interviews at the end of the learning process pointed out to the promoting of the method to their learning of geometry in different directions. The students' interest in geometry lesson increased following the activities. The contribution extends beyond the subject being studied, as they view it as an efficient method for learning other topics in geometry. Students developed a more positive attitude towards geometry as a result of studying the geometry in the context of geometric ornaments. Learning with tangible objects like ornaments contributed to the understanding of

geometric concepts as in line with previous researches (Murray, 2001; McNeil & Jarvin, 2007) and enhancing the geometric problems skills of the students (Nur et al., 2020b). This finding is in line with other researches (Ahmad & Chang, 2017; Holtzman & Susholtz, 2011; Kohlhase, 2017; Lavinia, 2011) who pointed out to the contribution to enhance learning outcomes.

Conclusion

In conclusion the findings indicate that school students enjoy learning geometry through analyzing and constructing with compass and straightedge geometric ornaments from their own culture and other cultures. They found it as a joyful learning and a beneficial method that help them acquire knowledge and understand geometric concept. This way they help students develop a positive attitude towards geometry. Geometric ornaments are rich patterns that can provide an opportunity for posing and solving geometric problems and such enhancing their problem-solving skills.

Recommendations

For further research it is recommended to examine the relationship between the use of ornaments and students' achievements in geometry in the ninth grade. And there is a need of additional research examining the potential impact of the cultural context represented by ornaments on the students' motivation and attitudes towards mathematics in general, and geometry in particular.

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